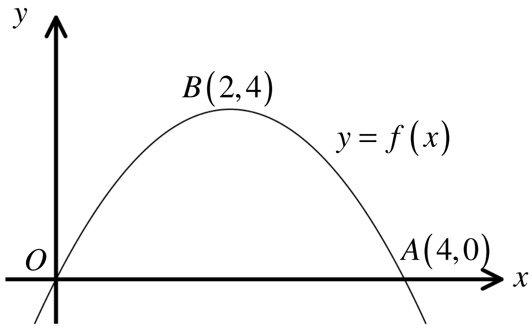
TRANSFORMATIONS OF GRAPHS & FUNCTIONS

EXAM QUESTIONS

Question 1 (**)



The figure above shows the graph of the curve with equation $y = f(x)$.

The curve crosses the x axis at $O(0,0)$ and at the point $A(4,0)$.

The curve has a maximum at $B(2,4)$.

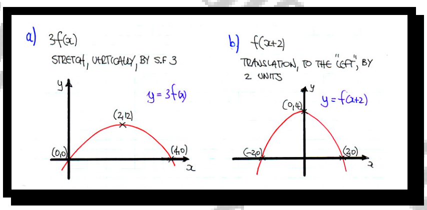
Sketch on a separate set of axes the graph of …

a) … $y = 3f(x)$.

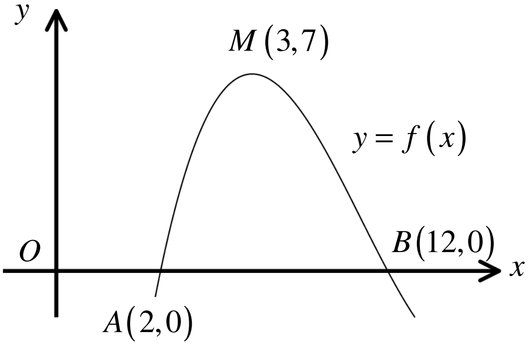
b) … $y = f(x+2)$.

Each sketch must include the coordinates of any points where the graph crosses the coordinate axes and the new coordinates of the maximum point of the curve.

graph



Question 2 (**)



The figure above shows the graph of the curve with equation $y = f(x)$.

The curve crosses the x axis at the points $A(2,0)$ and $B(12,0)$.

The curve has a maximum at $M(3,7)$.

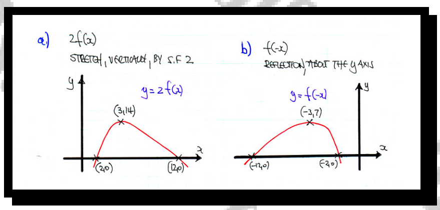
Sketch on a separate set of axes the graph of …

a) … $y = 2f(x)$.

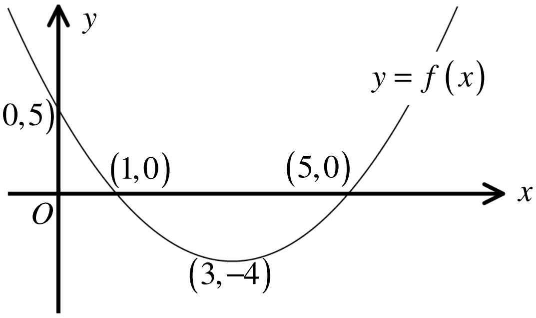
b) … $y = f(-x)$.

Each sketch must include the coordinates of any points where the graph crosses the coordinate axes and the new coordinates of the maximum point of the curve.

graph



Question 3 (**)



The figure above shows the graph of the curve with equation $y = f(x)$.

The curve crosses the x axis at the points $(1,0)$ and $(5,0)$, the y axis at $(0,5)$.

The curve has a minimum at $(3,-4)$.

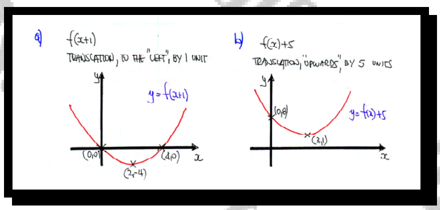
Sketch on a separate set of axes the graph of …

a) … $y = f(x+1)$.

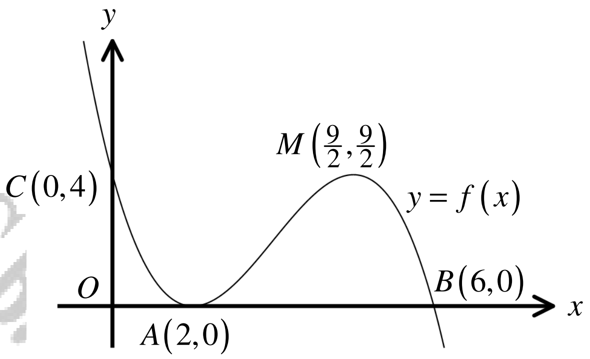
b) … $y = f(x)+5$.

Each sketch must include the coordinates of any points where the graph crosses the coordinate axes and the new coordinates of the minimum point of the curve.

graph



Question 4 (**)



The figure above shows the graph of the curve with equation $y = f(x)$.

The curve meets the x axis at $A(2,0)$ and at the point $B(6,0)$, the y axis at $C(0,4)$.

The curve has a maximum at $M\left(\frac{9}{2},\frac{9}{2}\right)$.

Sketch on a separate set of axes the graph of …

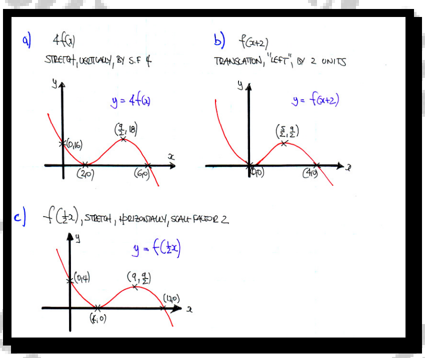
a) … $y = 4f(x)$.

b) … $y = f(x+2)$.

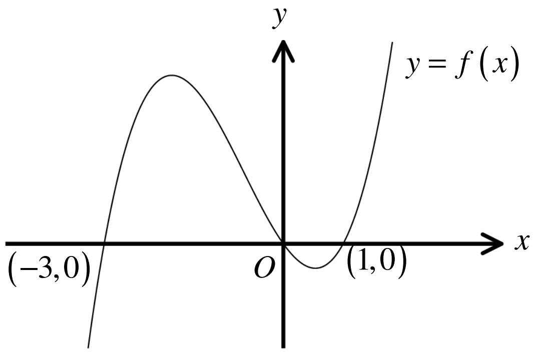
c) … $y = f\left(\frac{1}{2}x\right)$.

Each sketch must include the coordinates of any points where the graph meets the coordinate axes and the new coordinates of the maximum point of the curve.

graph



Question 5 (**)



The figure above shows the graph of the curve with equation $y = f(x)$.

The curve meets the x axis at $(-3,0)$, at $(1,0)$ and at the origin O.

Sketch on a separate set of axes the graph of …

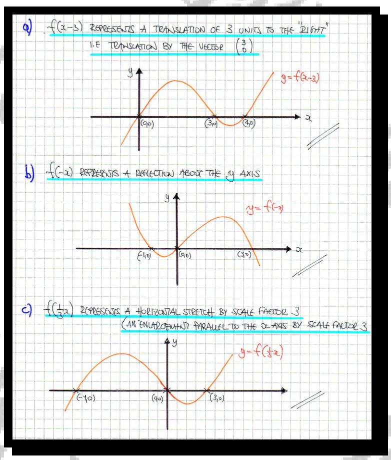
a) … $y = f(x-3)$.

b) … $y = f(-x)$.

c) … $y = f\left(\frac{1}{3}x\right)$.

Each sketch must include the coordinates of any points where the graph meets the coordinate axes.

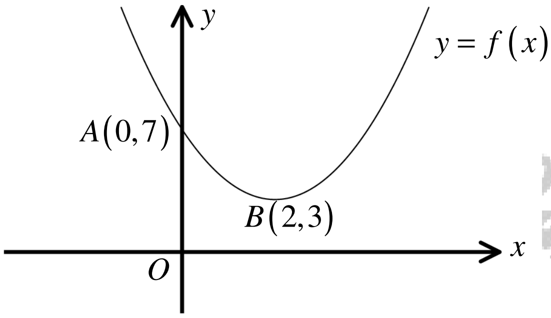
$\boxed{}$, $\boxed{\text{graph}}$



Question 6 (**)

The figure below shows the graph of the curve with equation $y = f(x)$.

The curve meets the y axis at $A(0,7)$ and has a minimum point at $B(2,3)$.



Sketch on a separate set of axes the graph of …

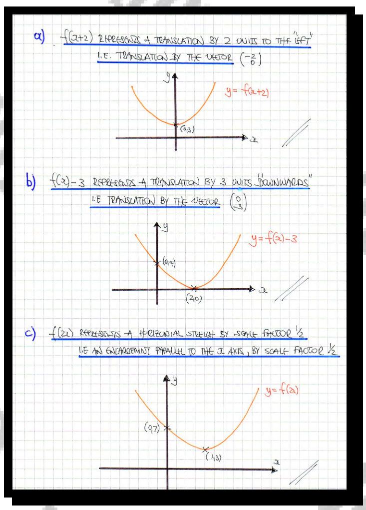
a) … $y = f(x+2)$.

b) … $y = f(x) - 3$.

c) … $y = f(2x)$.

Each sketch must include the coordinates of any points where the graph meets the coordinate axes and the new coordinates of the point B.

$\boxed{}$, $\boxed{\text{graph}}$



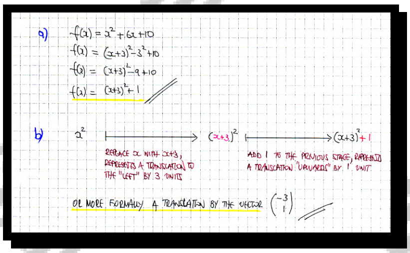
Question 7 (**)

$$f(x) = x^2 + 6x + 10, \ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $f(x) = (x+a)^2 + b$, where a and b are integers.

b) Describe geometrically the two transformations which map the graph of x^2 onto the graph of $f(x)$.

$\boxed{}$, $\boxed{a=3}$, $\boxed{b=1}$, $\boxed{\text{translation by } \begin{pmatrix} -3 \\ 1 \end{pmatrix}}$



a) $f(x) = x^2 + 6x + 10$

$f(x) = (x+3)^2 - 3^2 + 10$

$f(x) = (x+3)^2 - 9 + 10$

$f(x) = (x+3)^2 + 1$

b) $x^2 \longmapsto (x+3)^2 \longmapsto (x+3)^2 + 1$

REPLACE x WITH $x+3$, REPRESENTS A TRANSLATION TO THE "LEFT" BY 3 UNITS

ADD 1 TO THE PREVIOUS STAGE, REPRESENTS A TRANSLATION "UPWARDS" BY 1 UNIT

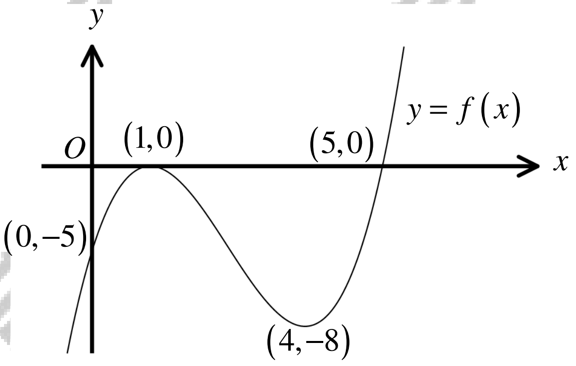
OR MORE FORMALLY A TRANSLATION BY THE VECTOR $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

Question 8 (**)

The figure below shows the graph of the curve with equation $y = f(x)$.

The curve crosses the x axis at the points $(1,0)$ and $(5,0)$, the y axis at $(0,-5)$.

The curve has a minimum at $(4,-8)$.



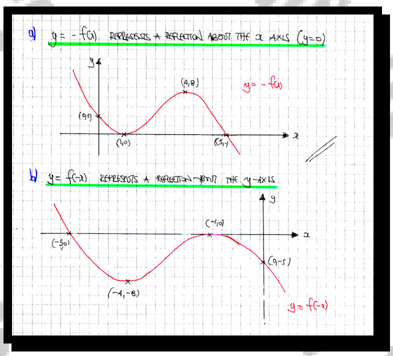
Sketch on separate diagrams the graph of …

a) … $y = -f(x)$.

b) … $y = f(-x)$.

Each sketch must include the coordinates of any points where the graph crosses the coordinate axes and the new coordinates of the minimum point of the curve.

graph

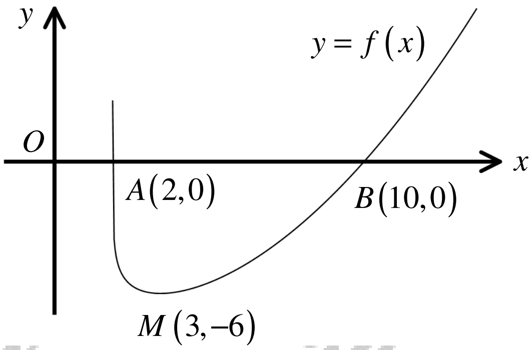


Question 9 (**)

The figure below shows the graph of the curve with equation $y = f(x)$.

The curve crosses the x axis at $A(2,0)$ and at the point $B(10,0)$.

The curve has a minimum at $M(3,-6)$.



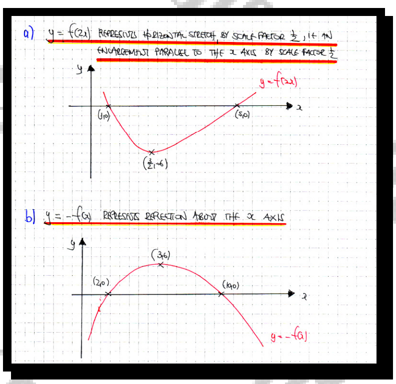
Sketch on separate diagrams the graph of …

a) … $y = f(2x)$.

b) … $y = -f(x)$.

Each sketch must include the coordinates of any points where the graph crosses the x axis and the new coordinates of the minimum point of the curve.

[ ], [graph]

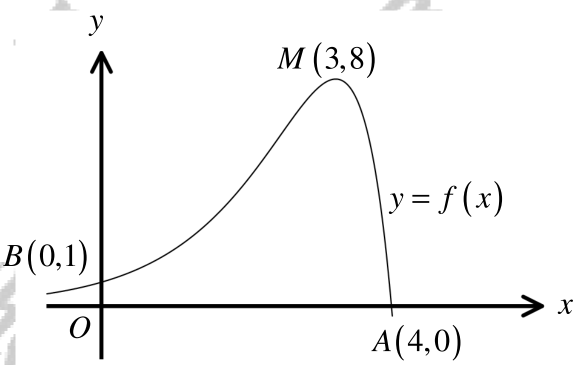


Question 10 (**)

The figure below shows the graph of the curve with equation $y = f(x)$.

The curve crosses the x axis at $A(4,0)$, the y axis at $B(0,1)$.

The curve has a maximum at $M(3,8)$.



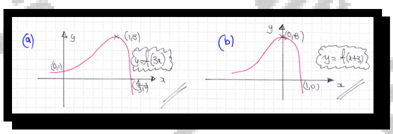
Sketch on separate diagrams the graph of …

a) … $y = f(3x)$.

b) … $y = f(x+3)$.

Each sketch must include the coordinates of any points where the graph crosses the coordinate axes and the new coordinates of the maximum point of the curve.

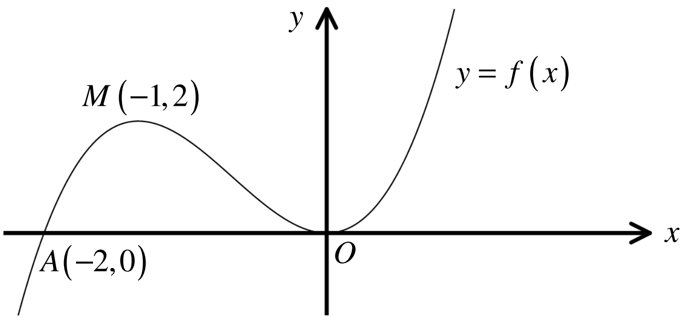
graph



Question 11 (**)

The figure below shows the graph of the curve with equation $y = f(x)$.

The curve meets the x axis at the origin and at the point $A(-2,0)$ and has a local maximum at $M(-1,2)$.



Sketch on separate diagrams the graph of …

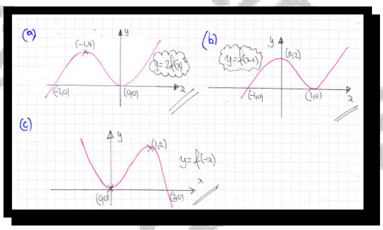
a) … $y = 2f(x)$.

b) … $y = f(x-1)$.

c) … $y = f(-x)$

Each sketch must include the coordinates of any points where the graph crosses the coordinate axes and the new coordinates of the maximum point of the curve.

graph

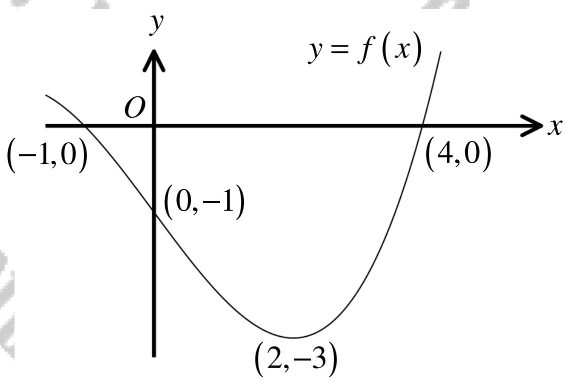


Question 12 (**)

The figure below shows the graph of the curve with equation $y = f(x)$.

The curve meets the x axis at the points with coordinates $(-1,0)$ and $(3,0)$, and the y axis at the point with coordinates $(0,-1)$.

The curve has a minimum at $(2,-3)$.



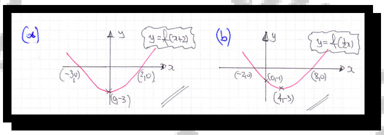
Sketch on separate diagrams the graph of …

a) … $y = f(x+2)$.

b) … $y = f\left(\frac{1}{2}x\right)$.

Each sketch must include the coordinates of any points where the graph meets the coordinate axes and the new coordinates of the minimum point of the curve.

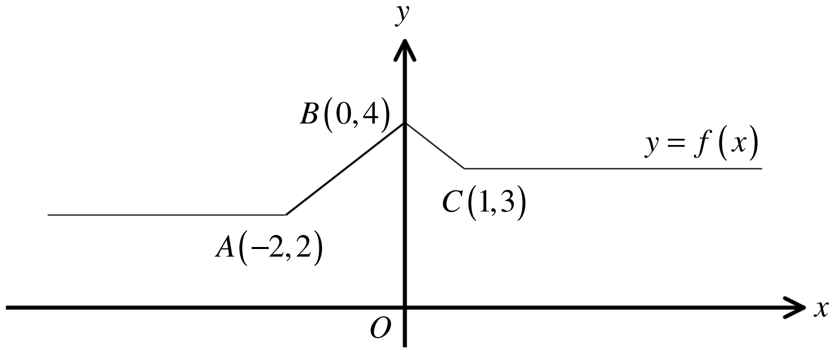
graph



Question 13 (**)

The figure below shows the graph of a function with equation $y = f(x)$.

