DATA ANALYSIS EXAM QUESTIONS
Question 1 (**)  
The number of phone text messages sent by 11 different students is given below:  
14, 25, 31, 36, 37, 41, 51, 52, 55, 79, 112.

a) Find the lower quartile, the median and the upper quartile of the data.

b) Show clearly that there is only one outlier in the data.

c) Draw a suitably labelled box plot for this data, clearly indicating any outliers.

d) Determine with justification the skewness of the data.

\[ Q_1 = 31, \quad Q_2 = 41, \quad Q_3 = 55, \quad 112 \text{ is the only outlier}, \quad \text{positive skew} \]
Question 2  (**)

The number of bottles of red wine sold by a local supermarket over a two week period is shown below.

22, 14, 11, 33, 32, 45, 4, 12, 13, 20, 27, 44, 30, 15.

a) Display the above data in an ordered stem and leaf diagram.

b) Calculate the mean and the standard deviation of the data.

c) Find the median and the quartiles of the data and use them to determine if there are any outliers.

d) Draw a suitably labelled box plot for this data.

e) Determine with justification the skewness of the data.

$\bar{x} = 23, \sigma = 12.11, Q_1 = 13, Q_2 = 21, Q_3 = 33,$ no outliers, positive skew.
Question 3  (***)

The concentration of lactic acid, in appropriate units, after a period of intense exercise was measured in the blood of 12 marathon runners.

<table>
<thead>
<tr>
<th>Athlete</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lactic Acid Concentration</td>
<td>180</td>
<td>172</td>
<td>110</td>
<td>175</td>
<td>256</td>
<td>140</td>
<td>241</td>
<td>450</td>
<td>205</td>
<td>375</td>
<td>402</td>
<td>195</td>
</tr>
</tbody>
</table>

a) Find the mean and the standard deviation of the data.

b) Determine the value of the median and the quartiles.

The skewness of data can be determined by the formula

\[ \text{skewness} = \frac{3(\text{mean} - \text{median})}{\text{standard deviation}} \]

c) Evaluate this expression for this data and hence state its skew.

d) Draw a suitably labelled box plot for this data.
You may assume that there are no outliers in this data.

\[ \bar{x} = 241.75, \ \sigma = 104.64, \ Q_1 = 173.5, \ Q_2 = 200, \ Q_3 = 315.5, \ 1.20, \ \text{positive skew} \]
Question 4  (***)

The \% marks, rounded to the nearest integer, of a recent Mathematics test taken by 16 students, were summarised in an ordered stem and leaf diagram.

\[
\begin{align*}
4 & \quad 7 \\
5 & \quad 2, 3, 8 \\
6 & \quad 0, 3, 4, a, b \quad \text{where } 52 = 52 \\
7 & \quad 3, 6, c, d, 8 \\
8 & \quad 1, 9
\end{align*}
\]

a) Determine the lower quartile of the data.

b) Given the median is 68 and \( a \neq b \), find the value of \( a \) and the value of \( b \).

It is further given that \( c \neq d \).

c) Find the possible values of the upper quartile.

\[Q_3 = 59, \quad a = 7, \quad b = 9, \quad 76.5, 77, 77.5\]
Question 5  (**+**)

A company decides to give their 23 employees a skills test in order to decide if any of these employees need to be retrained.

The maximum possible score in this test is 50 and the results are summarised in an ordered stem and leaf diagram.

```
0 5
1 9,9
2 1,6,8
3 3,4,5,7
4 2,3,4,4,8,9,9
5 0,0,0,0,0,0
```

where $29 = 29$.

a) Find the median score of the test.

b) Determine the interquartile range of the scores.

The company decides to retrain any employee whose score is less than the lower quartile minus the interquartile range.

c) Show clearly that only one employee will undergo retraining.

d) Draw a suitably labelled box plot for this data, clearly indicating any outliers, as found in part (c).

e) Determine with justification the skewness of the scores.

$Q_2 = 43$, $IQR = 22$, 05 is the only outlier, negative skew
Question 6  (**+**)

The following set of data shows the number of posts made, in a given day, in a social media site by a group of individuals:

1, 12, 13, 14, 16, 17, 20, 21, 23, 24, 26, 39, 55.

