Maggie's Revision

Question 8 (***)

The table below shows the average midday temperature x of a seaside town, in $°\text{C}$, and the number of people y, that used a certain restaurant in that town.

x	17	20	25	29	27	21	20	24
y	40	42	42	43	44	39	41	45

a) Find the value of S_{xx}, S_{yy} and S_{xy}, and hence calculate the product moment correlation coefficient between x and y.

b) State the value of the product moment correlation coefficient between x and y if the temperature was measured in degrees Fahrenheit instead of Centigrade.

c) Determine the equation of the regression line between x and y, giving the answer in the form

$$y = a + bx,$$

where a and b are constants.

d) State, with a reason, which is the explanatory variable in the above described scenario and state the statistical name of the other variable.

e) Interpret in the context of this question the physical meaning of b.

f) Use the equation of the regression line to estimate the value of y when …

i. … $x = 16$.

ii. … $x = 35$.

Comment further on the reliability of each of these two estimates.

$\boxed{}$, $\boxed{r \approx 0.670 \text{ regardless of units}}$, $\boxed{y = 34.4 + 0.331x}$, $\boxed{y_{16} \approx 40}$, $\boxed{y_{35} \approx 46}$

[solution overleaf]

a) USING THE CALCULATOR IN STAT MODE

$\Sigma x = 183$	$\Sigma x^2 = 4301$	$\Sigma xy = 7724$
$\Sigma y = 336$	$\Sigma y^2 = 14140$	$n = 8$

FIND THE VALUES OF S_{xx}, S_{yy}, S_{xy}

$$S_{xx} = \Sigma x^2 - \frac{\Sigma x \Sigma x}{n} = 4301 - \frac{183 \times 183}{8} = 114.875$$

$$S_{yy} = \Sigma y^2 - \frac{\Sigma y \Sigma y}{n} = 14140 - \frac{336 \times 336}{8} = 28$$

$$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 7724 - \frac{183 \times 336}{8}\ 38$$

CALCULATE THE P.M.C.C

$$r = \frac{S_{xy}}{S_{xx} S_{yy}} = \frac{38}{\sqrt{28 \times 114.875}} \approx 0.670$$

b) THE P.M.C.C WILL BE UNCHANGED, AS IT REMAINS UNAFFECTED BY SCALING OR CHOICE OF ORIGIN

c) COLLECT | CALCULATE ALL THE AUXILIARIES

- $b = \frac{S_{xy}}{S_{xx}} = \frac{38}{114.875} = \frac{304}{919} \approx 0.331$
- $\bar{x} = \frac{\Sigma x}{n} = \frac{183}{8} = 22.875$
- $\bar{y} = \frac{\Sigma y}{n} = \frac{336}{8} = 42$
- $a = \bar{y} - b\bar{x} = 42 - \left(\frac{304}{919}\right)(22.875) \approx 34.4330794|3\ldots$

$\therefore\ y = 34.4 + 0.331x$

d) THE EXPLANATORY VARIABLE (INDEPENDENT VARIABLE) IS THE TEMPERATURE x, AS IT IS THE TEMPERATURE THAT AFFECTS THE DINNERS & NOT THE OTHER WAY ROUND

y (NO OF DINNERS) IS CALLED THE RESPONSE VARIABLE

e) $b = 0.331$ IS THE GRADIENT OF LINE

IT REPRESENTS THE EXTRA NUMBER OF DINNERS PER DEGREE OF TEMPERATURE RISE

f) I) IF $x = 16$ $\qquad y = 34.4 + 0.331 \times 16 \simeq 40$

ALTHOUGH THE P.M.C.C IS NOT VERY STRONG, 16 IS ONLY 1 DEGREE LESS THAN THE SMALLEST VALUE OF x WHICH WAS USED TO CREATE THE REGRESSION LINE, SO IT COULD BE RELIABLE

II) IF $x = 35$ $\qquad y = 34.4 + 0.331 \times 35 \simeq 46$

GIVEN THAT THE P.M.C.C IS NOT STRONG, AND 35 IS "WAY ABOVE" THE HIGHEST TEMPERATURE WHICH WAS USED TO CREATE THE REGRESSION LINE, THE ESTIMATE WILL BE UNRELIABLE

Question 9 (***)

The table below shows the maximum temperature T °C on five different days and the corresponding ice cream sales, N, of a certain shop on those days.