The graph consists of four line segments joined at the points $A(-2,2)$, $B(0,4)$ and $C(1,3)$.



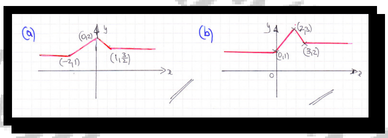
Sketch on separate diagrams the graph of …

a) … $y = \frac{1}{2}f(x)$.

b) … $y = f(x-2) - 1$.

Each sketch must include the new coordinates of the points A, B and C.

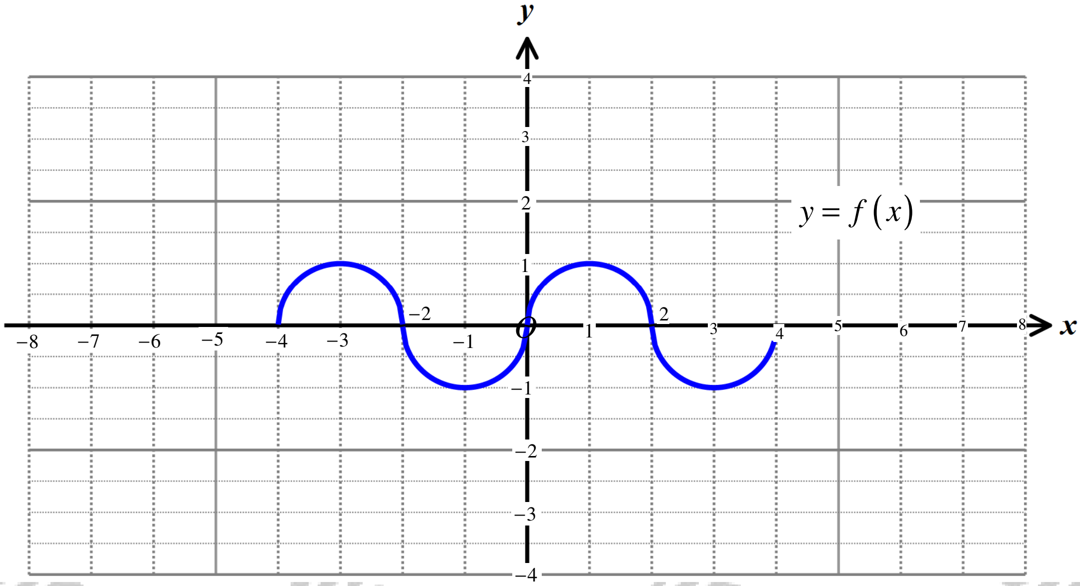
graph



Question 14 (**)

The figure below shows the graph of a function with equation $y = f(x)$, $-4 \le x \le 4$.

The graph consists of four identical semicircular arcs joined together.

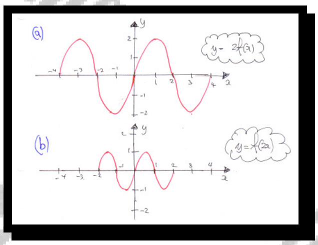


Sketch **accurately** on separate diagrams the graphs of …

a) … $y = 2f(x)$.

b) … $y = f(2x)$.

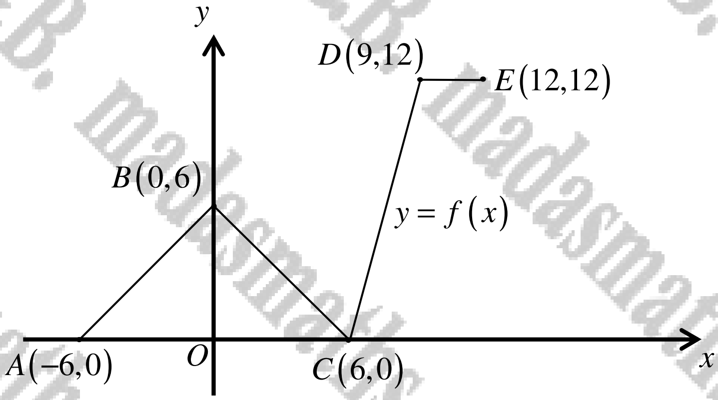
graph



Question 15 (**)

The figure below shows the graph of a function with equation $y = f(x)$.

The graph consists of four straight line segments joining the points $A(-6,0)$, $B(0,6)$, $C(6,0)$, $D(9,12)$ and $E(12,12)$.



Sketch on separate diagrams the graph of …

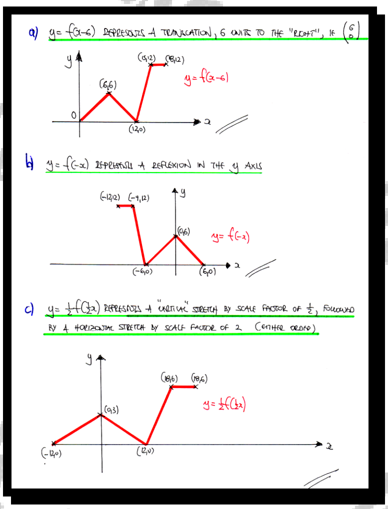
a) … $y = f(x-6)$.

b) … $y = f(-x)$.

c) … $y = \frac{1}{2} f\left(\frac{1}{2}x\right)$.

Each sketch must include the new coordinates of A, B, C, D and E.

, graph



Question 16 (+)**

The curve C has equation

$$y = (x-a)^2 + b,$$

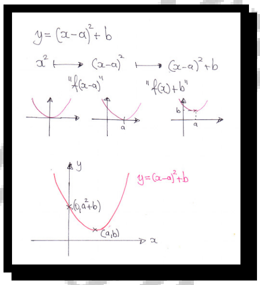
where a and b are positive constants.

By considering the two transformations that map the graph of $y = x^2$ onto the graph of C, or otherwise, sketch the graph of C.

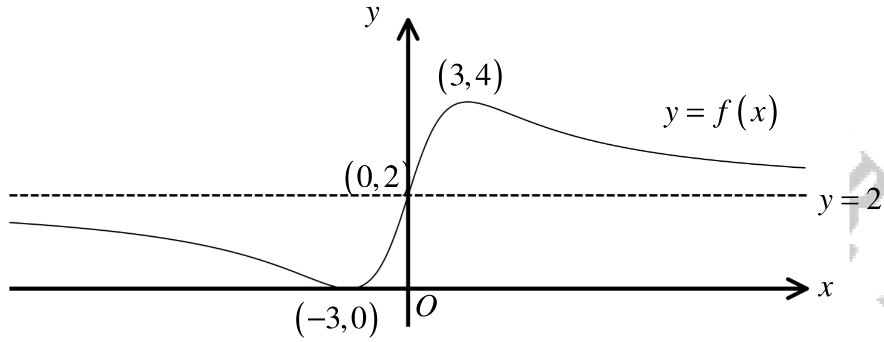
The sketch must include the coordinates, in terms of a and b, of …

- … all the points where the curve meets the coordinate axes.
- … the minimum point of the curve.

graph



Question 17 (**+)



The figure above shows the graph of a curve with equation $y = f(x)$. The curve meets the x axis at $(-3,0)$ and the y axis at $(0,2)$. The curve has a maximum at $(3,4)$ and a minimum at $(-3,0)$.

The line with equation $y = 2$ is a horizontal asymptote to the curve.

Sketch on separate diagrams the graph of …

a) … $y = f(x+3)$.

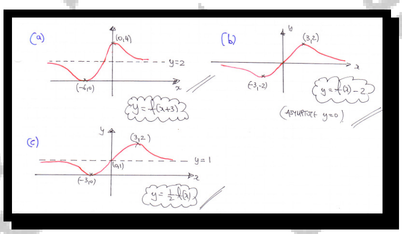
b) … $y = f(x) - 2$.

c) … $y = \frac{1}{2} f(x)$.

Each of the sketches must include

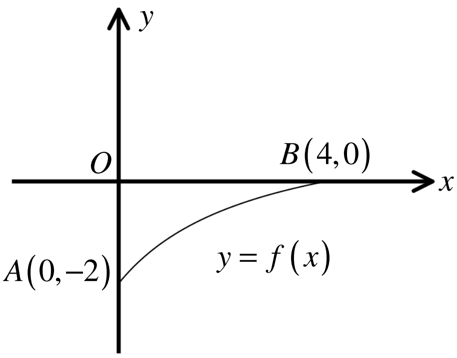
- the coordinates of any points where the graph meets the coordinate axes.
- the coordinates of any minimum or maximum points of the curve.
- any asymptotes to the curve, clearly labelled.

$\boxed{}$, $\boxed{\text{graph}}$



Question 18 (**+)

The figure below shows the graph of the curve with equation $y = f(x)$.



The curve meets the coordinate axes at the points $A(0,-2)$ and $B(4,0)$.

Sketch in a separate diagram the graph of …

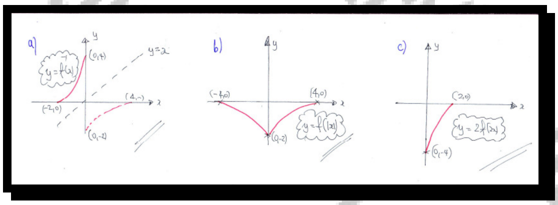
a) … $y = f^{-1}(x)$.

b) … $y = f(|x|)$.

c) … $y = 2f(2x)$.

Each sketch must indicate clearly the coordinates of any points where the graph meets the coordinate axes.

$\boxed{}$, $\boxed{\text{graph}}$



Question 19 (***)

The curve C has equation

$$y = 2^{x-3}.$$

a) Describe geometrically a single transformation that maps the graph of $y = 2^x$ onto the graph of C.

b) Describe geometrically a **different** transformation that can also map the graph of $y = 2^x$ onto the graph of C.

$\boxed{}$, $\boxed{\text{translation "right" by 3 units}}$, $\boxed{\text{enlargement, vertically, by scale factor } \frac{1}{8}}$

a) IF $f(x) = 2^x$ THEN $f(x-3) = 2^{(x-3)}$

HENCE THIS IS A TRANSLATION, IN THE POSITIVE x DIRECTION BY 3 UNITS

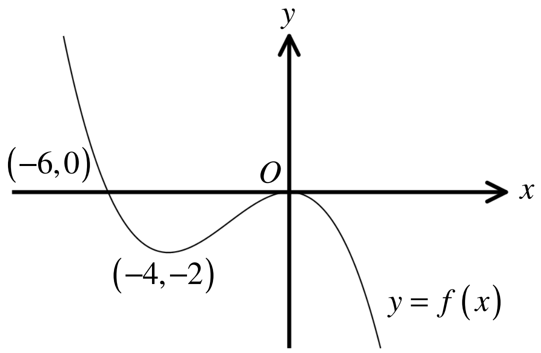
b) REWRITING AS FOLLOWS

$$y = 2^{x-3} = 2^x \times 2^{-3} = 2^x \times \frac{1}{2^3} = \frac{1}{8}(2^x)$$

HENCE IF $f(x) = 2^x$ THEN $\frac{1}{8}f(x) = \frac{1}{8}(2^x)$

THIS IS ALSO A VERTICAL STRETCH BY SCALE FACTOR OF $\frac{1}{8}$

Question 20 (***)



The figure above shows the graph of a function with equation $y = f(x)$.

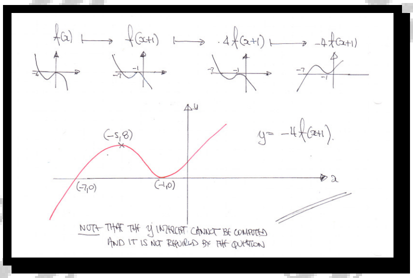
The graph of the function meets the x axis at $(-6,0)$ and has stationary points at the origin and at the point with coordinates $(-4,-2)$.

Sketch the graph of

$$y = -4f(x+1).$$

The sketch must include the coordinates of any points where the graph meets the x axis and the coordinates of the two stationary points.

$\boxed{}$, $\boxed{\text{graph}}$



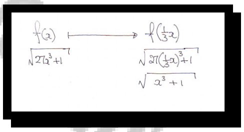
Question 21 (***)

$$f(x)=\sqrt{27x^3+1},\ x\geq -\tfrac{1}{3}.$$

The graph of $f(x)$ is stretched horizontally by scale factor 3, to produce the graph of $g(x)$.

Determine in its simplest form the equation of $g(x)$.

$\boxed{g(x)=\sqrt{x^3+1}}$



Question 22 (***)

$$f(x)=\sqrt{x},\ x\in\mathbb{R},\ x\geq 0.$$

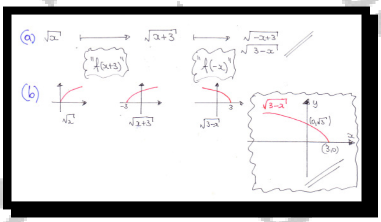
The graph of $f(x)$ is translated by 3 units in the negative x direction, followed by a reflection in the y axis, forming the graph of $g(x)$.

a) Find the equation of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include the coordinates of all the points where the curve meets the coordinate axes.

$\boxed{g(x)=\sqrt{3-x}}$



Question 23 (***)

The curve C has equation

$$y = x^3 - 9x\,.$$

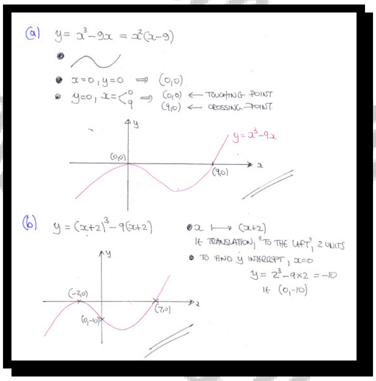
a) Sketch the graph of C.

b) Hence sketch on a separate diagram the graph of

$$y = (x+2)^3 - 9(x+2)\,.$$

Each of the two sketches must include the coordinates of all the points where the curve meets the coordinate axes.

$\boxed{}$, $\boxed{\text{graph}}$



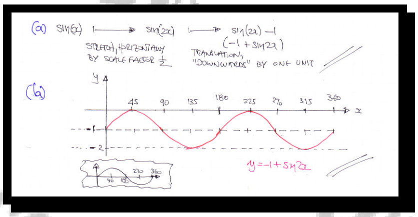
Question 24 (***)

$$y = -1 + \sin 2x^\circ, \quad 0 \le x \le 360.$$

a) Describe geometrically the two transformations that map the graph of $y = \sin x^\circ$ onto the graph of $y = -1 + \sin 2x^\circ$.

b) Sketch the graph of $y = -1 + \sin 2x^\circ$, $0 \le x \le 360$.

horizontal stretch by scale factor $\frac{1}{2}$,
followed by translation "downwards" by 1 unit



Question 25 (***)

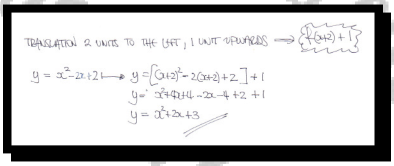
A curve C has equation

$$y = x^2 - 2x + 2, \quad x \in \mathbb{R}.$$

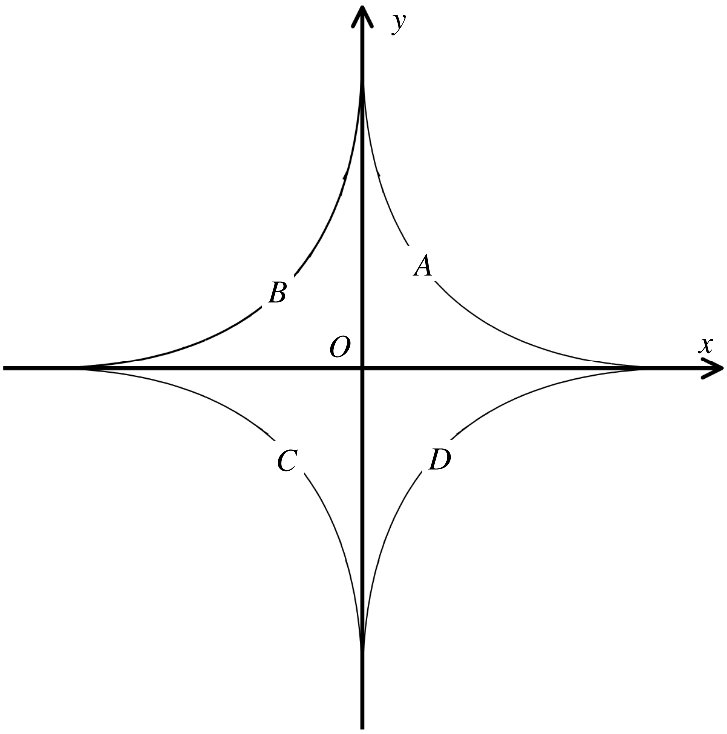
The graph of C is translated by the vector $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

Determine the equation of the translated graph, in its simplest form.

$\boxed{y = x^2 + 2x + 3}$



Question 26 (***)



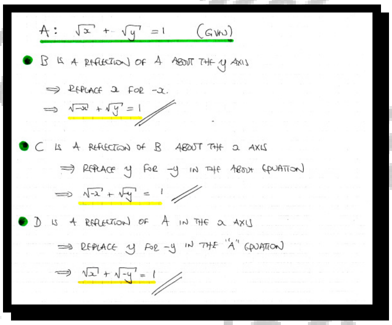
The figure above shows a star shaped curve consisting of four distinct sections, each in a separate quadrant, labelled as A, B, C and D.

The equation of A is

$$\sqrt{x}+\sqrt{y}=1, \quad 0 \le x \le 1, \quad 0 \le y \le 1.$$

Determine the equations for each of the remaining sections B, C and D.

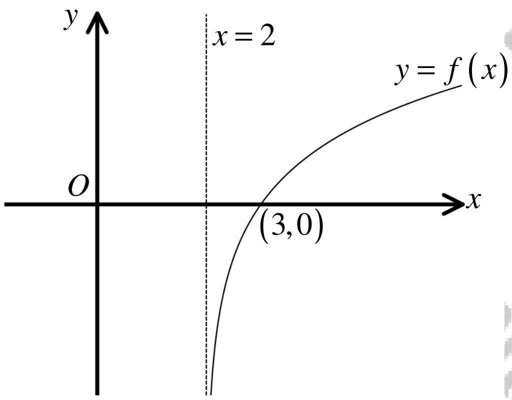
$\boxed{\text{SYN}}$, $\boxed{B:\sqrt{-x}+\sqrt{y}=1}$, $\boxed{C:\sqrt{-x}+\sqrt{-y}=1}$, $\boxed{D:\sqrt{x}+\sqrt{-y}=1}$



A: $\sqrt{x}+\sqrt{y}=1$ (GIVEN)

- B IS A REFLECTION OF A ABOUT THE y AXIS
  $\Rightarrow$ REPLACE x FOR $-x$.
  $\Rightarrow \sqrt{-x}+\sqrt{y}=1$
- C IS A REFLECTION OF B ABOUT THE x AXIS
  $\Rightarrow$ REPLACE y FOR $-y$ IN THE ABOVE EQUATION
  $\Rightarrow \sqrt{-x}+\sqrt{-y}=1$
- D IS A REFLECTION OF A IN THE x AXIS
  $\Rightarrow$ REPLACE y FOR $-y$ IN THE "A" EQUATION
  $\Rightarrow \sqrt{x}+\sqrt{-y}=1$

Question 27 (***)



The figure above shows the graph of the curve with equation

$$y = f(x), \ x \in \mathbb{R}, \ x > 2.$$

The curve meets the x axis at the point with coordinates $(3,0)$, and the straight line with equation $x = 2$ is an asymptote to the curve.

