# FUNCTION PRACTICE PRACTICE

# FUNCTION INTRODUCTION

### Question 1

Find the range for each of the following functions.

**a**) 
$$f(x) = x^2 + 1, x \in \mathbb{R}$$
.

**b**) 
$$g(x) = x^2 + 1, x \in \mathbb{R}, 1 < x \le 3$$

- c)  $h(x) = x^2 + 1, x \in \mathbb{R}, x \le -1.$ 
  - $f(x) \in \mathbb{R}, f(x) \ge 1, \quad g(x) \in \mathbb{R}, 2 < g(x) \le 10, \quad h(x) \in \mathbb{R}, h(x) \ge 2$



### **Question 2**

Find the range for each of the following functions.

**a**)  $f(x) = (x-4)^2 + 1, x \in \mathbb{R}, x > 4.$ 

**b**) 
$$g(x) = (x+3)^2 - 1, x \in \mathbb{R}, x \ge -4$$
.

c) 
$$h(x) = (x-5)^2 + 2, x \in \mathbb{R}, 0 < x < 6$$

$$f(x) \in \mathbb{R}, f(x) > 1, \quad g(x) \in \mathbb{R}, g(x) \ge -1, \quad h(x) \in \mathbb{R}, \ 2 \le h(x) < 27$$

### Question 3

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Find the range for each of the following functions.

**a**) 
$$f(x) = x^2 + 1, x \in \mathbb{R}, x > 0$$

- **b**)  $g(x) = x^2 8x + 13, x \in \mathbb{R}, x \ge 0$ .
- c)  $h(x) = x^2 + 2x + 2, x \in \mathbb{R}, -5 \le x < -2$ .
  - $f(x) \in \mathbb{R}, f(x) > 1, \quad g(x) \in \mathbb{R}, g(x) \ge -3, \quad h(x) \in \mathbb{R}, \quad 2 < h(x) \le 17$

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### Question 4

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Find the range for each of the following functions.

a) 
$$f(x) = x^2 - 6x + 6, x \in \mathbb{R}$$

**b**) 
$$g(x) = x^2 + 8x + 12, x \in \mathbb{R}, -3 \le x \le 0.$$

h(x) = 
$$x^2 - 10x + 26$$
,  $x \in \mathbb{R}$ ,  $x \ge 0$ .

 $f(x) \in \mathbb{R}, f(x) \ge -3, \quad g(x) \in \mathbb{R}, \quad -3 \le g(x) \le 12, \quad h(x) \in \mathbb{R}, \quad h(x) \ge 1$ 

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### Question 5

Find the range for each of the following functions.

a) 
$$f(x) = \sqrt{x+1}, x \in \mathbb{R}, x \ge 0$$
.

**b**) 
$$g(x) = \sqrt{x-2}, x \in \mathbb{R}, 6 \le x < 1$$

c)  $h(x) = 2 - \sqrt{x}, x \in \mathbb{R}, x \ge 4$ .

$$f(x) \in \mathbb{R}, f(x) \ge 1$$
,  $g(x) \in \mathbb{R}, 2 \le g(x) < 3$ ,  $h(x) \in \mathbb{R}, h(x) \le 0$ 



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### **Question 6**

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Find the range for each of the following functions.

**a**) 
$$f(x) = \frac{1}{x-2}, x \in \mathbb{R}, x > 2.$$

**b**) 
$$g(x) = \frac{2}{x+3}, x \in \mathbb{R}, x \ge 1.$$

c) 
$$h(x) = \frac{1}{x-1} + 2, x \in \mathbb{R}, x > 2.$$

$$f(x) \in \mathbb{R}, f(x) > 0$$
,  $g(x) \in \mathbb{R}, 0 < g(x) \le \frac{1}{2}$ ,  $h(x) \in \mathbb{R}, 2 < h(x) < 3$ 



### Question 7

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I.F.G.p

Find the range for each of the following functions.

a) 
$$f(x) = 15 - (x-2)^2$$
,  $x \in \mathbb{R}, 0 \le x \le 4$ 

**b**) 
$$g(x) = 8 - x^3, x \in \mathbb{R}, 0 \le x \le 2$$

c)  $h(x) = \frac{1}{x+3}, x \in \mathbb{R}, x \ge 0.$ 

 $f(x) \in \mathbb{R}, 11 \le f(x) \le 15$ ,  $g(x) \in \mathbb{R}, 0 \le g(x) \le 8$ ,  $h(x) \in \mathbb{R}, 0 < h(x) \le \frac{1}{3}$ 

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### **Question 8**

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Find the range for each of the following functions.

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a) 
$$f(x) = x^2 - 4x + 3, x \in \mathbb{R}, x > 2$$

**b**) 
$$g(x) = x^2 + 4x + 2, x \in \mathbb{R}, x \ge 0$$

**b**) 
$$g(x) = x^2 + 4x + 2, x \in \mathbb{R}, x \in \mathbb{R}, x \in \mathbb{R}, x \neq 2$$
.  
**c**)  $h(x) = \frac{1}{x-2}, x \in \mathbb{R}, x \neq 2$ .  
 $f(x) \in \mathbb{R}, f(x)$ 





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### Question 9

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Find the range for each of the following functions.

a) 
$$f(x) = \sqrt{x+2}, x \in \mathbb{R}, x \ge -1$$

**b**) 
$$g(x) = 2 - e^x, x \in \mathbb{R}, x \le 0$$

c) 
$$h(x) = \frac{1}{x+2}, x \in \mathbb{R}, x \ge 0.$$





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### Question 10

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Find the range for each of the following functions.

a) 
$$f(x) = 4 - \sqrt{x}, x \in \mathbb{R}, x \ge 0$$
.

- **b**)  $g(x) = 2 + e^{-x}$ ,  $x \in \mathbb{R}, x \leq 0.$
- $-2, \ x \in \mathbb{R}, \ x \ge 0 \, .$  $\mathbf{c}) \quad h(x) = \frac{1}{x+2}$ 
  - $\boxed{f(x) \in \mathbb{R}, f(x) \le 4}, \ \boxed{g(x) \in \mathbb{R}, g(x) \ge 3}, \ \boxed{h(x) \in \mathbb{R}, -2 < h(x) \le -2}$ 32

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### Question 11

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Find the range for each of the following functions.