T	15	20	25	30	35
N	79	145	182	255	302

a) Find the value of S_{TT}, S_{NN} and S_{TN}, and hence, determine the value of the product moment correlation coefficient between T and N.

b) State, with a reason, which is the explanatory variable in the above described scenario and state the statistical name of the other variable.

c) Determine the equation of the regression line between N and T, giving the answer in the form

$$N = a + bT ,$$

where a and b are constants.

d) Interpret in the context of this question the physical meaning of b.

e) Use the equation of the regression line to estimate the value of N when …

i. … $T = 18°\text{C}$.

ii. … $T = 37°\text{C}$.

iii. … $T = 45°\text{C}$

Comment further on the reliability of each of these estimates.

$\boxed{}$, $\boxed{S_{TT} = 250}$, $\boxed{S_{NN} = 31145.2}$, $\boxed{S_{TN} = 2780}$, $\boxed{r = 0.996}$, $\boxed{N = 11.12T - 85.4}$, $\boxed{N_{18} \approx 115}$, $\boxed{N_{37} \approx 326}$, $\boxed{T_{45} \approx 415}$

[solution overleaf]

a) OBTAIN SUMMARY STATISTICS

$\sum T = 125$	$\sum T^2 = 3375$	$\sum TN = 26855$
$\sum N = 963$	$\sum T^2 = 216619$	$n = 5$

- $S_{TT} = \sum T^2 - \frac{\sum T \sum T}{n} = 3375 - \frac{125 \times 125}{5} = 250$
- $S_{NN} = \sum N^2 - \frac{\sum N \sum N}{n} = 216619 - \frac{963 \times 963}{5} = 31145.2$
- $S_{TN} = \sum TN - \frac{\sum T \sum N}{n} = 26855 - \frac{125 \times 963}{5} \; 2780$

CALCULATING THE P.M.C.C

$$r = \frac{S_{TN}}{\sqrt{S_{TT} \, S_{NN}}} = \frac{2780}{\sqrt{250 \times 31145.2}} \simeq 0.99627\ldots \simeq 0.996$$

b) T (TEMPERATURE) IS THE EXPLANATORY VARIABLE (INDEPENDENT) AS IT IS THE TEMPERATURE WHICH AFFECTS THE ICE CREAM SALES AND NOT THE OTHER WAY ROUND.

THE ICE CREAM SALES (N) IS THE RESPONSE VARIABLE.

c) OBTAIN ALL THE AUXILIARIES

- $b = \frac{S_{TN}}{S_{TT}} = \frac{2780}{250} = 11.12$
- $\bar{T} = \frac{\sum T}{n} = \frac{125}{5} = 25$
- $\bar{N} = \frac{\sum N}{n} = \frac{963}{5} = 1926$
- $a = \text{"}\bar{y} - b\bar{x}\text{"} = \bar{N} - b\bar{T} = 192.6 - 11.12 \times 25 = -85.4$

$$\therefore N = 11.12T - 85.4$$

d) $b = 11.12$ IS THE GRADIENT

IT REPRESENTS THE NUMBER OF EXTRA ICE CREAMS TO BE SOLD, PER DEGREE RISE

e) I) IF $T = 18$, $N = 11.12 \times 18 - 85.4 \approx 115$

IT SHOULD BE RELIABLE AS THIS TEMPERATURE IS BETWEEN 15 & 35 DEGREES AND THE P.M.C.C IS VERY STRONG

IF $T = 37$, $N = 11.12 \times 37 - 85.4 \approx 326$

IT SHOULD BE RELIABLE AS 37 IS ONLY JUST ABOVE 35°C AND THE P.M.C.C IS VERY STRONG

IF $T = 45$, $N = 11.12 \times 45 - 85.4 \approx 415$

NOT LIKELY TO BE RELIABLY AS THIS TEMPERATURE IS WAY ABOUT 35° AND THERE IS NO EVIDENCE THAT THIS LINEAR PATTERN CONTINUES

Question 10 (***+)

The table below shows the amount spent per month by a car dealership on marketing and advertising m, in £1000, and the number of cars c sold that month.

m	7	8	9	10	11
c	7	12	10	11	13

a) Find the value of the product moment correlation coefficient between m and c.

b) Determine the equation of the regression line between m and c, giving the answer in the form

$$c = a + bm,$$

where a and b are constants.

c) Use the equation of the regression line to estimate the number of cars that are expected to be sold in a month where the amount spent on marketing and advertising is …

i. … £8,800.

ii. … £20,000.

Comment further on the reliability of each of these two estimates.

d) Interpret in the context of this question the physical meaning of a and b.

