COMBINATORIAL PROBABILITY

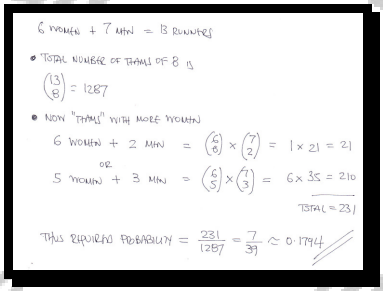
Question 1 (+)**

The Oakwood Jogging Club consists of 7 men and 6 women who go for a 5 mile run every Thursday.

It is decided that a team of 8 runners would be picked at random out of the 13 runners, to represent the club at a larger meeting.

Determine the probability that the team of 8 will have more women than men.

$$\boxed{\frac{7}{39}}$$



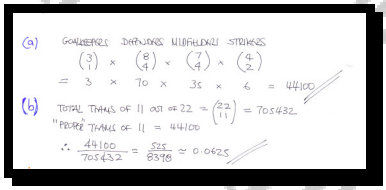
Question 2 (+)**

A football manager has available for selection 3 goalkeepers, 8 defenders, 7 midfielders and 4 strikers. He is planning to pick for the next game 1 goalkeeper, 4 defenders, 4 midfielders and 2 strikers.

a) Find the number of possible teams he can select, assuming that all players are equally likely to be picked up.

b) Given that the manager picks 11 players at random from the available 22, determine the probability that he picked 1 goalkeeper, 4 defenders, 4 midfielders and 2 strikers.

$$\boxed{44100}, \quad \boxed{\frac{525}{8398} \approx 0.0625}$$



Question 3 (+)**

An exam consists of two papers.

The first paper contains 10 questions of which a candidate must answer 6 questions.

a) Find the number of ways these 6 questions can be picked.

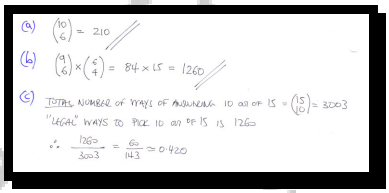
The second paper has two sections. Section A has 9 questions of which a candidate must answer 6 questions and section B has 6 questions of which a candidate must answer 4 questions.

b) Find the number of ways a candidate could pick the questions in this paper.

A candidate does not read the instructions properly and answers at random 10 questions in the second paper.

c) Determine the probability that he picked 6 questions from section A and 4 questions from section B.

$\boxed{210}$, $\boxed{1260}$, $\boxed{\frac{60}{143}}$



Question 4 (+)**

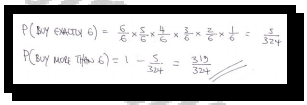
Packs of a certain brand of cereal contain free toy characters from a recent film.

There are 6 different characters to collect and these characters are distributed, with equal probability, randomly and independently in each pack.

William is about to start collecting these characters.

Find the probability that he will need to buy more than 6 packets in order to complete the set of the 6 different characters.

$$\frac{319}{324}$$

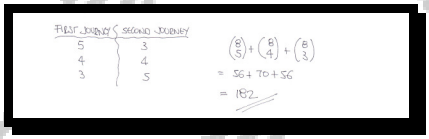


Question 5 (+)**

A taxi which can carry at most 5 passengers on any journey, makes two journeys in transporting 8 passengers from their hotel to the airport.

Determine the number of different ways in which the people for the first journey may be selected.

$$182$$



Question 6 (+)**

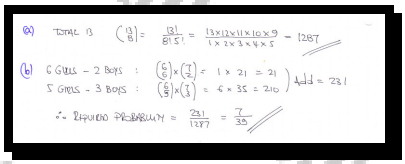
There are 7 boys and 6 girls in the student council of a school.

A committee of 8 people is to be selected from the members of this council to organize a sports day.

a) Find the number of different ways in which the committee can be selected if all the members are available.

b) Find the probability that the committee will have more girls than boys.

