

MODULUS FUNCTION PRACTICE

MODULUS EQUATIONS

Question 1

Solve the following equations.

a) $|2x+1|=9$

b) $|3-x|=6$

c) $3|4x-3|-1=14$

d) $3-|2x+3|=1$

$$x=4,-5, \quad x=-3,9, \quad x=-\frac{1}{2},2, \quad x=-\frac{1}{2},-\frac{5}{2}$$

Handwritten solutions for Question 1:

a) $|2x+1|=9$
 $2x+1=9 \Rightarrow 2x=8 \Rightarrow x=4$
 $2x+1=-9 \Rightarrow 2x=-10 \Rightarrow x=-5$ both ok

b) $|3-x|=6$
 $3-x=6 \Rightarrow -x=3 \Rightarrow x=-3$
 $3-x=-6 \Rightarrow -x=-9 \Rightarrow x=9$ both work

c) $3|4x-3|-1=14$
 $3|4x-3|=15$
 $|4x-3|=5$
 Hence:
 $4x-3=5 \Rightarrow 4x=8 \Rightarrow x=2$
 $4x-3=-5 \Rightarrow 4x=-2 \Rightarrow x=-\frac{1}{2}$ both work

d) $3-|2x+3|=1$
 $2=|2x+3|$
 $2x+3=2 \Rightarrow 2x=-1 \Rightarrow x=-\frac{1}{2}$
 $2x+3=-2 \Rightarrow 2x=-5 \Rightarrow x=-\frac{5}{2}$ both ok

Question 2

Solve the following equations.

a) $|2x+1|=5$

b) $|4-x|=2$

c) $4|2x-1|-3=9$

d) $3|x|+5=5|x|-1$

$x=2, -3$, $x=2, 6$, $x=2, -1$, $x=3, -3$

Handwritten solutions for the four equations:

a) $|2x+1|=5$
 $2x+1=5$ or $2x+1=-5$
 $2x=4$ or $2x=-6$
 $x=2$ or $x=-3$

b) $|4-x|=2$
 $4-x=2$ or $4-x=-2$
 $-x=-2$ or $-x=-6$
 $x=2$ or $x=6$

c) $4|2x-1|-3=9$
 $4|2x-1|=12$
 $|2x-1|=3$
 $2x-1=3$ or $2x-1=-3$
 $2x=4$ or $2x=-2$
 $x=2$ or $x=-1$

d) $3|x|+5=5|x|-1$
 $6=2|x|$
 $3=|x|$
 $x=3$ or $x=-3$

Question 3

Solve the following equations.

a) $|4-x| = |4x-1|$

b) $|2x+5| = 3-x$

c) $3|x+2|+8=5|x+2|-2$

d) $|3x-1| = 2x+1$

$$x = \pm 1, \quad x = -\frac{2}{3}, -8, \quad x = 3, -7, \quad x = 0, 2$$

Handwritten solutions for Question 3:

(a) $|4-x| = |4x-1|$
 $(4-x = 4x-1) \Rightarrow (-5x = -5) \Rightarrow x = 1$
 $(4-x = -4x+1) \Rightarrow (-3x = -3) \Rightarrow x = 1$ Both work

(b) $|2x+5| = 3-x$
 $(2x+5 = 3-x) \Rightarrow (3x = -2) \Rightarrow x = -\frac{2}{3}$
 $(2x+5 = -3-x) \Rightarrow (3x = -8) \Rightarrow x = -\frac{8}{3}$ Both are ok

(c) $3|x+2|+8=5|x+2|-2$
 $-2|x+2| = -10$
 $|x+2| = 5$
 $x+2 = 5 \Rightarrow x = 3$
 $x+2 = -5 \Rightarrow x = -7$ Both are ok

(d) $|3x-1| = 2x+1$
 $(3x-1 = 2x+1) \Rightarrow (x = 2)$
 $(3x-1 = -2x-1) \Rightarrow (5x = 0) \Rightarrow x = 0$ Both are ok

Question 4

Solve the following equations.

a) $|2x+1| = x+3$

b) $|4-2x| = \frac{1}{2}x$

c) $|2x-1| = \frac{1}{2}x+2$

d) $\left|\frac{3}{2}x+1\right| = |5-x|$

$$x = 2, -\frac{4}{3}, \quad x = \frac{8}{3}, \frac{8}{5}, \quad x = 2, -\frac{2}{5}, \quad x = -12, \frac{8}{5}$$

Handwritten solutions for Question 4:

(a) $|2x+1| = x+3$
 $2x+1 = x+3 \Rightarrow x=2$
 $2x+1 = -(x+3) \Rightarrow 2x+1 = -x-3 \Rightarrow 3x = -4 \Rightarrow x = -\frac{4}{3}$
 Both solutions work.