$\boxed{}$, $\boxed{r = 0.755}$, $\boxed{c = 0.7 + 1.1m}$, $\boxed{c_{8.8} \approx 10}$, $\boxed{c_{20} \approx 23}$

a) START BY OBTAINING THE SUMMARY STATISTICS

- $\sum m = 45$ • $\sum m^2 = 415$ • $\sum mc = 488$
- $\sum c = 53$ • $\sum c^2 = 583$ • $n = 5$

CALCULATING S_{mm}, S_{cc} & S_{mc}

$$S_{mm} = \sum m^2 - \frac{\sum m \sum m}{n} = 415 - \frac{45 \times 45}{5} = 10$$

$$S_{cc} = \sum c^2 - \frac{\sum c \sum c}{n} = 583 - \frac{53 \times 53}{5} = 21.2$$

$$S_{mc} = \sum mc - \frac{\sum m \sum c}{n} = 488 - \frac{45 \times 53}{5} = 11$$

FIND THE P.M.C.C

$$r = \frac{S_{mc}}{\sqrt{S_{mm} S_{cc}}} = \frac{11}{\sqrt{10 \times 21.2}} = 0.755 \quad \text{3 d.p.}$$

b) FIND ALL AUXILIARIES FIRST

$$\bar{m} = \frac{\sum m}{n} = \frac{45}{5} = 9$$

$$\bar{c} = \frac{\sum c}{n} = \frac{53}{5} = 10.6$$

$$b = \frac{S_{mc}}{S_{mm}} = \frac{11}{10} = 1.1$$

$$a = \bar{c} - b\bar{m} = 10.6 - 1.1(9) = 0.7$$

$$\therefore \; c = 0.7 + 1.1m$$

c) I) $c_{8.8} = 0.7 + 1.1 \times 8.8 = 10.38$ I.E AROUND 10 CARS

AS THE CORRELATION COEFFICIENT IS FAIRLY HIGH, AND 8.8 (£8800) LIES WITHIN THE RANGE OF VALUES OF m WHICH WAS USED FOR THE REGRESSION LINE, THE ESTIMATE SHOULD BE RELIABLE (IT WOULD HAVE BEEN MORE RELIABLE IF MORE POINTS WERE USED)

(INTERPOLATION)

II) $c_{20} = 0.7 + 1.1 \times 20 = 22.7$ IE AROUND 23 CARS

AS $m = 20$, IS "WAY ABOVE" THE LARGEST VALUE OF m, WHICH WAS USED TO PRODUCE THE REGRESSION LINE, THE ESTIMATE COULD NOT BE RELIABLE

(EXTRAPOLATION)

d)
- $a = 0.7$ ("y INTERCEPT")

NO OF CARS EXPECTED TO BE SOLD WHEN NO MONEY IS SPENT ON ADVERTISING

- $b = 1.1$ ("GRADIENT")

NO OF EXTRA CARS EXPECTED TO BE SOLD PER £1000 SPENT ON ADVERTISING

Question 8 (***)

The probability distribution of a discrete random variable X is given by

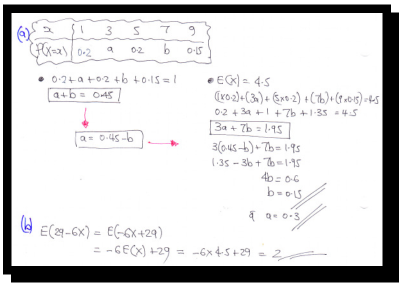
x	1	3	5	7	9
$\mathrm{P}(X=x)$	0.2	a	0.2	b	0.15

where a and b are positive constants.

a) Given that $\mathrm{E}(X)=4.5$, find the value of a and the value of b.

b) Determine $\mathrm{E}(29-6X)$.

$\boxed{a=0.3,\ \ b=0.15}$, $\boxed{\mathrm{E}(29-6X)=2}$



(a)

x	1	3	5	7	9
$P(X=x)$	0.2	a	0.2	b	0.15

- $0.2+a+0.2+b+0.15=1$

$\boxed{a+b=0.45}$

$\boxed{a=0.45-b}$

- $E(X)=4.5$

$(1\times0.2)+(3a)+(5\times0.2)+(7b)+(9\times0.15)=4.5$

$0.2+3a+1+7b+1.35=4.5$

$\boxed{3a+7b=1.95}$

$3(0.45-b)+7b=1.95$

$1.35-3b+7b=1.95$

$4b=0.6$

$b=0.15$

& $a=0.3$

(b) $E(29-6X)=E(-6X+29)$

$=-6E(X)+29=-6\times4.5+29=2$

Question 17 (*+)**

The discrete random variable X has the following probability distribution

x	0	2	3
$\mathrm{P}(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

a) Determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

A game in a fun fair consists of throwing 5 darts on a small target.