$\boxed{1287}$, $\boxed{\frac{7}{39}}$



Question 7 (***)

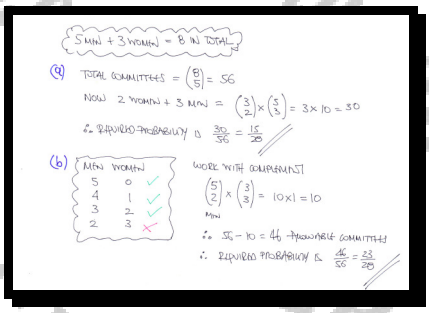
A **five** member committee is to be selected at random from a group consisting of 5 men and 3 women.

Find the probability that the selected committee will contain …

a) … exactly 2 women.

b) … no more than 2 women.

$\boxed{\frac{15}{28}}$, $\boxed{\frac{23}{28}}$



Question 8 (*+)**

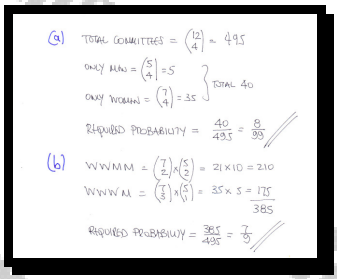
A committee of 4 people is to be chosen at random from a group of 5 men and 7 women.

Determine the probability that the committee will consist …

a) … of members of the same gender.

b) … of members of both genders but at least as many women as men.

$\boxed{\frac{8}{99}}$, $\boxed{\frac{7}{9}}$



Question 9 (*+)**

The five letters of the word T-E-A-C-H are written on five separate pieces of card.

a) Find the number of arrangements that can be made using these five letters.

b) Find the probability that in these five letter arrangements …

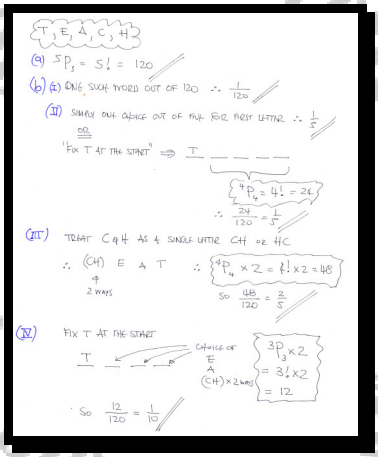
i. … the letters will spell the word C-H-E-A-T.

ii. … the first letter is T.

iii. … the letters C and H are next to each other.

iv. … the first letter is T **and** the letters C and H are next to each other.

$\boxed{120}$, $\boxed{\frac{1}{120}}$, $\boxed{\frac{1}{5}}$, $\boxed{\frac{2}{5}}$, $\boxed{\frac{1}{10}}$



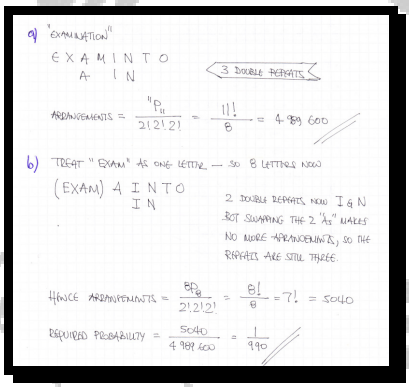
Question 10 (*+)**

The eleven letters of the word E-X-A-M-I-N-A-T-I-O-N are written on eleven separate pieces of card.

a) Find the number of arrangements that can be made using these eleven letters.

b) Find the probability that the four letter word E-X-A-M will appear in one of these eleven letter arrangements

$$\boxed{4989600}\ ,\ \boxed{\frac{1}{990}}$$



Question 11 (*+)**

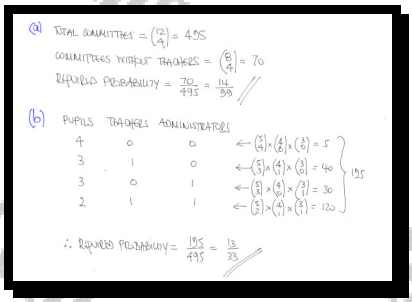
A committee of 4 people is to be chosen at random from the members of a school council which consists of 5 pupils, 4 teachers and 3 administrators.

Determine the probability that the committee will contain ...

a) … no teachers.

b) … at least 2 pupils, no more than 1 teacher and no more than 1 administrator.