Sketch, on separate diagrams, each of the following graphs.

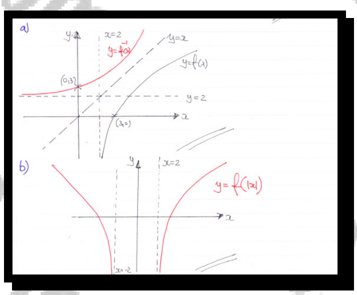
a) $y = f^{-1}(x)$.

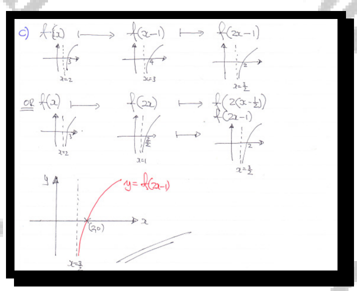
b) $y = f(|x|)$.

c) $y = f(2x-1)$.

Each sketch must include the coordinates of any points where the graph meets the coordinate axes and the equations of any asymptotes.

, graph





Question 28 (***)

A curve C has equation

$$y = \frac{1}{x^2}, \; x \in \mathbb{R}, \; x \neq 0.$$

a) Sketch the graph of C.

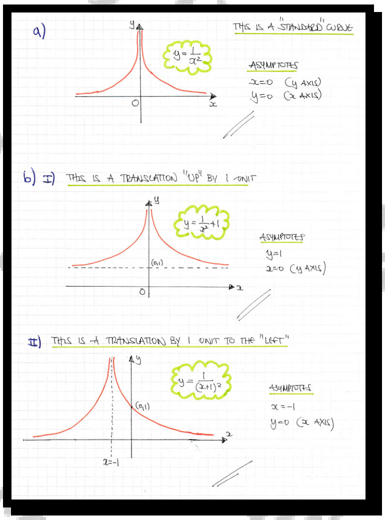
b) Sketch on separate set of axes the graph of ...

i. ... $y = \frac{1}{x^2} + 1, \; x \in \mathbb{R}, \; x \neq 0$.

ii. ... $y = \frac{1}{(x+1)^2}, \; x \in \mathbb{R}, \; x \neq -1$.

Mark clearly in each sketch the equations of any asymptotes to these curves and the coordinates of any intersections with the coordinate axes.

$\boxed{}$, $\boxed{\text{graph}}$



Question 29 (***)

A curve C has equation

$$f(x) = -\frac{1}{x^2}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

a) Sketch the graph of C.

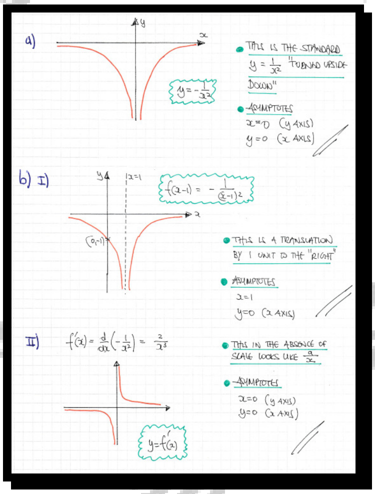
b) Sketch on separate set of axes the graph of …

i. … $f(x-1)$.

ii. … $f'(x)$.

Mark clearly in each sketch the equations of any asymptotes to these curves and the coordinates of any intersections with the coordinate axes.

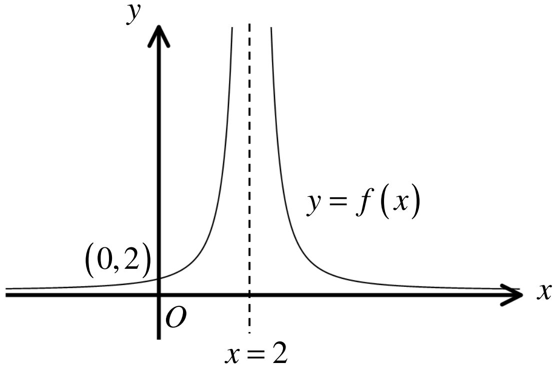
$\boxed{}$, $\boxed{\text{graph}}$



Question 30 (***+)

The figure below shows the graph of a function with equation $y = f(x)$.

The curve meets the y axis at $(0,2)$ while the lines with equations $x = 2$ and $y = 0$ are asymptotes to the curve.



Sketch on separate diagrams the graph of …

a) … $y = 2f(x+2)$.

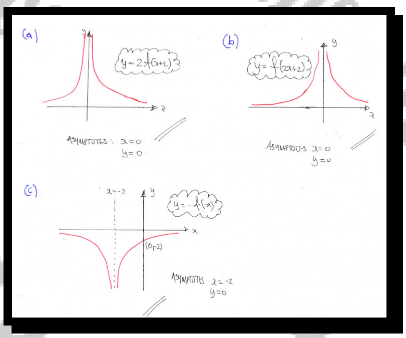
b) … $y = f(2x+2)$.

c) … $y = -f(-x)$.

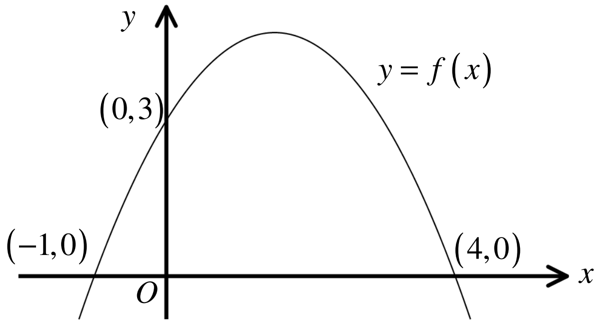
The sketches must include

- the coordinates of any points where the graph meets the coordinate axes.
- any asymptotes to the curve, clearly labelled.

C3, graph



Question 31 (***+)



The figure above shows the graph of a function with equation $y = f(x)$.

The curve meets the y axis at the point with coordinates $(0,3)$ and the x axis at the points with coordinates $(-1,0)$ and $(4,0)$.

Sketch on separate diagrams the graphs of

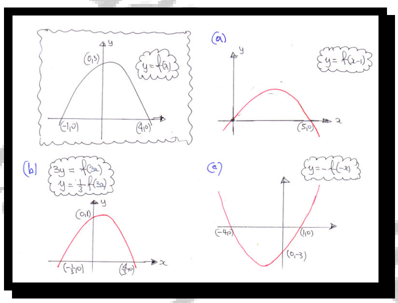
a) $y = f(x-1)$.

b) $3y = f(3x)$.

c) $y = -f(-x)$.

The sketches must include the coordinates of any points where the transformed graph meets the coordinate axes.

graph



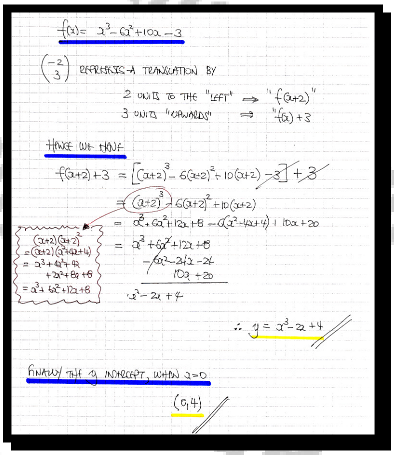
Question 32 (***+)

$$f(x) \equiv x^3 - 6x^2 + 10x - 3, \ x \in \mathbb{R}.$$

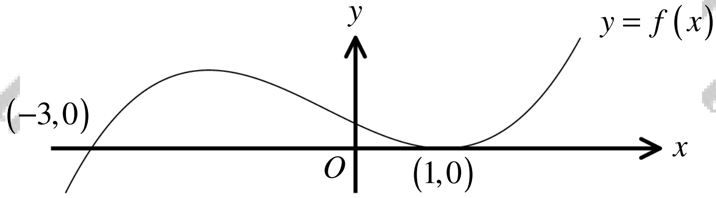
The graph of $f(x)$ is translated by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$, forming the graph of a new curve, denoted by C.

Find, in its simplest form, the equation of C, stating further the coordinates of the point where the graph of C crosses the y axis.

$\boxed{}$, $\boxed{y = x^3 - 2x + 4}$, $\boxed{(0,4)}$



Question 33 (***+)



The figure above shows the graph of the curve C with equation

$$f(x) = x^3 + ax^2 + bx + c,$$

where a, b and c are constants.

The curve crosses the x axis at $(-3,0)$ and touches the x axis at $(1,0)$.

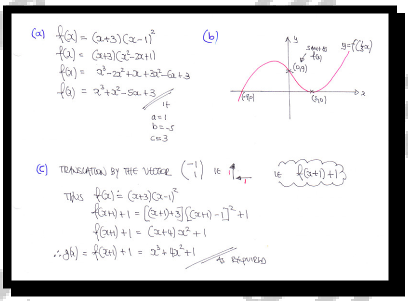
a) Find the value of a, b and c.

b) Sketch the graph of $y = f\left(\frac{1}{3}x\right)$, clearly marking the coordinates of any points of intersection with the coordinate axes.

The graph of C is translated by the vector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ to give the graph of $g(x)$.

c) Show clearly that $g(x) = x^3 + 4x + 1$.

$\boxed{}$, $\boxed{a=1}$, $\boxed{b=-5}$, $\boxed{c=3}$



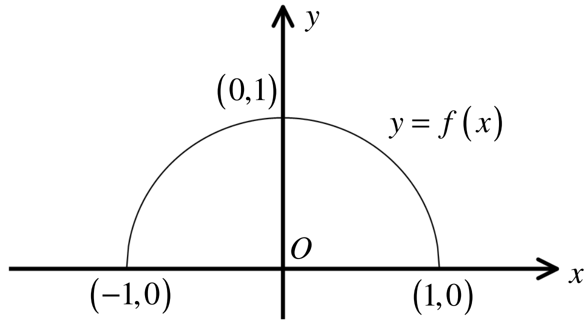
(a) $f(x) = (x+3)(x-1)^2$
$f(x) = (x+3)(x^2-2x+1)$
$f(x) = x^3-2x^2+x+3x^2-6x+3$
$f(x) = x^3+x^2-5x+3$
ie
$a=1$
$b=-5$
$c=3$

(b)

(c) TRANSLATION BY THE VECTOR $\binom{-1}{1}$ IE IE $f(x+1)+1$

THUS $f(x) = (x+3)(x-1)^2$
$f(x+1)+1 = [(x+1)+3][(x+1)-1]^2+1$
$f(x+1)+1 = (x+4)x^2+1$
$\therefore g(x) = f(x+1)+1 = x^3+4x^2+1$ AS REQUIRED

Question 34 (***+)



The figure above shows the graph of a function with equation $y = f(x)$.

The curve meets has a maximum point on the y axis at the point with coordinates $(0,1)$.

It meets the x axis at the points with coordinates $(-1,0)$ and $(1,0)$.

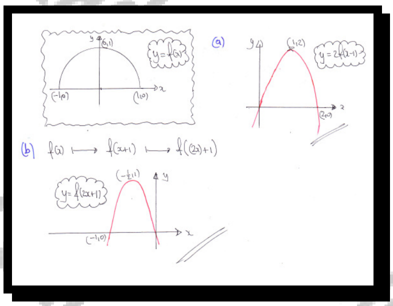
Sketch on separate diagrams the graphs of …

a) … $y = 2f(x-1)$.

b) … $y = f(2x-1)$.

The sketches must include the coordinates of any points where the transformed graph meets the coordinate axes, and the coordinates of its maximum point.

graph



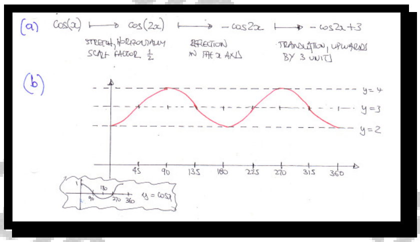
Question 35 (***+)

$$y = 3 - \cos 2x^\circ, \ 0 \le x \le 360.$$

a) Describe geometrically the three transformations that map the graph of $y = \cos x^\circ$ onto the graph of $y = 3 - \cos 2x^\circ$.

b) Sketch the graph of $y = 3 - \cos 2x^\circ$, $0 \le x \le 360$.

horizontal stretch by scale factor 2,
followed by reflection in the x axis,
followed by translation "upwards" by 3 units



Question 36 (***+)

$$x^3 - 4x + 1 = 0.$$

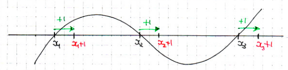
The above cubic equation has three real roots x_1, x_2 and x_3.

Use transformation arguments to find, in a simplified form, another cubic equation whose roots are

$$x_1 + 1, \qquad x_2 + 1, \qquad x_3 + 1.$$

$\boxed{\text{PLA}}$, $\boxed{x^3 - 3x^2 - x + 4 = 0}$

THERE IS NO NEED TO FIND THE ROOTS OF THE CUBIC



LOOKING IN THE ABOVE GRAPH

$f(x) = x^3 - 4x + 1$

TRANSLATION 1 UNIT TO THE "RIGHT"

$f(x-1) = (x-1)^3 - 4(x-1) + 1$

$f(x-1) = (x-1)(x-1)^2 - 4x + 4 + 1$

$f(x-1) = (x-1)(x^2 - 2x + 1) - 4x + 5$

$f(x-1) = \begin{array}{l} x^3 - 2x^2 + x \\ \quad - x^2 + 2x - 1 \end{array} \; - 4x + 5$

$f(x-1) = x^3 - 3x^2 + 3x - 1 - 4x + 5$

$f(x-1) = x^3 - 3x^2 - x + 4$

$\therefore \; x^3 - 3x^2 - x + 4 = 0$

Question 37 (***+)

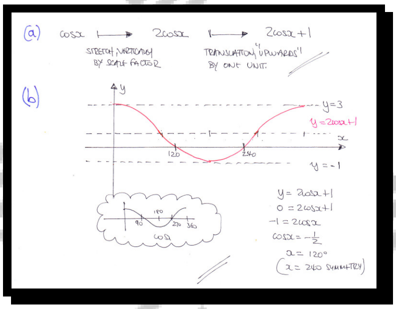
$$y = 1 + 2\cos x°,\ 0 \le x \le 360.$$

a) Describe geometrically the two transformations that map the graph of $y = \cos x°$ onto the graph of $y = 1 + 2\cos x°$.

b) Sketch the graph of $y = 1 + 2\cos x°$, $0 \le x \le 360$.

The sketch must include the coordinates of any points where the graph meets the coordinate axes.

vertical stretch by scale factor 2, followed by translation "upwards" by 1 unit



Question 38 (***+)

The curve C has equation

$$y = 9 - (x-2)^2 .$$

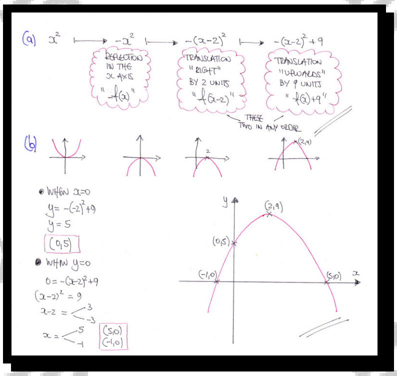
a) Describe geometrically the three transformations that map the graph of $y = x^2$ onto the graph of C.

b) Hence, sketch the graph of C.

The sketch must include the coordinates of

- … all the points where the curve meets the coordinate axes.
- … the coordinates of the maximum point of the curve.

$\boxed{\text{reflection in the } x \text{ axis}}$, $\boxed{\text{translation "right" by 2 units}}$, $\boxed{\text{translation "upwards" by 9 units}}$



Question 39 (***+)

$$f(x) = \frac{1}{x}, \ x \in \mathbb{R}, \ x \neq 0.$$

$$g(x) = \frac{1}{x+2} + 2, \ x \in \mathbb{R}, \ x \neq -2.$$

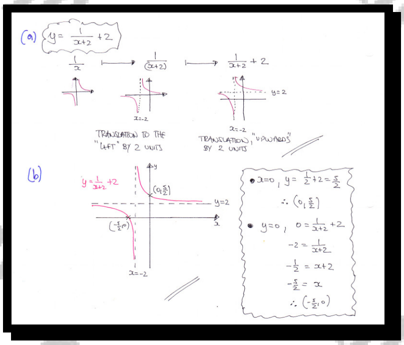
a) Describe mathematically the two transformations that map the graph of $f(x)$ onto the graph of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include the coordinates of …

- … all the points where the curves meet the coordinate axes.
- … the equations of any asymptotes of the curves.

$\boxed{\text{translation "left" by 2 units, followed by translation "upwards" by 2 unit}}$



Question 40 (***+)

A curve C has equation

$$y = x^2 + 8x + 12, \ x \in \mathbb{R}.$$

Describe fully a sequence of two transformations which map the graph of $y = x^2$ onto the graph of C.

$\boxed{}$, $\boxed{\text{translation by the vector } \begin{bmatrix} -4 \\ -4 \end{bmatrix}}$

$y = x^2 + 8x + 12$

$y = (x+4)^2 - 16 + 12$

$y = (x+4)^2 - 4$

"$y = f(x+4) - 4$"

$\therefore$ TRANSLATION BY THE VECTOR $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$

Question 41 (***+)

A curve C has equation

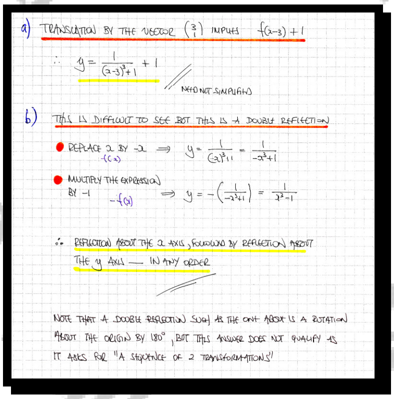
$$y = \frac{1}{x^3+1}, \quad x \in \mathbb{R}, x \neq -1.$$

a) Determine an equation of the curve which is obtained by translating C by the vector $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

b) Describe fully a sequence of two transformations which map the graph of C onto the graph with equation

$$y = \frac{1}{x^3-1}, \quad x \in \mathbb{R}, x \neq 1.$$

$\boxed{}$, $\boxed{y = \dfrac{1}{(x-3)^3+1}+1}$, $\boxed{\text{reflections in the } x \text{ axis and the } y \text{ axis, in either order}}$



a) TRANSLATION BY THE VECTOR $\binom{3}{1}$ INPUTS $f(x-3)+1$

$\therefore \quad y = \dfrac{1}{(x-3)^3+1}+1$

NEED NOT SIMPLIFIED

b) THIS IS DIFFICULT TO SEE BUT THIS IS A DOUBLE REFLECTION

- REPLACE x BY $-x$ $\quad f(-x) \quad \Rightarrow \quad y = \dfrac{1}{(-x)^3+1} = \dfrac{1}{-x^3+1}$
- MULTIPLY THE EXPRESSION BY -1 $\quad -f(x) \quad \Rightarrow \quad y = -\left(\dfrac{1}{-x^3+1}\right) = \dfrac{1}{x^3-1}$

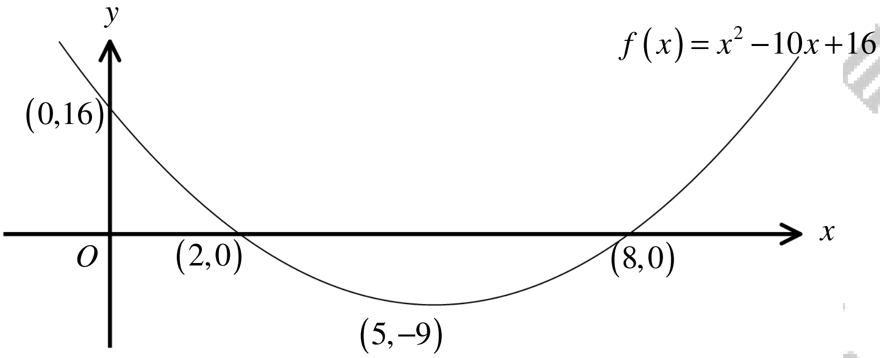
$\therefore$ REFLECTION ABOUT THE x AXIS, FOLLOWED BY REFLECTION ABOUT THE y AXIS — IN ANY ORDER

NOTE THAT A DOUBLE REFLECTION SUCH AS THE ONE ABOVE IS A ROTATION ABOUT THE ORIGIN BY 180°, BUT THIS ANSWER DOES NOT QUALIFY AS IT ASKS FOR "A SEQUENCE OF 2 TRANSFORMATIONS"

Question 42 (***+)

The figure below shows the graph of a function with equation $f(x) = x^2 - 10x + 16$.