If a dart lands on the central portion of the target the dart scores 3 points.

If a dart lands on the outer portion of the target the dart scores 2 points, otherwise the dart scores no points.

To win a prize, 10 or more points must be scored with 5 darts.

Paul has scored 6 points with his first 3 darts.

The likelihood of Paul scoring 0, 2 or 3 points is given by the probability distribution of part **(a)**.

b) Find the probability that Paul will win a prize after he throws his last 2 darts.

$\boxed{}$, $\boxed{\mathrm{E}(X) = \frac{7}{6}}$, $\boxed{\mathrm{Var}(X) = \frac{53}{36}}$, $\boxed{\frac{1}{4}}$

a)

x	0	2	3
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

- $E(X) = \sum x P(X=x) = \left(0 \times \frac{1}{2}\right) + \left(2 \times \frac{1}{3}\right) + \left(3 \times \frac{1}{6}\right) = \frac{7}{6}$
- $E(X^2) = \sum x^2 P(X=x) = \left(0^2 \times \frac{1}{2}\right) + \left(2^2 \times \frac{1}{3}\right) + \left(3^2 \times \frac{1}{6}\right) = \frac{17}{6}$
- $\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = \frac{17}{6} - \left(\frac{7}{6}\right)^2 = \frac{53}{36}$

b) TO WIN A PRIZE WITH HIS LAST 2 DARTS...

- HE MUST SCORE 4 OR MORE POINTS WITH 2 DARTS
- IE

$$\left.\begin{array}{ll} 2-2: & \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \\ 2-3: & \frac{1}{3} \times \frac{1}{6} = \frac{1}{18} \\ 3-2: & \frac{1}{6} \times \frac{1}{3} = \frac{1}{18} \\ 3-3: & \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \end{array}\right\} \text{ADDING GIVES } \frac{1}{4}$$

Question 18 (****)

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases} k(2-x) & x=0,1,2 \\ \frac{1}{4} & x=3 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $k=\frac{1}{4}$.

b) Find the value of $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$.

c) Determine $\mathrm{Var}(3-X)$.

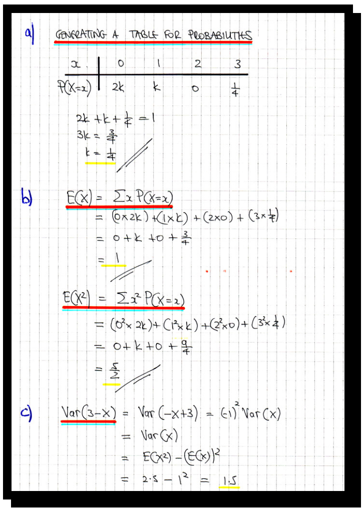
Two independent observations of X are made, denoted by X_1 and X_2.

d) Find the probability distribution of Y, where $Y=X_1+X_2$.

e) Calculate $P(1.5 \le Y \le 3.5)$.

$\boxed{}$, $\boxed{\mathrm{E}(X)=1}$, $\boxed{\mathrm{E}(X^2)=2.5}$, $\boxed{\mathrm{Var}(3-X)=1.5}$, $\boxed{\mathrm{P}(Y=y)=\begin{cases} \frac{1}{16} & y=2,6 \\ \frac{1}{8} & y=4 \\ \frac{1}{4} & y=0,1,3 \\ 0 & \text{otherwise} \end{cases}}$,