**a**) 
$$f(x) = \frac{1}{4-x}, x \in \mathbb{R}, x \ge 5$$
.

**b**) 
$$g(x) = 25 - (x - 4)^2, x \in \mathbb{R}, x \ge 0.$$

c) 
$$h(x) = x^3 - 2, x \in \mathbb{R}, x < 2.$$

 $f(x) \in \mathbb{R}, -1 \le f(x) < 0 , \quad g(x) \in \mathbb{R}, \quad g(x) \le 25 , \quad h(x) \in \mathbb{R}, \quad h(x) < 6$ 



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### Question 12

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Find the range for each of the following functions.

a) 
$$f(x) = e^x + 2, x \in \mathbb{R}$$
.

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**b**) 
$$g(x) = 4 - e^{-x}, x \in \mathbb{R}, x \ge 0$$
  
**c**)  $h(x) = 3 - e^{x+1}, x \in \mathbb{R}, x \ge -1$ 

) 
$$h(x) = 3 - e^{x+1}, x \in \mathbb{R}, x \ge -1.$$

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### Question 13

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Find the range for each of the following functions.

**a**) 
$$f(x) = \sqrt{4-x}, x \in \mathbb{R}, x < 0.$$

**b**) 
$$g(x) = \sqrt{2x+1}, x \in \mathbb{R}, -\frac{1}{2} \le x \le 0$$

) 
$$h(x) = \ln(12 - 4x), x \in \mathbb{R}, x < 2$$

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 $f(x) \in \mathbb{R}, f(x) > 2$ ,  $g(x) \in \mathbb{R}, 0 \le g(x) \le 1$ ,  $h(x) \in \mathbb{R}, h(x) > \ln 4$ 



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### Question 1

Find fg(x) and gf(x) if

$$f(x) = 2x + 1, \ x \in \mathbb{R}$$

$$g(x) = x^2 - 1, \ x \in \mathbb{R}$$

Simplify the answers as much as possible.



### Question 2

Find fg(x) and gf(x) if

$$f(x) = 4 - 3x, \ x \in \mathbb{R}$$

 $g(x) = \sqrt{x}, \quad x \in \mathbb{R}, \ x \ge 0.$ 

Simplify the answers as much as possible.

$$fg(x) = 4 - 3\sqrt{x}, \quad gf(x) = \sqrt{4 - 3x}$$

 $\begin{array}{l} \left( \begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right) = \begin{array}{c} \left( \begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right) = \begin{array}{c} \left( \begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right) = \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array}\right) = \begin{array}{c} \left( \begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right) = \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array}\right) = \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array}\right) = \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array}$ 

### Question 3

Find fg(x) and gf(x) if

$$f(x) = 3x - 8, x \in \mathbb{R}$$

$$g(x) = \frac{1}{x}, \quad x \in \mathbb{R}, \ x \neq 0.$$

Simplify the answers as much as possible.



•  $\beta \stackrel{\circ}{+} \left( \gamma = \vartheta \left( f(\sigma) \right) = \vartheta \left( \frac{\sigma}{\sigma} \cdot \sigma \right) = \frac{2\sigma - \delta}{\sigma}$ 

**Question 4** 

Find fg(x) and gf(x) if

$$f(x) = 4x - 1, \ x \in \mathbb{R}$$

 $g(x) = \frac{x}{x+1}, \quad x \in \mathbb{R}, \ x \neq -1$ 

Simplify the answers as much as possible.

$$fg(x) = \frac{3x-1}{x+1}$$
,  $gf(x) = \frac{4x-1}{4x}$ 

•  $\left\{ \begin{array}{ll} \underline{a}(s) &= \underline{A}(\underline{b}(s)) = \overline{A}(\frac{\infty}{(\lambda+1)}) = -\underline{A}(\frac{\infty}{(\lambda+1)}) - | &= -\frac{4\alpha}{(\lambda+1)} - | \\ &= -\frac{4\alpha}{(\lambda+1)} = -\frac{3\alpha}{(\lambda+1)} \\ \bullet & \underline{g}(\underline{b}) = -\underline{g}(\underline{b}(\underline{a})) = -\underline{g}(\underline{b}(\lambda-1)) = -\frac{4\alpha}{(d_{\lambda}-1)+1} = -\frac{4\alpha}{(d_{\lambda}-1)+1} \\ \end{array} \right\}$ 

### Question 5

Find fg(x) and gf(x) if

$$f(x) = 2x^2 + 1, \ x \in \mathbb{R}$$

$$g(x) = \sqrt{x}, \quad x \in \mathbb{R}, \ x \ge 0.$$

Simplify the answers as much as possible.



•  $f(g(x)) = f(f(x)) = f(\sqrt{x^2} + 1) = \sqrt{x^2 + 1}$ 

Question 6

Find fg(x) and gf(x) if

$$f(x) = (x+3)^2, \ x \in \mathbb{I}$$

 $g(x) = 2x, \quad x \in \mathbb{R}.$ 

Simplify the answers as much as possible.

 $fg(x) = (2x+3)^2$  $gf(x) = 2(x+3)^2$ 

### Question 7

Find fg(x) and gf(x) if

$$f(x) = 2x - 1, x \in \mathbb{R}$$

 $g(x) = \sqrt{x+3}, \quad x \in \mathbb{R}, \ x \ge -3.$ 

Simplify the answers as much as possible.

$$fg(x) = 2\sqrt{x+3} - 1$$
,  $gf(x) = \sqrt{2x+2}$ 

f(x) = g(f(x))

 $A(2x-1) = \sqrt{(2x-1)t_3} = \sqrt{2x+2^2}$ 

**Question 8** 

Find fg(x) and gf(x) if

$$f(x) = \sqrt{x}, \quad x \in \mathbb{R}, \quad x \ge 0$$

$$g(x) = \frac{2x^2}{x^2 - 1}, \quad x \in \mathbb{R}, \ x \neq \pm 1.$$

Simplify the answers as much as possible.

$$fg(x) = \sqrt{\frac{2x^2}{x^2 - 1}}, \quad gf(x) = \frac{2x}{x - 1}$$

### Question 9

Find fg(x) and gf(x) if

 $f(x) = 2x - 3, \ x \in \mathbb{R}$ 

 $g(x) = x - \frac{1}{x}, \quad x \in \mathbb{R}, \ x \neq 0.$ 

Simplify the answer as much as possible.