The curve meets the x axis at $(2,0)$ and at the point $(8,0)$, the y axis at $(0,16)$ and has a minimum at $(5,-9)$.



Sketch on separate diagrams the graphs of

a) $y = f(x+2)$.

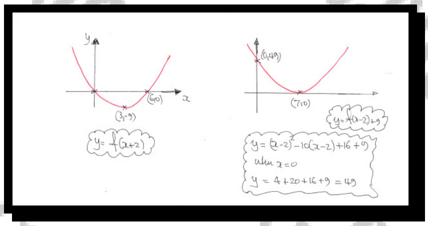
b) $y = f(x-2) + 9$.

The sketches must include

- the coordinates of any points where the graph meets the coordinate axes,
- the new coordinates of the minimum point of the curve.

Show detailed calculations on how the y intercept of the graph (b) was obtained.

$\boxed{\text{graph}}$, $\boxed{(0,49)}$



Question 43 (***+)

Sketch on separate diagrams the curve with equation …

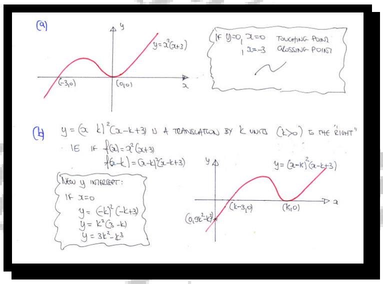
a) … $y = x^2(x+3)$.

b) … $y = (x-k)^2(x-k+3)$,

where k is a constant such that $k > 3$.

Both sketches must include the coordinates, in terms of k where appropriate, of any points where each of the curves meets the coordinate axes.

[ ], [graph]



Question 44 (***+)

A semi cubical parabola C has equation

$$y = \sqrt{x^3}, \; x \in \mathbb{R}, \; x \geq 0.$$

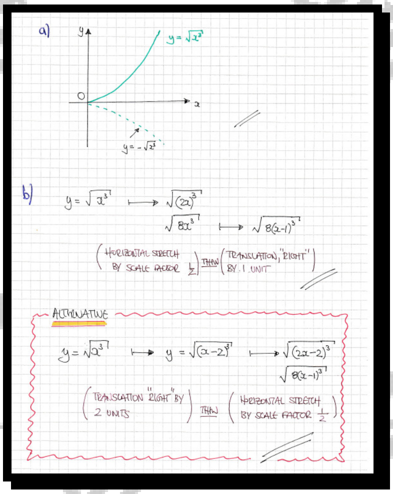
a) Sketch the graph of C.

b) Describe fully a sequence of two transformations which map the graph of C onto the graph with equation

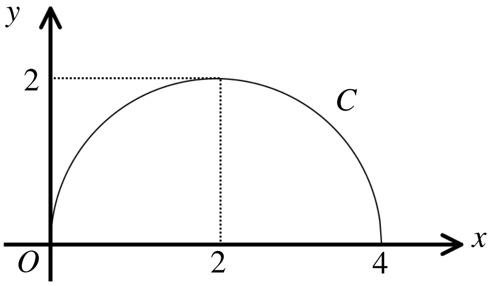
$$y = \sqrt{8(x-1)^3}, \; x \in \mathbb{R}, \; x \geq 1.$$

$\boxed{}$,

horizontal stretch by the scale factor $\frac{1}{2}$, followed by translation by the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$



Question 45 (***+)



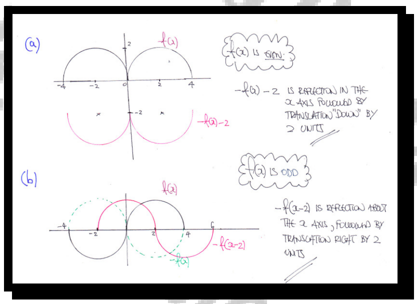
The figure above shows **part** of the curve C with equation

$$y = f(x), \ -4 \le x \le 4 .$$

a) Given that the graph of C is **even**, sketch the graph of $y = -2 - f(x)$.

b) Given that the graph of C is **odd**, sketch the graph of $y = -f(x-2)$.

graph



Question 46 (***+)

$$f(x) = 2x^2 - 8x + 14, \quad x \in \mathbb{R}.$$

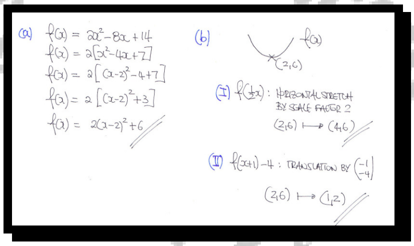
a) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are integer constants.

b) Find the coordinates of the minimum point on the curve with equation …

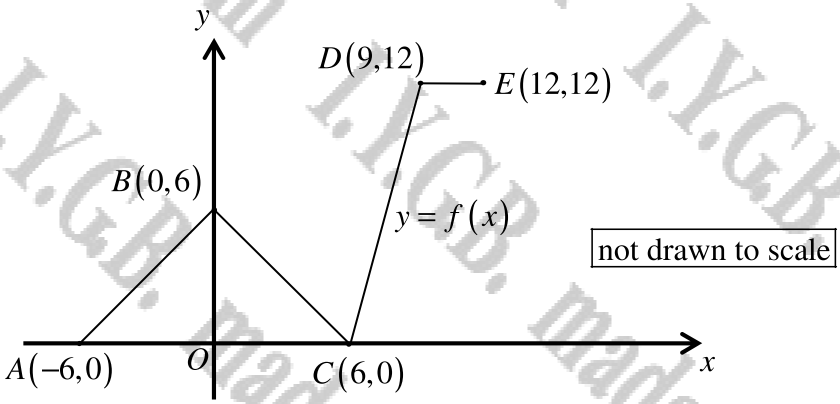
i. … $y = f\left(\frac{1}{2}x\right)$.

ii. … $y = f(x+1) - 4$.

$\boxed{f(x) = 2(x-2)^2 + 6}$, $\boxed{(4,6)}$, $\boxed{(1,2)}$



Question 47 (***+)



The figure above shows the graph of a function with equation $y = f(x)$.

The graph consists of four straight line segments joining the points $A(-6,0)$, $B(0,6)$, $C(6,0)$, $D(9,12)$ and $E(12,12)$.

a) Write down, with some justification, the number of roots of the equation …

i. … $f(x) = 2$.

ii. … $f(x) = x$

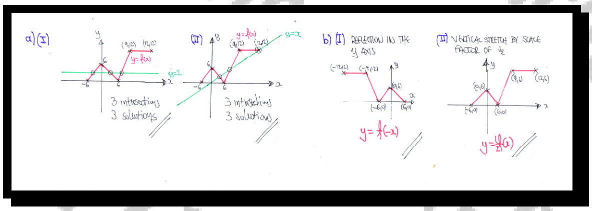
b) Sketch on separate diagrams the graph of …

i. … $y = f(-x)$.

ii. … $y = \frac{1}{2}f(x)$.

Each sketch must include the new coordinates of A, B, C, D and E.

$\boxed{}$, $\boxed{\text{3 roots}}$, $\boxed{\text{3 roots}}$, $\boxed{\text{graph}}$



Question 48 (***+)

A curve is defined as

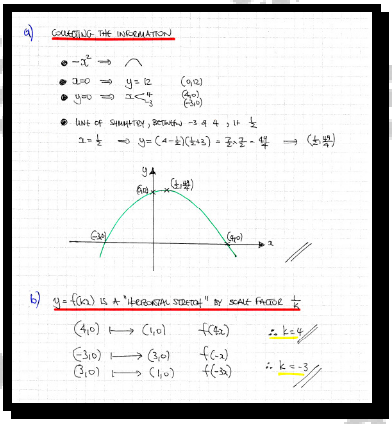
$$f(x) \equiv (4-x)(x+3) \quad , \quad x \in \mathbb{R} \, ,$$

a) Sketch the graph of $f(x)$, clearly indicating the coordinates of its vertex and the coordinates of any points where the graph meets the coordinate axes.

The graph of the curve with equation $y = f(kx)$, where k is a constant, passes through the point with coordinates $(1,0)$.

b) Determine the two possible values of k.

$\boxed{}$, $\boxed{k = 4 \ \cup \ k = -3}$



a) COLLECTING THE INFORMATION

- $-x^2 \Rightarrow \frown$
- $x=0 \Rightarrow y=12$ $(0,12)$
- $y=0 \Rightarrow x < {4 \atop -3}$ $(4,0)$ $(-3,0)$
- LINE OF SYMMETRY, BETWEEN -3 & 4, IE $\frac{1}{2}$

$x=\frac{1}{2} \Rightarrow y=(4-\frac{1}{2})(\frac{1}{2}+3) = \frac{7}{2}\times\frac{7}{2} = \frac{49}{4} \longrightarrow (\frac{1}{2},\frac{49}{4})$

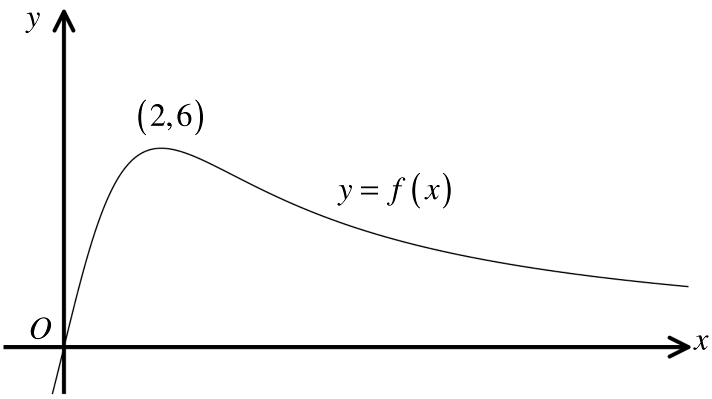
b) $y=f(kx)$ IS A "HORIZONTAL STRETCH" BY SCALE FACTOR $\frac{1}{k}$

$(4,0) \longmapsto (1,0)$ $f(4x)$ $\therefore k=4$

$(-3,0) \longmapsto (3,0)$ $f(-x)$

$(3,0) \longmapsto (1,0)$ $f(-3x)$ $\therefore k=-3$

Question 49 (***+)



The figure above shows part of the curve with equation $y = f(x)$, which has a local maximum at $(2,6)$.

The graph $y = f(x)$ is transformed onto the graph of $y = g(x)$, so that the graph of $y = g(x)$ has a **local minimum** at the origin.

Express $g(x)$ in terms of $f(x)$.

$\boxed{}$, $\boxed{g(x) = 6 - f(x+2)}$

LOOKING AT THE TRANSFORMATIONS OF $(2,6)$

- MAX $(2,6)$ — $y = f(x)$

TRANSLATION "LEFT" BY 2 UNITS

- MAX $(0,6)$ — $y = f(x+2)$

TRANSLATION "DOWN" BY 6 UNITS

- MAX $(0,0)$ — $y = f(x+2) - 6$

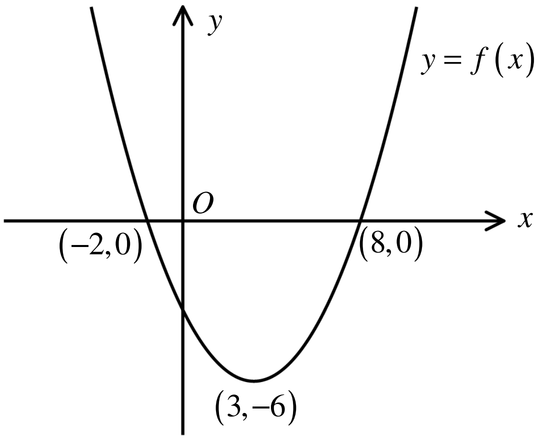
REFLECTION ABOUT THE x AXIS

- MIN $(0,0)$ — $y = -[f(x+2) - 6]$

$\therefore g(x) = 6 - f(x+2)$

Question 50 (***+)

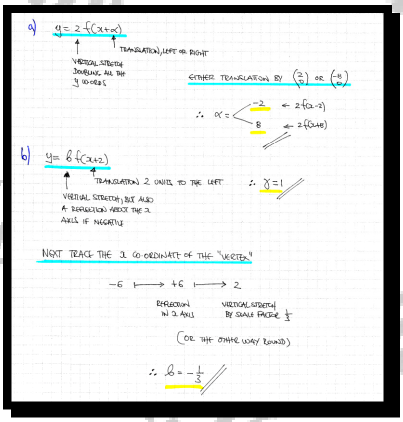
A curve with equation $y = f(x)$ meets the x axis at the points with coordinates $(-2,0)$ and $(8,0)$, and has a stationary point at $(3,-6)$, as shown in the figure below.



a) If the graph of $y = 2f(x+\alpha)$ passes through the origin, determine the possible values of α.

b) If the stationary point on the graph of $y = \beta f(x+2)$ has coordinates $(\gamma, 2)$, state the value of β and the value of γ.

$\boxed{}$, $\boxed{\alpha = -2 \cup \alpha = 8}$, $\boxed{\beta = -\frac{1}{3}}$, $\boxed{\gamma = 1}$



a) $y = 2f(x+\alpha)$

VERTICAL STRETCH DOUBLING ALL THE y CO-ORDS

TRANSLATION, LEFT OR RIGHT

EITHER TRANSLATION BY $\binom{2}{0}$ OR $\binom{-8}{0}$

$\therefore \alpha = -2 \leftarrow 2f(x-2)$, $\alpha = 8 \leftarrow 2f(x+8)$

b) $y = \beta f(x+2)$

VERTICAL STRETCH, BUT ALSO A REFLECTION ABOUT THE x AXIS IF NEGATIVE

TRANSLATION 2 UNITS TO THE LEFT $\therefore \gamma = 1$

NEXT TRACE THE x CO-ORDINATE OF THE "VERTEX"

$-6 \xmapsto{\text{REFLECTION IN } x \text{ AXIS}} +6 \xmapsto{\text{VERTICAL STRETCH BY SCALE FACTOR } \frac{1}{3}} 2$

(OR THE OTHER WAY ROUND)

$\therefore \beta = -\frac{1}{3}$

Question 51 (****)

$$f(x) = 3x^2 + 5x - 2, \ x \in \mathbb{R}.$$

a) Solve the equation $f(x) = 0$.

b) Sketch the graph of $f(x)$.

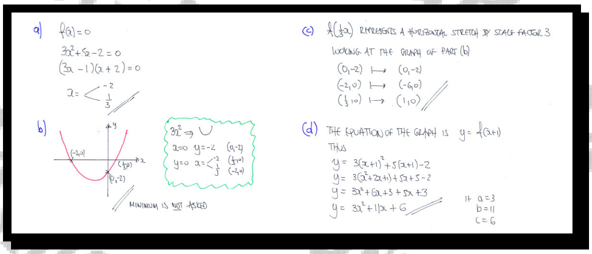
The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes.

c) Find the coordinates of any points where the graph of the curve with equation $y = f\left(\frac{1}{3}x\right)$ meets the coordinate axes.

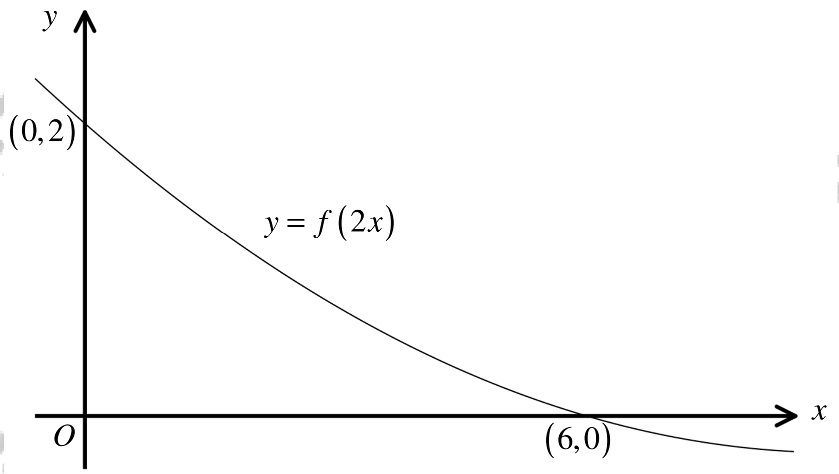
The graph of $y = f(x)$ is translated by 1 unit in the negative x direction onto the graph of the curve with equation $y = ax^2 + bx + c$, where a, b and c are constants.

d) Determine the value of a, b and c.

$\boxed{}$, $\boxed{x = -2, \ x = \frac{1}{3}}$, $\boxed{(-2,0), \left(\frac{1}{3},0\right), (0,-2)}$, $\boxed{(-6,0), (1,0), (0,-2)}$, $\boxed{a = 3, b = 11, c = 6}$



Question 52 (****)



The figure above shows part of the curve with equation $y = f(2x)$.

The curve meets the coordinate axes at $(6,0)$ and $(0,4)$.

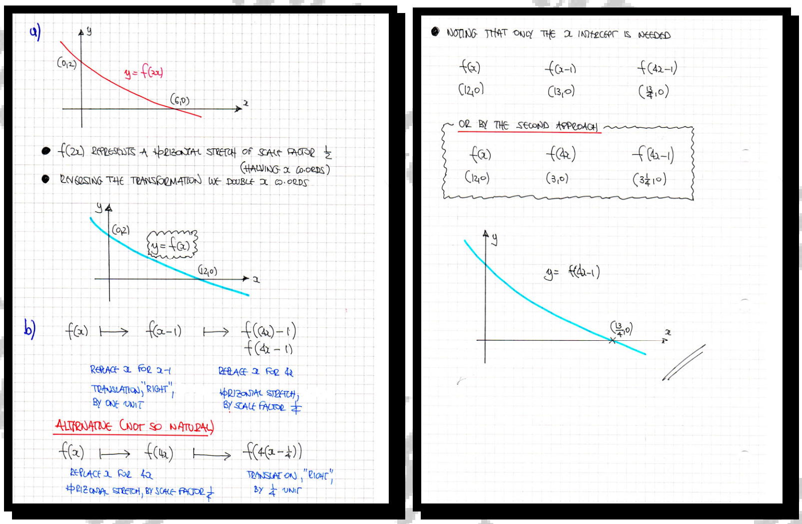
b) Sketch the graph of $y = f(x)$.