For this set of data, ...

a) ... determine the value of the median and the quartiles.

b) ... calculate the mean and the standard deviation.

c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

\[
(Q_1, Q_2, Q_3) = (14, 20, 26) \quad \text{or} \quad (Q_1, Q_2, Q_3) = (13.5, 20, 25), \quad \bar{x} = 21.6, \quad \sigma \approx 12.9, \quad 55 \text{ is an outlier}, \quad \text{no skew or positive skew depending on the method}
\]
A farmer keeps chicken and sells most of the eggs they lay.

The table below summarizes information about the number of eggs laid by his chickens every week, for a period of 47 weeks.

<table>
<thead>
<tr>
<th>Total number of eggs laid in a week</th>
<th>Number of weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td>53</td>
<td>4</td>
</tr>
<tr>
<td>54</td>
<td>7</td>
</tr>
<tr>
<td>55</td>
<td>10</td>
</tr>
<tr>
<td>56</td>
<td>11</td>
</tr>
<tr>
<td>57</td>
<td>8</td>
</tr>
<tr>
<td>58</td>
<td>5</td>
</tr>
<tr>
<td>59</td>
<td>1</td>
</tr>
</tbody>
</table>

a) Calculate the mean and the standard deviation of the eggs laid per week.

b) Determine the median and the quartiles for these data.

c) If the farmer only sells 45 eggs per week and keeps the rest for his family, find the mean and the standard deviation of the eggs he keeps for his family.

d) Use the median and mean to determine the skew of the above data, and hence determine whether this data can be modelled by a Normal distribution.
Question 8  (***)

The number of hours worked in a given week by a group of 64 individuals is summarized in the table below.

<table>
<thead>
<tr>
<th>Hours (nearest hour)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 10</td>
<td>5</td>
</tr>
<tr>
<td>11 – 20</td>
<td>16</td>
</tr>
<tr>
<td>21 – 25</td>
<td>14</td>
</tr>
<tr>
<td>26 – 30</td>
<td>17</td>
</tr>
<tr>
<td>31 – 40</td>
<td>10</td>
</tr>
<tr>
<td>41 – 59</td>
<td>2</td>
</tr>
</tbody>
</table>

a) Estimate, by linear interpolation, the value of the median.

b) Estimate the mean and the standard deviation of these data.

c) Establish, with justification, the skewness of the data.

d) Determine the possibility whether the data contain any outliers.

\[ Q_2 \approx 24.4, \quad \bar{x} \approx 23.88, \quad \sigma \approx 9.54, \quad \text{negative skew} \]
Question 9  (***)

A group of patients with a certain respiratory condition were asked to hold their breath for as long as they could.

The results are summarized in the table below.

<table>
<thead>
<tr>
<th>Time $t$ (in seconds)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; t \leq 10$</td>
<td>30</td>
</tr>
<tr>
<td>$10 &lt; t \leq 15$</td>
<td>35</td>
</tr>
<tr>
<td>$15 &lt; t \leq 18$</td>
<td>33</td>
</tr>
<tr>
<td>$18 &lt; t \leq 20$</td>
<td>20</td>
</tr>
<tr>
<td>$20 &lt; t \leq 30$</td>
<td>25</td>
</tr>
<tr>
<td>$30 &lt; t \leq 50$</td>
<td>10</td>
</tr>
</tbody>
</table>

a) Draw an accurate histogram to represent this data.

b) Use the histogram to estimate the number of patients that managed to hold their breath between 24 and 36 seconds.

c) Calculate estimates for the mean and standard deviation of this data.

\[
\bar{x} \approx 16.6, \quad \sigma \approx 8.85
\]
Question 10  (***)

The daily commuting distances of 125 individuals, rounded to the nearest mile, is summarised in the table below.

<table>
<thead>
<tr>
<th>Distance (nearest mile)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 9</td>
<td>12</td>
</tr>
<tr>
<td>10 – 19</td>
<td>22</td>
</tr>
<tr>
<td>20 – 29</td>
<td>48</td>
</tr>
<tr>
<td>30 – 39</td>
<td>26</td>
</tr>
<tr>
<td>40 – 49</td>
<td>8</td>
</tr>
<tr>
<td>50 – 59</td>
<td>5</td>
</tr>
<tr>
<td>60 – 69</td>
<td>3</td>
</tr>
<tr>
<td>70 – 79</td>
<td>1</td>
</tr>
</tbody>
</table>

a) Estimate the mean and the standard deviation of these commuting distances.

b) Use linear interpolation to estimate the value of the median.

c) Determine with justification the skewness of the data.

d) Explain which out of the mean and standard deviation or the median and the interquartile range are more appropriate measures to summarize this data.

\[ \bar{x} \approx 26.74, \quad \sigma \approx 13.85, \quad Q_2 = 25.3 \text{ to } 25.5, \quad \text{positive skew}, \quad \text{median & IQR} \]
Question 11 (***)

The ages of the residents of Arnold Street are denoted by \( x \), the ages of the residents of Benedict Street are denoted by \( y \).

