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PROBABILITY SAMPLING DISTRIBUTIONS

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Question 1 (*)**

A bag contains a **large** number of coins. Half of the coins are 10 pence pieces, one third are 20 pence pieces and the rest are 5 pence pieces.

A sample of two coins is selected at random.

Determine the sampling distribution of the mean of the two coins.

mean	5	7.5	10	12.5	15	20
P(mean)	$\frac{1}{36}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$

Handwritten solution for Question 1. It shows a table with columns for 'OUTCOMES', 'MEAN', and 'PROBABILITY'. The outcomes listed are (5,5), (5,10), (5,20), (10,5), (10,10), (10,20), (20,5), (20,10), and (20,20). The corresponding means are 5, 7.5, 12.5, 7.5, 10, 15, 12.5, 15, and 20. The probabilities are calculated as $\frac{1}{2} \times \frac{1}{2}$ for each pair. To the right, there is a diagram showing the distribution of means: 5, 7.5, 10, 12.5, 15, 20, with arrows indicating the relative frequencies: $\frac{1}{36}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{3}$, $\frac{1}{9}$.

Question 2 (*)**

A bag contains a **large** number of coins. Two thirds of the coins are 20 pence pieces and the rest are 50 pence pieces.

A sample of three coins is selected at random.

Find the sampling distribution of the median of the three coins.

median	20	50
P(median)	$\frac{20}{27}$	$\frac{7}{27}$

Handwritten solution for Question 2. It shows a table with columns for 'OUTCOMES', 'MEDIAN', and 'PROBABILITY'. The outcomes listed are (20,20,20), (20,20,50), (20,50,20), (50,20,20), (20,50,50), (50,20,50), (50,50,20), and (50,50,50). The corresponding medians are 20, 20, 20, 20, 50, 50, 50, and 50. The probabilities are calculated as $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$ for each outcome. To the right, there is a diagram showing the distribution of medians: 20 and 50, with arrows indicating the relative frequencies: $\frac{20}{27}$ and $\frac{7}{27}$.

Question 3 (*)**

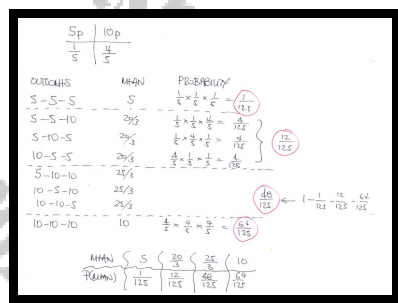
A bag contains a **large** number of coins, some 5 pence pieces and some 10 pence pieces.

The ratio of 5 pence pieces to the 10 pence pieces is 1:4.

A sample of three coins is selected at random.

Find the sampling distribution of the mean of the three coins.

mean	5	$\frac{20}{3}$	$\frac{25}{3}$	10
P(mean)	$\frac{1}{125}$	$\frac{12}{125}$	$\frac{48}{125}$	$\frac{64}{125}$



Question 4 (*)**

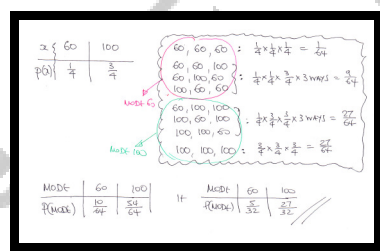
A **large** number of light bulbs are stored in the stock-room of an electrical shop.

The ratio of 60 watt bulbs to 100 watt bulbs is 1:3.

A sample of three light bulbs is selected at random.

Find the sampling distribution of the mode of the three bulbs.

mode	60	100
P(mode)	$\frac{5}{32}$	$\frac{27}{32}$



Question 5 (*)**

During hot days, an ice cream van sells a large number of ice cream cones containing either 1, 2 or 3 scoops of ice cream.

The respective probabilities of a customer buying a 1, 2 or 3 scoop ice cream cone are $\frac{1}{6}$, $\frac{1}{2}$ or $\frac{1}{3}$.

A random sample of 2 customers is examined, each customer having bought an ice cream cone from this van.

