CURVE SKETCHING EXAM QUESTIONS

Question 1 (**)

$$f(x) = x^2 + 6x + 10, \; x \in \mathbb{R}.$$

a) Express $f(x)$ in the form

$$f(x) = (x+a)^2 + b,$$

where a and b are integers.

b) Describe geometrically the transformations which map the graph of x^2 onto the graph of $f(x)$.

$\boxed{a=3}$, $\boxed{b=1}$, $\boxed{\text{translation by } \begin{pmatrix} -3 \\ 1 \end{pmatrix}}$

(a) $x^2+6x+10 = (x+3)^2-3^2+10 = (x+3)^2-9+10$
$= (x+3)^2+1$

(b) $x^2 \xmapsto[\text{TRANSLATION, "LEFT", BY 3 UNITS}]{} (x+3)^2 \xmapsto[\text{TRANSLATION, "UPWARDS", BY 1 UNIT}]{} (x+3)^2+1$

OR TRANSLATION BY VECTOR $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

Question 2 (+)**

The curve C has equation

$$y = (x-a)^2 + b,$$

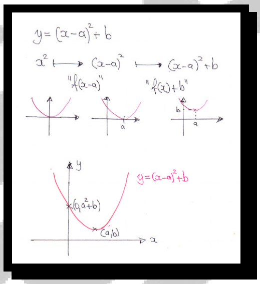
where a, b are positive constants.

By considering the two transformations that map the graph of $y = x^2$ onto the graph of C, or otherwise, sketch the graph of C.

The sketch must include the coordinates, in terms of a, b, of …

- … all the points where the curve meets the coordinate axes.
- … the maximum point of the curve.

graph



Question 3 (***)

$$f(x) = \sqrt{x}\,,\ x \in \mathbb{R}\,,\ x \geq 0\,.$$

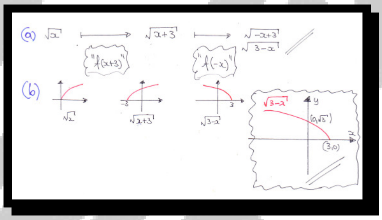
The graph of $f(x)$ is translated by 3 units in the negative x direction, followed by a reflection in the y axis, forming the graph of $g(x)$.

a) Find the equation of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include the coordinates of all the points where the curve meets the coordinate axes.

$$\boxed{g(x) = \sqrt{3-x}}$$



Question 4 (***)

The curve C has equation

$$y = x^3 - 9x\,.$$

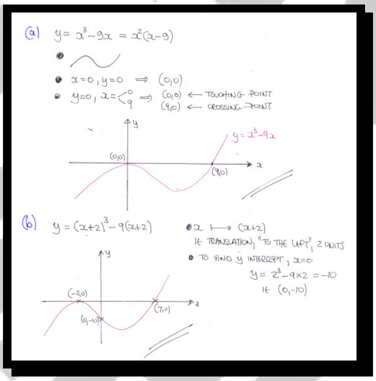
a) Sketch the graph of C.

b) Hence sketch on a separate diagram the graph of

$$y = (x+2)^3 - 9(x+2)\,.$$

Both sketches must include the coordinates of all the points where each of the curves meets the coordinate axes.

graph



Question 5 **(***)**

A curve is defined by the equation

$$f(x) = (x-a)(x-b)^2(x-c)^3, \; x \in \mathbb{R},$$

where a, b and c are constants.

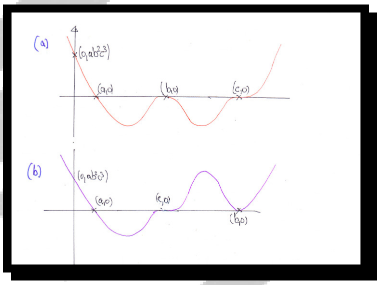
Sketch the graph of $f(x)$ in each of the following cases.

a) $0 < a < b < c$

b) $0 < a < c < b$.