(b) $|4-2x| = \frac{1}{2}x$
 $4-2x = \frac{1}{2}x \Rightarrow 4 = \frac{5}{2}x \Rightarrow x = \frac{8}{5}$
 $4-2x = -\frac{1}{2}x \Rightarrow 4 = \frac{3}{2}x \Rightarrow x = \frac{8}{3}$
 Both solutions work.

(c) $|2x-1| = \frac{1}{2}x+2$
 $2x-1 = \frac{1}{2}x+2 \Rightarrow \frac{3}{2}x = 3 \Rightarrow x=2$
 $2x-1 = -(\frac{1}{2}x+2) \Rightarrow 2x-1 = -\frac{1}{2}x-2 \Rightarrow \frac{5}{2}x = -1 \Rightarrow x = -\frac{2}{5}$
 Both solutions work.

(d) $|\frac{3}{2}x+1| = |5-x|$
 $\frac{3}{2}x+1 = 5-x \Rightarrow \frac{5}{2}x = 4 \Rightarrow x = \frac{8}{5}$
 $\frac{3}{2}x+1 = -(5-x) \Rightarrow \frac{3}{2}x+1 = -5+x \Rightarrow \frac{1}{2}x = -6 \Rightarrow x = -12$
 Both solutions work.

Question 5

Solve the following equations.

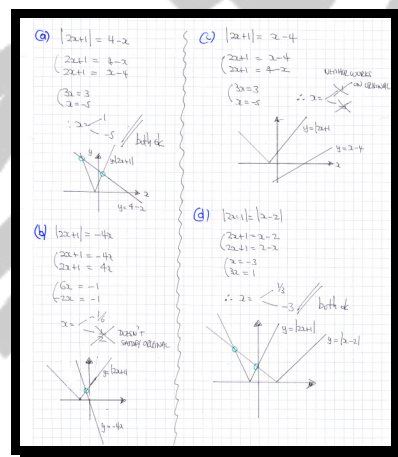
a) $|2x+1| = 4-x$

b) $|2x+1| = -4x$

c) $|2x+1| = x-4$

d) $|2x+1| = |x-2|$

$x = 1, -5$, $x = -\frac{1}{6}$, **no solutions**, $x = -3, \frac{1}{3}$



Question 6

Solve the following equations.

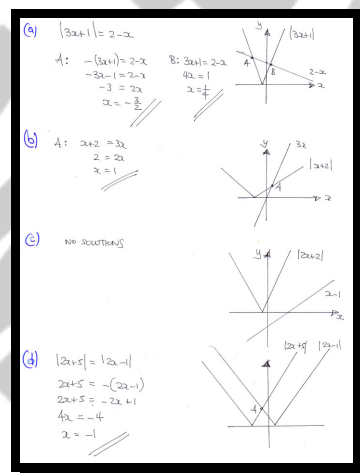
a) $|3x+1| = 2-x$

b) $|x+2| = 3x$

c) $|2x+2| = x-1$

d) $|2x+5| = |2x-1|$

$x = \frac{1}{4}, -\frac{3}{2}$, $x = 1$, no solutions, $x = -1$



Question 7

Solve the following equations.