 $fg(x) = \frac{2x^2 - 3x - 2}{x}, \quad gf(x) = \frac{4x^2 - 12x + 8}{2x - 3}$ 

(a)  $f_{\frac{1}{2}}(x) = f_{\frac{1}{2}}(x) = f_{\frac{1}{2}}(x) = 2(x-\frac{1}{2}) - 3 = 2x - \frac{2}{2} - 3$ (b)  $g_{\frac{1}{2}}(x) = g_{\frac{1}{2}}(x) = g_{\frac{1}{2}}(x-3) = (2x-3) - \frac{1}{2x-3}$  $= \frac{(2x-3)^{2}-1}{2x-3} = \frac{4x^{2}-12x+8}{2x-3}$ 

Question 10

Find fg(x) and gf(x) if

$$f(x) = x^3 - 1, \ x \in \mathbb{R}$$

$$g(x) = \frac{1}{\sqrt{x}}, \quad x \in \mathbb{R}, \ x > 0$$

Simplify the answers as much as possible.

 $\frac{fg(x) = \frac{1}{x\sqrt{x}} - 1}{x\sqrt{x}}, \quad gf(x) = \frac{1}{x\sqrt{x}}$ 

•  $f_{\mathfrak{B}}(s) = f_{\mathfrak{B}}(u) = f_{\mathfrak{C}}(\frac{1}{\sqrt{n}}) = \left(\frac{1}{\sqrt{n}}\right)^3 - 1 = \frac{1}{2\sqrt{n}} - 1$ •  $g_{\mathfrak{B}}(u) = g_{\mathfrak{C}}(u) = g_{\mathfrak{C}}(u) - 1 = \frac{1}{\sqrt{n}}$ 

 $x^{3} - 1$ 

### Question 11

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Find fg(x) and gf(x) if

$$f(x) = 6 - x^2, \ x \in \mathbb{R}$$

 $g(x) = \frac{x+1}{x}$  $x \in \mathbb{R}, \ x \neq 0.$ 

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Simplify the answers as much as possible.

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$$fg(x) = \frac{5x^2 - 2x - 1}{x^2}, \quad gf(x) = \frac{7 - x^2}{6 - x^2}$$

 $f_{\mathfrak{g}}(\mathfrak{z}) = f(\mathfrak{g}(\mathfrak{z})) = f(\frac{\mathfrak{z}+1}{\mathfrak{z}}) = 6 - (\frac{\mathfrak{z}+1}{\mathfrak{z}})^2 = 6 - \frac{\mathfrak{z}+\mathfrak{z}+1}{\mathfrak{z}^2}$  $\frac{(2^2 - (\chi^2 + 2\chi + i))}{\chi^2} = \frac{(2^2 - \chi^2 - 2\chi - i)}{\chi^2} = \frac{5\chi^2 + 2\chi - i}{\chi^2}$  $\mathfrak{G} \overset{\text{p}}{\to} \overset{p}}{\to} \overset{p}{\to} \overset{p}{\to} \overset{p}}{\to} \overset{p}}{\to} \overset{p}{\to} \overset{p}}{\to$ 

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### Question 12

The following functions are defined by

$$f(x) = 2x+3, x \in \mathbb{R}.$$
$$g(x) = 1-x^2, x \in \mathbb{R}.$$
$$h(x) = \frac{1}{x}, x \in \mathbb{R}, x \neq 0.$$

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Find all six possible two-fold compositions for the above functions, simplifying the final answers as much as possible.

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$$fg(x) = 5 - 2x^{2} \\ gf(x) = -4x^{2} - 12x - 8 \\ hf(x) = \frac{1}{2x + 3} \\ hg(x) = \frac{1}{1 - x^{2}} \\ hg(x) = \frac{1}{1 - x^{2$$

-f(g(u))=f(0-x)= 2(-x)+3=2-2x2	
$(\theta, \theta) = g(2x+3) = 1 - (2x+3)^2 - 1 - (4x^2-12x+9) = -4x^2 + 12x - 8$	,
$f(h(a)) = f(x) = 2(x) + 3 = \frac{2}{3} + 3$	
$h(f(\alpha)) = h(2\alpha+3) = \frac{1}{22+3}$	
$g(h(a)) = g(\frac{1}{a}) = 1 - \frac{1}{2^{2}} = \frac{2^{2}-1}{2^{2}}$	
$h(g(x)) = h(1-x^2) = \frac{1}{1-x^2}$	
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### Question 13

The following functions are defined by

$$f(x) = 2x+1, x \in \mathbb{R}.$$
$$g(x) = e^{x}, x \in \mathbb{R}.$$
$$h(x) = \sin x, x \in \mathbb{R}.$$

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Find all six possible two-fold compositions for the above functions simplifying the final answers as much as possible.

$$\begin{array}{c} fg(x) = 2e^{x} + 1\\ gf(x) = e^{2x+1} \end{array}, \quad \begin{array}{c} fh(x) = 2\sin x + 1\\ hf(x) = \sin(2x+1) \end{array}, \quad \begin{array}{c} gh(x) = e^{\sin x}\\ hg(x) = \sin(e^{x}) \end{array} \end{array}$$

fg(2)= f(g(2))= f(1) = 1 - (-1) $gf(q) = g(f(q)) = g(r-q) = g(z) = 3^{-}_{z} z = q$ (6)  $f(h(q)) = f(h(q)) = -f(\sqrt{q^{-1}}) =$ -f(3) = 0 (d)  $h f(-15) = h(f(-15)) = h(1+15) = h(16) = \sqrt{16} = 4$  $\frac{1}{2}h(q) = g(h(q)) = g(nq) = g(2) = 2^2 - 5 = -1$ (e)  $h(q(3) = h(q(3)) = h(3^2 - 5) = h(4) = \sqrt{4^2} = 2$ 

### Question 14

The following functions are defined by

$$f(x) = 1 - 2x, x \in \mathbb{R} .$$
$$g(x) = e^{x}, x \in \mathbb{R} .$$
$$h(x) = \sqrt{x}, x \in \mathbb{R}, x \ge 0.$$

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Find all six possible two-fold compositions for the above functions simplifying the final answers as much as possible.

$$\begin{bmatrix}
fg(x) = 1 - 2e^{x} \\
gf(x) = e^{1 - 2x}
\end{bmatrix}, \quad
\begin{bmatrix}
fh(x) = 1 - 2\sqrt{x} \\
hf(x) = \sqrt{1 - 2x}
\end{bmatrix}, \quad
\begin{bmatrix}
gh(x) = e^{\sqrt{x}} \\
hg(x) = e^{\frac{1}{2}x}
\end{bmatrix}$$

$f_g(x) = f(g(x)) = f(e^x) = 1 - 2e^x$	
$g(f_{a}) = g(f_{a}) = g(1-2x) = e^{1-2x}$	
$f(y) = f(y) = f(y) = f(y) = 1 - 2y^{-1}$	
$\int_{\Omega} f(x) = \int_{\Omega} \left( \frac{f(x)}{f(x)} \right) = \int_{\Omega} \left( \frac{1-2x}{x} \right) = \sqrt{1-2x^{2}}$	
$g_{\lambda}h(x) = g(h(x)) = g(\sqrt{x^{2}}) = e^{\sqrt{x^{2}}}$	
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### Question 15

The following functions are defined by

$$f(x) = 1 - x, x \in \mathbb{R}$$
.  
 $g(x) = x^2 - 5, x \in \mathbb{R}$ 

$$h(x) = \sqrt{x}, x \in \mathbb{R}, x \ge 0.$$

Evaluate the following function compositions.

