DISCRETE RANDOM VARIABLES
Question 1 (**)  
The probability distribution of a discrete random variable \( X \) is given by:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>( \frac{3}{8} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{4} )</td>
<td>( a )</td>
<td>( \frac{1}{24} )</td>
</tr>
</tbody>
</table>

where \( a \) is a positive constant.

a) Explain why \( a = 0 \).

b) Find the value of \( E(X) \).

c) Calculate \( \text{Var}(X) \).

\( \sum P(X = x) = 1 \), \( E(X) = 1 \), \( \text{Var}(X) = 1 \)
Question 2 (***)

The probability distribution of a discrete random variable $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{1}{12}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
</tbody>
</table>

Find, showing full workings where appropriate, the value of

a) $P(1 < X \leq 3)$.

b) $F(1.8)$.

c) $E(X)$.

d) $\text{Var}(X)$.

e) $E(2X - 3)$.

f) $\text{Var}(2X - 3)$.

$\frac{2}{3}$, $F(1.8) = \frac{1}{3}$, $E(X) = \frac{23}{12}$, $\text{Var}(X) = \frac{131}{144} \approx 0.910$, $E(2X - 3) = \frac{5}{6}$, $\text{Var}(2X - 3) = \frac{131}{36} \approx 3.639$
Question 3 (**)

The probability distribution of a discrete random variable $X$ is given by

$$P(X = x) = \begin{cases} \frac{kx^2}{3}, & x = 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$$

a) Determine the value of the constant $k$.

b) Find the value of ...

i. ... $E(X)$.

ii. ... $\text{Var}(X)$.

c) Determine ...

i. ... $E(5X - 4)$.

ii. ... $\text{Var}(5X - 4)$.

$\boxed{k = \frac{1}{50}, \quad E(X) = 4.32, \quad \text{Var}(X) = 0.5776, \quad E(5X - 4) = 17.6, \quad \text{Var}(5X - 4) = 14.44}$
Question 4  (**)

The discrete random variable $X$ has mean 7 and variance 11.

a) Calculate $E(X^2)$.

b) Given that $Y = 2X - 4$, determine the mean and variance of $Y$.

\[ E(X^2) = 60, \quad E(Y) = 10, \quad \text{Var}(Y) = 44 \]
Question 5 (**)

The probability distribution of a discrete random variable \( X \) is given by

\[
P(X = x) = \begin{cases} k(2-x)^2 & \text{if } x = -2, -1, 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}
\]

a) Determine the value of the constant \( k \).

b) Find the value of …
   
   i. \( E(X) \).
   
   ii. \( E(X^2) \).

b) Determine …
   
   i. \( E(1-15X) \).
   
   ii. \( \text{Var}(1-15X) \).

\[
\begin{align*}
\text{FS1-P} & = 30, \\
k & = \frac{1}{30}, \\
E(X) & = \frac{-4}{3}, \\
E(X^2) & = \frac{37}{15}, \\
E(1-15X) & = 21. \\
\text{Var}(1-15X) & = 155
\end{align*}
\]
Question 6 (**)  
The probability distribution of a discrete random variable $X$ is given by

$$P(X = x) = \begin{cases} kx(5-x) & x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

a) Determine the value of the constant $k$.  
b) State the value of $E(X)$.  
c) Calculate $\text{Var}(X)$.  
d) Determine the value of $E(4X - 5)$.  

$$k = \frac{1}{20}, \quad E(X) = 2.5, \quad \text{Var}(X) = 1.05, \quad E(4X - 5) = 5$$
Question 7 (***)

The probability distribution of a discrete random variable \( X \) is given by

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>0.5</td>
<td>0.35</td>
<td>( a )</td>
<td>( b )</td>
</tr>
</tbody>
</table>

where \( a \) and \( b \) are positive constants.

a) Given that \( E(X) = 0.67 \), find the value of \( a \) and the value of \( b \).

b) Determine the variance of \( X \).

c) Calculate \( \text{Var}(5+10X) \).