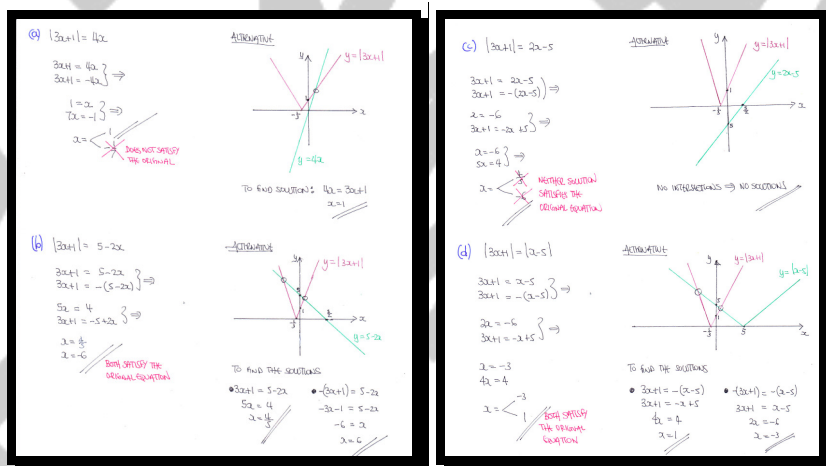
a) $|3x+1| = 4x$

b) $|3x+1| = 5-2x$

c) $|3x+1| = 2x-5$

d) $|3x+1| = |x-5|$

$x=1$, $x=-6, \frac{4}{5}$, no solutions, $x=-3, 1$



Question 8

Solve the following equations.

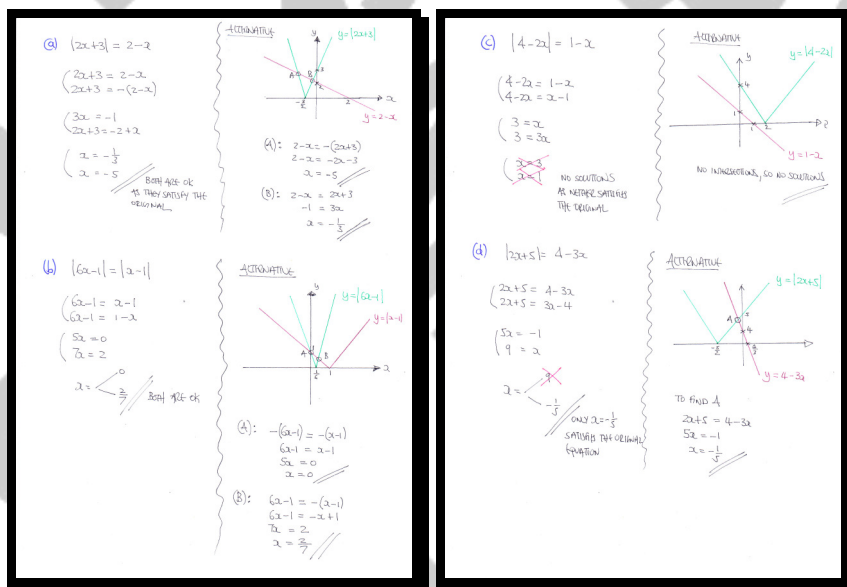
a) $|2x+3| = 2-x$

b) $|6x-1| = |x-1|$

c) $|4-2x| = 1-x$

d) $|2x+5| = 4-3x$

$x = -5, -\frac{1}{3}$, $x = 0, \frac{2}{7}$, no solutions, $x = -\frac{1}{5}$



Question 9

Solve the following equations.

a) $|e^x - 2| = 1$

b) $|13 - 2x^2| = 5$

c) $|20 - x^2| = 16$

d) $2|\sin 2x| = 1, \quad 0 \leq x \leq \pi$

$x = 0, \ln 3, \quad x = \pm 2, \pm 3, \quad x = \pm 2, \pm 6, \quad x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

Handwritten solutions for Question 9:

a) $|e^x - 2| = 1$
 $e^x - 2 = 1 \Rightarrow e^x = 3 \Rightarrow x = \ln 3$
 $e^x - 2 = -1 \Rightarrow e^x = 1 \Rightarrow x = 0$
 Both solutions are valid.

b) $|13 - 2x^2| = 5$
 $13 - 2x^2 = 5 \Rightarrow -2x^2 = -8 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$
 $13 - 2x^2 = -5 \Rightarrow -2x^2 = -18 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$
 All solutions are valid.

c) $|20 - x^2| = 16$
 $20 - x^2 = 16 \Rightarrow -x^2 = -4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$
 $20 - x^2 = -16 \Rightarrow -x^2 = -36 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$
 All solutions are valid.

d) $2|\sin 2x| = 1$
 $|\sin 2x| = \frac{1}{2}$
 $\sin 2x = \frac{1}{2}$
 $2x = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{6} + 2n\pi$
 $x = \frac{\pi}{12} + n\pi \quad \text{or} \quad x = \frac{5\pi}{12} + n\pi$
 For $0 \leq x \leq \pi$, the solutions are $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$.