$\boxed{\frac{14}{99}}$, $\boxed{\frac{13}{33}}$

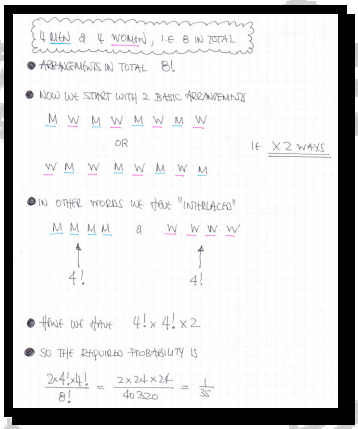


Question 12 (*+)**

4 men and 4 women are going to stand next to each other for a group photograph.

Given that the way they stand next to each other is completely random, determine the probability that no 2 men and no 2 women stand next to each other.

$$\frac{1}{35}$$



Question 13 (****)

Alex, Beth and Cain are 3 students in a class which consists of a total of 8 students.

4 students are selected at random.

a) Find the probability that Alex and Beth will be selected but Cain is not selected.

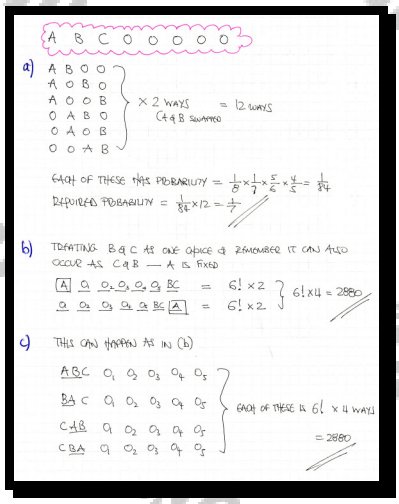
Next all 8 students are standing next to each for a group photo.

b) Determine the number of arrangements in which ...

i. ... Alex is standing at one end and Beth and Cain are standing next to each other.

ii. ... Alex and Beth are standing next to each other and Cain is standing next to them.

$\boxed{\frac{1}{7}}$, $\boxed{2880}$, $\boxed{2880}$



Question 14 (****)

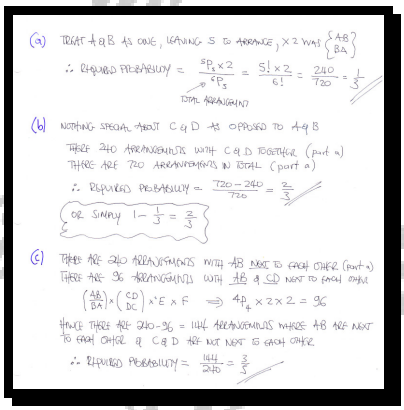
Six books labelled as A, B, C, D, E and F are arranged at random on a shelf.

a) Determine the probability that A and B are placed next to each other.

b) Determine the probability that C and D are not placed next to each other.

c) Given that A and B are placed next to each other, determine the probability that C and D are not placed next to each other.

$$\boxed{\frac{1}{3}}, \boxed{\frac{2}{3}}, \boxed{\frac{3}{5}}$$



Question 15 (****)

A group of 7 pupils consists of 3 girls and 4 boys. The names of two of the boys are Argi and Bargi.

The seven students sit in random order on a bench.

a) Determine the probability that …

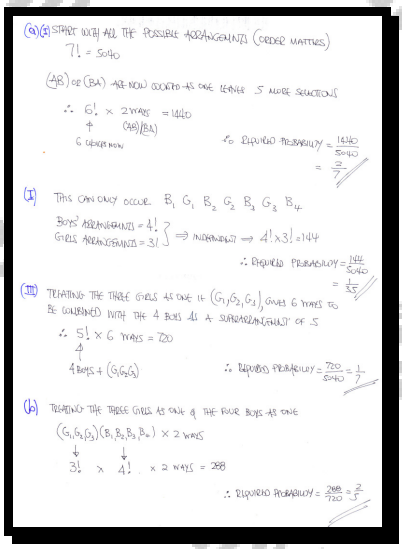
i. … Argi and Bargi sit next to each other.

ii. … no two boys sit next to each other.

iii. … the three girls sit next to each other.