\[ a = 0.13, \quad b = 0.02 \quad \text{Var}(X) = 0.6011, \quad \text{Var}(5+10X) = 60.11 \]
Question 8  (***)

The probability distribution of a discrete random variable $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.2</td>
<td>$a$</td>
<td>0.2</td>
<td>$b$</td>
<td>0.15</td>
</tr>
</tbody>
</table>

where $a$ and $b$ are positive constants.

a) Given that $E(X) = 4.5$, find the value of $a$ and the value of $b$.

b) Determine $E(29 - 6X)$.

$$a = 0.3, \quad b = 0.15, \quad E(29 - 6X) = 2$$
Question 9  (***)

Two fair spinners, both numbered with 0, 1, 2 and 3, are spun together and the product of their scores is recorded.

The discrete random variable $X$ represents the product of the scores of these spinners and its probability distribution is summarized in the table below:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

a) Find the value of $a$, $b$ and $c$.

b) Determine $E(X)$.

c) Find the value of $\text{Var}(X)$.

d) Calculate $E(4X - 1)$.

e) Calculate $\text{Var}(4X - 1)$.

\[
a = \frac{1}{8}, \quad b = \frac{1}{8}, \quad c = \frac{1}{16}, \quad E(X) = \frac{9}{4}, \quad \text{Var}(X) = \frac{115}{16}, \quad E(4X - 1) = 8, \quad \text{Var}(4X - 1) = 115
\]
Question 10 (***)

The probability distribution of a discrete random variable $X$ is summarised in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

a) Find the value of ...
   i. ... $E(X)$.
   ii. ... $E(X^2)$.
   iii. ... $\text{Var}(X)$.

b) Calculate ...
   i. ... $E(3 - X)$.
   ii. ... $\text{Var}(3 - X)$

c) Determine the value of

$$P[4X - 3 \geq 2(X + 1)].$$

$$E(X) = 3.25, \quad E(X^2) = 12.65, \quad \text{Var}(X) = 2.0875, \quad E(3 - X) = -0.25, \quad \text{Var}(3 - X) = 2.0875, \quad P[4X - 3 \geq 2(X + 1)] = 0.7$$
Question 11 (***)

The cumulative distribution \( F(x) \), of a discrete random variable \( X \) is given by

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(x) )</td>
<td>0.25</td>
<td>0.40</td>
<td>0.55</td>
<td>0.65</td>
<td>0.75</td>
<td>0.85</td>
<td>0.95</td>
<td>1</td>
</tr>
</tbody>
</table>

a) Find the value …

i. … \( E(X) \).

ii. … \( \text{Var}(X) \).

The discrete random variable \( Y \) is defined as \( Y = 5X - 3 \).

b) Determine the mean and variance of \( Y \).

\( E(Y) = 15 \), \( \text{Var}(Y) = 126 \)
Question (***)

The probability distribution of a discrete random variable $X$ is given by

$$P(X = x) = \begin{cases} \frac{1}{12} & x = 1, 2, 3, \ldots, 12 \\ 0 & \text{otherwise} \end{cases}$$

Determine $P(2 < X < 3X - 4 \leq 2X + 7)$.

\[
P(X + 2 < 3X - 4 \leq 2X + 7) = \frac{4}{3}
\]
Question (***+)

A sixth form class consists of 6 boys and 4 girls.

Three students are selected at random from this class and the variable $X$ represents the number of girls selected.

Show that the probability distribution of $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P($X = x$)</td>
<td>$\frac{5}{30}$</td>
<td>$\frac{15}{30}$</td>
<td>$\frac{9}{30}$</td>
<td>$\frac{1}{30}$</td>
</tr>
</tbody>
</table>
Question 12 (***)

The cumulative distribution of a discrete random variable $X$ is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{2k+3}{20} & 1 \leq x < 2 \\ \frac{k+5}{10} & 4 \leq x < 5 \\ \frac{k+2}{4} & x \geq 5 \\ \end{cases}$$

where $k$ is a positive constant.

a) Show clearly that $2k = 2$.

b) Find the value of ...

i. $\ldots E(X)$.

ii. $\ldots E(X^2)$.

c) Calculate $\text{Var}(20X - 2)$.

$$E(X) = 3.45, \quad E(X^2) = 14.05, \quad \text{Var}(20X - 2) = 859$$
**Question 13 (***+)**

The probability distribution of a discrete random variable $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.05</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
<td>0.25</td>
<td>0.2</td>
<td>0.05</td>
</tr>
</tbody>
</table>

(a) Find the value of $E(X)$.