These are summarized in the following back to back stem and leaf diagram.

\[
\begin{array}{c|c}
\text{Arnold Street} & \text{Benedict Street} \\
\hline
50 & 50 \\
55,53,31 & 99,125 \\
98,65,5,4,322213 & 678 \\
6410000 & 4122348 \\
951,44,4,4,5,8,8 & 61344599 \\
7269 & \\
\end{array}
\]

where 32 in Arnold Street and 39 in Benedict Street.

a) Find separately for the residents of Arnold Street and Benedict Street, ...

i. ... the mode.

ii. ... the lower quartile, the median and the upper quartile.

iii. ... the mean and the standard deviation.

You may assume \( \sum x = 866 \), \( \sum x^2 = 31514 \), \( \sum y = 1516 \), \( \sum y^2 = 86880 \).
A coefficient of skewness is defined as
\[
\frac{\text{mean} - \text{mode}}{\text{standard deviation}}.
\]

b) Evaluate this coefficient for the ages in each street.

c) Compare the distribution of the ages between the two streets.

<table>
<thead>
<tr>
<th>Street 1</th>
<th>Street 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode = 40</td>
<td>mode = 54</td>
</tr>
<tr>
<td>$Q_1 = 29$</td>
<td>$Q_1 = 42.5$</td>
</tr>
<tr>
<td>$Q_2 = 34$</td>
<td>$Q_2 = 54$</td>
</tr>
<tr>
<td>$Q_3 = 40$</td>
<td>$Q_3 = 64$</td>
</tr>
<tr>
<td>$\bar{x} \approx 32.07$</td>
<td>$\bar{y} \approx 54.14$</td>
</tr>
<tr>
<td>$\sigma_x = 11.77$</td>
<td>$\sigma_y = 13.09$</td>
</tr>
<tr>
<td>skew $\approx -0.67$</td>
<td>skew $= 0.01$</td>
</tr>
</tbody>
</table>
Question 12  (***)

The number of hours worked in a given week by a group of 64 freelance electricians is summarized in the table below.

<table>
<thead>
<tr>
<th>Hours (nearest hour)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 10</td>
<td>5</td>
</tr>
<tr>
<td>11 – 20</td>
<td>16</td>
</tr>
<tr>
<td>21 – 25</td>
<td>14</td>
</tr>
<tr>
<td>26 – 30</td>
<td>17</td>
</tr>
<tr>
<td>31 – 40</td>
<td>10</td>
</tr>
<tr>
<td>41 – 59</td>
<td>2</td>
</tr>
</tbody>
</table>

a) Draw an accurate histogram to represent this data.

b) Use the histogram to estimate the number of freelance electricians that worked between 15 and 37 hours during that week.

c) Estimate the median of the data.

\[
\text{Median} \approx 48, \quad Q_2 \approx 24.4
\]
Question 13  (***)

The number of hours spent on homework by 70 students, in a particular week, is summarized in the table below.

<table>
<thead>
<tr>
<th>Hours (nearest hour)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 – 3</td>
<td>6</td>
</tr>
<tr>
<td>4 – 6</td>
<td>18</td>
</tr>
<tr>
<td>7 – 9</td>
<td>15</td>
</tr>
<tr>
<td>10 – 11</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>13 – 15</td>
<td>6</td>
</tr>
</tbody>
</table>

a) Draw an accurate histogram to represent this data.

b) Use the histogram to estimate the number of students that spent between 7.75 and 13.5 hours during that week.

c) Estimate the median of the data.

\[ MMS-E \approx 36, \quad Q_2 \approx 7.72 \]
Question 14  (***)
The times taken to complete a 3 mile run, in minutes, by the members of a jogging club are summarized in the table below.