b) Given the three girls sat next to each other find the probability that the four boys also sat next to each other.

$$\boxed{\frac{2}{7}}, \boxed{\frac{1}{35}}, \boxed{\frac{1}{7}}, \boxed{\frac{2}{5}}$$



Question 16 (****)

The numbers 1, 2, 3 and 4 are to be used to make a four digit password.

a) Calculate the number of the four digit passwords that can be created if …

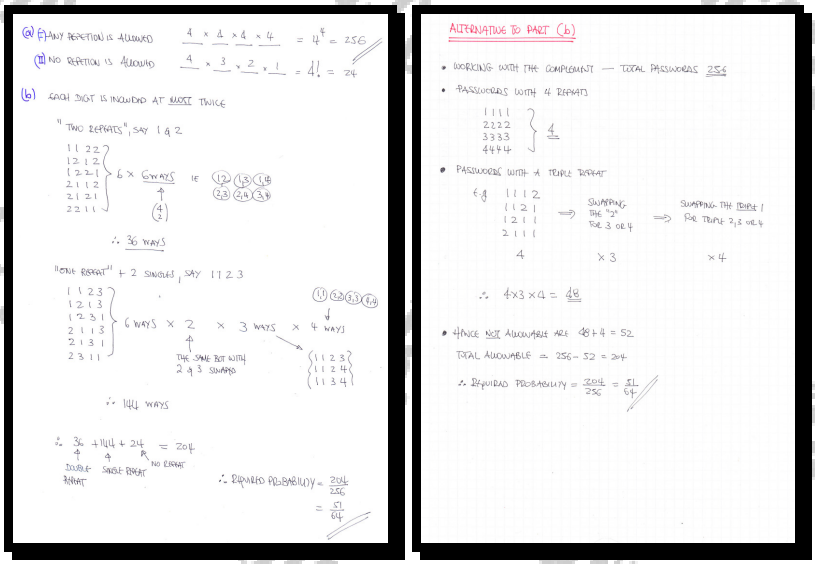
i. … any repetitions are allowed.

ii. … no repetitions are allowed.

A security requirement is such so that a digit can be repeated at most twice. A four digit password is generated at random.

b) Determine the probability that this four digit password will comply with the security requirement.

$\boxed{4^4 = 256}$, $\boxed{4! = 24}$, $\boxed{\frac{51}{64}}$



Question 17 (****)

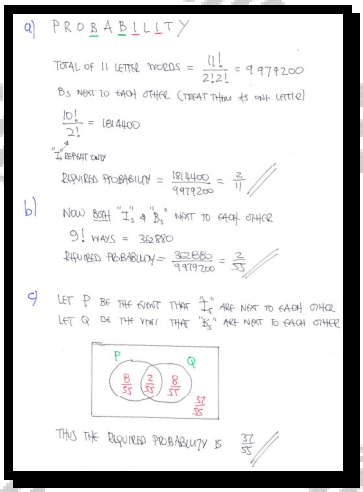
The 11 letters of the word *PROBABILITY* are written on 11 separate pieces of card. These cards are selected at random and arranged in a line next to each other.

a) Determine the probability that the two cards with the letter *B* will appear next to each other.

b) Find the probability that the two cards with the letter *B* will appear next to each other **and** the two cards with the letter *I* will appear next to each other.

c) Hence deduce the probability that the two cards with the letter *B* will **not** appear next to each other **and** the two cards with the letter *I* will **not** appear next to each other.

$$\boxed{\frac{2}{11}}\,,\ \boxed{\frac{2}{55}}\,,\ \boxed{\frac{37}{55}}$$



Question 18 (****)

$$S\ S\ S\ T\ T\ T\ C\ I\ I\ A$$

The 10 letters above, are written on 10 separate pieces of card. These cards are selected at random and arranged in a line next to each other.

a) Find the probability that the 10 letter arrangement will spell $STATISTICS$.