The sketch must include the coordinates of any points where the graph meets the coordinate axes.

c) Sketch on separate diagram the graph of $y = f(4x-1)$.

The sketch must include the coordinates of the point where the graph meets the x axis.

$\boxed{}$, $\boxed{\text{graph}}$



Question 53 (****)

A function f is defined by

$$f(x) = 4x(x-1), \ x \in \mathbb{R}.$$

The graph of $g(x)$ is obtained by translating the graph of $f(x)$ by 1 unit in the positive x direction, followed by a horizontal stretch by scale factor of $\frac{2}{3}$.

a) Determine an equation for $g(x)$.

The graph of $f(x)$ is obtained by translating the graph of $h(x)$ by 1 unit in the positive x direction, followed by a vertical stretch by scale factor of 2.

b) Determine an equation for $h(x)$.

$\boxed{}$, $\boxed{g(x) = 9x^2 - 18x + 8}$, $\boxed{h(x) = 2x^2 + 2x}$

a) IDENTIFYING THE TRANSFORMATIONS

- TRANSLATION BY 1 UNIT TO THE "RIGHT" : $x \mapsto x-1$

$$4x(x-1) \longmapsto 4(x-1)[(x-1)-1]$$
$$4(x-1)(x-2)$$

- HORIZONTAL STRETCH BY SCALE FACTOR $\frac{2}{3}$: $x \mapsto \frac{3}{2}x$

$$4(x-1)(x-2) \longmapsto 4\left[\left(\tfrac{3}{2}x\right)-1\right]\left[\left(\tfrac{3}{2}x\right)-2\right]$$
$$4\left(\tfrac{3}{2}x-1\right)\left(\tfrac{3}{2}x-2\right)$$
$$4\left(\tfrac{9}{4}x^2-3x-\tfrac{3}{2}x+2\right)$$
$$9x^2-12x-6x+8$$

$$\therefore \ g(x) = 9x^2-18x+8$$

b) IDENTIFYING AND REVERSING THE TRANSFORMATIONS

- REVERSING A VERTICAL STRETCH BY SCALE FACTOR OF 2

WE HAVE A VERTICAL STRETCH BY SCALE FACTOR $\frac{1}{2}$

$$4x(x-1) \longmapsto \tfrac{1}{2}[4x(x-1)] = 2x(x-1)$$

- REVERSING A TRANSLATION BY 1 UNIT TO THE "RIGHT"

WE NEED A TRANSLATION BY 1 UNIT TO THE "LEFT"

$$2x(x-1) \longmapsto 2(x+1)[(x+1)-1]$$
$$\longmapsto 2x(x+1)$$

$$\therefore \ h(x) = 2x(x+1) = 2x^2+2x$$

Question 54 (****)

The curve C has equation

$$f(x) = (x-a)(x+b), \ x \in \mathbb{R},$$

where a and b are constants such that $a > b > 0$.

Sketch, in separate sets of axes, the graph of ...

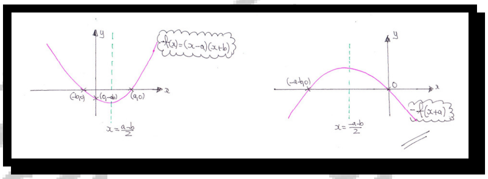
a) ... $y = f(x)$.

b) ... $y = -f(x+a)$.

Each of the graphs must show clearly …

- … the coordinates of any points where the curve meets the coordinates axes.
- … the equation of the line of symmetry of the curve.

graph



Question 55 (****)

$$f(x) = 8^x, \; x \in \mathbb{R}.$$

a) Describe the geometric transformation which maps the graph of $f(x)$ onto the graph of …

i. … $y = \left(\frac{1}{8}\right)^x$.

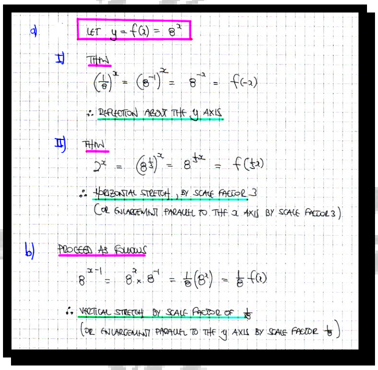
ii. … $y = 2^x$.

The graph of $f(x)$ is mapped onto the graph of $y = 8^{x-1}$ by a single geometric transformation T, which is **not a translation**.

b) Describe T geometrically.

[ ], [reflection in the y axis], [horizontal stretch, scale factor 3],

[vertical stretch, scale factor $\frac{1}{8}$]



a) $\boxed{\text{LET } y = f(x) = 8^x}$

I) THEN

$\left(\frac{1}{8}\right)^x = \left(8^{-1}\right)^x = 8^{-x} = f(-x)$

∴ REFLECTION ABOUT THE y AXIS

II) THEN

$2^x = \left(8^{\frac{1}{3}}\right)^x = 8^{\frac{1}{3}x} = f\left(\frac{1}{3}x\right)$

∴ HORIZONTAL STRETCH, BY SCALE FACTOR 3

(OR ENLARGEMENT PARALLEL TO THE x AXIS BY SCALE FACTOR 3)

b) PROCEED AS FOLLOWS

$8^{x-1} = 8^x \times 8^{-1} = \frac{1}{8}\left(8^x\right) = \frac{1}{8}f(x)$

∴ VERTICAL STRETCH BY SCALE FACTOR OF $\frac{1}{8}$

(OR ENLARGEMENT PARALLEL TO THE y AXIS BY SCALE FACTOR $\frac{1}{8}$)

Question 56 (****)

The curve C has equation

$$y = \frac{2x+3}{x-2}, \quad x \in \mathbb{R}, \; x \neq 2.$$

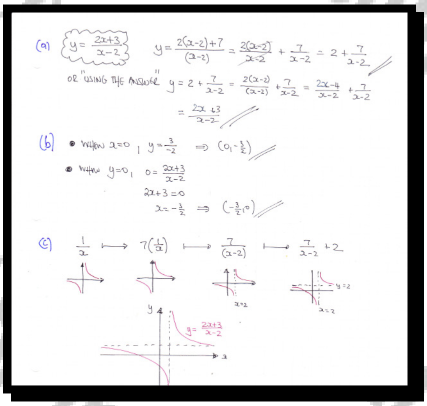
a) Show clearly that

$$\frac{2x+3}{x-2} \equiv 2 + \frac{7}{x-2}.$$

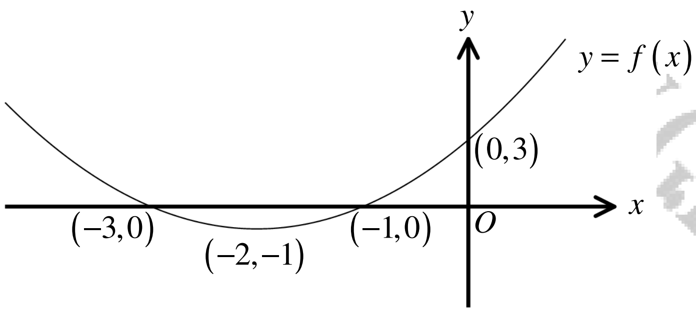
b) Find the coordinates of the points where C meets the coordinate axes.

c) Sketch the graph of C showing clearly the equations of any asymptotes.

$$\boxed{\left(0,-\frac{3}{2}\right), \left(-\frac{3}{2},0\right)}$$



Question 57 (****)



The figure above shows the graph of a function with equation

$$f(x) = x^2 + 4x + 3, \; x \in \mathbb{R}.$$

The curve meets the x axis at the points $(-3,0)$ and $(-1,0)$, the y axis at $(0,3)$ and has a minimum point at $(-2,-1)$.

Sketch on separate diagrams the graphs of …

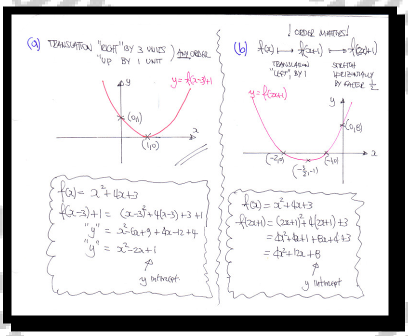
a) … $y = f(x-3)+1$.

b) … $y = f(2x+1)$.

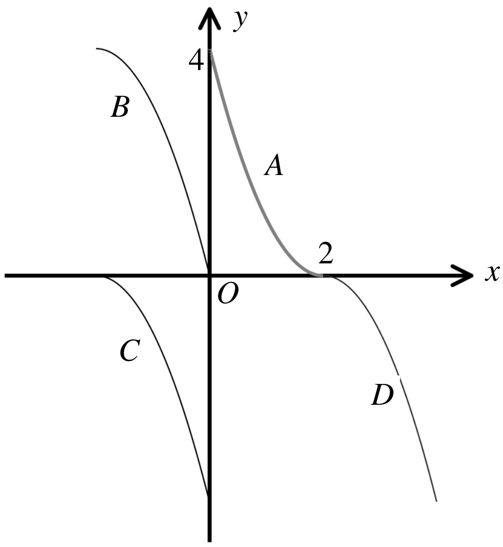
The graphs must show …

- … the coordinates of any points where the graph meets the coordinate axes.
- … the new coordinates of the minimum point of the curve.
- … detailed calculations for finding the y coordinate of each graph.

graph



Question 58 (****)



The figure above shows four distinct graphs, each located within a separate quadrant, labelled as A, B, C and D.

The equation of A is

$$y = (x-2)^2, \quad 0 \le x \le 2, \quad 0 \le y \le 4.$$

Find the equations for each of the remaining sections, B, C and D, giving each of the equations in a simplified form $y = f(x)$.

$\boxed{B: y = 4-(x+2)^2 = -x^2-4x}$, $\boxed{C: y = -(x+2)^2 = -x^2-4x-4}$, $\boxed{D: y = -(2-x)^2 = -x^2+4x-4}$

APPROACH THE TRANSFORMATIONS AS FOLLOWS

- A > REFLECTION ABOUT x THAN REFLECTION ABOUT y > C

$(x-2)^2 \longmapsto -(x-2)^2 \longmapsto -(-x-2)^2 = -(x^2+4x+4)$
$= \underline{\underline{-x^2-4x-4}}$

- C > TRANSLATION, "UPWARDS", BY 4 UNITS > B

$-x^2-4x-4 \longmapsto (-x^2-4x-4)+4 = \underline{\underline{-x^2-4x}}$

- C > TRANSLATION, "RIGHT", BY 4 UNITS > D

$-x^2-4x-4 \longmapsto -(x-4)^2-4(x-4)-4 = -x^2+8x\cancel{-16}-4x\cancel{+16}-4$
$= \underline{\underline{-x^2+4x-4}}$

SUMMARIZING WE HAVE

- A : $y=(x-2)^2$ $\quad 0 \le x \le 2 \quad 0 \le y \le 4$
- B : $y=-(x-2)^2$
  $y=-x^2-4x-4$ $\quad -2 \le x \le 0 \quad 0 \le y \le 4$
- C : $y=4-(x-2)^2$
  $y=-x^2-4x$ $\quad -2 \le x \le 0 \quad -4 \le y \le 0$
- D : $y=-(2-x)^2$
  $y=-x^2+4x-4$ $\quad 2 \le x \le 4 \quad -4 \le y \le 0$

Question 59 (****)

A curve C has equation

$$y = 3^x + 1, \; x \in \mathbb{R}.$$

a) Sketch the graph of C, clearly indicating the equation of the asymptote to the curve and the coordinates of any intercepts with the coordinate axes.

b) Find an equation of the curve which is obtained by reflecting the graph of C in the x axis followed by reflection of the graph of C in the y axis.

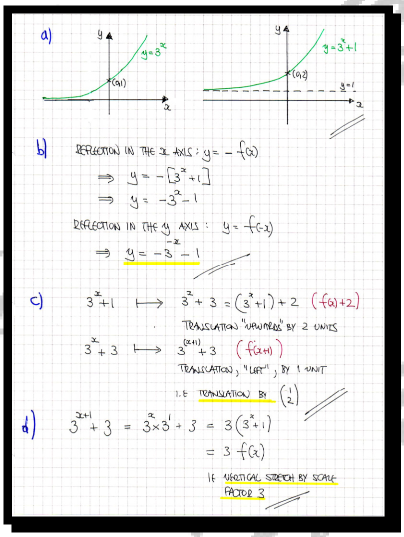
c) Describe fully a sequence of two transformations which map the graph of C onto the graph with equation

$$y = 3^{x+1} + 3, \; x \in \mathbb{R}.$$

d) Describe fully a **single** transformation which map the graph of C onto the graph with equation

$$y = 3^{x+1} + 3, \; x \in \mathbb{R}.$$

$\boxed{}$, $\boxed{y = -3^{-x} - 1}$, $\boxed{\text{translation by the vector } \begin{bmatrix} -1 \\ 2 \end{bmatrix}}$, $\boxed{\text{vertical stretch by the scale factor } 3}$

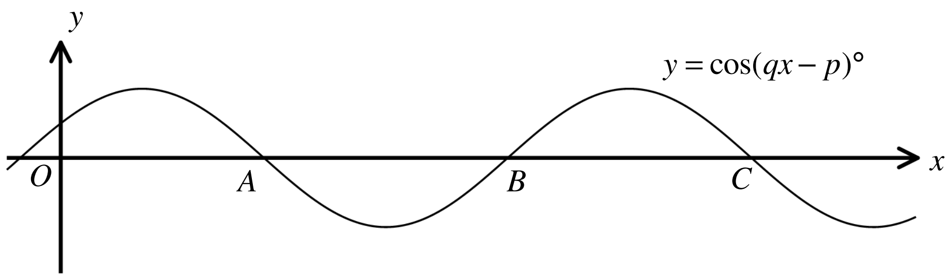


Question 60 (****)

The figure below shows part of the graph of

$$y = \cos(qx - p)°, \; x \in \mathbb{R},$$

where q and p are positive constants.

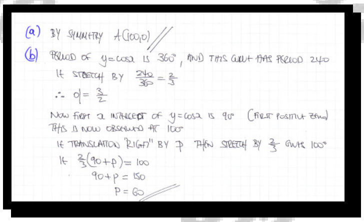


The graph of $y = \cos(qx - p)°$ crosses the x axis at the points A, $B(220,0)$ and $C(340,0)$.

a) State the coordinates of A.

b) Determine the values of q and p.

$\boxed{A(100,0)}$, $\boxed{q = \frac{3}{2}, p = 60}$


(a) BY SYMMETRY $A(100,0)$

(b) PERIOD OF $y = \cos x$ IS $360°$, AND THIS CURVE HAS PERIOD 240

IE STRETCH BY $\frac{240}{360} = \frac{2}{3}$

$\therefore q = \frac{3}{2}$

NOW FIRST x INTERCEPT OF $y = \cos x$ IS $90°$ (FIRST POSITIVE ZERO)

THIS IS NOW OBSERVED AT $100°$

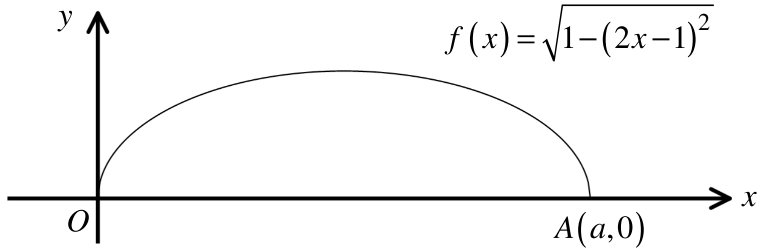
IE TRANSLATION "RIGHT" BY p THEN STRETCH BY $\frac{2}{3}$ GIVES $100°$

IE $\frac{2}{3}(90+p) = 100$

$90 + p = 150$

$p = 60$

Question 61 (****)



The figure above shows the graph of the function

$$f(x) \equiv \sqrt{1-(2x-1)^2}\,,\ x \in \mathbb{R},\ 0 \le x \le a\,.$$

a) Find the value of the constant a.

b) State the range of $f(x)$.

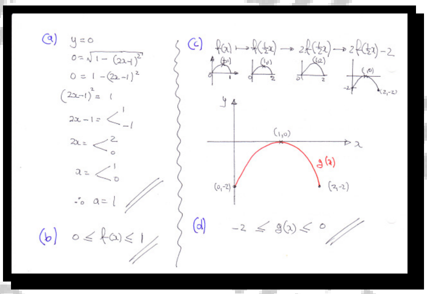
The function g is suitably defined by

$$g(x) = 2f\left(\tfrac{1}{2}x\right) - 2\,.$$

c) Sketch the graph of $g(x)$.

d) State the domain and range of $g(x)$.

$\boxed{a=1}$, $\boxed{0 \le f(x) \le 1}$, $\boxed{0 \le x \le 2,\quad -2 \le g(x) \le 0}$



Question 62 (****)

A cubic curve C has equation

$$y = (3-x)(4+x)^2 .$$

a) Sketch the graph of C.

The sketch must include any points where the graph meets the coordinate axes.

b) Sketch in separate diagrams the graph of …

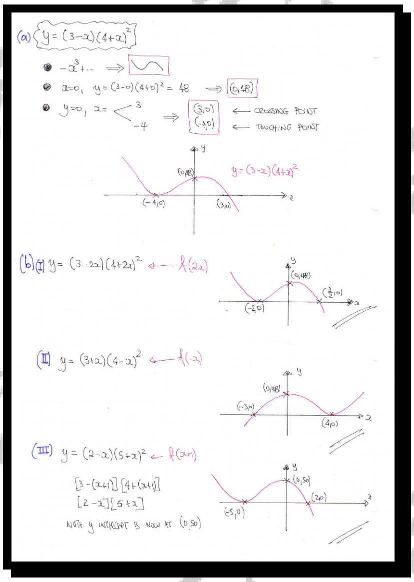
i. … $y = (3-2x)(4+2x)^2$.

ii. … $y = (3+x)(4-x)^2$.

iii. … $y = (2-x)(5+x)^2$.

Each of the sketches must include any points where the graph meets the coordinate axes.

, graph



Question 63 (****)

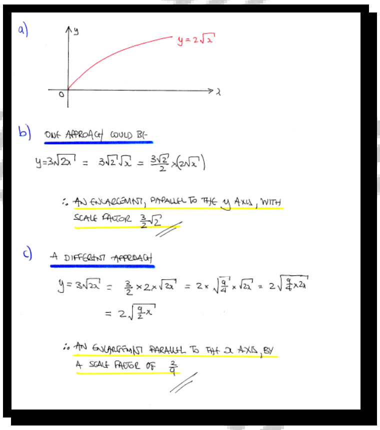
a) Sketch the graph of $y = 2\sqrt{x}$ in its largest real domain.

b) Describe a **single** geometric transformation which maps the graph of $y = 2\sqrt{x}$ onto the graph of $y = 3\sqrt{2x}$.

c) Describe a **single** geometric transformation which maps the graph of $y = 2\sqrt{x}$ onto the graph of $y = 3\sqrt{2x}$, other than the one described in part (**b**).