<table>
<thead>
<tr>
<th>Times (nearest hour)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 – 14</td>
<td>24</td>
</tr>
<tr>
<td>15 – 17</td>
<td>24</td>
</tr>
<tr>
<td>18 – 19</td>
<td>19</td>
</tr>
<tr>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>21 – 23</td>
<td>21</td>
</tr>
<tr>
<td>24 – 28</td>
<td>15</td>
</tr>
</tbody>
</table>

a) Estimate the mean and standard deviation of this data.

b) Estimate, by linear interpolation, the median of this data.

c) Draw an accurate histogram to represent this data.

d) Find the proportion of data which lies within 3 standard deviations of the mean.

e) Discuss briefly whether this data could be modelled by a Normal distribution.

\[ \bar{x} = 18.5, \quad \sigma = 4.33, \quad Q_2 = 18.4, \quad 100\% \]
Question 15  (***)

The monthly mileages of a sales rep are summarised in the table below.

<table>
<thead>
<tr>
<th>Mileages (m)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3250 \leq m &lt; 3300$</td>
<td>19</td>
</tr>
<tr>
<td>$3300 \leq m &lt; 3350$</td>
<td>45</td>
</tr>
<tr>
<td>$3350 \leq m &lt; 3400$</td>
<td>16</td>
</tr>
<tr>
<td>$3400 \leq m &lt; 3450$</td>
<td>5</td>
</tr>
<tr>
<td>$3450 \leq m &lt; 3500$</td>
<td>2</td>
</tr>
</tbody>
</table>

By using the coding

$$y = \frac{x - 3325}{50},$$

where $x$ represents the midpoint of each class, estimate the mean and the standard deviation of this data.

[$\mu = 3332$, $\sigma = 45.2$]

\[\text{Diagram}\]
The histogram above shows the distribution of the heights, to the nearest cm, of some plants in a garden centre. It is further given that there were 18 plants with a height between 5 cm and 8 cm, rounded to the nearest cm.

a) Use the histogram to estimate the median.

b) Estimate, by calculation, the mean and the standard deviation of the heights of these plants.

\[ \text{median} \approx 13.6, \quad \bar{x} \approx 13.48, \quad \sigma \approx 3.45 \]
Question 17  (***)
In a histogram the commuting times of a group of individuals, correct to the nearest minute, are plotted on the \( x \) axis.

In this histogram the class 47 – 50 has a frequency of 48 and is represented by a rectangle of base 6 cm and height 3.6 cm.

In the same histogram the class 51 – 55 has a frequency of 30.

Determine the measurements, in cm, of the rectangle that represents the class 51 – 55.

\[
\text{[base = 7.5 cm], [height = 1.8 cm]}
\]
The diameters of fine sand particles, in mm, are summarised in the table below.

<table>
<thead>
<tr>
<th>Diameters ($d$)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.02 &lt; d \leq 0.04$</td>
<td>25</td>
</tr>
<tr>
<td>$0.04 &lt; d \leq 0.06$</td>
<td>76</td>
</tr>
<tr>
<td>$0.06 &lt; d \leq 0.08$</td>
<td>111</td>
</tr>
<tr>
<td>$0.08 &lt; d \leq 0.10$</td>
<td>255</td>
</tr>
<tr>
<td>$0.10 &lt; d \leq 0.12$</td>
<td>33</td>
</tr>
</tbody>
</table>

a) By using the coding

$$y = 50(x - 0.09),$$

where $x$ represents the midpoint of each class, estimate the mean and the standard deviation of this data.

b) Estimate, by linear interpolation, the median diameter of these sand particles.

c) Describe, with justification, the skewness of the data.

$$M = 0.0778, \quad \sigma = 0.0197, \quad Q_2 = 0.08298$$
Question 19  (***+)  

In a histogram the weights of apples, $W$ grams, are plotted on the $x$ axis.

In this histogram the class $125 \leq W < 130$ has a frequency of 75 and is represented by a rectangle of base 1.8 cm and height 12 cm.

In the same histogram the class $150 \leq W < 170$ has a frequency of 40.

Find the measurements, in cm, of the rectangle that represents the class $150 \leq W < 170$.

\[
\text{base} = 7.2 \text{ cm}, \text{ height} = 1.6 \text{ cm}
\]
Question 20  (***)

The masses, \( x \) kg, of 40 students were measured and the results were summarized using the notation below.

\[
\sum_{n=1}^{40} (x_n - 50) = 140 \quad \text{and} \quad \sum_{n=1}^{40} (x_n - 50)^2 = 4490.
\]

Calculate the mean and standard deviation of the masses of these 40 students.

\[\bar{x} = 53.5 \quad \sigma = 10\]
Question 21  (***)
In a histogram the weights of peaches, correct to the nearest gram, are plotted on the x axis.

