GEOMETRIC DISTRIBUTION

Question 1 (**+)

The discrete random variable X is modelled as being geometrically distributed with parameter 0.2.

a) State two conditions that must be satisfied by X, so that the geometric model is valid.

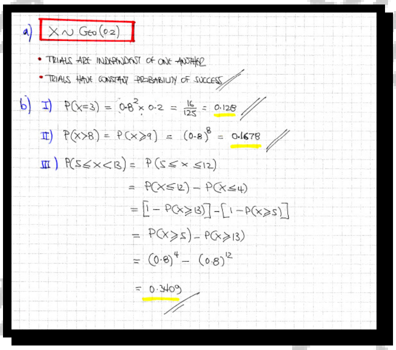
b) Showing full workings, where appropriate, calculate the value of …

i. … $\mathrm{P}(X=3)$.

ii. … $\mathrm{P}(X>8)$.

iii. … $\mathrm{P}(5 \le X < 13)$.

$\boxed{}$, $\boxed{0.128}$, $\boxed{0.1678}$, $\boxed{0.3409}$



a) $\boxed{X \sim \mathrm{Geo}(0.2)}$

- TRIALS ARE INDEPENDENT OF ONE ANOTHER
- TRIALS HAVE CONSTANT PROBABILITY OF SUCCESS

b) I) $P(X=3) = 0.8^2 \times 0.2 = \frac{16}{125} = 0.128$

II) $P(X>8) = P(X \geqslant 9) = (0.8)^8 = 0.1678$

III) $P(5 \leqslant X < 13) = P(5 \leqslant X \leqslant 12)$

$= P(X \leqslant 12) - P(X \leqslant 4)$

$= [1 - P(X \geqslant 13)] - [1 - P(X \geqslant 5)]$

$= P(X \geqslant 5) - P(X \geqslant 13)$

$= (0.8)^4 - (0.8)^{12}$

$= 0.3409$

Question 2 (+)**

It is known that in a certain town 30% of the people own an *Apfone*.

A researcher asks people at random whether they own an *Apfone*.

The random variable X represents the number of people asked up to and including the first person who owns an *Apfone*.

Determine that …

a) … $\mathrm{P}(X = 4)$.

b) … $\mathrm{P}(X > 4)$.

c) … $\mathrm{P}(X < 6)$.

$\boxed{0.1029}$, $\boxed{0.2401}$, $\boxed{0.8319}$

$X \sim \mathrm{Geo}(0.3)$, WHERE X = FIRST PERSON WHO OWNS AN APFONE

a) $P(X=4) = 0.7 \times 0.7 \times 0.7 \times 0.3 = 0.1029$

b) $P(X>4) = P(X \geqslant 5) = P($FIRST 4 PEOPLE DO NOT OWN AN APFONE$)$

$= 0.7 \times 0.7 \times 0.7 \times 0.7 = 0.2401$

c) $P(X<6) = P(X \leqslant 5) = P(X=1,2,3,4,5)$

$= 0.3 + 0.3 \times 0.7 + 0.3 \times 0.7^2 + 0.3 \times 0.7^3 + 0.3 \times 0.7^4$

$= 0.3\left[1 + 0.7 + 0.7^2 + 0.7^3 + 0.7^4\right]$

$= 0.8319$

Question 3 (+)**

Arthur and Henry are rolling a fair six sided die and the winner of their game will be the first person to get a "six".

Arthur rolls the die first.

Determine the probability that …

a) … Arthur wins on his second throw.

b) … Arthur wins on his third throw.

c) … Arthur wins the game.

$\boxed{\dfrac{25}{216}}, \boxed{\dfrac{625}{7776}}, \boxed{\dfrac{6}{11}}$

(a) $\cancel{A}\cancel{H}A: \ \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{25}{216}$

(b) $\cancel{A}\cancel{H}\cancel{A}\cancel{H}A: \ \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{625}{7776}$

(c) $A + \cancel{AH}A + \cancel{AHAH}A + \cancel{AHAHAH}A + \dots$

$= \frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \dots$

$= \frac{1}{6} + \frac{1}{6}\times\frac{25}{36} + \frac{1}{6}\times\left(\frac{25}{36}\right)^2 + \frac{1}{6}\times\left(\frac{25}{36}\right)^3 + \dots$

$= \dfrac{\frac{1}{6}}{1-\frac{25}{36}} \quad \leftarrow S_\infty = \dfrac{a}{1-r}$

$= \frac{6}{11}$

Question 4 (***)

Nigel is playing tournament chess against a computer program and the probability he wins against the program at any given game is 0.25.