 , $\boxed{\text{vertical stretch by scale factor of } \frac{3}{2}\sqrt{2}}$,

$\boxed{\text{horizontal stretch by scale factor of } \frac{2}{9}}$



a)

b) ONE APPROACH COULD BE

$y = 3\sqrt{2x} = 3\sqrt{2}\sqrt{x} = \frac{3\sqrt{2}}{2} \times \left(2\sqrt{x}\right)$

∴ AN ENLARGEMENT, PARALLEL TO THE y AXIS, WITH SCALE FACTOR $\frac{3}{2}\sqrt{2}$

c) A DIFFERENT APPROACH

$y = 3\sqrt{2x} = \frac{3}{2} \times 2 \times \sqrt{2x} = 2 \times \sqrt{\frac{9}{4}} \times \sqrt{2x} = 2\sqrt{\frac{9}{4} \times 2x}$

$= 2\sqrt{\frac{9}{2}x}$

∴ AN ENLARGEMENT PARALLEL TO THE x AXIS, BY A SCALE FACTOR OF $\frac{2}{9}$

Question 64 (****)

Three geometric transformations are defined as follows.

- R is a reflection in the y axis.
- S is a stretch parallel to the x axis, by scale factor of $\frac{1}{2}$.
- T is a translation by 3 units in the negative x direction .

The transformation T, followed by S, followed by R is applied to the graph of the curve with equation $y = \sqrt{x}$.

a) Determine a simplified equation for the transformed graph.

b) Determine a sequence of transformations in terms of R, S and T **only**, which map the graph of $y = \sqrt{x}$ onto the graph of $y = \sqrt{4x+3}$.

$\boxed{y = \sqrt{3-2x}}$, $\boxed{T - S - S}$

(a) $\sqrt{x} \xmapsto{T} \sqrt{(x+3)} = \sqrt{x+3} \xmapsto{S} \sqrt{(2x)+3} = \sqrt{2x+3} \xmapsto{R} \sqrt{2(-x)+3} = \sqrt{3-2x}$

$\therefore y = \sqrt{3-2x}$

(b) $\sqrt{x} \xmapsto{T} \sqrt{(x+3)} = \sqrt{x+3} \xmapsto{S} \sqrt{(2x)+3} = \sqrt{2x+3} \xmapsto{S} \sqrt{2(2x)+3} = \sqrt{4x+3}$

$\therefore$ THE SEQUENCE IS T, S, S

Question 65 (****)

$$f(x) = \frac{x-2}{x-3}, \; x \in \mathbb{R}, \; x \neq 3.$$

a) Express $f(x)$ in the form

$$f(x) = a + \frac{1}{x+b},$$

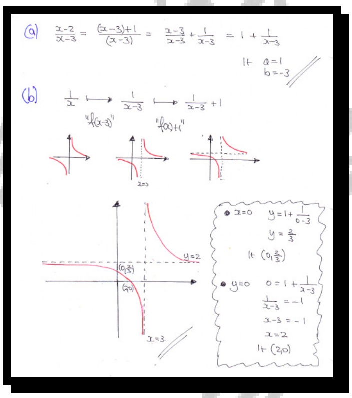
where a and b are integers.

b) By considering a series of transformations which map the graph of $\frac{1}{x}$ onto the graph of $f(x)$, sketch the graph of $f(x)$.

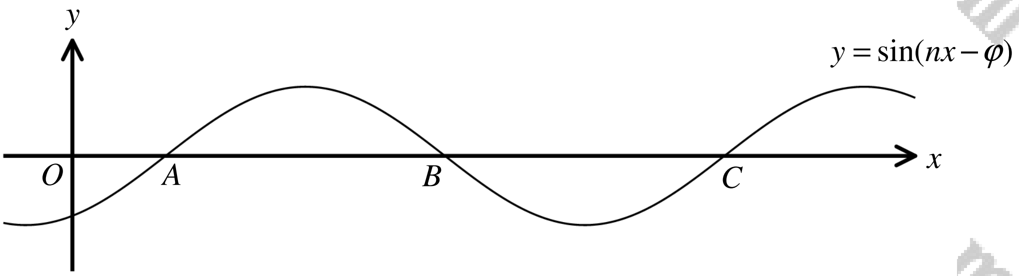
The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

$\boxed{a=1}$, $\boxed{b=-3}$



Question 66 (****)



The figure below shows part of the graph of

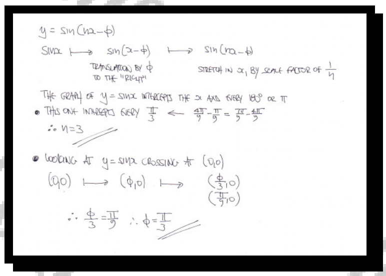
$$y = \sin(nx - \varphi),$$

where n and φ are positive constants, with $0 \le \varphi < \frac{\pi}{2}$.

The graph crosses the x axis at the points A, B and C with respective coordinates $\left(\frac{\pi}{9}, 0\right)$, $\left(\frac{4\pi}{9}, 0\right)$ and $\left(\frac{7\pi}{9}, 0\right)$.

Determine the value of n and the value of φ.

$\boxed{n = 3}$, $\boxed{\varphi = \frac{\pi}{3}}$



Question 67 (****)

$$f(x) = x^2 - 2x - 8, \ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $f(x) = (x+a)^2 + b$, where a and b are integers.

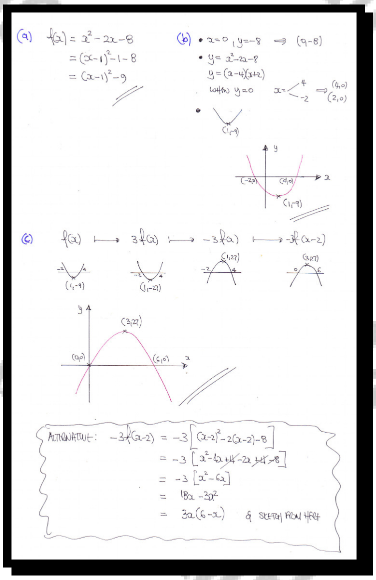
b) Sketch the graph of $f(x)$.

a) By considering a series of three geometrical transformations, or otherwise, sketch the graph of $y = -3f(x-2)$.

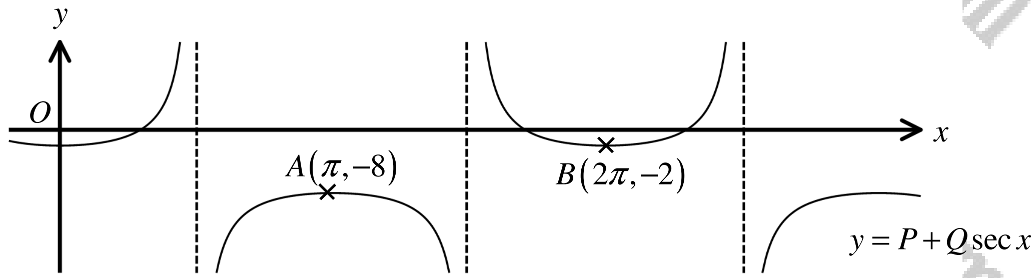
Both sketches must include the coordinates of …

- … all the points where the curves meets the coordinate axes.
- … the minimum or maximum points of the curves.

$\boxed{a = -1}$, $\boxed{b = -9}$



Question 68 (****)



The figure above shows part of the graph of

$$y = P + Q\sec x,$$

where P and Q are non zero constants.

The graph has turning points at $A(\pi,-8)$ and $B(2\pi,-2)$.

By using transformation considerations **only**, find the value of P and the value of Q.

$\boxed{}$, $\boxed{P=-5}$, $\boxed{Q=3}$

- SECx exist between −1 & 1, ie A "GAP" OF 2
- THIS GRAPH HAS A GAP OF 6 (FROM −2 to −8), SO IT MUST HAVE BEEN STRETCHED BY FACTOR OF 3 IN THE y DIRECTION
- BUT THIS MEANS IT SHOULD HAVE A GAP BETWEEN −3 & 3 BUT IT HAS A GAP BETWEEN −8 & −2, SO IT MUST HAVE BEEN TRANSLATED BY 5 UNITS DOWN

$\therefore y = -5 + 3\sec x$

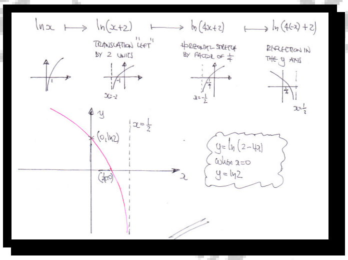
Question 69 (****)

By considering a sequence of three transformations, or otherwise, sketch the graph of

$$y = \ln(2-4x),\ x \in \mathbb{R},\ x \le \frac{1}{2}.$$

The sketch must include the coordinates of any points where the graph meets the coordinate axes and the equations of any asymptotes.

$\boxed{}$, $\boxed{\text{graph}}$



Question 70 (****)

A curve has equation

$$y = 2^{3x}, \ x \in \mathbb{R}.$$

a) Describe the single geometric transformation which map the graph of $y = 2^{3x}$ onto the graph of $y = 2^{3x+4}$.

b) Describe a **different** geometric transformation which map the graph of $y = 2^{3x}$ onto the graph of $y = 2^{3x+4}$.

$\boxed{}$, $\boxed{\text{appropriate description}}$

a) MANIPULATING THE EXPONENTIAL AS FOLLOWS

$2^{3x+4} = 2^{3\left(x+\frac{4}{3}\right)}$ i.e. "$f\left(x+\frac{4}{3}\right)$"

$\therefore$ TRANSLATION BY THE VECTOR $\begin{pmatrix} -\frac{4}{3} \\ 0 \end{pmatrix}$

b) CREATING AN ENLARGEMENT AS FOLLOWS

$2^{3x+4} = 2^{3x} \times 2^{4} = 16 \times 2^{3x}$ i.e. "$16f(x)$"

$\therefore$ STRETCH PARALLEL TO THE y AXIS BY SCALE FACTOR 16

Question 71 (****+)

A curve has equation

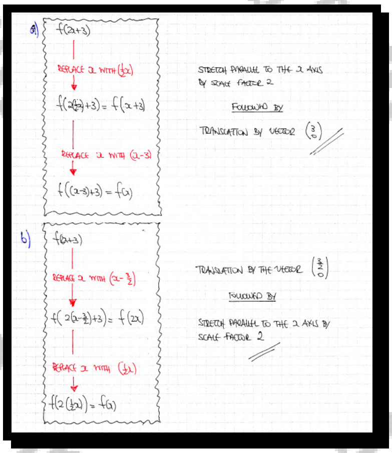
$$y = f(2x+3).$$

a) Describe the two geometric transformations which map the graph of $y = f(2x+3)$ onto the graph of $y = f(x)$.

b) Describe a **different** set of two geometric transformations which map the graph of $y = f(2x+3)$ onto the graph of $y = f(x)$.

The description must be formal, clearly indicating the order in which the two transformations take place.

$\boxed{}$, $\boxed{\text{appropriate description}}$



a) $f(2x+3)$

REPLACE x WITH $\left(\frac{1}{2}x\right)$

$f\left(2\left(\frac{1}{2}x\right)+3\right) = f(x+3)$

REPLACE x WITH $(x-3)$

$f((x-3)+3) = f(x)$

STRETCH PARALLEL TO THE x AXIS BY SCALE FACTOR 2

FOLLOWED BY

TRANSLATION BY VECTOR $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

b) $f(2x+3)$

REPLACE x WITH $\left(x-\frac{3}{2}\right)$

$f\left(2\left(x-\frac{3}{2}\right)+3\right) = f(2x)$

REPLACE x WITH $\left(\frac{1}{2}x\right)$

$f\left(2\left(\frac{1}{2}x\right)\right) = f(x)$

TRANSLATION BY THE VECTOR $\begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}$

FOLLOWED BY

STRETCH PARALLEL TO THE x AXIS BY SCALE FACTOR 2

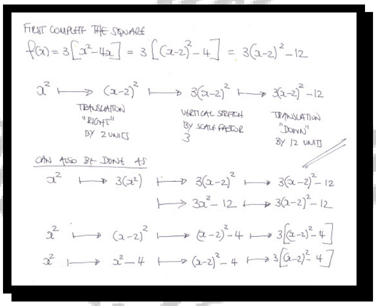
Question 72 (****+)

$$f(x) = 3x^2 - 12x, \ x \in \mathbb{R}.$$

The curve with equation $y = x^2$ is mapped onto the curve with equation $y = f(x)$ through a sequence of three geometric transformations.

Describe these three transformations geometrically, clearly indicating the order in which they occur.

translation "right" by 2 units,
vertical stretch by scale factor 3,
translation "down" by 12 units


FIRST COMPLETE THE SQUARE

$f(x) = 3[x^2 - 4x] = 3[(x-2)^2 - 4] = 3(x-2)^2 - 12$

$x^2 \longmapsto (x-2)^2 \longmapsto 3(x-2)^2 \longmapsto 3(x-2)^2 - 12$

TRANSLATION "RIGHT" BY 2 UNITS; VERTICAL STRETCH BY SCALE FACTOR 3; TRANSLATION "DOWN" BY 12 UNITS

CAN ALSO BE DONE AS

$x^2 \longmapsto 3(x^2) \longmapsto 3(x-2)^2 \longmapsto 3(x-2)^2 - 12$

$\longmapsto 3x^2 - 12 \longmapsto 3(x-2)^2 - 12$

$x^2 \longmapsto (x-2)^2 \longmapsto (x-2)^2 - 4 \longmapsto 3[(x-2)^2 - 4]$

$x^2 \longmapsto x^2 - 4 \longmapsto (x-2)^2 - 4 \longmapsto 3[(x-2)^2 - 4]$

Question 73 (****+)

$$f(x) \equiv \sqrt{8x^3 - 15}, \ x \geq \frac{\sqrt[3]{15}}{8}.$$

a) Describe the geometric transformation which maps the graph of $f(x)$ onto the graph of $\sqrt{x^3 - 15}$.

The graph of $g(x)$ is a translation of $f(x)$ by the vector $\begin{bmatrix} 1 \\ 15 \end{bmatrix}$.

b) Evaluate $g(3)$.

$\boxed{}$, $\boxed{\text{horizontal stretch by scale factor 2}}$, $\boxed{g(3) = 22}$

a) $f(x) = \sqrt{8x^3 - 15}$

$f(\frac{1}{2}x) = \sqrt{8(\frac{1}{2}x)^3 - 15} = \sqrt{8(\frac{1}{8}x^3) - 15} = \sqrt{x^3 - 15}$

$\therefore$ HORIZONTAL STRETCH BY SCALE FACTOR 2

b) TRANSLATION BY THE VECTOR $\binom{1}{15}$ IS $f(x-1) + 15$

HENCE

$g(x) = f(x-1) + 15$

$g(3) = f(3) + 15 = \sqrt{8 \times 2^3 - 15} + 15 = \sqrt{49} + 15 = 22$

Question 74 (****+)

The curve with equation $y = f(x)$ is translated by the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, followed by a horizontal stretch of scale factor $\frac{1}{2}$, to give the graph of the curve with equation

$$y = 8x^2 - 22x + 10.$$

Show clearly that

$$f(x) = 2x^2 - 7x + 1.$$

$\boxed{}$, $\boxed{\text{proof}}$

$$f(x) \longmapsto \cdots\cdots \longmapsto \boxed{8x^2 - 22x + 10}$$

TRANSLATION BY $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$: $x \mapsto (x-1)$

STRETCH, HORIZONTALLY, BY S.F. $\frac{1}{2}$: $x \mapsto 2x$

REVERSE THE TRANSFORMATIONS

- $x \mapsto \frac{1}{2}x$

$$8\left(\tfrac{1}{2}x\right)^2 - 22\left(\tfrac{1}{2}x\right) + 10 = 8\left(\tfrac{1}{4}x^2\right) - 11x + 10 = 2x^2 - 11x + 10$$

- $x \mapsto (x+1)$

$$2(x+1)^2 - 11(x+1) + 10 = 2(x^2 + 2x + 1) - 11x - 11 + 10 = 2x^2 + 4x + 2 - 11x - 11 + 10 = 2x^2 - 7x + 1$$

Question 75 (****+)

The functions f and g are defined for all x and are defined by

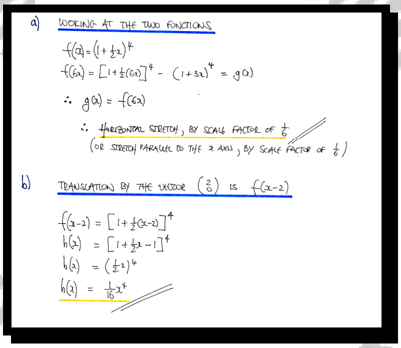
$$f(x) = \left(1 + \tfrac{1}{2}x\right)^4 \quad \text{and} \quad g(x) = (1 + 3x)^4 .$$

a) Describe the geometrical transformation which maps the graph of $f(x)$ onto the graph of $g(x)$.

The graph of $f(x)$ is translated by the vector $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ to give the graph of $h(x)$.

b) Find an expression for $h(x)$ in its simplest form.

$\boxed{\text{horizontal stretch by scale factor of } \frac{1}{6}}$, $\boxed{h(x) = \frac{1}{16}x^4}$



a) LOOKING AT THE TWO FUNCTIONS

$f(x) = \left(1 + \frac{1}{2}x\right)^4$

$f(6x) = \left[1 + \frac{1}{2}(6x)\right]^4 = (1 + 3x)^4 = g(x)$

$\therefore \ g(x) = f(6x)$

$\therefore$ HORIZONTAL STRETCH, BY SCALE FACTOR OF $\frac{1}{6}$

(OR STRETCH PARALLEL TO THE x AXIS, BY SCALE FACTOR OF $\frac{1}{6}$)

b) TRANSLATION BY THE VECTOR $\binom{2}{0}$ IS $f(x-2)$

$f(x-2) = \left[1 + \frac{1}{2}(x-2)\right]^4$

$h(x) = \left[1 + \frac{1}{2}x - 1\right]^4$

$h(x) = \left(\frac{1}{2}x\right)^4$

$h(x) = \frac{1}{16}x^4$

Question 76 (****+)

The curve C_1, with equation $y = f(x)$, undergoes 3 transformations in the order given below.

1. A translation of 2 units, in the negative x direction.

2. An enlargement parallel to the x axis, with scale factor 2.

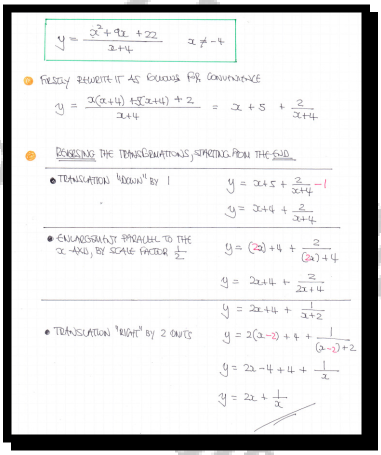
3. A translation of 1 unit in the positive y direction.

The resulting curve C_2 has equation

$$y = \frac{x^2 + 9x + 22}{x+4}, \quad x \in \mathbb{R}, \; x \neq -4 .$$

Determine in its simplest form an equation for C_1.

$\boxed{}$, $\boxed{y = 2x + \frac{1}{x}}$



$$y = \frac{x^2+9x+22}{x+4} \qquad x \neq -4$$

FIRSTLY REWRITE IT AS FOLLOWS FOR CONVENIENCE

$$y = \frac{x(x+4)+5(x+4)+2}{x+4} = x+5+\frac{2}{x+4}$$

REVERSING THE TRANSFORMATIONS, STARTING FROM THE END

- TRANSLATION "DOWN" BY 1

$$y = x+5+\frac{2}{x+4} - 1$$
$$y = x+4+\frac{2}{x+4}$$

- ENLARGEMENT PARALLEL TO THE x-AXIS, BY SCALE FACTOR $\frac{1}{2}$

$$y = (2x)+4+\frac{2}{(2x)+4}$$
$$y = 2x+4+\frac{2}{2x+4}$$
$$y = 2x+4+\frac{1}{x+2}$$

- TRANSLATION "RIGHT" BY 2 UNITS

$$y = 2(x-2)+4+\frac{1}{(x-2)+2}$$
$$y = 2x-4+4+\frac{1}{x}$$
$$y = 2x+\frac{1}{x}$$

Question 77 (****+)

$$f(x) = 2\log_4 x, \ x \in \mathbb{R}, \ x > 0.$$

$$g(x) = 1 + 2\log_4 x, \ x \in \mathbb{R}, \ x > 0.$$

a) State the translation vector that maps the graph of $f(x)$ onto the graph of $g(x)$.