MODULUS INEQUALITIES

Question 1

Solve the following inequalities.

a) $|3x-1| < 5$

b) $|1-2x| \leq 5$

c) $|4x+1| \geq 3$

d) $|2x-5| > x$

$$\boxed{-\frac{4}{3} < x < 2}, \quad \boxed{-2 \leq x \leq 3}, \quad \boxed{x \leq -1 \text{ or } x \geq \frac{1}{2}}, \quad \boxed{x < \frac{5}{3} \text{ or } x > 5}$$

Handwritten solutions for the four inequalities:

- (a) $|3x-1| < 5$** : Solve equation $|3x-1|=5$.
 $3x-1=5 \Rightarrow 3x=6 \Rightarrow x=2$
 $3x-1=-5 \Rightarrow 3x=-4 \Rightarrow x=-\frac{4}{3}$
 $-\frac{4}{3} < x < 2$
 Graph: Number line from $-\frac{4}{3}$ to 2 with open circles at both ends.
- (b) $|1-2x| \leq 5$** : Solve equation $|1-2x|=5$.
 $1-2x=5 \Rightarrow -2x=4 \Rightarrow x=-2$
 $1-2x=-5 \Rightarrow -2x=-6 \Rightarrow x=3$
 $\therefore -2 \leq x \leq 3$
 Graph: Number line from -2 to 3 with closed circles at both ends.
- (c) $|4x+1| \geq 3$** : Solve equation $|4x+1|=3$.
 $4x+1=3 \Rightarrow 4x=2 \Rightarrow x=\frac{1}{2}$
 $4x+1=-3 \Rightarrow 4x=-4 \Rightarrow x=-1$
 $\therefore x < -1 \text{ or } x > \frac{1}{2}$
 Graph: Number line with open circles at -1 and $\frac{1}{2}$, and arrows pointing outwards.
- (d) $|2x-5| > x$** : Solve equation $|2x-5|=x$.
 $2x-5=x \Rightarrow x=5$
 $2x-5=-x \Rightarrow 3x=5 \Rightarrow x=\frac{5}{3}$
 $\therefore x < \frac{5}{3} \text{ or } x > 5$
 Graph: Number line with open circles at $\frac{5}{3}$ and 5 , and arrows pointing outwards.

Question 2

Solve the following inequalities.

a) $|2x+1| < x+2$

b) $|1-2x| \leq |x-3|$

c) $|2x-3| > |x+1|$

d) $|3x-1| < 2-x$

$$\boxed{-1 < x < 1}, \quad \boxed{-2 \leq x \leq \frac{4}{3}}, \quad \boxed{x < \frac{2}{3} \text{ or } x > 4}, \quad \boxed{-\frac{1}{2} < x < \frac{3}{4}}$$

Handwritten solutions for the four inequalities:

a) $|2x+1| < x+2$, solve equation $|2x+1| = x+2$
 $2x+1 = x+2 \Rightarrow x=1$
 $2x+1 = -x-2 \Rightarrow 3x = -3 \Rightarrow x = -1$
 Both are OK
 $\therefore -1 < x < 1$
 If $x=0 \Rightarrow |2x+1| < x+2$
 $\Rightarrow |1| < 2$
 $\Rightarrow 1 < 2$
 zero values

b) $|1-2x| \leq |x-3|$, solve equation $|1-2x| = |x-3|$
 $1-2x = x-3 \Rightarrow -3x = -4 \Rightarrow x = \frac{4}{3}$
 $1-2x = -x+3 \Rightarrow -x = 2 \Rightarrow x = -2$
 Both are OK
 $\therefore -2 \leq x \leq \frac{4}{3}$
 If $x=0 \Rightarrow |1-2x| \leq |x-3|$
 $\Rightarrow |1| \leq | -3|$
 $\Rightarrow 1 \leq 3$
 zero values