Nigel is playing several practice games every day, one after the other.

a) Find the probability that on a given day ...

i. ... Nigel wins for the first time on the 4^{th} game played.

ii. … Nigel has to play more than 4 games before he wins for the first time.

If Nigel does not win any of the first 5 games played in a given day, he plays no more games in that day. Nigel starts training on a Monday on a given week.

b) Determine the probability that Nigel wins his first game on the Thursday of that week.

$\boxed{\frac{27}{256} \approx 0.105}$, $\boxed{\frac{81}{256} \approx 0.316}$, $\boxed{\approx 0.0102}$

X = NO OF GAMES PLAYED UNTIL A WIN IS ACHIEVED
$X \sim \text{Geo}(0.25)$

a) $P(X=4) = (0.75)^3(0.25) = \frac{27}{256} \approx 0.105$

b) $P(X>4) = P(X \geqslant 5) = P(\text{Lose the first } 4)$
$= \left(\frac{3}{4}\right)^4 = \frac{81}{256} \approx 0.316$

c) FIRST FIND $P(\text{lose the first } 5) = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$

$\therefore$ "WINS BEFORE 5" IS $1 - \frac{243}{1024} = \frac{781}{1024}$

$\therefore$ MON TUE WED THU

LOSES ALL 5, LOSES ALL 5, LOSES ALL 5, WINS WITHIN 5

$\left(\frac{243}{1024}\right)^3 \times \frac{781}{1024} \approx 0.0102$

Question 5 (***)

In a statistical experiment, a token is placed at the origin $(0,0)$ of a square grid.

A fair six sided die is rolled repeatedly until a "*six*" is obtained.

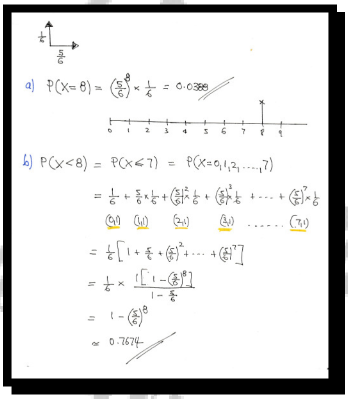
- Every time a "*six*" **is not** obtained, the token is moved by one unit in the positive x direction.
- When a "*six*" **is** obtained, the token is moved by one unit in the positive y direction and the experiment is over, with the token at the point with coordinates $(X,1)$.

Determine …

a) … $\mathrm{P}(X=8)$.

b) … $\mathrm{P}(X<8)$

$\boxed{\approx 0.0388}$, $\boxed{\approx 0.7674}$



a) $P(X=8) = \left(\frac{5}{6}\right)^8 \times \frac{1}{6} = 0.0388$

b) $P(X<8) = P(X \leqslant 7) = P(X=0,1,2,\dots,7)$

$= \frac{1}{6} + \frac{5}{6}\times\frac{1}{6} + \left(\frac{5}{6}\right)^2\times\frac{1}{6} + \left(\frac{5}{6}\right)^3\times\frac{1}{6} + \dots + \left(\frac{5}{6}\right)^7\times\frac{1}{6}$

(0,1) (1,1) (2,1) (3,1) …… (7,1)

$= \frac{1}{6}\left[1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \dots + \left(\frac{5}{6}\right)^7\right]$

$= \frac{1}{6} \times \frac{1\left[1-\left(\frac{5}{6}\right)^8\right]}{1-\frac{5}{6}}$

$= 1 - \left(\frac{5}{6}\right)^8$

≈ 0.7674

Question 6 (***+)

The small central section on a standard dart board is called the bull's eye.

When Albert aim for the bull's eye the probability he hits it is 0.3.

When Buckle aim for the bull's eye the probability he hits it is 0.2.

One day the two players decide to play a game aiming a single dart at the bull's eye in alternative fashion, starting with Buckle. The winner is the first to hit the bull's eye.

Assuming that all probabilities are constant, show that Buckle is less likely to win the game compared with Albert.

$$\boxed{\mathrm{P}(\text{Buckle}) = \frac{5}{11} < \frac{1}{2}}$$

$P(\text{Albert}) = 0.3$
$P(\text{Buckle}) = 0.2$

BUCKLE WINS IN 1 : $0.2 = 0.2$

BUCKLE WINS IN 2 : $(0.8 \times 0.7) \times 0.2 = 0.2 \times 0.56^1$

BUCKLE WINS IN 3 : $(0.8 \times 0.7) \times (0.8 \times 0.7) \times 0.2 = 0.2 \times 0.56^2$