- a) fg(2)
- **b**) gf(4)
- c) fh(9)
- **d**) hf(-15)
- e) gh(4)
- **f**) hg(3)

# fg(2) = 2, gf(4) = 4, fh(9) = -2, hf(-15) = 4, gh(4) = -1, hg(3) = 2

(a)  $\frac{1}{2}g(x) = \frac{1}{2}(g(x)) - \frac{1}{2}(z^2x) - \frac{1}{2}(y^{-1}) - \frac{1}{2}(-(-)) = 2$ (b)  $\frac{1}{2}g(x) - \frac{1}{2}(g(x)) - \frac{1}{2}(z^2x) - \frac{1}{2}(y^{-1}) - \frac{1}{2}(y^{-1})$ 

### Question 16

The following functions are defined by

$$f(x) = 2x + 5, x \in \mathbb{R}$$

$$g(x) = \frac{4}{x}, \quad x \in \mathbb{R}, \ x \neq 0$$

$$h(x) = \sqrt{x+2}, \quad x \in \mathbb{R}, \ x \ge -2$$

Evaluate the following function compositions.

- **a**)  $fg\left(\frac{1}{2}\right)$ **b**) gf(-2)
- c) hf(1)
- d) fh(2)
- e)  $hg\left(\frac{2}{7}\right)$

- **f**)  $gh\left(-\frac{7}{4}\right)$
- g) gfh(-1)
- **h**) fgf(-2)

i)  $fff\left(\frac{1}{4}\right)$ 

- $fg\left(\frac{1}{2}\right) = 21, gf\left(-2\right) = 4, hf\left(1\right) = 3, fh(2) = 9, hg\left(\frac{2}{7}\right) = 4, gh\left(-\frac{7}{4}\right) = 8, gfh(1) = \frac{4}{7}, fgf\left(-2\right) = 13, fff\left(\frac{1}{4}\right) = 37$ 
  - $\begin{array}{l} \textbf{(a)} \quad & \int_{1}^{1} \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(b)} \quad & \frac{1}{2} = \frac{1}{1} \frac{1}{2} = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} = \frac{1}{1} \frac{1}{2} = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \right) \\ \textbf{(c)} \quad & \frac{1}{2} \left( \frac{1}{2} \right) = \left( \frac{1}{2} \left( \frac{1}{2} \right) \right) \\ \textbf{(c)} \quad & \frac{1}{2} \left( \frac{1}{2} \right) \\ \textbf{(c)} \quad$

### Question 17

The following functions are defined by

$$f(x) = x - 2, x \in \mathbb{R}$$
$$g(x) = \ln x, x \in \mathbb{R}, x > 0$$
$$h(x) = e^{2x}, x \in \mathbb{R}.$$

Find simplified expressions the following function compositions, stating in each case the domain and range.



### Question 17

The following functions are defined by

$$f(x) = 2x - 1, x \in \mathbb{R}, x \le 18$$
$$g(x) = x^2 + 2, x \in \mathbb{R}, x \ge 1$$
$$h(x) = \sqrt{x}, x \in \mathbb{R}, x \ge 0.$$

Find simplified expressions for each of the following function compositions, stating in each case the domain and range.



# FUNCTION TREES FUNCTIC INVERSES INVE

### Question 1

>

For each of the following functions find an expression for its inverse.

a) 
$$f(x) = 4x - 1, x \in \mathbb{R}$$
.

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a) 
$$f(x) = 4x - 1, x \in \mathbb{R}$$
.  
b)  $g(x) = 1 + \sqrt{x}, x \in \mathbb{R}, x \ge 0$   
c)  $h(x) = 1 - \sqrt{x - 5}, x \in \mathbb{R}, x \ge 1$   
 $f^{-1}(x)$ 

a) 
$$f(x) = 4x - 1, x \in \mathbb{R}$$
.  
b)  $g(x) = 1 + \sqrt{x}, x \in \mathbb{R}, x \ge 0$ .  
c)  $h(x) = 1 - \sqrt{x - 5}, x \in \mathbb{R}, x \ge 5$ .  

$$\boxed{f^{-1}(x) = \frac{x + 1}{4}}, \boxed{g^{-1}(x) = 0}$$

C)	$h(x) = 1 - \sqrt{x-5}, x \in \mathbb{R}, x \ge 5.$	48	0		
1	$f^{-1}(x) = \frac{x}{2}$	$\frac{+1}{4}, g^{-1}(x) = (x)$	$[-1)^2$ , $h^{-1}(x) = 5 + 1$	$\frac{(1-x)^2}{(1-x)^2}$	2
Co	Sinaths .	naths	$\begin{array}{c} (\mathbf{a}) - \{\theta_1\} = \{b_{n-1}, x_n \} \\ = g_1 = [b_{n-1}] \\ \Rightarrow g_2 = [b_{n-1}] \\ \Rightarrow g_2 = [b_{n-1}] \\ \Rightarrow g_2 = [b_n] \\ \Rightarrow g_1 = [b_n] \\ \Rightarrow g_2 = $	$ \begin{aligned} & \begin{pmatrix} (s) = (-\sqrt{k-s}^{-1}) \\ & g_{2,1} = \sqrt{k+s}^{-1} \\ & g_{2,1} = \sqrt{k+s}^{-1} \\ & g_{2,2} = (-g) \\ & g_{2,3} = (-g)^{-2} \\ & g_{3,2} = (-g)^{-2} \\ & g_{3,3} = (-g)^{-2} \\$	4
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2	n. 1120	·Gp	``G ps	3.172	
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-1) <sup>2</sup> , <i>k</i>	$t^{-1}(x) = 5$	$5 + (1 - x)^2$	1.1.6.1
(a) f(1)=42-1, xtR => y= b2-1	(b) g(2) = (+12, 2≥0 ⇒ y=1+12	(c) $h(x) = (-\sqrt{x-x^{-1}})$ $= 3 - 1 - \sqrt{x-x^{-1}}$	1211
$\Rightarrow y_{+1} = 4x$ $\Rightarrow x_{=} \ddagger (y_{+1})$ $\therefore \nexists (y_{1}) = \frac{1}{4} (x_{+1})$	$ \begin{array}{c} \Longrightarrow  y_{-1} = \sqrt{x} \\ \Longrightarrow  (y_{-1})^2 = x \\ \therefore  \overline{g}(y_{-1}) = (x_{-1})^2 \end{array} $		~~~,

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### **Question 2**

For each of the following functions find an expression for its inverse.

a) 
$$f(x) = 5 - 2x, x \in \mathbb{R}$$
.