Each sketch must clearly show any intercepts with the coordinate axes, in terms of a, b and c, where appropriate.

graph



Question 6 (*+)**

$$f(x) = x^2 - 2x - 8, \ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $f(x) = (x+a)^2 + b$, where a and b are integers.

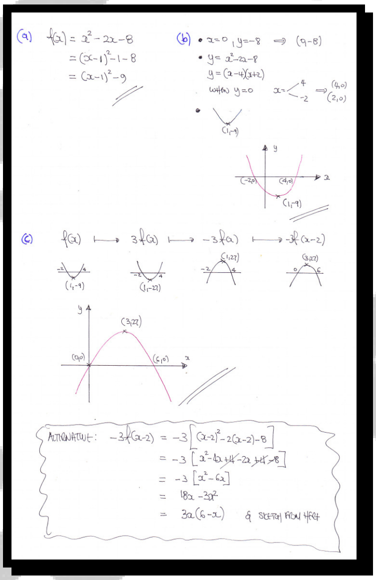
b) Sketch the graph of $f(x)$.

a) By considering a series of three geometrical transformations, sketch the graph of $y = -3f(x-2)$.

Both sketches must include the coordinates of …

- … all the points where the curves meets the coordinate axes.
- … the minimum or maximum points of the curves.

$\boxed{a = -1}$, $\boxed{b = -9}$



Question 7 **(***+)**

The curve C has equation

$$y = 9 - (x-2)^2 .$$

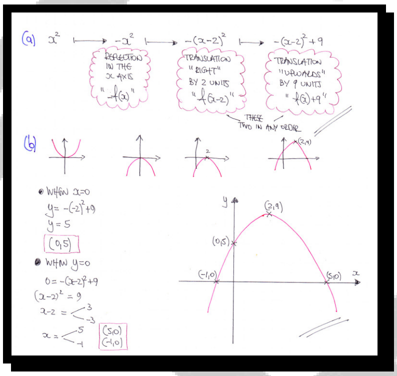
a) Describe geometrically the three transformations that map the graph of $y = x^2$ onto the graph of C.

b) Hence, sketch the graph of C.

The sketch must include the coordinates of

- … all the points where the curve meets the coordinate axes.
- … the maximum point of the curve.

$\boxed{\text{reflection in the } x \text{ axis}}$, $\boxed{\text{translation "right" by 2 units}}$,
$\boxed{\text{translation "upwards" by 9 units}}$



Question 8 (*+)**

The curve C has equation

$$y = \frac{2x+3}{x-2}, \; x \in \mathbb{R}, \; x \neq 2.$$

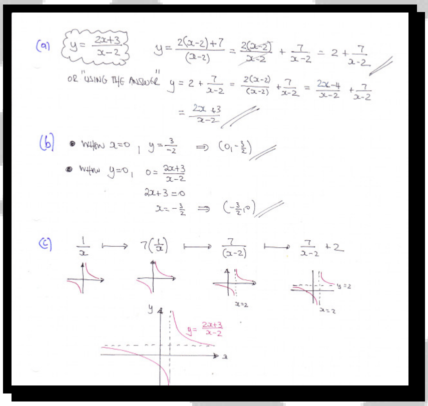
a) Show clearly that

$$\frac{2x+3}{x-2} \equiv 2 + \frac{7}{x-2}.$$

b) Find the coordinates of the points where C meets the coordinate axes.

c) Sketch the graph of C showing clearly the equations of any asymptotes.