BUCKLE WINS IN 4 : $(0.8 \times 0.7) \times (0.8 \times 0.7) \times (0.8 \times 0.7) \times 0.2 = 0.2 \times 0.56^3$

$$\therefore P(\text{BUCKLE WINS}) = 0.2 + 0.2 \times 0.56 + 0.2 \times 0.56^2 + 0.2 \times 0.56^3$$
$$= 0.2\left[1 + 0.56 + 0.56^2 + 0.56^3 + \dots\right]$$
$$= 0.2 \times \frac{1}{1-0.56} \quad \leftarrow \quad S_\infty = \frac{a}{1-r}$$
$$= \frac{0.2}{0.44} = \frac{5}{11} < \frac{1}{2}$$

$\therefore$ BUCKLE IS LESS LIKELY TO WIN THE GAME

Question 7 **(****)**

Two cricket players, Markus and Dean, decide to throw balls at a wicket, in alternate fashion, starting with Markus. The winner is the player who is first to hit the wicket.

The probability that Markus hits the wicket is 0.2 for any of his throws.

The probability that Dean hits the wicket is p for any of his throws.

If Markus throws first, the probability he wins the game is $\frac{5}{13}$.

Determine the value of p.

$\boxed{}$, $\boxed{p = 0.4}$

- LET THE PROBABILITY OF SUCCESS OF DEAN BE p, $0 < p < 1$
- LET MARKUS GO FIRST

$P(X=1) = 0.2$

$P(X=2) = 0.8(1-p) \times 0.2$

$P(X=3) = 0.8(1-p)\,0.8(1-p) \times 0.2$

$P(X=4) = 0.8(1-p)\,0.8(1-p)\,0.8(1-p) \times 0.2$

ETC

- NOW FORM AN EXPRESSION FOR A MARKUS WIN, KNOWN TO BE $\frac{5}{13}$

$$\Rightarrow P(X=1) + P(X=2) + P(X=3) + P(X=4) + \dots = \frac{5}{13}$$

$$\Rightarrow 0.2 + 0.8(1-p) \times 0.2 + [0.8(1-p)]^2 \times 0.2 + [0.8(1-p)]^3 \times 0.2 + \dots = \frac{5}{13}$$

$$\Rightarrow 0.2\Big[\underbrace{1 + 0.8(1-p) + [0.8(1-p)]^2 + [0.8(1-p)]^3 + \dots}_{\text{THIS IS A G.P WITH } a=1,\ r=0.8(1-p),\ S_\infty = \frac{a}{1-r}}\Big] = \frac{5}{13}$$

$$\Rightarrow \frac{1}{5} \times \frac{1}{1-0.8(1-p)} = \frac{5}{13}$$

$$\Rightarrow \frac{1}{1-0.8+0.8p} = \frac{25}{13}$$

$$\Rightarrow \frac{1}{0.2+0.8p} = \frac{25}{13}$$

$$\Rightarrow 0.8p + 0.2 = \frac{13}{25}$$

$$\Rightarrow 0.8p = \frac{8}{25}$$

$$\therefore p = 0.4$$

Question 8 (****)

A coin is biased so that the probability of obtaining "heads" in any toss is p, $p \neq \frac{1}{2}$.

The coin is tossed repeatedly until a "head" is obtained.

The probability of obtaining "heads" after an even number of tosses is $\frac{2}{5}$.

Determine the value of p.

$\boxed{}$, $\boxed{p = \frac{1}{3}}$

WORKING AS FOLLOWS

- LET p BE THE PROBABILITY OF OBTAINING HEADS
- LET X BE THE NUMBER OF TOSSES UNTIL WE OBTAIN A HEAD

$\Rightarrow P(X=2) = (1-p)p$
$\Rightarrow P(X=4) = (1-p)^3 p$
$\Rightarrow P(X=6) = (1-p)^5 p \ldots$

$\Rightarrow p(1-p) + p(1-p)^3 + p(1-p)^5 + p(1-p)^7 + \ldots = \frac{2}{5}$
$\Rightarrow p\underbrace{\left[(1-p) + (1-p)^3 + (1-p)^5 + (1-p)^7 + \ldots\right]}_{\text{THIS IS A G.P WITH } a = 1-p,\ r = (1-p)^2} = \frac{2}{5}$

USING THE SUM TO INFINITY FORMULA $S_\infty = \frac{a}{1-r}$

$\Rightarrow p\left(\frac{1-p}{1-(1-p)^2}\right) = \frac{2}{5}$
$\Rightarrow \frac{p-p^2}{1-(1-2p+p^2)} = \frac{2}{5}$
$\Rightarrow \frac{p-p^2}{2p-p^2} = \frac{2}{5}$

AS $p \neq 0$ WE MAY DIVIDE IT THROUGH IN THE L.H.S

$\Rightarrow \frac{1-p}{2-p} = \frac{2}{5}$
$\Rightarrow 5 - 5p = 4 - 2p$
$\Rightarrow 1 = 3p$
$\Rightarrow p = \frac{1}{3}$

Question 9 (****)

A bag contains a large number of coins, of which some are pound coins and some are two pound coins. A coin is selected at random from the bag with replacement, **until a two pound coin** is selected.

