

Created by T. Madas

TRANSFORMATIONS OF GRAPHS

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Question 1

Describe geometrically each of the following transformations.

- | | |
|---------------------------------|---|
| a) $f(x) + 2$ | Translation “upwards” by 2 units |
| b) $f(x + 2)$ | Translation “to the left” by 2 units |
| c) $2f(x)$ | Stretch in the y direction by a factor of 2 |
| d) $f(2x)$ | Stretch in the x direction by a factor of $\frac{1}{2}$ |
| e) $-f(x)$ | Reflection about the x axis |
| f) $f(x) - 4$ | Translation “downwards” by 4 units |
| g) $f(x - 3)$ | Translation “to the right” by 3 units |
| h) $3f(x)$ | Stretch in the y direction by a factor of 3 |
| i) $f(-x)$ | Reflection about the y axis |
| j) $f\left(\frac{1}{4}x\right)$ | Stretch in the x direction by a factor of 4 |

Question 2

Describe geometrically each of the following transformations.

- | | |
|---|---|
| a) $f(x+1)-2$ | Translation, “to the left” by 1 unit, followed by translation, “downwards” by 2 units. |
| b) $4f(x-1)$ | Translation “to the right” by 1 unit, followed by stretch in the y direction by a factor of 4. |
| c) $3f(x)-1$ | Stretch in the y direction by a factor of 3, followed by translation “downwards” by 1 unit. |
| d) $f(2x+3)$ | Translation “to the left” by 3 units, followed by stretch in the x direction by a factor of $\frac{1}{2}$. |
| e) $4-f(x)$ | Reflection about the x axis, followed by translation “upwards” by 4 units. |
| f) $f(-3x)$ | Reflection about the y axis, followed by stretch in the x direction by a factor of $\frac{1}{3}$. |
| g) $-f\left(\frac{1}{2}x\right)$ | Reflection about the x axis, followed by stretch in the x direction by a factor of 2. |
| h) $2[f(x)+3]$ | Translation “upwards” by 3 units, followed by stretch in the y direction by a factor of 2. |
| i) $-4f(x)$ | Reflection about the x axis, followed by stretch in the y direction by a factor of 4. |
| j) $f(2-x)$ | Translation “to the left” by 2 units, followed by reflection in the y axis. |

Question 3

Describe geometrically each of the following transformations.

a) $2f(x-1)$

Translation “to the right” by 1 unit, followed by stretch in the y direction by factor of 2.

b) $f(3x+1)$

Translation “to the left” by 1 unit, followed by stretch in the x direction by a factor of $\frac{1}{3}$.

c) $f(4-x)$

Translation “to the left” by 4 units, followed by reflection in the y axis.

d) $\frac{1}{2}f\left(\frac{1}{2}x+1\right)$

Translation “to the left” by 1 unit, followed by stretch in the x direction by a factor of 2, followed by stretch in the y direction by a factor of $\frac{1}{2}$.

e) $f\left(\frac{1}{2}x-2\right)$

Translation “to the right” by 2 units, followed by stretch in the x direction by a factor of 2.

f) $f(5-x)$

Translation “to the left” by 5 units, followed by reflection in the y axis.

g) $f(4-2x)$

Translation “left” by 4 units, followed by stretch in the x direction by a factor of $\frac{1}{2}$ and reflection in the y axis in either order.

h) $-4f(x)$

Reflection about the x axis, followed by stretch in the y direction by a factor of 4.

i) $f(2-x)$

Translation “to the left” by 2 units, followed by reflection in the y axis.

Question 4

Write each of the transformations described below in f notation.

a) Translation “upwards” by 5 units

$$f(x) + 5$$

b) Translation “to the left” by 4 units

$$f(x + 4)$$

c) Stretch in the y direction by a factor of 3

$$3f(x)$$

d) Stretch in the x direction by a factor of $\frac{1}{2}$

$$f(2x)$$

e) Reflection about the x axis

$$-f(x)$$

f) Translation “downwards” by 1 unit

$$f(x) - 1$$

g) Translation “to the right” by 5 units

$$f(x - 5)$$

h) Stretch in the y direction by a factor of 4

$$4f(x)$$

i) Reflection about the y axis

$$f(-x)$$

j) Stretch in the x direction by a factor of 3

$$f\left(\frac{1}{3}x\right)$$

Question 5

Write each of the transformations described below in f notation.

- a) Translation “to the left” by 2 units, followed by translation “downwards” by 4 units
- b) Translation “to the right” by 3 units, followed by stretch in the y direction by a factor of 2
- c) Stretch in the y direction by a factor of 6, followed by translation “downwards” by 5 units
- d) Translation “to the left” by 5 units, followed by stretch in the x direction by a factor of $\frac{1}{2}$
- e) Reflection about the x axis, followed by translation “upwards” by 2 units
- f) Reflection about the y axis, followed by stretch in the x direction by a factor of 3
- g) Reflection about the x axis, followed by stretch in the x direction by a factor of 2
- h) Translation “upwards” by 1 unit, followed by stretch in the y direction by a factor of 3
- i) Reflection about the x axis, followed by stretch in the y direction by a factor of 3
- j) Translation “to the left” by 4 units, followed by reflection about the y axis

a) $f(x+2)-4$

b) $2f(x-3)$

c) $6f(x)-5$

d) $f(2x+5)$

e) $2-f(x)$

f) $f(-\frac{1}{3}x)$

g) $-f(\frac{1}{2}x)$

h) $3[f(x)+1]$

i) $-3f(x)$

j) $f(4-x)$

Question 6

Describe the transformation or set of transformation for each of the following mappings.

a) $\ln x \mapsto 2\ln(x-3)$

b) $\sin x \mapsto \sin\left(2x + \frac{\pi}{6}\right)$

c) $\sqrt{x} \mapsto \sqrt{3x-6}$

d) $e^x \mapsto 4e^{3x} - 1$

e) $\frac{1}{x} \mapsto \frac{1}{2-x}$

f) $\ln(x^2+1) \mapsto \ln(4x^2+1)$

g) $\frac{1}{x^2+1} \mapsto \frac{4}{x^2+4}$ (hint: it is a single stretch)

a) stretch in the y direction, followed by translation “to the right” by 3 units.
(either order)

b) translation “to the left” by $\frac{\pi}{6}$ units, followed by stretch in the x direction by factor of $\frac{1}{2}$.

c) translation “to the right” by 6 units, followed by stretch in the x direction by factor of $\frac{1}{3}$.

d) stretch in the y direction by factor of 4, followed by translation “downwards” by 1 unit, followed by stretch in the x direction by factor of $\frac{1}{3}$.

e) translation “to the left” by 2 units, followed by reflection in the y axis.

f) stretch in the y direction by factor of $\frac{1}{3}$.

g) stretch in the x direction by factor of 2.