$\boxed{P(1.5 \le Y \le 3.5)=\frac{5}{16}}$



a) GENERATING A TABLE FOR PROBABILITIES

x	0	1	2	3
$P(X=x)$	$2k$	k	0	$\frac{1}{4}$

$2k+k+\frac{1}{4}=1$

$3k=\frac{3}{4}$

$k=\frac{1}{4}$

b) $E(X)=\sum x P(X=x)$

$=(0\times 2k)+(1\times k)+(2\times 0)+(3\times \frac{1}{4})$

$=0+k+0+\frac{3}{4}$

$=1$

$E(X^2)=\sum x^2 P(X=x)$

$=(0^2\times 2k)+(1^2\times k)+(2^2\times 0)+(3^2\times \frac{1}{4})$

$=0+k+0+\frac{9}{4}$

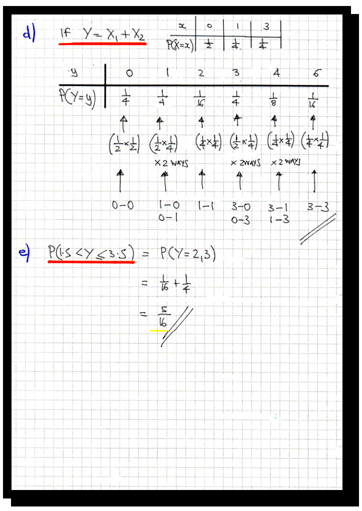
$=\frac{5}{2}$

c) $\mathrm{Var}(3-X)=\mathrm{Var}(-X+3)=(-1)^2\,\mathrm{Var}(X)$

$=\mathrm{Var}(X)$

$=E(X^2)-(E(X))^2$

$=2.5-1^2=1.5$



d) If $Y=X_1+X_2$

x	0	1	3
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

y	0	1	2	3	4	6
$P(Y=y)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
	$(\frac{1}{2}\times\frac{1}{2})$	$(\frac{1}{2}\times\frac{1}{4})$ ×2 WAYS	$(\frac{1}{4}\times\frac{1}{4})$	$(\frac{1}{2}\times\frac{1}{4})$ ×2 WAYS	$(\frac{1}{4}\times\frac{1}{4})$ ×2 WAYS	$(\frac{1}{4}\times\frac{1}{4})$
	0–0	1–0, 0–1	1–1	3–0, 0–3	3–1, 1–3	3–3

e) $P(1.5<Y\le 3.5)=P(Y=2,3)$

$=\frac{1}{16}+\frac{1}{4}$

$=\frac{5}{16}$

Question 19 (****)

The discrete random variable X has the following probability distribution

x	0	1	3
$\mathrm{P}(X = x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

a) Determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

Two independent observations of X are made, denoted by X_1 and X_2.

b) Find the probability distribution of $X_1 + X_2$.

c) Calculate $P(X_1 > X_2)$.

$\boxed{\mathrm{E}(X) = \frac{11}{6}}$, $\boxed{\mathrm{Var}(X) = \frac{53}{36}}$, $\boxed{\mathrm{P}(X_1 + X_2 = r) = \begin{cases} \frac{1}{36} & r = 0 \\ \frac{1}{9} & r = 1,2 \\ \frac{1}{4} & r = 6 \\ \frac{1}{3} & r = 4 \\ 0 & \text{otherwise} \end{cases}}$,

$\boxed{P(X_1 > X_2) = \frac{11}{36}}$

a)

x	0	1	3
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

$$E(X) = \sum x P(X=x)$$
$$= \left(0 \times \tfrac{1}{6}\right) + \left(1 \times \tfrac{1}{3}\right) + \left(3 \times \tfrac{1}{2}\right)$$
$$= 0 + \tfrac{1}{3} + \tfrac{3}{2}$$
$$= \tfrac{11}{6}$$

$$E(X^2) = \sum x^2 P(X=x)$$
$$= \left(0^2 \times \tfrac{1}{6}\right) + \left(1^2 \times \tfrac{1}{3}\right) + \left(3^2 \times \tfrac{1}{2}\right)$$
$$= 0 + \tfrac{1}{3} + \tfrac{9}{2}$$
$$= \tfrac{29}{6}$$

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$$
$$= \tfrac{29}{6} - \left(\tfrac{11}{6}\right)^2$$
$$= \tfrac{29}{6} - \tfrac{121}{36}$$
$$= \tfrac{53}{36}$$

b) DETERMINE ALL THE OUTCOMES FOR $X_1 + X_2$

$0+0 = 0 \quad \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
$0+1 = 1 \quad \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$
$0+3 = 3 \quad \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
$1+0 = 1 \quad \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
$1+1 = 2 \quad \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
$1+3 = 4 \quad \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$
$3+0 = 3 \quad \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
$3+1 = 4 \quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
$3+3 = 6 \quad \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

LET $Y = X_1 + X_2$

y	0	1	2	3	4	6
$P(Y=y)$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$

c) USING THE OUTCOMES FROM PART (b)

$$\left.\begin{array}{ll} 1,0 : & \frac{1}{18} \\ 3,0 : & \frac{1}{12} \\ 3,1 : & \frac{1}{6} \end{array}\right\} \text{Adding} = \frac{11}{36}$$

Question 5 (***)