It is given that the probability it will take …

… exactly 2 attempts until a two pound coin is selected is $\frac{3}{16}$.

… more than 3 attempts until a two pound coin is selected is $\frac{27}{64}$.

Determine the probability that a two pound coin will be selected for the first time on the fifth attempt.

$\boxed{}$, $\boxed{\frac{81}{1024} \approx 0.0791}$

LET X = NUMBER OF PICKS UNTIL A £2 COIN IS SELECTED

LET p = THE PROBABILITY OF PICKING A £2 COIN

HENCE $X \sim \text{Geo}(p)$

$P(X=2) = \frac{3}{16}$

$(1-p)p = \frac{3}{16}$

$P(X>3) = P(X \geq 4) = \frac{27}{64}$

$P(X = 4,5,6,7,\dots) = \frac{27}{64}$

$P(X = 1,2,3) = 1 - \frac{27}{64}$

$p + (1-p)p + (1-p)^2 p = \frac{37}{64}$

SOLVING THE FIRST EQUATION FOR p

$\Rightarrow (1-p)p = \frac{3}{16}$

$\Rightarrow p - p^2 = \frac{3}{16}$

$\Rightarrow 16p - 16p^2 = 3$

$\Rightarrow 0 = 16p^2 - 16p + 3$

$\Rightarrow (4p-3)(4p-1) = 0$

$\Rightarrow p = \frac{3}{4}, \frac{1}{4}$

VERIFY EACH VALUE WITH THE SECOND EQUATION

- IF $p = \frac{3}{4}$: $\frac{3}{4} + \frac{1}{4}\times\frac{3}{4} + \frac{1}{16}\times\frac{3}{4} = \frac{63}{64} \neq \frac{37}{64}$
- IF $p = \frac{1}{4}$: $\frac{1}{4} + \frac{3}{4}\times\frac{1}{4} + \frac{9}{16}\times\frac{1}{4} = \frac{37}{64}$

$\therefore p = \frac{1}{4}$

FINALLY WE HAVE

$P(X=5) = (1-p)^4 p = \left(\frac{3}{4}\right)^4 \times \frac{1}{4} = \frac{81}{1024} \approx 0.0791$

(P.T.O)

ALTERNATIVE SOLUTION OF THE EQUATIONS

$p(1-p) = \frac{3}{16}$

$p + (1-p)p + (1-p)^2 p = \frac{37}{64}$

$p + p(1-p) + p(1-p)(1-p) = \frac{37}{64}$

$p + \frac{3}{16} + \frac{3}{16}(1-p) = \frac{37}{64}$

$p + \frac{3}{16} + \frac{3}{16} - \frac{3}{16}p = \frac{37}{64}$

$\frac{13}{16}p = \frac{13}{64}$

$\frac{1}{16}p = \frac{1}{64}$

$p = \frac{1}{4}$

(THIS APPROACH GIVES $\frac{1}{4}$ WITHOUT THE NEED FOR VERIFICATION)

Question 10 (*****)

A discrete random variable X is geometrically distributed with parameter p.

Show that …

a) … $\mathrm{E}(X) = \dfrac{1}{p}$.

b) … $\mathrm{Var}(X) = \dfrac{1-p}{p^2}$.

$\boxed{}$, $\boxed{\text{proof}}$

a) $\boxed{\text{LET } X \sim \text{Geo}(p) \quad 0 < p < 1}$

x	1	2	3	4	5	....
$P(X=x)$	p	$(1-p)p$	$(1-p)^2p$	$(1-p)^3p$	$(1-p)^4p$	....

$$E(X) = p + 2p(1-p) + 3p(1-p)^2 + 4p(1-p)^3 + 5p(1-p)^4 + \dots$$
$$E(X) = p\left[1 + 2(1-p) + 3(1-p)^2 + 4p(1-p)^3 + 5p(1-p)^4 + \dots\right.$$

MULTIPLY THE ABOVE LINE BY $-(1-p)$

$$-(1-p)E(X) = p\left[\quad -(1-p) - 2(1-p)^2 - 3(1-p)^3 - 4(1-p)^4 - \dots\right]$$
$$E(X) = p\left[1 + 2(1-p) + 3(1-p)^2 + 4(1-p)^3 + 5(1-p)^4 + \dots\right]$$