**b**) 
$$g(x) = \frac{3}{x} - 2, \quad x \in \mathbb{R}, x \neq 0$$

a) 
$$f(x) = 5 - 2x, x \in \mathbb{R}$$
.  
b)  $g(x) = \frac{3}{x} - 2, x \in \mathbb{R}, x \neq 0$ .  
c)  $h(x) = \sqrt{\frac{x}{2} - 1}, x \in \mathbb{R}, x \ge 2$ .  
 $f^{-1}(x) = \frac{1}{2}$ 

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$$f^{-1}(x) = \frac{5-x}{2}, \quad g^{-1}(x) = \frac{3}{x+2}, \quad h^{-1}(x) = 2x^2 + 2$$



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### **Question 3**

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For each of the following functions find an expression for its inverse.

a) 
$$f(x) = \frac{x+2}{x}, \quad x \in \mathbb{R}, x \neq 0.$$

a) 
$$f(x) = \frac{x+2}{x}, \quad x \in \mathbb{R}, x \neq 0.$$
  
b)  $g(x) = \frac{2x-3}{x+4}, \quad x \in \mathbb{R}, x \neq -4.$   
c)  $h(x) = \frac{x-2}{2x-1}, \quad x \in \mathbb{R}, x \neq \frac{1}{2}.$ 

c) 
$$h(x) = \frac{x-2}{2x-1}, \quad x \in \mathbb{R}, x \neq \frac{1}{2}.$$

 $f^{-1}(x) = \frac{2}{x-1}$  $g^{-1}(x) = \frac{4x+3}{2-x}, h^{-1}(x) = \frac{x-2}{2x-1}$ 

Allo Also	(a) $\int_{0} \int_{-\frac{1}{2}} \frac{1}{24} g_{240}$ (b) $g_{30} = \frac{2x-3}{2x+4} + g_{14}$ (c) $(g_{30}) = \frac{2x-2}{2x-1}$ $= \int_{0}^{1} g_{2} = \frac{2x+3}{2x} = 2 \int_{0}^{1} g_{1} = \frac{2x-3}{2x+4}$ $= \int_{0}^{1} g_{1-2x+2} = 2 \int_{0}^{1} g_{1+2} g_{2+2} = 2 \int_{0}^{1} g_{2+2} g_{2+2} = 2 \int_{0$
Con	$\begin{array}{cccc} \Rightarrow \mathbf{X}(y_{1}-y_{2}) = -\lambda - \frac{1}{2y_{1}} & \Rightarrow \mathbf{X}(y_{2}-y_{1}) = -\lambda - \frac{1}{2y_{1}} & \Rightarrow \mathbf{X}(y_{2}-y_{1}) = \frac{1}{2y_{2}} \\ \Rightarrow \mathbf{X} = \frac{\pi}{y_{1}-1} & \Rightarrow \mathbf{X} = -\frac{1-y_{2}}{y_{2}-1} & \frac{1}{2y_{2}} & \Rightarrow \mathbf{X} = \frac{4y_{2}-1}{y_{2}-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \uparrow (y_{1}) = \frac{\pi}{y_{2}-1} & & & & & \\ \vdots & & & & & & \\ \vdots & & & & &$
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### **Question 4**

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For each of the following functions find an expression for its inverse.

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a) 
$$f(x) = 3 - 4x, x \in \mathbb{R}$$
.

**b**) 
$$g(x) = \frac{1}{x} + 2, \quad x \in \mathbb{R}, \ x \neq 0.$$
  
**c**)  $h(x) = \sqrt{x+5}, \quad x \in \mathbb{R}, \ x \ge -5$ 

) 
$$h(x) = \sqrt{x+5}, x \in \mathbb{R}, x \ge -5$$

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### Question 5

For each of the following functions find an expression for its inverse.

a) 
$$f(x) = 20 - 4x, x \in \mathbb{R}$$
.

**b**) 
$$g(x) = 5 - \frac{2}{x}, \quad x \in \mathbb{R}, \ x \neq 0.$$

b) 
$$h(x) = \sqrt{x-2}, x \in \mathbb{R}, x \ge 0$$
.

For each of the following functions find an expression for its inverse.  
a) 
$$f(x) = 20 - 4x$$
,  $x \in \mathbb{R}$ .  
b)  $g(x) = 5 - \frac{2}{x}$ ,  $x \in \mathbb{R}$ ,  $x \neq 0$ .  
c)  $h(x) = \sqrt{x} - 2$ ,  $x \in \mathbb{R}$ ,  $x \neq 0$ .  

$$\int \frac{1}{y^{-1}(x) = 5 - \frac{1}{4}x}, \quad \int \frac{1}{y^{-1}(x) = (x - 2)^{2}} h(x^{-1}(x) - (x - 2)^{2})$$

$$\int \frac{1}{y^{-1}(x) - \frac{1}{2}x^{-1}} h(x^{-1}(x) - (x - 2)^{2}) h(x^{-1}(x) - (x - 2)^{2})$$

$$\int \frac{1}{y^{-1}(x) - \frac{1}{2}x^{-1}} h(x^{-1}(x) - (x - 2)^{2}) h(x^{-1}(x) - (x - 2)^{2})$$

$$\int \frac{1}{y^{-1}(x) - \frac{1}{2}x^{-1}} h(x^{-1}(x) - (x - 2)^{2}) h(x^{-1}(x) - (x - 2)^{$$

for its inverse.	"COL	
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m.	- 12	1
$1(1)$ 2 $1^{-1}$	$(u) (u + 2)^2$	2
$(x) = \frac{1}{5-x}, \underline{n}$	(x) = (x+2)	Sh.
100		100.
		- 912
9=20-42 9= 5-2 9= 5-2	$y = \sqrt{2} - 2$	
$f_{2} = 20 - 4$ $\frac{2}{3} = 2 - 4$ $x = 5 - 44$ $\frac{2}{3} = \frac{1}{5 - 4}$	$(\underline{y}+2) = \sqrt{\chi}$	
$x = \frac{x}{2}$	$(1 + 1)^{-1} (1 + 2)^{-2}$	
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### Question 6