(b) Calculate $\text{Var}(X)$.

(c) Determine $P(\mu - \sigma \leq X \leq \mu + \sigma)$.

(d) Find the value of $E\left(4X^2 - 3.2\right)$.

\[ E(X) = 3.3, \quad \text{Var}(X) = 2.41, \quad P(\mu - \sigma \leq X \leq \mu + \sigma) = 0.6, \quad E\left(4X^2 - 3.2\right) = 50 \]
Question 14 (***)

A box contains three blue discs and two red discs.

Three discs are selected at random from the box without replacement.

The variable $X$ represents the number of blue discs selected.

a) Show that the probability distribution of $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(X = x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{3}{10}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{6}{10}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{10}$</td>
</tr>
</tbody>
</table>

b) Determine $E(X)$ and $\text{Var}(X)$.

$E(X) = \frac{9}{5} = 1.8$, $\text{Var}(X) = \frac{9}{25} = 0.36$
Question 15  (***)

A sixth form class consists of 6 boys and 4 girls.

Three students are selected at random from this class and the variable $X$ represents the number of girls selected.

a) Show that the probability distribution of $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{5}{30}$</td>
<td>$\frac{15}{30}$</td>
<td>$\frac{9}{30}$</td>
<td>$\frac{1}{30}$</td>
</tr>
</tbody>
</table>

b) Determine $E(X)$ and $\text{Var}(X)$.

\[
E(X) = \frac{6}{\cancel{30}} \cdot \frac{\cancel{30}}{\cancel{30}} = 1.2, \quad \text{Var}(X) = \frac{14}{25} = 0.56
\]
Question (***+)

The probability distribution of the discrete random variable \( X \) is given by

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>0.4 (-a)</td>
<td>2(a)</td>
<td>0.6 (-a)</td>
</tr>
</tbody>
</table>

where \( a \) is a constant.

a) State the range of the possible values of \( a \).

Two independent observations of \( X \), denoted by \( X_1 \) and \( X_2 \) are considered.

a) Determine, in terms of \( a \), a simplified expression for \( P(X_1 + X_2 = 6) \).

\[
P(X_1 + X_2 = 6) = 6a^2 - 2a + 0.48
\]
Question 16  (***)

Two standard fair cubical dice, numbered 1 to 6 are such rolled and the random variable $X$ represents the sum of the scores of the two dice.

Determine the value of $\text{Var}(X)$.

\[ \text{Var}(X) = \frac{35}{6} \]
Question 17 (***)

The discrete random variable $X$ has the following probability distribution

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

a) Determine $E(X)$ and $\text{Var}(X)$.

A game in a fun fair consists of throwing 5 darts on a small target.

If a dart lands on the central portion of the target the dart scores 3 points.

If a dart lands on the outer portion of the target the dart scores 2 points, otherwise the dart scores no points.

To win a prize, 10 or more points must be scored with 5 darts.

Paul has scored 6 points with his first 3 darts.

The likelihood of Paul scoring 0, 2 or 3 points is given by the probability distribution of part (a).

b) Find the probability that Paul will win a prize after he throws his last 2 darts.
Question 18 (***)

The probability distribution of a discrete random variable $X$ is given by

$$P(X = x) = \begin{cases} 
  k(2-x) & x = 0, 1, 2 \\
  \frac{1}{4} & x = 3 \\
  0 & \text{otherwise}
\end{cases}$$

a) Show that $k = \frac{1}{4}$.

b) Find the value of $E(X)$ and $E(X^2)$.

c) Determine $\text{Var}(3 - X)$.