c) $|2x-3| > |x+1|$, solve equation $|2x-3| = |x+1|$
 $2x-3 = x+1 \Rightarrow x=4$
 $2x-3 = -x-1 \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$
 Both are OK
 $\therefore x < \frac{2}{3} \text{ or } x > 4$
 If $x=0 \Rightarrow |2x-3| > |x+1|$
 $\Rightarrow | -3| > |1|$
 $\Rightarrow 3 > 1$
 zero values

d) $|3x-1| < 2-x$, solve equation $|3x-1| = 2-x$
 $3x-1 = 2-x \Rightarrow 4x = 3 \Rightarrow x = \frac{3}{4}$
 $3x-1 = -2-x \Rightarrow 4x = -1 \Rightarrow x = -\frac{1}{4}$
 Both are OK
 $\therefore -\frac{1}{4} < x < \frac{3}{4}$
 If $x=0 \Rightarrow |3x-1| < 2-x$
 $\Rightarrow | -1| < 2$
 $\Rightarrow 1 < 2$
 zero values

Question 3

Solve the following inequalities.

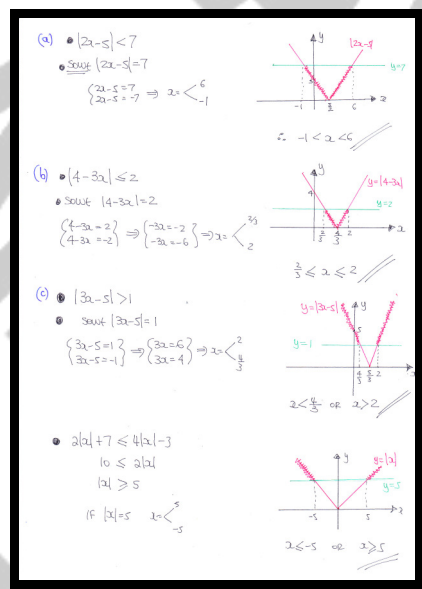
a) $|2x-5| < 7$

b) $|4-3x| \leq 2$

c) $|3x-5| > 1$

d) $2|x|+7 \leq 4|x|-3$

$$\boxed{-1 < x < 6}, \quad \boxed{\frac{2}{3} \leq x \leq 2}, \quad \boxed{x < \frac{4}{3} \text{ or } x > 2}, \quad \boxed{x \leq -5 \text{ or } x \geq 5}$$



Question 4

Solve the following inequalities.

a) $|2x+3| < 5$

b) $|4x-3| > x$

c) $|6x| \geq |x+1|$

d) $|x-1| \geq 2|x+2|$

$$-4 < x < 1, \quad x < \frac{3}{5} \text{ or } x > 1, \quad x < -\frac{1}{7} \text{ or } x > \frac{1}{5}, \quad -5 < x < -1$$

③ $|2x+3| < 5$; Solve $2x+3=5$
 $2x+3=5 \Rightarrow 2x=2 \Rightarrow x=1$ Both work
 $2x+3=-5 \Rightarrow 2x=-8 \Rightarrow x=-4$ Both work
 $\therefore -4 < x < 1$

④ $|4x-3| > x$; solve $|4x-3|=x$
 $4x-3=x \Rightarrow 3x=3 \Rightarrow x=1$ Both work
 $4x-3=-x \Rightarrow 5x=3 \Rightarrow x=\frac{3}{5}$ Both work
 $\therefore x < \frac{3}{5}$ or $x > 1$

⑤ $|6x| \geq |x+1|$; solve equation $|6x|=|x+1|$
 $6x=x+1 \Rightarrow 5x=1 \Rightarrow x=\frac{1}{5}$ Both work
 $6x=-(x+1) \Rightarrow 7x=-1 \Rightarrow x=-\frac{1}{7}$ Both work
 $\therefore x \leq -\frac{1}{7}$ or $x \geq \frac{1}{5}$