It is given that the graph of $g(x)$ can also be obtained from the graph of $f(x)$ by a single transformation, but this transformation is **not** a translation.

b) Describe this transformation geometrically.

$\boxed{}$, $\boxed{\begin{pmatrix}0\\1\end{pmatrix}}$, $\boxed{\text{horizontal stretch by scale factor } \frac{1}{2}}$

a) $f(x) = 2\log_4 x$

$g(x) = 1 + 2\log_4 x = 1 + f(x) = f(x) + 1$

$\therefore$ TRANSLATION BY $\begin{pmatrix}0\\1\end{pmatrix}$

b) $g(x) = 1 + 2\log_4 x$

$g(x) = \log_4 4 + \log_4 x^2$

$g(x) = \log_4 (4x^2)$

$g(x) = \log_4 [(2x)^2]$

$g(x) = 2\log_4 (2x)$

$g(x) = f(2x)$

OR

$g(x) = \log_4 4 + 2\log_4 x$

$g(x) = \log_4 2^2 + 2\log_4 x$

$g(x) = 2\log_4 2 + 2\log_4 x$

$g(x) = 2[\log_4 2 + \log_4 x]$

$g(x) = 2\log(2x)$

$\therefore$ HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{2}$

(STRETCH PARALLEL TO THE x AXIS, BY S.F $\frac{1}{2}$)

Question 78 (****+)

$$y = \sqrt{x^2+16}\,, \quad x \in \mathbb{R}\,.$$

a) Describe the geometric transformation which maps the graph of $y=\sqrt{x^2+16}$ onto the graph of $y = 4\sqrt{x^2+1}$.

When the graph of $y=\sqrt{x^2+16}$ is translated by the vector $\begin{bmatrix} k \\ 0 \end{bmatrix}$, where k is a non zero constant, the image of the transformed graph passes through the point $(6,5)$.

b) Determine the possible values of k.

$\boxed{}$, $\boxed{\text{horizontal stretch by scale factor } \frac{1}{4}}$, $\boxed{k=3 \ \cup \ k=9}$

$\boxed{y = \sqrt{x^2+16} \qquad x \in \mathbb{R}}$

a) LET $y = f(x)$

THEN $y = f(4x) = \sqrt{(4x)^2+16} = \sqrt{16x^2+16} = \sqrt{16(x^2+1)}$

$= \sqrt{16}\sqrt{x^2+1} = 4\sqrt{x^2+1}$

$\therefore$ HORIZONTAL STRETCH, BY SCALE FACTOR OF $\frac{1}{4}$

b) LET THE TRANSLATION BY THE VECTOR $\binom{k}{0}$ BE $y = f(x-k)$

(NOTE THAT IF k IN NEGATIVE THEN THE TRANSLATION WILL BE TO THE "LEFT")

$y = f(x-k) = \sqrt{(x-k)^2+16}$

NOW THIS CURVE PASSES THROUGH $(6,5)$

$\Rightarrow 5 = \sqrt{(6-k)^2+16}$

$\Rightarrow 25 = (6-k)^2+16$

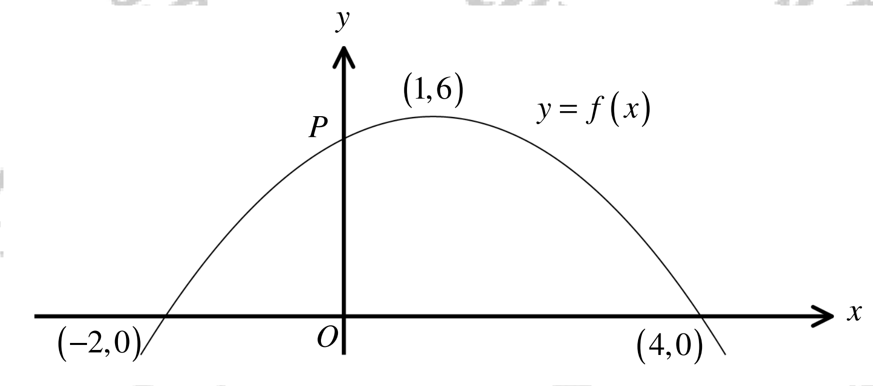
$\Rightarrow 9 = (6-k)^2$

$\Rightarrow 9 = (k-6)^2$

$\Rightarrow k-6 = \begin{cases} 3 \\ -3 \end{cases}$

$\Rightarrow k = \begin{cases} 9 \\ 3 \end{cases}$

Question 79 (****+)



The figure above shows the graph of the **quadratic** curve with equation $y = f(x)$ that meets the x axis at the points $(-2,0)$ and $(-4,0)$, and the y axis at the point P. The curve has a maximum at $(1,6)$.

a) Sketch the graph of

$$y = -3f(x+2),$$

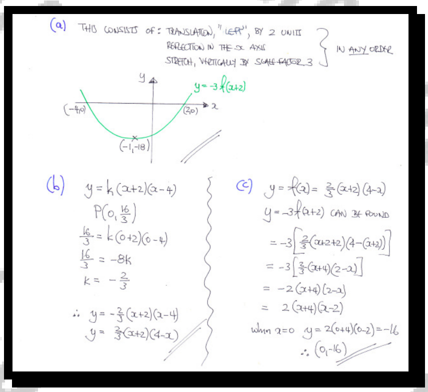
showing clearly the coordinates of any points where the graph meets the x axis, and the new coordinates of the maximum point of the curve.

The point P has coordinates $\left(0, \frac{16}{3}\right)$.

b) Find an equation for $y = f(x)$.

c) Hence, or otherwise, determine the y intercept of $y = -3f(x+2)$.

, $\boxed{y = \frac{2}{3}(x+2)(4-x)}$, $\boxed{P'(0,-16)}$



Question 80 (****+)

A quadratic has equation

$$y = A + Bx - x^2, \ x \in \mathbb{R}.$$

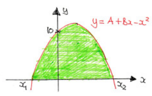
The image of the curve, when reflected in the y axis, is identical to the image of the curve when translated by the vector $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$.

Given further that the curve meets the y axis at $(0,10)$, determine the area of the finite region bounded by the curve and the x axis.

$\boxed{\text{area} = \dfrac{343}{6}}$

- STARTING WITH A SKETCH
- BY INSPECTION $A=10$
- WRITING IN f NOTATION

$f(x) = 10 + Bx - x^2$



- "WHEN THE CURVE IS REFLECTED IN THE y AXIS IT IS THE SAME AS TRANSLATING THE CURVE BY 3 UNITS TO THE LEFT"

$\Rightarrow f(-x) = f(x+3)$

$\Rightarrow \cancel{10} + B(-x) - (-x)^2 = \cancel{10} + B(x+3) - (x+3)^2$

$\Rightarrow -Bx \cancel{- x^2} = Bx + 3B \cancel{- x^2} - 6x - 9$

$\Rightarrow 0 = 2Bx - 6x + 3B - 9$

$\Rightarrow 0 = 2(B-3)x + 3(B-3)$

$\therefore B = 3$

- HENCE WE HAVE $f(x) = 10 + 3x - x^2$

$-f(x) = x^2 - 3x - 10$

$-f(x) = (x+2)(x-5)$

$f(x) = (x+2)(5-x)$

- FINALLY THE AREA CAN BE FOUND

$\text{AREA} = \int_{-2}^{5} 10 + 3x - x^2 \, dx = \left[10x + \frac{3}{2}x^2 - \frac{1}{3}x^3\right]_{-2}^{5}$

$= \left(50 + \frac{75}{2} - \frac{125}{3}\right) - \left(-20 + 6 + \frac{8}{3}\right) = \frac{343}{6}$

Question 81 (****+)

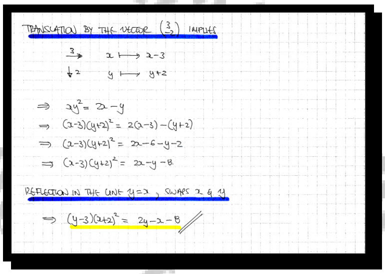
A curve has equation

$$xy^2 = 2x - y \ .$$

This curve is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by reflection about the line with equation $y = x$.

Find an equation of this curve after these transformations.

$\boxed{}$, $\boxed{(y-3)(x+2)^2 = 2y - x - 8}$



TRANSLATION BY THE VECTOR $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ IMPLIES

$\xrightarrow{3}$ $x \longmapsto x-3$

$\downarrow 2$ $y \longmapsto y+2$

$\Rightarrow xy^2 = 2x - y$

$\Rightarrow (x-3)(y+2)^2 = 2(x-3) - (y+2)$

$\Rightarrow (x-3)(y+2)^2 = 2x - 6 - y - 2$

$\Rightarrow (x-3)(y+2)^2 = 2x - y - 8$

REFLECTION IN THE LINE $y=x$, SWAPS x & y

$\Rightarrow (y-3)(x+2)^2 = 2y - x - 8$

Question 82 (****+)

The curve C_1, with equation $y = f(x)$, undergoes 3 transformations in the order given below.

1. A translation of 2 units, in the positive x direction.

2. A reflection about the y axis.

3. A translation of 1 unit in the positive y direction.

The resulting curve C_2 has equation

$$y = \frac{x^2+3x+3}{x^2+4x+5}, \quad x \in \mathbb{R}.$$

Determine an equation for C_1, giving the answer in the form $y = g(x)$, where $g(x)$ is a single simplified fraction.

$\boxed{}$, $\boxed{y = \dfrac{x}{1+x^2}}$

REVERSE THE ORDER AS WELL AS THE TRANSFORMATIONS THEMSELVES

- "DOWN TRANSLATION BY 1 UNIT", IE "$f(x) - 1$"

$$y = \frac{x^2+3x+3}{x^2+4x+5} \longmapsto y = \frac{x^2+3x+3}{x^2+4x+5} - 1$$

- "REFLECTION ABOUT THE y AXIS", AS IT INVERSES ITSELF, IE $f(-x)$

$$y = \frac{x^2+3x+3}{x^2+4x+5} - 1 \longmapsto y = \frac{(-x)^2+3(-x)+3}{(-x)^2+4(-x)+5} - 1$$

$$y = \frac{x^2-3x+3}{x^2-4x+5} - 1$$

- "TRANSLATION LEFT, BY 2 UNITS", IE $f(x+2)$

$$y = \frac{x^2-3x+3}{x^2-4x+5} - 1 \longmapsto y = \frac{(x+2)^2-3(x+2)+3}{(x+2)^2-4(x+2)+5} - 1$$

FINALLY TIDYING UP

$$y = \frac{x^2+4x+4-3x-6+3}{x^2+4x+4-4x-8+5} - 1 = \frac{x^2+x+1}{x^2+1} - 1$$

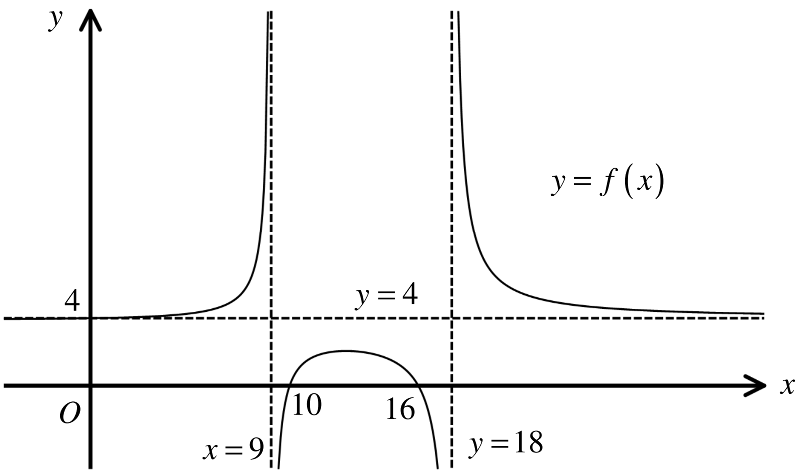
$$= \frac{x^2+x+1-(x^2+1)}{x^2+1}$$

$$= \frac{x^2+x+1-x^2-1}{x^2+1}$$

$$= \frac{x}{x^2+1}$$

$$\therefore \ y = \frac{x}{x^2+1}$$

Question 83 (****+)

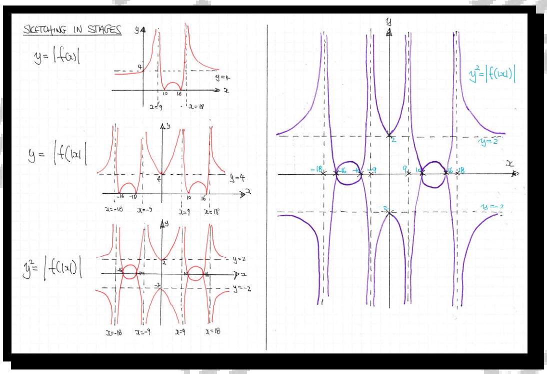


The figure above shows the curve with equation $y = f(x)$.

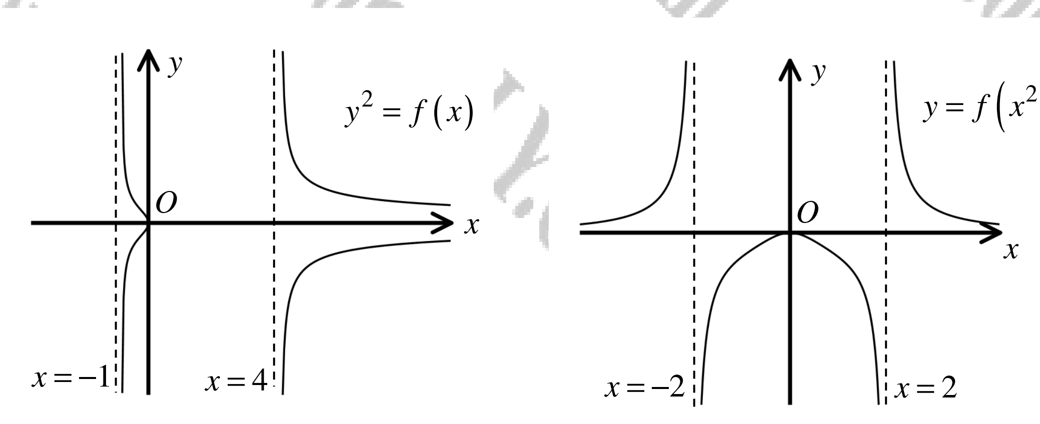
The equations of the three asymptotes to the curve, and the three intercepts of the curve with the coordinate axes are marked in the figure.

Sketch a detailed graph of $y^2 = |f(|x|)|$.

, graph



Question 84 (*****)



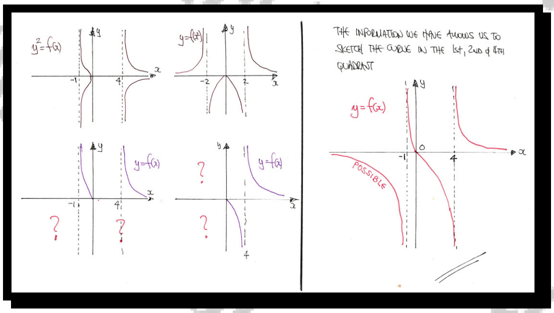
The figures above show two transformations of a function with equation $y = f(x)$, the graph of $y^2 = f(x)$ in the first set of axes, and the graph of $y = f(x^2)$ in the second set of axes.

The equations of the vertical asymptotes for each graph are included in the figures.

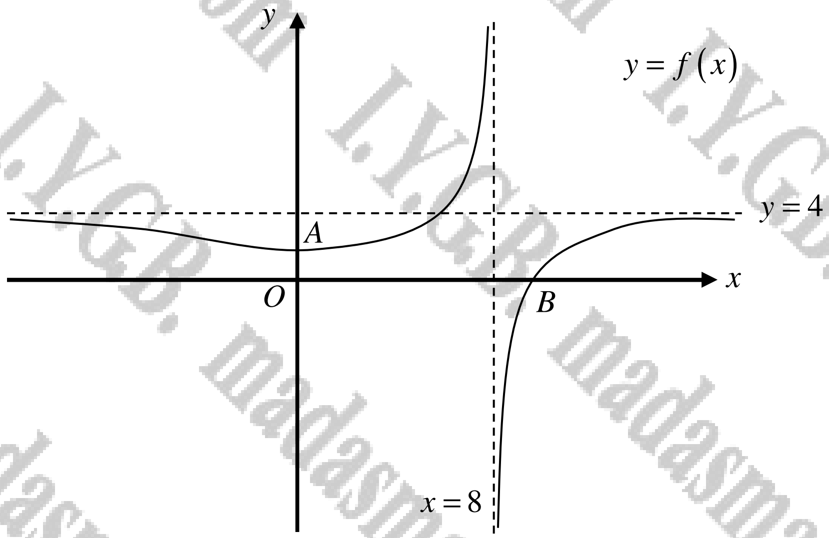
The x axis is a horizontal asymptote for both graphs.

Sketch a possible graph of $y = f(x)$, showing all relevant details.

, graph



Question 85 (*****)



The figure above shows the curve with equation $y = f(x)$.

The curve has a local minimum at $A(0,2)$ and crosses the x axis at $B(9,0)$.

The straight lines with equations $y = 4$ and $x = 8$ are asymptotes to the curve.

Sketch on separate set of axes the graph of …

a) … $y = \dfrac{1}{f(x)}$.

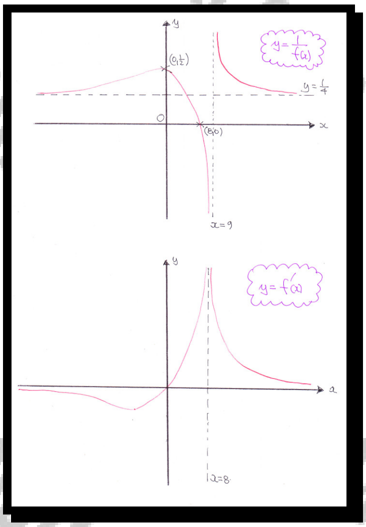
b) … $y = f'(x)$.

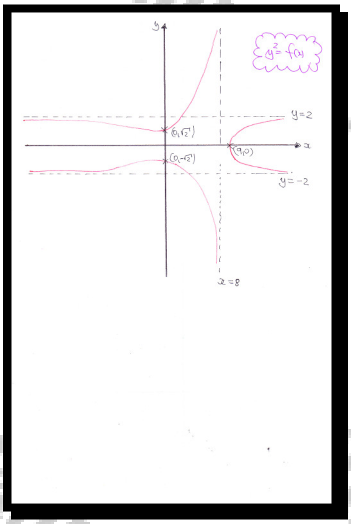
c) … $y^2 = f(x)$.

In each case, give if possible, the equations of any asymptotes, the coordinates of any stationary points and the coordinates of any points where the curve meets the coordinate axes.