Two independent observations of $X$ are made, denoted by $X_1$ and $X_2$.

d) Find the probability distribution of $Y$, where $Y = X_1 + X_2$.

e) Calculate $P(1.5 \leq Y \leq 3.5)$.
Question 19  (***)

The discrete random variable \( X \) has the following probability distribution:

\[
\begin{array}{c|c|c|c}
 x & 0 & 1 & 3 \\
 P(X = x) & \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\
\end{array}
\]

a) Determine \( E(X) \) and \( \text{Var}(X) \).

Two independent observations of \( X \) are made, denoted by \( X_1 \) and \( X_2 \).

b) Find the probability distribution of \( X_1 + X_2 \).

c) Calculate \( P(X_1 > X_2) \).

\[
P(X_1 + X_2 = r) = \begin{cases} 
\frac{1}{36} & r = 0 \\
\frac{1}{9} & r = 1, 2 \\
\frac{1}{4} & r = 3 \\
\frac{1}{3} & r = 4 \\
0 & \text{otherwise} 
\end{cases}
\]

\[
P(X_1 > X_2) = \frac{11}{36}
\]
The probability distribution of a discrete random variable $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$0.3 - k$</td>
<td>$2k$</td>
<td>$0.7 - k$</td>
</tr>
</tbody>
</table>

a) Find the range of possible values of the constant $k$.

b) Determine $E(X)$.

c) Given that $\text{Var}(X) = 0.72$, find the value of $k$.

$X_1$ and $X_2$ are two independent observations of $X$.

d) Find $P(X_1 = X_2)$.
Question 21  (****)

The probability distribution of the discrete random variable $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$0.4 - a$</td>
<td>$2a$</td>
<td>$0.6 - a$</td>
</tr>
</tbody>
</table>

where $a$ is a constant.

b) State the range of the possible values of $a$.

c) Show that $E(X)$ is independent of $a$.

d) Given that $\text{Var}(X) = 0.56$ show that $a = 0.2$.

Two independent observations of $X$, denoted by $X_1$ and $X_2$ are considered.

e) Calculate $P(X_1 + X_2 = 6)$.

\[ 0 \leq a \leq 0.4, \quad P(X_1 + X_2 = 6) = 0.32 \]
Question 22  (****)

A biased spinner can show whole numbers from 1 to 8.

The probability of showing an 8 is 0.05 and the probability of showing a 7 is 0.11.

The probabilities of showing any of the other six whole numbers are all equal to one another.

Players in a gambling parlour pay £5 for a single spin.

A score of 8 wins the player £50, a score of 7 wins the player £20, otherwise the player wins no money.

In a typical day, a gambling addict has 150 spins on this spinner.

Find the expected loss of the gambling addict in a typical day.

\[
\text{Expected loss} = (150 \times £0) - (150 \times £5) + (150 \times £20) - (150 \times £50) = £45
\]
Question 23 \((****)\)

The probability distribution of a discrete random variable \(X\) is given by

\[
P(X = x) = \begin{cases} 
\frac{1}{20} & x = 1, 2, 3, 4, 5 \\
\frac{1}{4} & x = 6 \\
0 & \text{otherwise}
\end{cases}
\]

a) Find \(P(X > 4)\).

b) Calculate \(E\left(\frac{1}{X}\right)\).

c) Show that \(\text{Var}\left(\frac{1}{X}\right) = \frac{173}{4800}\).

The discrete random variable \(Y\) is defined as \(Y = \frac{X + 3}{X}\).

d) Determine the value of \(E(Y)\) and the value of \(\text{Var}(Y)\).

\[
\begin{align*}
P(X > 4) &= 0.5, \\
E\left(\frac{1}{X}\right) &= \frac{7}{24}, \\
E(Y) &= \frac{15}{8}, \\
\text{Var}(Y) &= \frac{519}{1600}
\end{align*}
\]
Question 24 (***)

A sixth form class consists of 3 boys and 7 girls.

Three students are selected at random from this class and the variable \( X \) represents the number of boys selected.