⑥ $|x-1| \geq 2|x+2|$; solve equation $|x-1|=2|x+2|$
 $x-1=2(x+2) \Rightarrow x-1=2x+4 \Rightarrow -5=x \Rightarrow x=-5$ Both work
 $x-1=-2(x+2) \Rightarrow x-1=-2x-4 \Rightarrow 3x=-3 \Rightarrow x=-1$ Both work
 $\therefore -5 < x < -1$

Question 5

Solve the following inequalities.

a) $|2x-3| > |x+1|$

b) $|4x-3| \leq |2x+1|$

c) $6|x| \geq |2-3x|$

d) $|x-3| > 2|x+1|$

$$x < \frac{2}{3} \text{ or } x > 4, \quad \frac{1}{3} \leq x \leq 2, \quad x \leq -\frac{2}{3} \text{ or } x \geq \frac{2}{9}, \quad -5 < x < \frac{1}{3}$$

Handwritten solutions for the four inequalities:

a) $|2x-3| > |x+1|$, solve equation $|2x-3| = |x+1|$
 $2x-3 = x+1 \Rightarrow x = 4$
 $2x-3 = -(x+1) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$ (both ok)
 $\therefore x < \frac{2}{3}$ or $x > 4$

b) $|4x-3| \leq |2x+1|$, solve equation $|4x-3| = |2x+1|$
 $4x-3 = 2x+1 \Rightarrow 2x = 4 \Rightarrow x = 2$ (both ok)
 $4x-3 = -(2x+1) \Rightarrow 6x = 2 \Rightarrow x = \frac{1}{3}$ (both ok)
 $\therefore \frac{1}{3} \leq x \leq 2$

c) $6|x| \geq |2-3x|$, solve equation $6|x| = |2-3x|$
 $6x = 2-3x \Rightarrow 9x = 2 \Rightarrow x = \frac{2}{9}$ (both ok)
 $6x = -(2-3x) \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$ (both ok)
 $\therefore x \leq -\frac{2}{3}$ or $x \geq \frac{2}{9}$

d) $|x-3| > 2|x+1|$, solve equation $|x-3| = 2|x+1|$
 $x-3 = 2x+2 \Rightarrow -x = 5 \Rightarrow x = -5$ (both ok)
 $x-3 = -2(x+1) \Rightarrow 3x = -1 \Rightarrow x = -\frac{1}{3}$ (both ok)
 $\therefore -5 < x < -\frac{1}{3}$

Question 6

Solve the following inequalities.

a) $|2x-5| \leq 9$

b) $|x-1| \leq 4$

c) $8x > |x-8|$

d) $2|x+1| + x > 1$

$$-2 \leq x \leq 7, \quad -3 \leq x \leq 5, \quad x > \frac{8}{9}, \quad x < -3 \text{ or } x > -\frac{1}{3}$$

Handwritten solutions for the inequalities:

a) $|2x-5| \leq 9$; Solve equation $|2x-5| = 9$
 $2x-5=9 \Rightarrow 2x=14 \Rightarrow x=7$
 $2x-5=-9 \Rightarrow 2x=-4 \Rightarrow x=-2$
 $\therefore -2 \leq x \leq 7$

b) $|x-1| \leq 4$; Solve equation $|x-1| = 4$
 $x-1=4 \Rightarrow x=5$
 $x-1=-4 \Rightarrow x=-3$
 $\therefore -3 \leq x \leq 5$

c) $8x > |x-8|$; Solve equation $8x = |x-8|$
 $8x = x-8 \Rightarrow 7x = -8 \Rightarrow x = -\frac{8}{7}$
 $8x = 8-x \Rightarrow 9x = 8 \Rightarrow x = \frac{8}{9}$
 $\therefore x > \frac{8}{9}$

d) $2|x+1| + x > 1$; Solve equation $2|x+1| + x = 1$
 $2(x+1) = 1-x \Rightarrow 2x+2 = 1-x \Rightarrow 3x = -1 \Rightarrow x = -\frac{1}{3}$
 $2(x-1) = 1-x \Rightarrow 2x-2 = 1-x \Rightarrow 3x = 3 \Rightarrow x = 1$
 $\therefore x < -\frac{1}{3} \text{ or } x > 1$