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For each of the following functions find an expression for its inverse.

a) 
$$f(x) = \frac{4}{x+1}, x \in \mathbb{R}, x \neq -1.$$

a) 
$$f(x) = \frac{4}{x+1}$$
,  $x \in \mathbb{R}, x \neq -1$ .  
b)  $g(x) = \frac{2x}{x+1}$ ,  $x \in \mathbb{R}, x \neq -1$ .  
c)  $h(x) = \frac{x+2}{x-4}$ ,  $x \in \mathbb{R}, x \neq 4$ .

c) 
$$h(x) = \frac{x+2}{x-4}, x \in \mathbb{R}, x \neq 4.$$

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$$f^{-1}(x) = \frac{4}{x} - 1, \quad g^{-1}(x) = \frac{x}{2 - x}, \quad h^{-1}(x) = \frac{4x + 2}{x - 1}$$
(9)  $\frac{\sqrt{3}}{2} = \frac{4}{2x+1}, \quad x \in \mathbb{R}, x + 1, \quad (k) = \frac{4}{3} = \frac{4}{3x+1}, \quad x \in \mathbb{R}, x \neq -1$ 
(9)  $\frac{\sqrt{3}}{2} = \frac{4}{2x+1}, \quad x \in \mathbb{R}, x \neq -1$ 
(9)  $\frac{\sqrt{3}}{2} = \frac{4}{3x+1}, \quad x \in \mathbb{R}, x \neq -1$ 



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### Question 7

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6

For each of the following functions find an expression for its inverse.

a) 
$$f(x) = \frac{x}{x-1}, x \in \mathbb{R}, x \neq 1$$

- $x \in \mathbb{R}$ . b) g(x) =
- c)  $h(x) = \ln(5-x), x \in \mathbb{R}, x > 5.$

2  
= ln(5-x), 
$$x \in \mathbb{R}, x > 5$$
.  
$$\int f^{-1}(x) = \frac{x}{x-1}, \quad g^{-1}(x) = \frac{1}{2} \ln 2x, \quad h^{-1}(x) = 5 - e^{x}$$
$$\begin{cases} 0 & \frac{1}{2} 0 - \frac{2}{2\pi}, & x \in \mathbb{R} \\ 0 & \frac{2}{3\pi}, & 0 & \frac{1}{2} e^{x}, & x \in \mathbb{R} \\ 0 & \frac{2}{3\pi}, & 0 & \frac{1}{2} e^{x}, & x \in \mathbb{R} \\ 0 & \frac{2}{3\pi}, & 0 & \frac{1}{2} e^{x}, & x \in \mathbb{R} \\ 0 & \frac{2}{3\pi}, & 0 & \frac{1}{2} e^{x}, & x \in \mathbb{R} \\ 0 & \frac{2}{3\pi}, & 0 & \frac{1}{2} e^{x}, & x \in \mathbb{R} \\ 0 & \frac{2}{3\pi}, & 0 & \frac{1}{2} e^{x}, & x \in \mathbb{R} \\ 0 & \frac{2}{3\pi}, & 0 & \frac{1}{3} e^{x}, & \frac{1}$$

U.S.	10.		2
1211	Th.	(a) $f(x) = \frac{x}{x-1}$ , $x \in \mathbb{R}^{1}$ , $x \notin 1$ • $f(x) = \frac{x}{x-1}$ • $f(x) = x$	(b) $g(\lambda) = \frac{1}{2} e^{2\lambda}$ , $x \in \mathbb{R}$ • $g = \frac{1}{2} e^{2\lambda}$
18	· · C	$y_{2,-2} = y$ $x(y_{-1}) = x$ $z = \frac{y_{-1}}{y_{-1}}$	$l_{n}(2g) = 2\lambda$ $\lambda = \frac{1}{2} l_{n}(2g)$ • $g^{2}(2) = \frac{1}{2} l_{n}(2\lambda)$
. "0)	2	• $f(a) = \frac{1}{2a-1}$ (c) $h(a) = h_1(s-a), x \in \mathbb{R}_1 \rightarrow 5^{-1}$	
1. 1. "	·	• $q = (p (2 - 3))$ • $q = (p (2 - 3))$	
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### Question 8

For each of the following functions find an expression for its inverse.

a) 
$$f(x) = 1 + 2e^{-x}, x \in \mathbb{R}$$

**b**) 
$$g(x) = 2 - \ln(x+1), x \in \mathbb{R}, x > -1.$$

) 
$$h(x) = \sqrt{e^x - 2}, \quad x \in \mathbb{R}, \ x \ge \ln 2.$$

![](_page_35_Figure_6.jpeg)

(a) $f(x) = 1 + 2e^{-x}$	(b) g(x) = 2-m(x+1)
$\implies 0 = 1 + 3e_x$	$\Rightarrow$ $y = 2 - \ln(\alpha + 1)$
⇒9-l= ve <sup>x</sup>	$\Rightarrow$ $\ln(\alpha_{H}) = 2-y$
$\Rightarrow \frac{1}{2}(y_{-1}) = e^{-2}$	= 2+1 = e <sup>2-y</sup>
$\implies -\pi = \mu\left(\frac{\pi}{\beta-1}\right)$	-) a= -1+p
$\implies \alpha = -\ln\left(\frac{y-1}{2}\right)$	
*: X(x) = - ly(2-1)	$i g(x) = -(+e^{2-x})$
	<i>F</i>
(G) h(x)= ve-2	
=> y = Ne-2	
$\Rightarrow y^2 = e^2 - 2$	
$\Rightarrow y^2 + 2 = -e^{\lambda}$	[12] h(-2.2) //
$\implies \ln(y^2+2) = \lambda$	$\cdots \eta(q) \equiv \eta(q+2)$

### **Question 9**

For each of the following functions find an expression for its inverse.