ADDING THE TWO LINES ABOVE WE OBTAIN

$$\left[-(1-p)+1\right]E(X) = p\left[1 + (1-p) + (1-p)^2 + (1-p)^3 + (1-p)^4 + \dots\right]$$
$$\cancel{p}E(X) = \cancel{p}\left[1 + (1-p) + (1-p)^2 + (1-p)^3 + (1-p)^4 + \dots\right]$$
$$E(X) = \underbrace{1 + (1-p) + (1-p) + (1-p)^3 + (1-p)^4 + \dots}_{\text{THIS IS A GEOMETRIC PROGRESSION WITH}}$$

$a = 1$
$r = 1-p$
$S_\infty = \dfrac{a}{1-r}$

$$E(X) = \frac{1 \; \leftarrow a}{1-(1-p) \; \leftarrow 1-r}$$
$$E(X) = \frac{1}{p} \quad \text{// AS REQUIRED}$$

ALTERNATIVE VARIATION

HAVING FORMED AN EXPRESSION FOR EXPECTATION AS BEFORE

$$E(X) = p\left[1 + 2(1-p) + 3(1-p)^2 + 4(1-p)^3 + \dots\right]$$

LET $q = 1-p$

$$\Rightarrow E(X) = (1-q)\left[1 + 2q + 3q^2 + 4q^3 + \dots\right]$$
$$\Rightarrow E(X) = (1-q)\frac{d}{dq}\left[\underbrace{q + q^2 + q^3 + q^4 + \dots}_{\text{CONVERGENT G.P WITH}}\right]$$

$a = q$
$r = q^2$
$S_\infty = \dfrac{a}{1-r}$

$$\Rightarrow E(X) = (1-q)\frac{d}{dq}\left[\frac{q}{1-q}\right]$$
$$\Rightarrow E(X) = (1-q) \times \frac{(1-q)\times 1 - q(-1)}{(1-q)^2}$$
$$\Rightarrow E(X) = (1-q) \times \frac{1}{(1-q)^2}$$
$$\Rightarrow E(X) = \frac{1}{1-q}$$
$$\Rightarrow E(X) = \frac{1}{p} \quad //$$

b) • NEXT THE VARIANCE

$$E(X^2) = \left[1^2 \times p\right] + \left[2^2 \times p(1-p)\right] + \left[3^2 \times p(1-p)^2\right] + \left[4^2 \times p \times (1-p)^3\right] + \dots$$
$$E(X^2) = p\left[1 + 4(1-p) + 9(1-p)^2 + 16(1-p)^3 + 25(1-p)^4 + \dots\right]$$

• MULTIPLY THE EXPRESSION BY $-(1-p)$

$$-(1-p)E(X^2) = p\left[-(1-p) - 4(1-p)^2 - 9(1-p)^3 - 16(1-p)^4 - 25(1-p)^5 - \dots\right]$$
$$E(X^2) = p\left[1 + 4(1-p) + 9(1-p)^2 + 16(1-p)^3 + 25(1-p)^4 + 36(1-p)^5 + \dots\right]$$

• ADDING THE TWO LINES ABOVE

$$\left[1-(1-p)\right]E(X^2) = p\left[1 + 3(1-p) + 5(1-p)^2 + 7(1-p)^3 + 9(1-p)^4 + \dots\right]$$
$$\cancel{p}E(X^2) = \cancel{p}\left[1 + 3(1-p) + 5(1-p)^2 + 7(1-p)^3 + 9(1-p)^4 + \dots\right]$$
$$E(X^2) = 1 + 3(1-p) + 5(1-p)^2 + 7(1-p)^3 + 9(1-p)^4 + \dots$$

• MULTIPLY THE ABOVE LINE AGAIN BY $-(1-p)$

$$-(1-p)E(X^2) = \quad -(1-p) - 3(1-p)^2 - 5(1-p)^3 - 7(1-p)^4 - 9(1-p)^4 - \dots$$
$$E(X^2) = 1 + 3(1-p) + 5(1-p)^2 + 7(1-p)^3 + 9(1-p)^4 + 11(1-p)^4 \dots$$

• ADDING AGAIN THE TWO LINES ABOVE

$$\left[1-(1-p)\right]E(X^2) = 1 + 2(1-p) + 2(1-p)^2 + 2(1-p)^3 + 2(1-p)^4 + \dots$$
$$pE(X^2) = 1 + 2\left[\underbrace{(1-p) + (1-p)^2 + (1-p)^3 + (1-p)^4 + \dots}_{\text{THIS IS A CONVERGENT G.P}}\right]$$