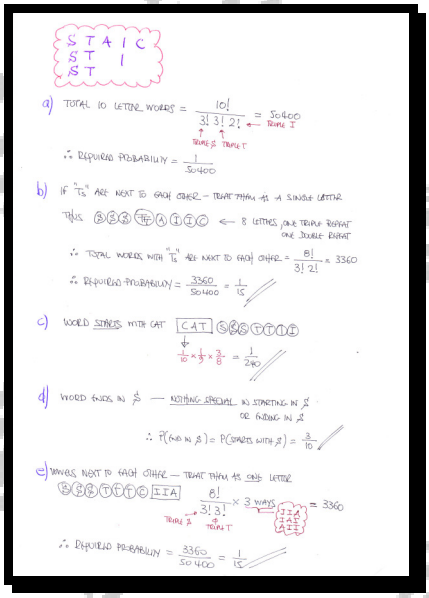
b) Determine the probability that in the 10 letter arrangement the 3 cards with the letter T will be next to one another.

c) Calculate the probability that the 10 letter arrangement will start with CAT , in that order.

d) Find the probability that the 10 letter arrangement will end with the letter S .

e) Determine the probability that in the 10 letter arrangement the 3 cards showing a vowel will be next to one another.

$$\boxed{\frac{1}{50400}}\ ,\ \boxed{\frac{1}{15}}\ ,\ \boxed{\frac{1}{240}}\ ,\ \boxed{\frac{3}{10}}\ ,\ \boxed{\frac{1}{15}}$$



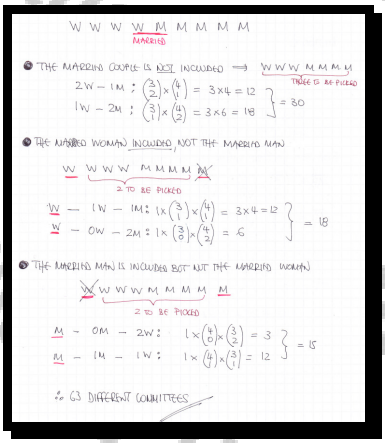
Question 19 (****)

A committee of 3 people is to be picked from 9 individuals, of which 4 are women and 5 are men. One of the 4 women is married to one of the 5 men.

The selection rules state that the committee must have at least a member from each gender and no married couple can serve together in a committee.

Determine the number of possible committees which can be picked from these 9 individuals.

63



Question 20 (****)

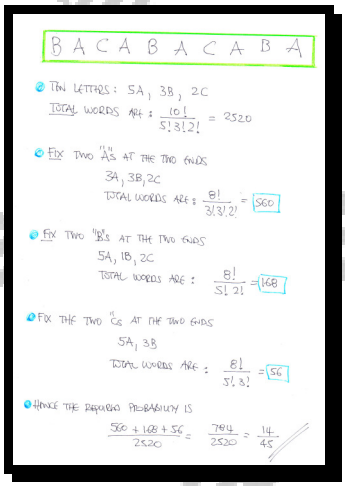
The 10 letters of the word

$$B\ A\ C\ A\ B\ A\ C\ A\ B\ A$$

are written on 10 separate pieces of card. These cards are selected at random and arranged in a line next to each other.

Determine the probability that the resulting arrangement will start and finish with the same letter.

$$\frac{14}{45}$$



Question 21 (****)

Coloured pegs are to be placed in 4 holes which are drilled in a straight line, next to each other. These coloured pegs are identical in size and 2 of them are red, 2 of them are green, 2 of them are brown, 2 of them are orange, 2 of them are pink and 2 of them are blue.

6 pegs, one from each of the 6 colours, are picked from the 12 pegs and four are placed in the holes.

a) Determine the number of different arrangements which can be made.

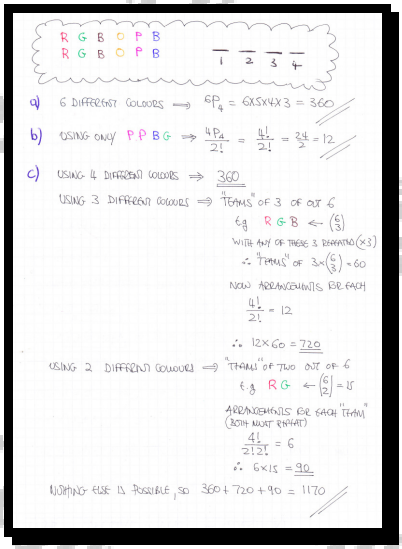
Next 4 pegs, 2 pink, 1 blue and 1 green are picked from the 12 pegs and are placed in the holes.

b) Find the number of different arrangements which can now be made.