**a**) 
$$f(x) = \ln(x-2) + 3, x \in \mathbb{R}, x > 2$$

**b**) 
$$g(x) = \frac{1}{2}(e^{x-4}+3), x \in \mathbb{R}$$
.

## $f^{-1}(x) = e^{x-3}+2$ , $g^{-1}(x) = 4 + \ln(2x-3)$

- The second sec	
a) -f(a) = ln(x-2)+3	(b) $g(a) = \frac{1}{2} \left( e^{a-4} + 3 \right)$
3= m(x-2)+3	$\mathcal{Y} = \neq (e^{2} + 3)$
y-3 = m(2-2)	2y= e+3
e <sup>9-3</sup> = x - 2	$2y - 3 = e^{x - 4}$
$5 + 6_{3-3} = \mathcal{I}$	ln(2y-3) = 2-4
: f(a) = e +2	$4 + \ln(2y-3) = 3$
	: g(a) = 4 + ln(21-3)

### Question 10

A function f is defined by

$$(x) = x^2 - 9, x \in \mathbb{R}, x \ge 0.$$

- **a**) Find an expression for  $f^{-1}(x)$
- **b**) Find the domain and range of  $f^{-1}(x)$ .

![](_page_36_Figure_6.jpeg)

Question 11

A function f is defined by

 $f(x) = (x-1)^2, x \in \mathbb{R}, x \ge 1.$ 

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Find the domain and range of  $f^{-1}(x)$ .

 $f^{-1}(x) \ge 1$  $|x| = 1 + \sqrt{x}$ ,  $x \ge 0$ ,

Con las on the set		,
(a) +(a) = (2-1) 1 221	(6)	-L
$= 4 = (\alpha - 1)^2$	(90)	
-> ±1/4=2-1	(91)	
7+19=2-1	y=x + +'.	
- 7-12 Ju	D 221 220	Spanen: 200
E. Oll	R (0)>0 +0)>1	RANGE: AGN >1

### Question 12

A function f is defined by

 $f(x) = \sqrt{x+4}, \ x \in \mathbb{R}, \ 0 \le x < 5.$ 

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Find the domain and range of  $f^{-1}(x)$ .

![](_page_37_Figure_6.jpeg)

19.0	<u>.</u>	
(a) f(a)=1/2+4, 05x<5	4 (AS) .	
$\Rightarrow y_{e} \sqrt{x_{e+4}}$ $\Rightarrow y_{e}^{2} = x_{e+4}$ $\Rightarrow \sqrt{y_{e-4}} = x$	600 / 2+1 22+0 22+1	
$\therefore  \chi_{(0)}^{p-1} = \chi_{-+}^{2}$	D pears 2523 R 26/10/3 06/60/55	" Darmin: 262<3 Range of A(1) 5

Question 13

A function f is defined by

$$f(x) = e^{2x} - 1, \ x \in \mathbb{R}.$$

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Find the domain and range of  $f^{-1}(x)$ .

![](_page_37_Picture_13.jpeg)

fa)= e-1, xel	4 y / fee)
$\Rightarrow y = e^{2\lambda}$	
$\Rightarrow g + i = 2\lambda$	
$\Longrightarrow \alpha = \frac{1}{2} \ln(y + i)$	$D = \frac{1}{2 \in \mathbb{R}} \frac{1}{\alpha > 1}$
$\therefore f(x) = \pm ln(x+1)$	R(40)>-1(0)+R(
	Domain: 2>-1 Range : fg)ER

### Question 14

A function f is defined by

$$f(x) = \frac{1}{2}e^{x} + 1, x \in \mathbb{R}, x \le 0.$$

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Find the domain and range of  $f^{-1}(x)$ .

![](_page_38_Picture_6.jpeg)

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Question 15

A function f is defined by

$$f(x) = x^2 + 1, \ x \in \mathbb{R}, \ x \ge 0.$$

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Find the domain and range of  $f^{-1}(x)$

# $f^{-1}(x) = \sqrt{x-1}$ , $x \ge 1$ , $f^{-1}(x) \ge 0$

$\begin{array}{cccc} f(0) = \alpha^2 + 1 &, \lambda \geqslant 0 &, y_A \\ \hline & & y_G = \alpha^2 + 1 &, \lambda \geqslant 0 &, y_A \\ \hline & & & y_G = 1 = \alpha^2 &, y_G = 1 &, \lambda \geqslant 0 &, y_A \\ \hline & & & & & y_G = 1 &, \lambda \geqslant 0 &, y_A \\ \hline & & & & & & & & & & & \\ \hline & & & & &$			
$ \begin{array}{c} \vdots \\ f(x) = \sqrt{2\pi}^{-1} \\ R \\ f(x) = 1 \\ \hline \\ R \\ f(x) \geq 1 \\ \hline \\ \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$ \begin{array}{l} f(t) = x^{2} + 1 \ , \ x \geqslant 0 \\ \hline \begin{array}{l} \hline \end{array} \\ \hline \begin{array}{l} \hline \end{array} \\ \hline \begin{array}{l} \hline \end{array} \\ \end{array} \\ \begin{array}{l} \hline \end{array} \\ \begin{array}{l} \hline \end{array} \\ \end{array} \\ \begin{array}{l} \hline \end{array} \\ \begin{array}{l} \hline \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{l} \hline \end{array} \\ \end{array} \end{array} \\ \begin{array}{l} \hline \end{array} \end{array} \\ \end{array} \\ \begin{array}{l} \hline \end{array} \end{array} \\ \end{array} \\ \end{array} \end{array} \\ \begin{array}{l} \hline \end{array} \end{array} \\ \end{array} \end{array} \\ \end{array} \end{array} \\ \begin{array}{l} \hline \end{array} \end{array} \\ \begin{array}{l} \hline \end{array} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \end{array} \\ \end{array} \end{array} \\ \end{array} \end{array} \\ \end{array} \\ \end{array} \end{array} \\ \\ \end{array} \\ \end{array} \\ \end{array} \\ \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \\ \end{array} \\ \end{array} \\ \end{array} \\ \\ \\ \end{array} \\ \\ \end{array} \\ \\ \\ \end{array} \\ \\ \\ \end{array} \\ \\ \\ \end{array} \\ \\ \end{array} \\ \\ \\ \end{array} \\ \\ \\ \end{array} \\ \\ \end{array} \\ \\ \end{array} \\ \\ \end{array} \\ \\ \\ \end{array} \\ \\ \\ \\ \end{array} \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\$	4 10 10 10 10 10 10 10 10 10 10	
Douthus : 2≥1 RASCE : £67≥9/		Donatin): 2≥1 RANCE: \$G7>9	/

### Question 16

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A function f is defined by

$$f(x) = (x+2)^2, x \in \mathbb{R}, x \ge -2.$$

 $^{-1}(x) = -2 + \sqrt{x}$ 

- **a**) Find an expression for  $f^{-1}(x)$
- **b**) Sketch in the same diagram the graphs of f(x) and  $f^{-1}(x)$ .
- c) Find the domain and range of  $f^{-1}(x)$ .