In this histogram the class 146–150 has a frequency of 75 and is represented by a rectangle of base 2.8 cm and height 7.5 cm.

In the same histogram a different class is represented by a rectangle of base 5.6 cm and height 10.5 cm.

Determine the frequency of this class.

\[ f = 210 \]
Question 22 (***+)

The following information about 5 observations of \( x \) is shown below.

\[
\sum_{i=1}^{5} \left( \frac{x_i - 255}{2} \right) = 50 \quad \text{and} \quad \sum_{i=1}^{5} \left( \frac{x_i - 255}{2} \right)^2 = 1650.
\]

Calculate the mean and standard deviation of \( x \).

\[\bar{x} = 275, \quad \sigma = 2\sqrt{230} \approx 30.3\]
Question 23  (***+) 
In a histogram the heights, \( h \) cm, of primary school pupils are plotted on the \( x \) axis.

In this histogram the class \( 120 \leq h < 130 \) has a frequency of 72 and is represented by a rectangle of base 4.2 cm and height 9 cm.

In the same histogram a different class is represented by a rectangle of base 2.1 cm and height 8 cm.

Determine the frequency of this class.

\[
\text{Frequency } f = 32
\]
Question 24  (***)

The table below shows the length of time, rounded to the nearest minute, spent by a group of patients for their dentist’s check up visit.

One of the frequencies is given as a positive constant $k$.

<table>
<thead>
<tr>
<th>Time (nearest minute)</th>
<th>2 - 6</th>
<th>7 - 11</th>
<th>12 - 16</th>
<th>17 - 31</th>
<th>32 - 36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Patients</td>
<td>6</td>
<td>15</td>
<td>$k$</td>
<td>24</td>
<td>12</td>
</tr>
</tbody>
</table>

Determine the standard deviation of these times, given that the mean of these times is 18.6 minutes.

\[
\sigma \approx 9.37
\]
Question 25  (***/+*)

In a histogram the weights of baby hamsters, correct to the nearest gram, are plotted on the $x$-axis.

In this histogram the class $24 - 30$ has a frequency of 63 and is represented by a rectangle of base 2.8 cm and height 6 cm.

In the same histogram the class $31 - 35$ has a frequency of 60.

Determine the measurements, in cm, of the rectangle that represents the class $31 - 35$.

[Diagram showing measurements]

**base = 2 cm, height = 8 cm**
Question 26 (***)

The distances rounded to the nearest mile, of 64 journeys covered by a taxi driver during a given week, is summarized in the table below.

<table>
<thead>
<tr>
<th>Distance (nearest mile)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 – 5</td>
<td>12</td>
</tr>
<tr>
<td>6 – 7</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
</tr>
<tr>
<td>9 – 11</td>
<td>13</td>
</tr>
<tr>
<td>12 – 17</td>
<td>6</td>
</tr>
</tbody>
</table>

a) Estimate the mean and the standard deviation of these weekly distances.

b) Estimate, by linear interpolation, the median value.

In a histogram drawn for the above data, the class 3 – 5 is represented by a rectangle of base length 1.2 cm and height 5 cm.

c) Find the base length and height of the rectangle representing the class 12 – 17 in the same histogram.

It is further given that the lower and upper quartiles of these distances are 6.07 and 9.19, respectively.

d) Investigate the possibility of any outliers.

e) By considering the skewness using the averages, discuss briefly whether the above set of data can be modelled by a normal distribution.

\[ \bar{x} = 7.94, \quad \sigma \approx 2.87, \quad Q_2 = 7.82, \quad \text{base} = 2.4 \text{ cm}, \quad \text{height} = 1.25 \text{ cm} \]
The weights, in kg, of the 164 bags packed by supermarket customers is summarized in the table above.

a) Estimate the mean and the standard deviation of these weights.

b) Estimate, by linear interpolation, the median value and hence determine with justification, the skewness of the data.

In a histogram drawn for the above data, the $1 \leq w < 3$ class is represented by a rectangle of base length 2.4 cm and height 2.5 cm.

c) Find the base length and height of the rectangle representing the $6.5 \leq w < 7$ class in the same histogram.