Determine the sampling distribution of T , where T represents the total number of scoops of ice cream bought by these 2 customers.

t	2	3	4	5	6
$P(T=t)$	$\frac{1}{36}$	$\frac{6}{36}$	$\frac{13}{36}$	$\frac{12}{36}$	$\frac{4}{36}$

No of scoops	1	2	3
Probability	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

OUTCOME ALL THE OUTCOMES AND THEIR CORRESPONDING PROBABILITIES

OUTCOMES	TOTAL SCORES	PROBABILITY
1-1	2	$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
1-2	3	$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
1-3	4	$\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$
2-1	3	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
2-2	4	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
2-3	5	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
3-1	4	$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
3-2	5	$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$
3-3	6	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

THUS WE HAVE

t	2	3	4	5	6
$P(T=t)$	$\frac{1}{36}$	$\frac{6}{36}$	$\frac{13}{36}$	$\frac{12}{36}$	$\frac{4}{36}$

Question 6 (***)

A bag contains a large number of 5 pence coins and 10 pence coins.

There are twice as many 10 pence coins as there are 5 pence coins.

A random sample of size 2 is taken from the bag and the value of each coin is denoted as X_1 and X_2 .

A statistic Y based on this sample is defined as

$$Y = \frac{1}{5}[3X_1 + 2X_2].$$

Determine the sampling distribution of Y and hence or otherwise find $\text{Var}(Y)$.

$$\boxed{\quad}, \quad \begin{array}{c|c|c|c} y & 5 & 7 & 8 & 10 \\ \hline P(Y=y) & \frac{1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{4}{9} \end{array}, \quad \text{Var}(Y) = \frac{26}{9}$$

• START BY CALCULATING ALL THE OUTCOMES AND THEIR CORRESPONDING PROBABILITIES.
 5 PENCE PROBATION $\frac{1}{3}$
 10 PENCE PROBATION $\frac{2}{3}$

X_1	X_2	$Y = \frac{1}{5}[3X_1 + 2X_2]$	PROBABILITY
5	5	$\frac{1}{5}(3 \times 5 + 2 \times 5) = 5$	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
5	10	$\frac{1}{5}(3 \times 5 + 2 \times 10) = 7$	$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$
10	5	$\frac{1}{5}(3 \times 10 + 2 \times 5) = 8$	$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$
10	10	$\frac{1}{5}(3 \times 10 + 2 \times 10) = 10$	$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

• HAVE THE PROBABILITY DISTRIBUTION OF Y IS

Y	5	7	8	10
$P(Y=y)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{4}{9}$

• TO FIND THE VARIANCE OF Y

$$E(Y) = (5 \times \frac{1}{9}) + (7 \times \frac{2}{9}) + (8 \times \frac{2}{9}) + (10 \times \frac{4}{9})$$

$$= \frac{5}{9} + \frac{14}{9} + \frac{16}{9} + \frac{40}{9}$$

$$= \frac{75}{9}$$

$$= \frac{25}{3}$$

• $E(Y^2) = (5^2 \times \frac{1}{9}) + (7^2 \times \frac{2}{9}) + (8^2 \times \frac{2}{9}) + (10^2 \times \frac{4}{9})$

$$= \frac{25}{9} + \frac{98}{9} + \frac{128}{9} + \frac{400}{9} = \frac{651}{9}$$

$$= \frac{217}{3}$$

• $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$

$$= \frac{217}{3} - (\frac{25}{3})^2$$

$$= \frac{26}{9}$$

ALTERNATIVE TO FIND VARIANCE

X	5	10
$P(X=x)$	$\frac{1}{3}$	$\frac{2}{3}$

• $E(X) = (5 \times \frac{1}{3}) + (10 \times \frac{2}{3}) = \frac{5}{3} + \frac{20}{3} = \frac{25}{3}$
 • $E(X^2) = (5^2 \times \frac{1}{3}) + (10^2 \times \frac{2}{3}) = \frac{25}{3} + \frac{200}{3} = \frac{225}{3} = 75$
 • $\text{Var}(X) = E(X^2) - [E(X)]^2 = 75 - (\frac{25}{3})^2 = \frac{26}{9}$

• $Y = \frac{1}{5}[3X_1 + 2X_2] \Rightarrow \text{Var}(Y) = (\frac{3}{5})^2 \text{Var}(X_1) + (\frac{2}{5})^2 \text{Var}(X_2)$

$$\Rightarrow \text{Var}(Y) = \frac{9}{25} \text{Var}(X) + \frac{4}{25} \text{Var}(X)$$

$$= \frac{13}{25} \text{Var}(X)$$

$$= \frac{13}{25} \times \frac{26}{9}$$

$$= \frac{26}{9} \quad \text{as before}$$