$a = 1-p$
$r = 1-p$
$S_\infty = \dfrac{a}{1-r}$

$$pE(X^2) = 1 + 2 \times \frac{1-p}{1-(1-p)}$$

• HENCE, FURTHER TIDYING UP

$$pE(X^2) = 1 + \frac{2(1-p)}{p}$$
$$p^2E(X^2) = p + 2 - 2p$$
$$p^2E(X^2) = 2 - p$$
$$E(X^2) = \frac{2-p}{p^2}$$

• FINALLY THE VARIANCE

$$\begin{aligned}\mathrm{Var}(X) &= E(X^2) - \left[E(X)\right]^2 \\ &= \frac{2-p}{p^2} - \left(\frac{1}{p}\right)^2 \\ &= \frac{2-p}{p^2} - \frac{1}{p^2} \\ &= \frac{1-p}{p^2} \quad //\end{aligned}$$

NEGATIVE BINOMIAL DISTRIBUTION

Question 1 (**)

A discrete random variable X has negative binomial distribution, with 6 successes required each with probability of success 0.4.

Determine the value of …

a) … $\mathrm{E}(X)$.

b) … $\mathrm{Var}(X)$.

c) … $\mathrm{P}(X=12)$.

$\boxed{}$, $\boxed{\mathrm{E}(X)=15}$, $\boxed{\mathrm{Var}(X)=22.5}$, $\boxed{\mathrm{P}(X=12)\approx 0.0883}$

$X \sim NB(6, 0.4)$, $r=6$ $p=0.4$

a) $E(X) = \frac{r}{p} = \frac{6}{0.4} = 15$

b) $\mathrm{Var}(X) = \frac{r(1-p)}{p^2} = \frac{6(1-0.4)}{0.4^2} = \frac{6\times 0.6}{0.16} = 22.5$

c) $P(X=12) = \binom{11}{5} 0.4^5 0.6^6 \times 0.4 = \binom{11}{5} 0.4^6 0.6^6$

$= 462 \times 0.24^6$

$= 0.0883$

Question 2 (**+)

Justin is playing "*Alien Shooter*" on the internet. It is a game where you battle against randomly drawn opponents where the reward of a battle is a "*game ticket*".

The probability of Justin winning a battle is thought to be 0.4.

a) Showing detailed workings where appropriate, calculate the probability of Justin …

i. … winning his first battle on his third attempt.

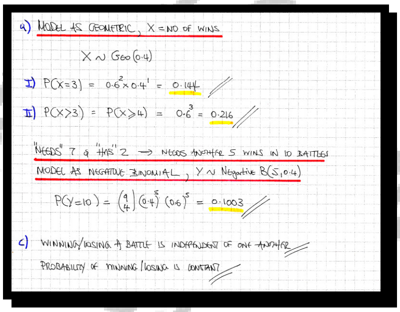
ii. … winning his first battle after his third attempt.

In Alien Shooter when you collect 7 game tickets you can upgrade your spaceship. Justin has already collected 2 game tickets from the previous day's play. He starts playing today, hoping to upgrade his spaceship.

b) Determine the probability he will have to have 10 battles by the time he is able to upgrade his spaceship.

c) State two conditions that must be satisfied in this scenario if the calculations are to be valid.

$\boxed{}$, $\boxed{0.144}$, $\boxed{0.216}$, $\boxed{0.1003}$



a) MODEL AS GEOMETRIC, X = NO OF WINS

$X \sim \text{Geo}(0.4)$

I) $P(X=3) = 0.6^2 \times 0.4^1 = 0.144$

II) $P(X>3) = P(X \geqslant 4) = 0.6^3 = 0.216$

"NEEDS" 7 & "HAS" 2 $\Longrightarrow$ NEEDS ANOTHER 5 WINS IN 10 BATTLES

MODEL AS NEGATIVE BINOMIAL, $Y \sim$ Negative $B(5, 0.4)$

$P(Y=10) = \binom{9}{4}(0.4)^5(0.6)^5 = 0.1003$

c) WINNING/LOSING A BATTLE IS INDEPENDENT OF ONE ANOTHER

PROBABILITY OF WINNING/LOSING IS CONSTANT

Question 3 (**+)

A discrete random variable X has negative binomial distribution, with $\mathrm{E}(X)=12$ and $\mathrm{Var}(X)=4$.

Determine the value of $\mathrm{P}(X=12)$.