It is further given that the lower and upper quartiles of these distances are 4.68 and 6.43, respectively.

d) Investigate the possibility of any outliers.

e) Discuss briefly whether the above set of data can be modelled by a normal distribution.

$$\bar{x} = 5.5, \sigma = 1.64, Q_2 = 5.80, \text{negative skew}, \quad \text{base} = 0.6 \text{ cm}, \quad \text{height} = 14 \text{ cm}$$
Question 28  (**+**)

The masses of 68 cows, in kg, are summarised in the table below.

<table>
<thead>
<tr>
<th>Mass ($m$)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600 &lt; m \leq 625$</td>
<td>11</td>
</tr>
<tr>
<td>$625 &lt; m \leq 650$</td>
<td>14</td>
</tr>
<tr>
<td>$650 &lt; m \leq 675$</td>
<td>28</td>
</tr>
<tr>
<td>$675 &lt; m \leq 700$</td>
<td>7</td>
</tr>
<tr>
<td>$700 &lt; m \leq 725$</td>
<td>5</td>
</tr>
<tr>
<td>$725 &lt; m \leq 750$</td>
<td>2</td>
</tr>
<tr>
<td>$750 &lt; m \leq 775$</td>
<td>1</td>
</tr>
</tbody>
</table>

a) By using the coding

\[ y = \frac{x - 662.5}{25}, \]

where $x$ represents the midpoint of each class, estimate the mean and standard deviation of this data.

b) Estimate, by the method of linear interpolation, the median mass of these cows.

\[ \bar{x} \approx 659.19, \quad \sigma \approx 32.91, \quad Q_2 \approx 658.0 \]
Question 29  (****)

The histogram below shows the distribution of the marks of 250 students.

a) Estimate how many students scored between 52 and 74 marks.

b) Use the histogram to estimate the median.

c) Calculate estimates for the mean and standard deviation of the marks of these students.

\[ \bar{x} \approx 51.8, \quad \sigma \approx 22.22 \]
Question 30    (****)

The mean and standard deviation of 20 observations \( x_1, x_2, x_3, \ldots, x_{20} \) are

\[
\bar{x} = 18.5 \quad \text{and} \quad \sigma_x = 6.5.
\]

The mean and standard deviation of 12 observations \( y_1, y_2, y_3, \ldots, y_{12} \) are

\[
\bar{y} = 25 \quad \text{and} \quad \sigma_y = 7.5.
\]

Determine the mean and the standard deviation of all 32 observations.

\[
\text{Mean} = 20.94, \quad \text{Standard deviation} = 7.58
\]
Question 31  (****)

The mean and standard deviation of the test marks of 40 pupils in a Mathematics class are 65 and 18, respectively.

The mean and standard deviation of the test marks of the 24 boys of the class are 72 and 20, respectively.

Find the mean and standard deviation of the test marks of the 16 girls of the class.

\[ \text{Mean} = 54.5, \quad \text{Standard deviation} = 5.12 \]
Question 32  (****)

The masses, $x$ kg, of 40 students were measured and the results were summarized using the notation below.

\[ \sum_{n=1}^{40} (x_n - 50) = 150 \quad \text{and} \quad \sum_{n=1}^{40} (x_n - 50)^2 = 4650. \]

Determine the value of \( \sum_{n=1}^{40} (x_n)^2 \).
Question 33 (****+)

It is given that for a sample of data \( x_1, x_2, x_3, x_4, x_5, \ldots, x_n \), the mean \( \bar{x} \) and standard deviation \( \sigma \) are

\[
\bar{x} = \frac{1}{n} \sum_{r=1}^{n} x_r = 2 \quad \text{and} \quad \sigma = \sqrt{\frac{1}{n} \sum_{r=1}^{n} (x_r)^2 - \left( \frac{1}{n} \sum_{r=1}^{n} x_r \right)^2} = 3.
\]

Determine, in terms of \( n \), the value of

\[
\sum_{r=1}^{n} (x_r + 1)^2.
\]

\[
\sum_{r=1}^{n} (x_r + 1)^2 = 18n
\]
Question 34  (****+)

The test marks, \( x \), of 20 students were coded and their results were summarized as

\[
\sum (x-10) = 220 \quad \text{and} \quad \sum (x-10)^2 = 2720.
\]

a) Use a detailed method to show that

\[
\sum x^2 = 9120.
\]

b) Calculate the mean and standard deviation of the test marks of these students.

\[
\overline{x} = 21, \quad \sigma = \sqrt{15} \approx 3.87
\]