Show clearly that \( E(X) = 0.9 \).
Question 25 (****)

The probability distribution of a discrete random variable $X$ is given by

\[ P(X = x) = \begin{cases} 
\frac{1}{10} & x = 1, 2, 3, \ldots, 10 \\
0 & \text{otherwise}
\end{cases} \]

(a) Find the value of $E(X^2)$.

(b) Determine $P(X + 2 < 3X - 4 < X + 7)$.

It is further given that $E(kX + 5) = 6.1$, where $k$ is a constant.

c) Find the value of $\text{Var}(kX + 5)$

$$E(X^2) = 38.5, \quad P(X + 2 < 3X - 4 < X + 7) = \frac{1}{5}, \quad \text{Var}(kX + 5) = 0.33$$
The probability distribution of a discrete random variable $X$ is given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$k$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>$0.4 - k$</td>
</tr>
</tbody>
</table>

where $k$ is a positive constant.

d) Determine the range of values of $E(X)$.

e) Given that $\text{Var}(X) = 1.36$, find the value of $k$.

\[
2.4 \leq E(X) \leq 4, \quad k = 0.05
\]
Question 27 (***)

Luke has 6 chocolates of which 2 have a hazelnut at their centre.

Luke eats his chocolates one after the other.

The random variable $X$ represents the number of chocolates Luke eats, up and including the first chocolate with a hazelnut at its centre.

Show, with detailed workings, that $\text{Var}(X) = \frac{14}{9}$
Question 28  (*****)

The probability distribution of a discrete random variable $X$ is given by

$$P(X = x) = \begin{cases} \frac{1}{7} & x = 1, 2, 3, \ldots, 7 \\ 0 & \text{otherwise} \end{cases}$$

The probability distribution of another discrete random variable $Y$ is given by

$$P(Y = y) = \begin{cases} \frac{1}{3} & x = 2, 3, 6 \\ 0 & \text{otherwise} \end{cases}$$

Two observations of $X$ are made, denoted by $X_1$ and $X_2$, and one observation of $Y$, denoted by $Y_1$ are considered.

Assuming these three observations are independent, calculate $P(X_1 + X_2 \geq 9 + Y_1)$.

$$P(X_1 + X_2 \geq 9 + Y_1) = \frac{1}{7}$$
Question 29 \( (****) \)

A biased six sided die has the following probability distribution

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

where the random variable \( X \) represents the number shown on its uppermost face when it comes to rest after it is rolled.

The die is rolled twice and the two independent observations of \( X \), \( X_1 \) and \( X_2 \), produce the score \( Y \) defined as

\[
Y = \begin{cases} 
6 & \text{if } X_1 = 6 \\
X_1 + X_2 & \text{if } X_1 \neq 6
\end{cases}
\]

a) Find the value of \( P(Y = 6) \).

b) Determine the probability distribution of \( Y \) and hence calculate the \( E(Y) \).

c) Find the value of \( P(Y < 7 | Y > 4) \).

\[ P(Y = 6) = 0.55 \quad E(Y) = 6.75 \quad P(Y < 7 | Y > 4) = \frac{59}{94} \]
Question 30  (*****)

The probability distribution of a discrete random variable $X$ is given by

$$P(X = x) = \begin{cases} 
  k & x = 1 \\
  \frac{1}{2} P(X = x-1) & x = 2, 3, 4 \\
  0 & \text{otherwise}
\end{cases}$$

where $k$ is a positive constant.

Three independent observations of $X$ are made, denoted by $X_1$, $X_2$ and $X_3$, and the variable $Y$ is defined as $Y = X_1 + X_2 + X_3$.

If $Y$ is an even number, determine the probability that $Y$ is greater than 9.

Answer: $\frac{1}{65}$
Question 31 (*****)

The probability distribution of a discrete random variable $X$ is given by

$$P(X = r + 1) = \begin{cases} \frac{2}{3}P(X = r) & r = 1, 2, 3, 4, 5, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Determine $P(2 \leq X \leq 4)$. 

\[ P(2 \leq X \leq 4) = \frac{38}{31} \]