Finally 4 pegs are picked at random from the total of 12 pegs and placed in the holes.

c) Determine the number of different arrangements which can be made on this occasion.

$\boxed{360}$, $\boxed{12}$, $\boxed{1170}$

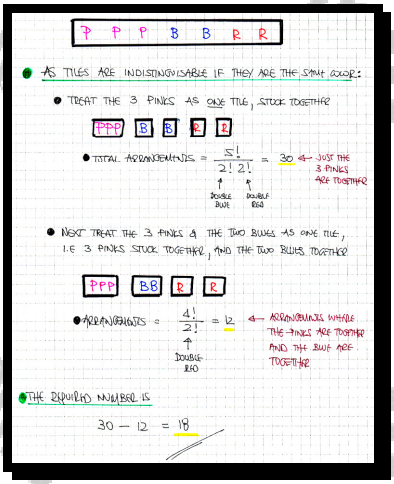


Question 22 (****)

Seven rectangular tiles, of which 3 are pink, 2 are blue and 2 are red, are placed in a straight line, next to each other.

Find the number of arrangements where the pink tiles are next to each other and the blue tiles are **not** next to each other.

18



Question 23 (**+)**

1, 2, 3, 4, 5, 6, 7, 8, 9

The above nine single digit numbers are written on nine separate pieces of card.

Four of these cards are picked at random and placed next to each other to form a four digit number.

a) Find the total different number arrangements of …

i. … four digit numbers that can be formed.

ii. … four digit **odd** numbers that can be formed.

iii. … four digit numbers that can be formed, whose all four digits are **odd**.

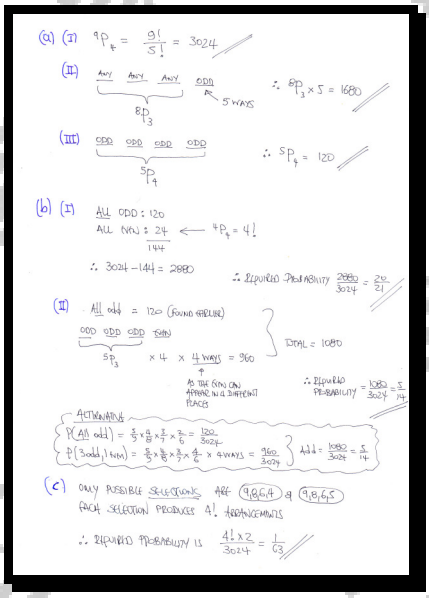
b) Determine the probability that the four digit number that will be formed …

i. … has odd and even digits.

ii. … will have **at least** three odd digits.

c) Find the probability that the **sum** of the four digit of the number that will be formed is 28 .

$\boxed{3024}$, $\boxed{1680}$, $\boxed{120}$, $\boxed{\frac{20}{21}}$, $\boxed{\frac{5}{14}}$, $\boxed{\frac{1}{63}}$



Question 24 (**+)**

The six letters of the word *RADIAN* are written on six separate pieces of card.

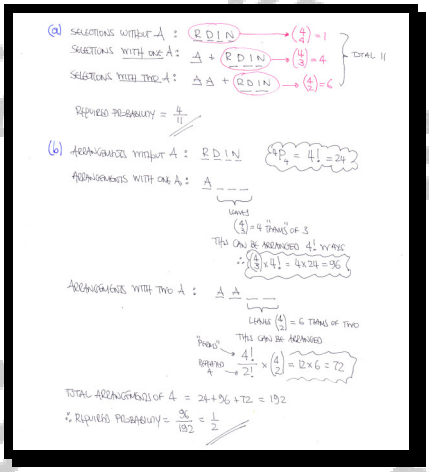
In an experiment, four cards are selected at random.

a) Find the probability only one of the two letters A will be included.

The experiment is repeated again, and four new cards are selected at random, forming a four letter arrangement.

b) Find the probability that only one of the two letters A will be included the four letter arrangement.