$\boxed{}$, $\boxed{\mathrm{P}(X=12)\approx 0.1936}$

USING STANDARD RESULTS FOR $X \sim NB(r,p)$

$E(X) = \frac{r}{p}$ $\quad$ $\mathrm{Var}(X) = \frac{r(1-p)}{p^2}$

$12 = \frac{r}{p}$ $\quad$ $4 = \frac{r(1-p)}{p^2}$

COMBINE EQUATIONS

$\Rightarrow 4 = \frac{r}{p}\left(\frac{1-p}{p}\right)$

$\Rightarrow 4 = 12\left(\frac{1-p}{p}\right)$

$\Rightarrow \frac{1}{3} = \frac{1-p}{p}$

$\Rightarrow p = 3-3p$

$\Rightarrow 4p = 3$

$\Rightarrow p = \frac{3}{4}$

USING $12 = \frac{r}{p}$

$\Rightarrow 12 = \frac{r}{3/4}$

$\Rightarrow r = 9$

FINALLY WE HAVE

$X \sim NB(9, 0.75)$

$\Rightarrow P(X=12) = \underbrace{\binom{11}{8}0.75^8\,0.25^3}_{\text{8 SUCCESSES IN THE FIRST 11 TRIALS}} \times 0.75$ SUCCESS IN THE 12^{TH}

$= \binom{11}{8}0.75^9 \times 0.25^3$

$= 0.1936$

Question 4 (***+)

It has been established over a long period of time that the probability that an electricity company operative will be able to take a reading from a house meter, due to the resident being at home, is 0.4.

a) Determine the probability that the first reading to be taken will be on, or before, the 7^{th} house visit.

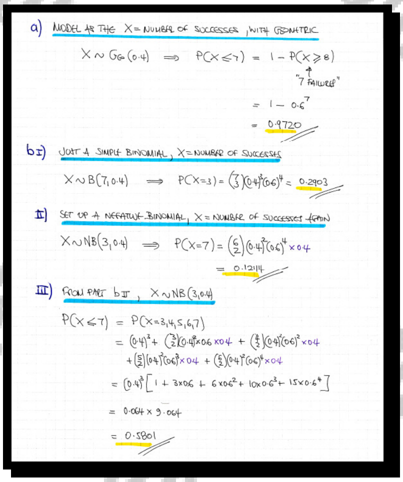
b) Find the probability that the operative will be able to take …

i. … exactly 3 readings in his first 7 visits.

ii. … his 3^{rd} reading on his 7^{th} visit.

iii. … his 3^{rd} reading on, or before his 7^{th} visit.

$\boxed{}$, $\boxed{\approx 0.9720}$, $\boxed{\approx 0.2903}$, $\boxed{\approx 0.1244}$, $\boxed{\approx 0.5801}$



a) MODEL AS THE X = NUMBER OF SUCCESSES, WITH GEOMETRIC

$X \sim Geo(0.4) \implies P(X \leqslant 7) = 1 - P(X \geqslant 8)$

"7 FAILURES"

$= 1 - 0.6^7$

$= 0.9720$

bI) JUST A SIMPLE BINOMIAL, X = NUMBER OF SUCCESSES

$X \sim B(7, 0.4) \implies P(X=3) = \binom{7}{3}(0.4)^3(0.6)^4 = 0.2903$

II) SET UP A NEGATIVE BINOMIAL, X = NUMBER OF SUCCESSES AGAIN

$X \sim NB(3, 0.4) \implies P(X=7) = \binom{6}{2}(0.4)^2(0.6)^4 \times 0.4$

$= 0.1244$

III) FROM PART bII, $X \sim NB(3, 0.4)$

$P(X \leqslant 7) = P(X = 3,4,5,6,7)$

$= (0.4)^3 + \binom{3}{2}(0.4)^2 \times 0.6 \times 0.4 + \binom{4}{2}(0.4)^2(0.6)^2 \times 0.4$

$+ \binom{5}{2}(0.4)^2(0.6)^3 \times 0.4 + \binom{6}{2}(0.4)^2(0.6)^4 \times 0.4$

$= (0.4)^3\left[1 + 3 \times 0.6 + 6 \times 0.6^2 + 10 \times 0.6^3 + 15 \times 0.6^4\right]$

$= 0.064 \times 9.064$

$= 0.5801$

Question 5 (*+)**

Steve is filming and uploading videos on the internet, and **every week** he plans to upload 5 videos.

The probability that a video has no significant faults and so it is deemed to be suitable for uploading to the internet, is 0.25. Once Steve has uploaded 5 videos he stops filming for that week.

a) Find the probability that, in a given week, Steve will have to film 11 videos.

b) Determine the number of videos Steve will be expected to film in a given week in order to meet his weekly uploading target of 5 videos.

In a season Steve plans to film for 40 weeks.

c) Estimate the probability that the **mean** number of videos Steve has to film per week, is greater than 18.