![](_page_39_Picture_7.jpeg)

 $-1(x) \ge -2$ 

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 $x \ge 0$ ,

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### Question 17

.G.B.

I.C.B.

A function f is defined by

 $f(x) = 1 + \sqrt{x-2}, \ x \in \mathbb{R}, x \ge 6.$ 

**a**) Find an expression for  $f^{-1}(x)$ .

**b**) Sketch in the same diagram the graphs of f(x) and  $f^{-1}(x)$ .

c) Find the domain and range of  $f^{-1}(x)$ .

 $f^{-1}(x) = 2 + (x-1)^2$ ,  $x \ge 3$ ,  $f^{-1}(x) \ge 0$ 

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![](_page_40_Picture_8.jpeg)

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### Question 18

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A function f is defined by

$$f(x) = 2 + \frac{1}{x+1}, x \in \mathbb{R}, x \ge 0$$

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Sketch in the same diagram the graphs of f(x) and  $f^{-1}(x)$ .
- c) Find the domain and range of  $f^{-1}(x)$ .

![](_page_41_Figure_7.jpeg)

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 $f^{-1}(x) = \frac{3-x}{x-2}, \ 2 < x \le 3, \ f^{-1}(x) \ge 0$ 

### Question 19

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A function f is defined by

$$f(x) = 4 - \frac{1}{x - 1}, x \in \mathbb{R}, x > 1$$

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Find the domain and range of  $f^{-1}(x)$ .

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![](_page_42_Picture_6.jpeg)

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Created by T. Madas

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### Question 20

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A function f is defined by

$$f(x) = 2\ln(1-x), x < 1.$$

 $f^{-1}(x) = 1 - e^{\frac{1}{2}x}$ 

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Sketch in the same diagram the graphs of f(x) and  $f^{-1}(x)$ .
- c) Find the domain and range of  $f^{-1}(x)$ .

$ \begin{array}{l} (\mathbf{\hat{a}})  \frac{1}{2} (\mathbf{\hat{a}}) = 2  \mathbf{\hat{a}} _{1}(-\mathbf{x})_{1} _{2} < 1  (\mathbf{\hat{b}}) \\ \Rightarrow \mathbf{\hat{b}} = 2 \mathbf{\hat{b}}(1-\mathbf{x}) \\ \Rightarrow \mathbf{\hat{b}} = 2 \mathbf{\hat{b}}(1-\mathbf{x}) \\ \Rightarrow \mathbf{\hat{c}} = 1 - \mathbf{x} \\ \Rightarrow \mathbf{x} = 1 - \mathbf{e}^{\frac{N}{2}} \\ \Rightarrow \mathbf{\hat{c}}^{\frac{N}{2}} = 1 - \mathbf{x} \\ \Rightarrow \mathbf{\hat{c}}^{\frac{N}{2}} = 1 - \mathbf{\hat{c}} \\ \end{array} $	$\begin{array}{c} \ln \alpha_{\star} & \begin{array}{c} & & \\ \downarrow & \\ h_{(2x1)} & & \\ \downarrow & \\ h_{(-x1)} & \\ \end{pmatrix} & \begin{array}{c} & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & \\ & & \\ & & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ \end{pmatrix} & \begin{array}{c} & & \\ & \\ & $
$\begin{array}{c} (C)  \begin{array}{c} \downarrow & \downarrow^{-1} \\ \hline D  2 < (  2 \in \mathbb{R} \\ \hline \mathbb{R}  10 \in \mathbb{R}  10 < 1 \\ \hline D  10 \text{ MM} \text{ i}  2 \in \mathbb{R} \\ \hline \text{Rhote:}  10 < 1 \end{array}$	10 5 1 9 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

,  $x \in \mathbb{R}$ 

 $f^{-1}(x) < 1$ 

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### Question 21

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I.C.B.

A function f is defined by

$$f(x) = 2 - 3\sin x, \quad -\frac{\pi}{2} \le x < \frac{\pi}{2}.$$

- **a**) Find an expression for  $f^{-1}(x)$ .
- **b**) Find the domain and range of  $f^{-1}(x)$ .

![](_page_44_Figure_6.jpeg)

F.G.B.

(a) {(a)=2-30M2	(b)	f	$\mathcal{Q}^{\rightarrow}$
⇒ y = 2-35142 ⇒ 35142 = 7-4	D	$-\frac{1}{2}\leq\lambda\leq\frac{1}{2}$	-1=3=2
$\Rightarrow$ SIND = $\frac{2-y}{3}$	R	$-1 \leq f(\sigma) \leq 2$	- <u>7</u> EA(6) E #
$\Rightarrow f_{(2)}^{(2)} = \arg(M(\frac{2-3}{3}))$		No Do Muhol : -1 RANCE - <u>∏</u>	$\leq \lambda \leq S$
			<i>.</i>

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### Question 22

F.G.B.

I.C.B.

A function f is defined by

$$f(x) = 2x^2 + 5, x \in \mathbb{R}, x \ge 0.$$

**a**) Find an expression for  $f^{-1}(x)$ 

**b**) Sketch in the same diagram the graphs of f(x) and  $f^{-1}(x)$ .

c) Find the domain and range of  $f^{-1}(x)$ .

$(a)  f(a) = 3t_{a} + 2^{1}  x \ge 0$	(b) y 100, ~,
$\Rightarrow y = 3a^2 + s$ $\Rightarrow y - s = 3a^2$	(a)
$\Rightarrow \mathcal{I}_{z} = \mp \sqrt{\frac{\pi}{2}(\beta - 2)}$	r (5(0))
$\Rightarrow x = \pm \sqrt{\frac{y-1}{2}}$	(c) 0 0-1
=> +(1) = V ==	D 20 2 ass
	R +(3)>5 +(3)>0 Douted: 2>5
	RANG: (a) >0

 $x \ge 5, \quad f^{-1}(x) \ge 0$ 

I.C.P.

 $\frac{x-\overline{5}}{2}$ 

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 $f^{-1}(x) = \sqrt{1}$ 

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### Question 23

Ĉ.B.

A function f is defined by

 $f(x) = x^2 - 4x - 1, x \in \mathbb{R}, x < 0.$ 

a) By completing the square, or otherwise, find an expression for  $f^{-1}(x)$ .

**b**) Sketch in the same diagram the graphs of f(x) and  $f^{-1}(x)$ .

c) Find the domain and range of  $f^{-1}(x)$ .

# $f^{-1}(x) = 2 - \sqrt{x+5}$ , x > -1, $f^{-1}(x) < 0$

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i.G.B.

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![](_page_46_Picture_8.jpeg)