$\boxed{\frac{4}{11}}$, $\boxed{\frac{1}{2}}$

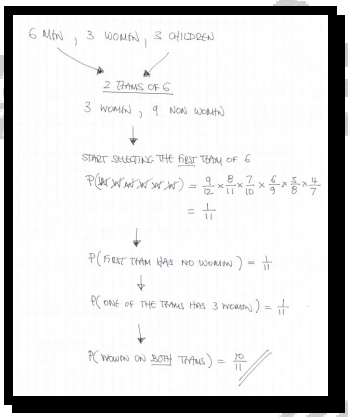


Question 25 (**+)**

From a total of 6 men, 3 women and 3 children, two teams of six people are selected at random.

Find the probability that both teams contain women.

$\frac{10}{11}$



Question 26 (**+)**

1, 1, 2, 2, 3, 3, 4, 4

The above eight single digit numbers are written on eight separate pieces of card.

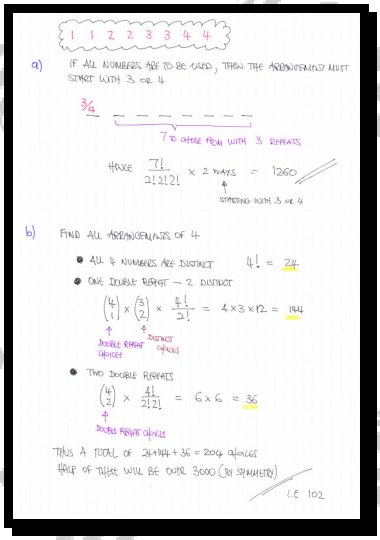
These cards are placed next to each other at random, forming an eight digit number.

a) Determine the total different eight digit numbers that can be formed.

Four cards are next picked at random from the eight original cards and placed next to each other to form a four digit number.

b) Find the number of four digit numbers which can be formed that exceed 3000.

$\boxed{1260}$, $\boxed{102}$



Question 27 (*****)

The six letters of the word *BUTTER* are written on six separate pieces of card.

In an experiment four cards are selected at random, forming a four letter arrangement.

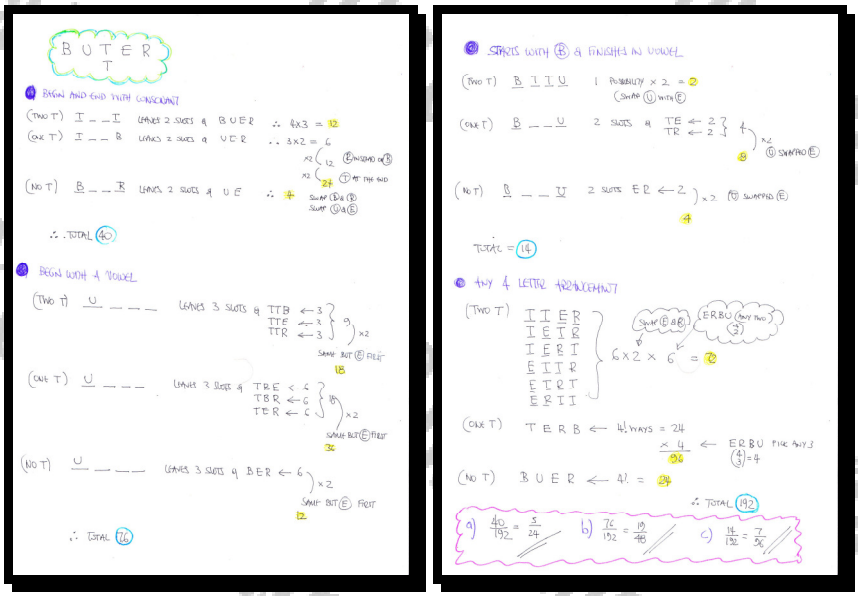
Find the probability that the four letter arrangement ...

a) … will begin and end with a consonant.

b) … will begin with a vowel.

c) … will start with B and end with a vowel.

$$\boxed{\frac{5}{24}}\ ,\ \boxed{\frac{19}{48}}\ ,\ \boxed{\frac{7}{96}}$$



Question 28 (*****)