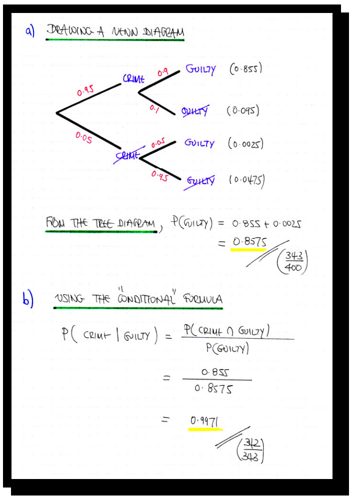
In a certain Crown Court 95% of the defendants being tried have actually committed the crime they are being tried for.

For those who committed the crime the probability of being found guilty is 90% and for those who did not commit the crime the probability of being found guilty is 5%.

a) Find the probability that a randomly chosen defendant will be found guilty.

b) Given that a randomly chosen defendant was found guilty, find the probability that the defendant committed the crime.

$\boxed{}$, $\boxed{\frac{343}{400}}$, $\boxed{\frac{342}{343}}$



a) DRAWING A VENN DIAGRAM

FROM THE TREE DIAGRAM, P(GUILTY) = 0.855 + 0.0025

= 0.8575 $\left(\frac{343}{400}\right)$

b) USING THE "CONDITIONAL" FORMULA

$$P(\text{CRIME} \mid \text{GUILTY}) = \frac{P(\text{CRIME} \cap \text{GUILTY})}{P(\text{GUILTY})}$$

$$= \frac{0.855}{0.8575}$$

$$= 0.9971 \quad \left(\frac{342}{343}\right)$$

Question 6 (***)

A test is developed to determine whether someone has or has not got a disease, which is known to be present in 3% of the population.

Given a person has the disease the test is positive with probability of 98%.

Given a person does not have the disease the test is positive with probability of 5%.

a) Draw a tree diagram to represent this information.

A person is selected at random from the population and tested for the disease.

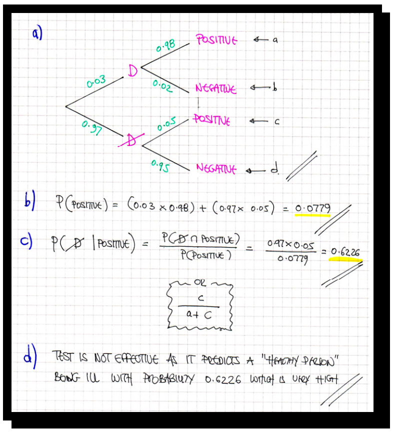
b) Find the probability that this person's test is positive.

A person who tested positive is selected.

c) Find the probability that the person does not have the disease.

d) Comment on the effectiveness of this test with reference to the answer given in part **(c)**.

$\boxed{}$, $\boxed{0.0779}$, $\boxed{0.6226}$



Question 9 (***+)

Markus is a health fanatic.

On a given day, the probabilities that he goes for a run, he uses the gym or he cycles are 0.5, 0.4 and 0.1, respectively.

Markus sometimes uses the sauna after these activities.

The probability he uses the sauna after he goes for a run is 0.1. The respective probabilities for using the sauna after using the gym or cycling are 0.6 and 0.3.

Find the probability that on a given day Markus …

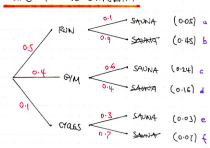
a) … will use the sauna.

b) … used the gym, given he used the sauna.

c) … did not go for a run, given he did not use the sauna.

$\boxed{}$, $\boxed{0.32}$, $\boxed{0.75}$, $\boxed{\frac{23}{68} \approx 0.3382}$

a) USING A TREE DIAGRAM



P(SAUNA) = "a + c + e" = 0.05 + 0.24 + 0.03 = 0.32

b) $P(\text{GYM} \mid \text{SAUNA}) = \frac{P(\text{GYM} \cap \text{SAUNA})}{P(\text{SAUNA})} = \frac{c}{a+c+e} = \frac{0.24}{0.32} = 0.75$

c) $P(\text{NO RUN} \mid \text{NO SAUNA}) = \frac{P(\text{NO RUN} \cap \text{NO SAUNA})}{P(\text{NO SAUNA})}$

$= \frac{d+f}{b+d+f}$

$= \frac{0.16 + 0.07}{0.16 + 0.07 + 0.45}$ ← OR 1 − 0.32

$= \frac{23}{68} \approx 0.3382$