$\boxed{}$, $\boxed{\approx 0.0365}$, $\boxed{\mathrm{E}(X)=20}$, $\boxed{\approx 0.949}$

a) MODEL AS NEGATIVE BINOMIAL

X = NUMBER OF VIDEOS STEVE HAS TO FILM

$X \sim NB(5, 0.25)$

- $P(X=11) = \underbrace{\binom{10}{4}(0.25)^4(0.75)^6}_{\text{4 "GOOD" VIDEOS IN THE FIRST 10}} \times \underbrace{0.25}_{\text{11TH VIDEO IS GOOD}} = 0.0365$

b) THIS IS JUST THE EXPECTATION OF $X \sim NB(r,p)$

- $E(X) = \frac{r}{p} = \frac{5}{0.25} = 20$

c) FIRSTLY CALCULATE THE VARIANCE

$\mathrm{Var}(X) = \frac{r(1-p)}{p^2} = \frac{5 \times 0.75}{0.25^2} = 60$

AS THE SAMPLE SIZE IS LARGE $(n > 30)$, BY THE C.L.T

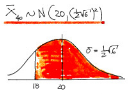
$\bar{X}_{40} \sim N\left(20, \frac{60}{40}\right)$ ie $\bar{X}_{40} \sim N\left(20, \left(\tfrac{1}{2}\sqrt{6}\right)^2\right)$

- $P(\bar{X}_{40} > 18)$

$= P\left(z > \frac{18-20}{\frac{1}{2}\sqrt{6}}\right)$

$= \Phi(-1.6330)$

$= 0.949$



Question 6 (****+)

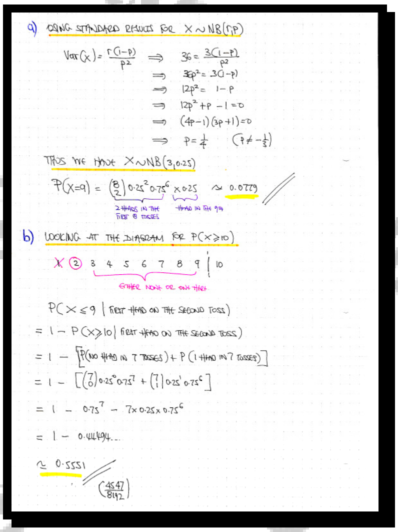
The discrete random variable X represents the number of times a biased coin must be tossed until 3 "*heads*" have been obtained.

It is further given that $\text{Var}(X) = 36$.

a) Calculate $\text{P}(X = 9)$.

b) If the first "head" was obtained on the second toss, find $\text{P}(X \le 9)$

$\boxed{\text{[illegible]}}$, $\boxed{\approx 0.0779}$, $\boxed{\approx 0.5551}$



a) USING STANDARD RESULTS FOR $X \sim NB(r,p)$

$$\text{Var}(X) = \frac{r(1-p)}{p^2} \Rightarrow 36 = \frac{3(1-p)}{p^2}$$
$$\Rightarrow 36p^2 = 3(1-p)$$
$$\Rightarrow 12p^2 = 1-p$$
$$\Rightarrow 12p^2 + p - 1 = 0$$
$$\Rightarrow (4p-1)(3p+1) = 0$$
$$\Rightarrow p = \frac{1}{4} \quad \left(p \neq -\frac{1}{3}\right)$$

THUS WE HAVE $X \sim NB(3, 0.25)$

$$P(X=9) = \underbrace{\binom{8}{2} 0.25^2 0.75^6}_{\text{2 HEADS IN THE FIRST 8 TOSSES}} \times \underbrace{0.25}_{\text{HEAD IN THE 9TH}} \simeq 0.0779$$

b) LOOKING AT THE DIAGRAM FOR $P(X \geqslant 10)$

$$\cancel{X} \quad 2 \quad \underbrace{3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9}_{\text{EITHER NONE OR ONE HERE}} \mid 10$$

$P(X \leqslant 9 \mid \text{FIRST HEAD ON THE SECOND TOSS})$

$= 1 - P(X \geqslant 10 \mid \text{FIRST HEAD ON THE SECOND TOSS})$

$= 1 - [P(\text{NO HEAD IN 7 TOSSES}) + P(\text{1 HEAD IN 7 TOSSES})]$

$= 1 - \left[\binom{7}{0} 0.25^0 0.75^7 + \binom{7}{1} 0.25^1 0.75^6\right]$

$= 1 - 0.75^7 - 7 \times 0.25 \times 0.75^6$

$= 1 - 0.44494\ldots$

$\simeq 0.5551$

$\left(\frac{4547}{8192}\right)$