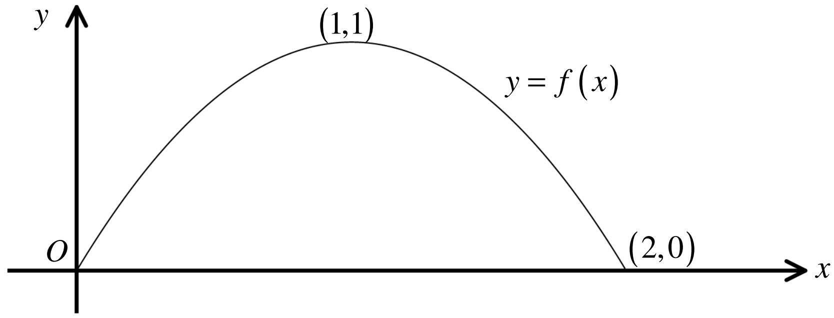
$\boxed{}$, $\boxed{\text{graph}}$

[solution overleaf]


y
$y = \frac{1}{f(x)}$
$(0, \frac{1}{2})$
$y = \frac{1}{4}$
O
x
(8, 0)
$x = 9$
y
$y = f'(x)$
x
$x = 8$


y
$y^2 = f(x)$
$y = 2$
$(0, \sqrt{2})$
x
(9, 0)
$(0, -\sqrt{2})$
$y = -2$
$x = 8$

Question 86 (*****)



The figure above shows **part** of the graph of the function $y = f(x)$.

The graph meets the x axis at $(2,0)$ and at the origin, and has a maximum at $(1,1)$.

It is given that $f(x)$ is defined for $0 \le x \le 4$ and $f(4-x) = -f(x)$.

Sketch on separate diagrams the graph of …

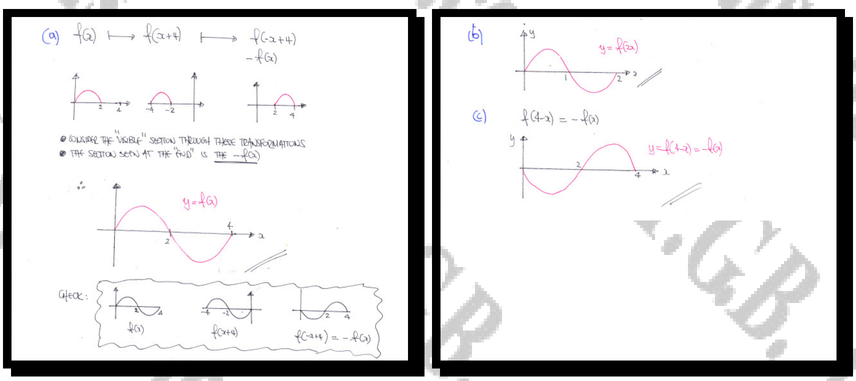
a) … $y = f(x)$, $0 \le x \le 4$.

b) … $y = f(2x)$, $0 \le x \le 2$.

c) … $y = f(4-x)$, $0 \le x \le 4$.

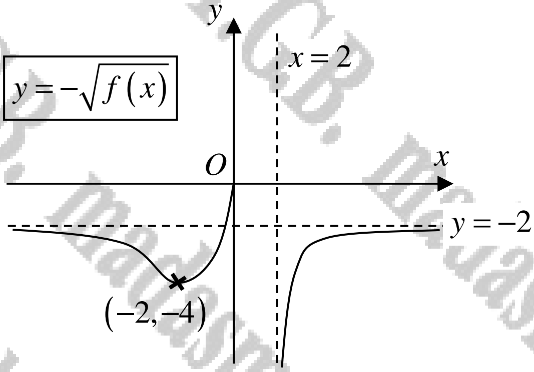
The sketches must include the coordinates of any points where each of the graphs meets the coordinate axes and the coordinates of any minimum or maximum points.

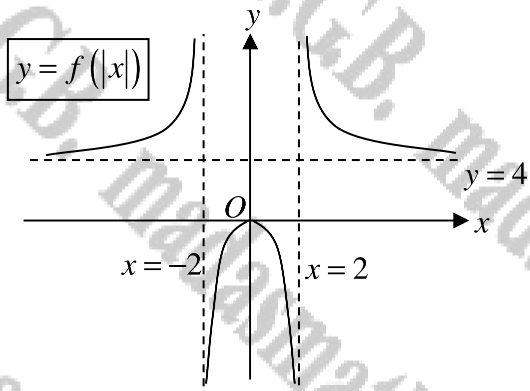
$\boxed{}$, $\boxed{\text{graph}}$



Question 87 (*****)

The graph of $y = -\sqrt{f(x)}$ and the graph of $y = f(|x|)$ are shown below, in two separate set of axes.



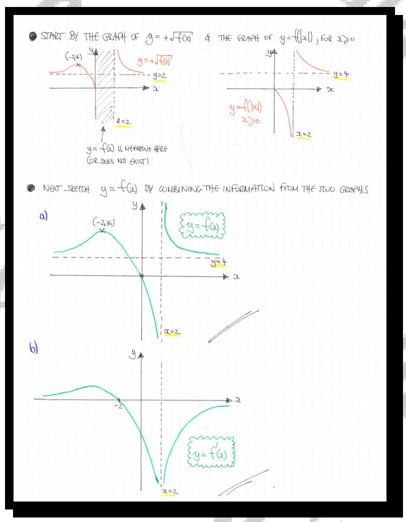


Sketch on separate set of axes a detailed graph of …

a) … $y = f(x)$.

b) … $y = f'(x)$.

$\boxed{}$, $\boxed{\text{graph}}$



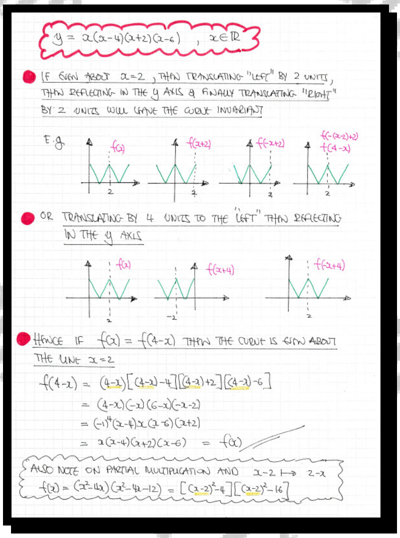
Question 88 (*****)

A quartic curve C has the following equation.

$$y = x(x-4)(x+2)(x-6), \quad x \in \mathbb{R}.$$

By considering suitable transformations, show that C is even about the straight line with equation $x = 2$.

$\boxed{}$, $\boxed{\text{proof}}$



$y = x(x-4)(x+2)(x-6)$, $x \in \mathbb{R}$

- IF EVEN ABOUT $x=2$, THEN TRANSLATING "LEFT" BY 2 UNITS, THEN REFLECTING IN THE y AXIS & FINALLY TRANSLATING "RIGHT" BY 2 UNITS WILL LEAVE THE CURVE INVARIANT

E.g.

- OR TRANSLATING BY 4 UNITS TO THE "LEFT" THEN REFLECTING IN THE y AXIS

- HENCE IF $f(x) = f(4-x)$ THEN THE CURVE IS EVEN ABOUT THE LINE $x=2$

$$\begin{aligned} f(4-x) &= (4-x)[(4-x)-4][(4-x)+2][(4-x)-6] \\ &= (4-x)(-x)(6-x)(-x-2) \\ &= (-1)^4(x-4)x(x-6)(x+2) \\ &= x(x-4)(x+2)(x-6) = f(x) \end{aligned}$$

ALSO NOTE ON PARTIAL MULTIPLICATION AND $x-2 \mapsto 2-x$

$f(x) = (x^2-4x)(x^2-4x-12) = [(x-2)^2-4][(x-2)^2-16]$

Question 89 (*****)

A curve is defined in the largest real domain by the equation

$$f(x) = -x^2 + 8x - 12.$$

Sketch on separate set of axes detailed graph of ...

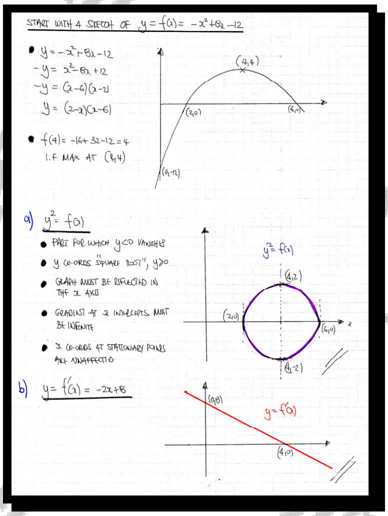
a) ... $y^2 = f(x)$.

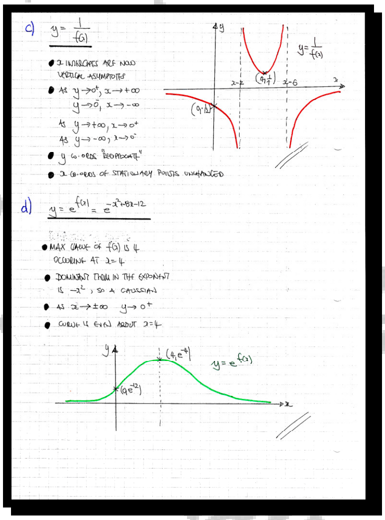
b) ... $y = f'(x)$.

c) ... $y = \dfrac{1}{f(x)}$.

d) ... $y^2 = e^{f(x)}$.

$\boxed{}$, $\boxed{\text{graph}}$





Question 90 (*****)

A cubic curve with equation

$$y = x^3 - 3x^2 - 9x + 3, \quad x \in \mathbb{R},$$

is odd about some point P.

Find the coordinates of P and use transformation arguments to justify the assertion that the curve is odd about P.

$\boxed{}$, $\boxed{P(1,-8)}$

$$y = x^3 - 3x^2 - 9x + 3, \quad x \in \mathbb{R}$$

USING THE FACT THAT ALL CUBICS HAVE ROTATIONAL SYMMETRY ABOUT THEIR POINT OF INFLEXION WE PROCEED AS FOLLOWS

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

$$\frac{d^2y}{dx^2} = 6x - 6$$

BY INSPECTION THE CURVE HAS A POINT OF INFLEXION AT $x = 1$

$$\therefore y = 1 - \cancel{3} - 9 + \cancel{3} = -8 \qquad \therefore P(1,-8)$$

TO JUSTIFY THE ODDITY ABOUT P, TRANSLATE THE CURVE TO THE ORIGIN & INVESTIGATE ODDITY ABOUT O

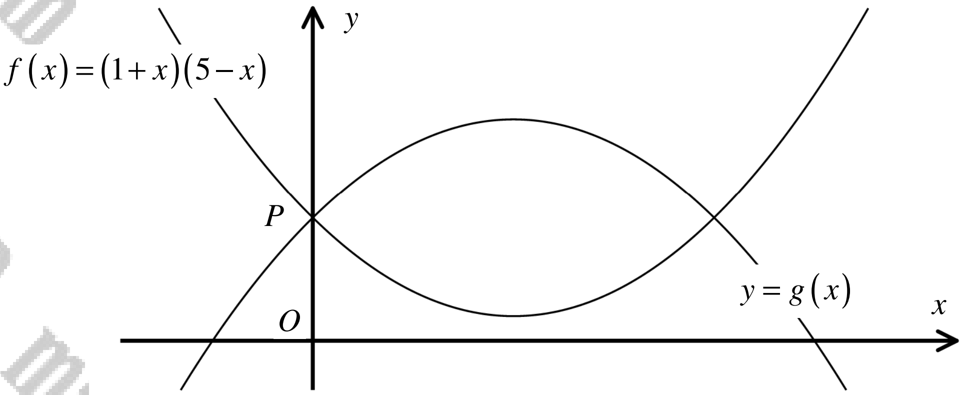
- "UP BY 8" $\implies y = (x^3 - 3x^2 - 9x + 3) + 8$
  $\implies y = x^3 - 3x^2 - 9x + 11$
- "LEFT BY 1" $\implies y = (x+1)^3 - 3(x+1)^2 - 9(x+1) + 11$
  $\implies y = x^3 + \cancel{3x^2} + 3x + \cancel{1} - \cancel{3x^2} - 6x - \cancel{3} - 9x - \cancel{9} + \cancel{11}$
  $\implies y = x^3 - 12x$

EVIDENTLY THIS IS ODD, AS

$$f(x) = x^3 - 12x$$
$$f(-x) = (-x)^3 - 12(-x)$$
$$= -x^3 + 12x$$
$$= -f(x)$$

CONSEQUENTLY "OUR CURVE" IS ODD ABOUT $P(1,-8)$

Question 91 (*****)



The figure above shows the curve with equation

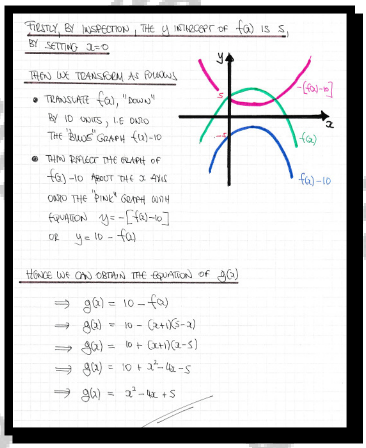
$$f(x)=(1+x)(5-x),$$

and the curve with equation $y=g(x)$.

The curve with equation $y=g(x)$ can be obtained by **two** transformations of the curve with equation $y=f(x)$. It is further given that these two transformations **do not** include any stretches, shears or rotations.

If both curves meet the y axis at the same point P, find an equation for $y=g(x)$.

$\boxed{g(x)=x^2-4x+5}$



FIRSTLY, BY INSPECTION, THE y INTERCEPT OF $f(x)$ IS 5, BY SETTING $x=0$

THEN WE TRANSFORM AS FOLLOWS

- TRANSLATE $f(x)$, "DOWN" BY 10 UNITS, I.E ONTO THE "BLUE" GRAPH $f(x)-10$
- THEN REFLECT THE GRAPH OF $f(x)-10$ ABOUT THE x AXIS ONTO THE "PINK" GRAPH WITH EQUATION $y=-[f(x)-10]$ OR $y=10-f(x)$

HENCE WE CAN OBTAIN THE EQUATION OF $g(x)$

$\Rightarrow g(x) = 10 - f(x)$

$\Rightarrow g(x) = 10 - (x+1)(5-x)$

$\Rightarrow g(x) = 10 + (x+1)(x-5)$

$\Rightarrow g(x) = 10 + x^2 - 4x - 5$

$\Rightarrow g(x) = x^2 - 4x + 5$

Question 92 (*****)

A curve has equation

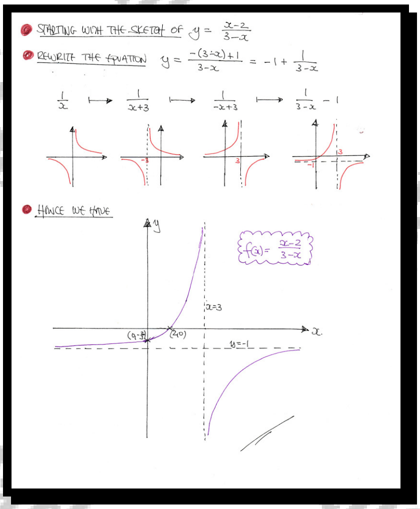
$$f(x) \equiv \frac{x-2}{3-x}, \qquad x \in \mathbb{R},\ x \neq 3.$$

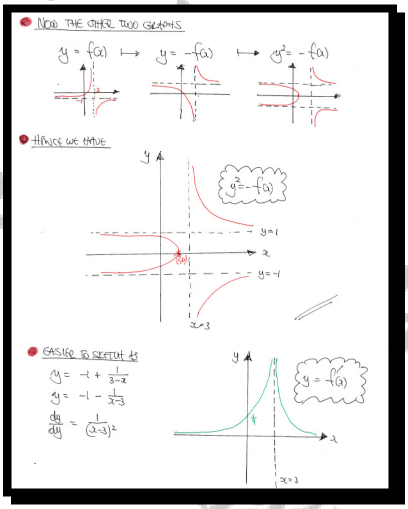
Sketch in separate set of axes the graph of

- $y = f(x)$
- $y^2 = -f(x)$
- $y = f'(x)$

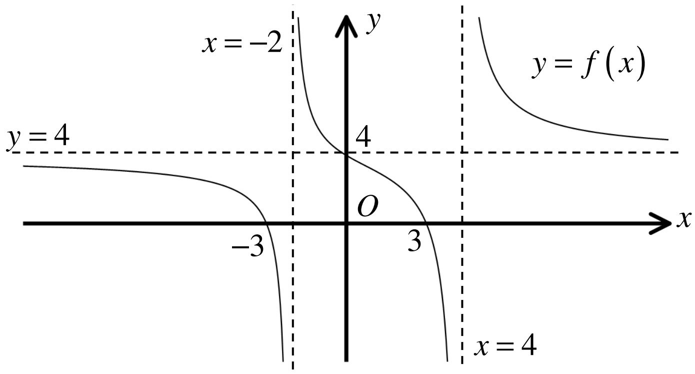
You must show in each case the equations of any asymptotes and the coordinates of any intersections with the coordinate axes.

[ ], [graph]





Question 93 (*****)



A sketch of the curve with equation $y = f(x)$ is shown above.

Important information about the curve, such as the equations of its asymptotes and its intercepts with the coordinate axes are marked in the diagram.

Sketch on separate detailed diagrams the graph of …

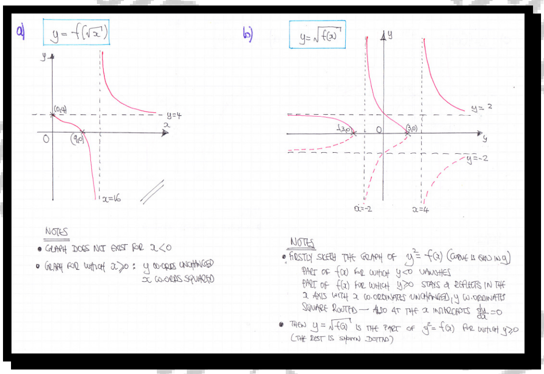
a) … $y = f\left(\sqrt{x}\right)$.

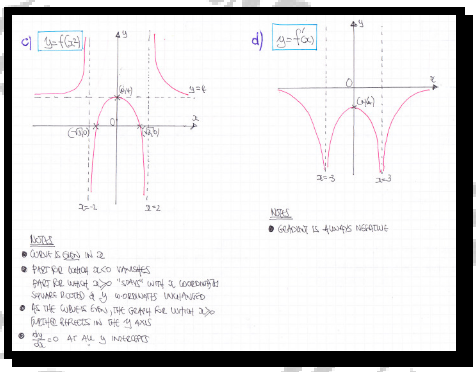
b) … $y = \sqrt{f(x)}$.

c) … $y = f\left(x^2\right)$.

d) … $y = f'(x)$.

$\boxed{}$, $\boxed{\text{graph}}$





Question 94 (*****)

A curve has equation

$$f(x) \equiv \begin{cases} x^2 - 6x + 8, & x \in \mathbb{R},\ 2 \le x \le 4 \\ f(x) + f(4-x) = 0, & x \in \mathbb{R} \\ f(x) - f(4+x) = 0, & x \in \mathbb{R} \end{cases}$$

Sketch a detailed graph of $f(1-2x)$, $x \in \mathbb{R}$, $0 \le x \le 4$.

$\boxed{}$, $\boxed{\text{graph}}$