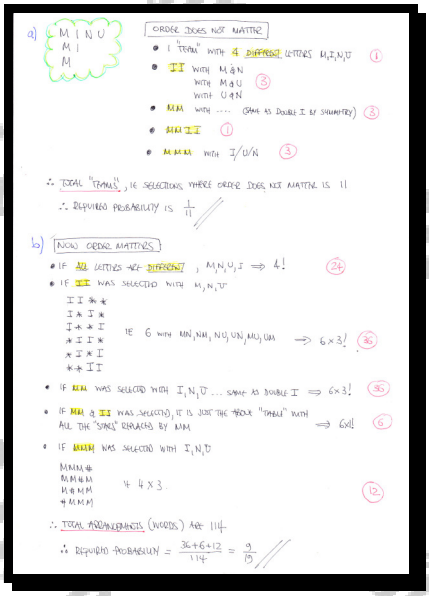
The 7 letters of the word *MINIMUM* are written on 7 separate pieces of card.

Four of these cards are picked at random, one after the other.

a) Determine the probability that all the cards will show a different letter.

b) If instead the four cards were arranged into a four letter word, in the order they were picked, determine the probability that the resulting arrangement will contain at least 2 cards with the letter *M*.

$$\boxed{\frac{1}{11}}, \quad \boxed{\frac{9}{19}}$$



Question 29 (*****)

Five 1^{st} year students and three 2^{nd} year students are standing next to each other, for a photograph to be taken.

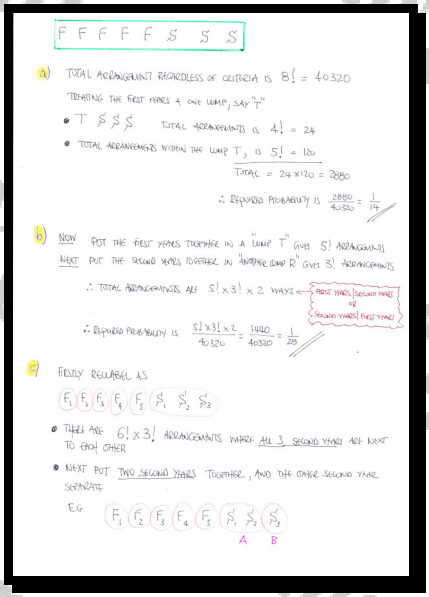
It assumed that the eight students positioned themselves at random.

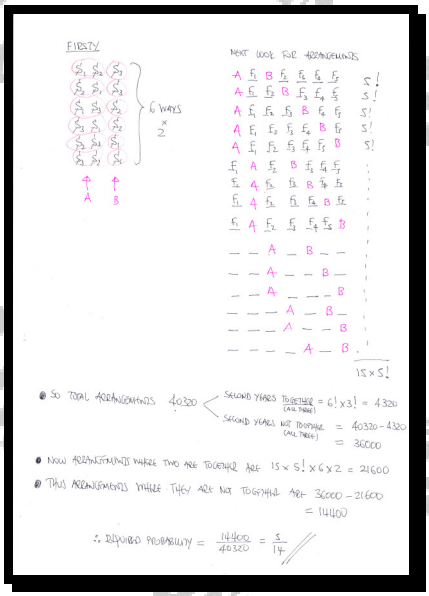
a) Find the probability that all the 1^{st} year students are standing next to each other.

b) Determine the probability that all the 1^{st} year students are standing next to each other and all the 2^{nd} year students are standing next to each other.

c) Find the probability that no 2^{nd} year students are standing next to each other.

$$\boxed{\frac{1}{14}}\ ,\ \boxed{\frac{1}{28}}\ ,\ \boxed{\frac{5}{14}}$$





Question 30 (*****)

5 adults and 6 children go to the cinema and sit at random next to each other, in a row which contains 11 empty consecutive seats.

a) Find the probability that no two adults sit next to each other.

Another 3 adults and 8 children go to the cinema and sit at random next to each other, in a row which also contains 11 empty consecutive seats.

b) Find the probability that no two adults sit next to each other.

$\boxed{\frac{1}{22}}$, $\boxed{\frac{5}{11}}$