$$\boxed{\left(0,-\frac{3}{2}\right),\left(-\frac{3}{2},0\right)}$$



Question 9 (***+)

$$f(x) = \frac{1}{x}, \; x \in \mathbb{R}, \; x \neq 0.$$

$$g(x) = \frac{1}{x+2} + 2, \; x \in \mathbb{R}, \; x \neq -2.$$

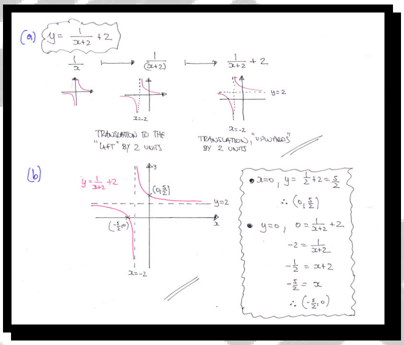
a) Describe mathematically the two transformations that map the graph of $f(x)$ onto the graph of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of any asymptotes of the curve.

translation "left" by 2 units, followed by translation "upwards" by 2 unit



Question 10 (***+)

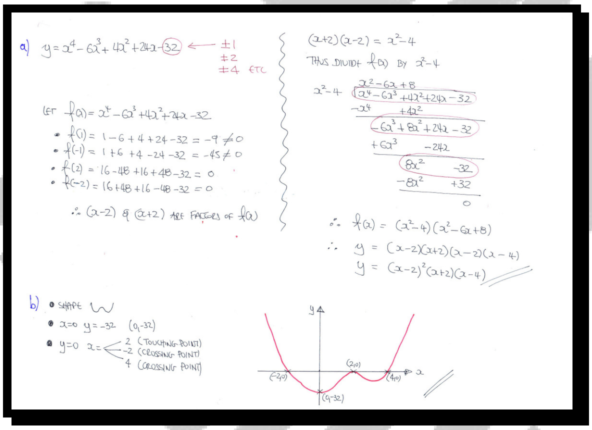
The curve C has equation

$$y = x^4 - 6x^3 + 4x^2 + 24x - 32 .$$

a) Express y as the product of four linear factors.

b) Hence the graph of C, showing clearly the coordinates of any points where the graph of C meets the coordinate axes.

$$\boxed{y = (x+2)(x-4)(x-2)^2}$$



a) $y = x^4 - 6x^3 + 4x^2 + 24x - 32 \leftarrow \pm 1, \pm 2, \pm 4$ ETC

LET $f(x) = x^4 - 6x^3 + 4x^2 + 24x - 32$

- $f(1) = 1 - 6 + 4 + 24 - 32 = -9 \neq 0$
- $f(-1) = 1 + 6 + 4 - 24 - 32 = -45 \neq 0$
- $f(2) = 16 - 48 + 16 + 48 - 32 = 0$
- $f(-2) = 16 + 48 + 16 - 48 - 32 = 0$

$\therefore (x-2)$ & $(x+2)$ ARE FACTORS OF $f(x)$

$(x+2)(x-2) = x^2 - 4$

THUS DIVIDE $f(x)$ BY $x^2 - 4$

$$\begin{array}{r l}
 & x^2 - 6x + 8 \\
x^2 - 4 \,\big) & x^4 - 6x^3 + 4x^2 + 24x - 32 \\
 & -x^4 \qquad\quad + 4x^2 \\
 & \quad -6x^3 + 8x^2 + 24x - 32 \\
 & \quad +6x^3 \qquad\quad - 24x \\
 & \qquad\qquad 8x^2 \qquad - 32 \\
 & \qquad\quad -8x^2 \qquad + 32 \\
 & \qquad\qquad\qquad\qquad 0
\end{array}$$

$\therefore f(x) = (x^2 - 4)(x^2 - 6x + 8)$

$\therefore y = (x-2)(x+2)(x-2)(x-4)$

$y = (x-2)^2(x+2)(x-4)$

b)
- SHAPE W
- $x = 0 \quad y = -32 \quad (0, -32)$
- $y = 0 \quad x = 2$ (TOUCHING POINT), -2 (CROSSING POINT), 4 (CROSSING POINT)

Question 11 (****)

$$f(x) = \frac{x-2}{x-3}, \ x \in \mathbb{R}, \ x \neq 3.$$

b) Express $f(x)$ in the form

$$f(x) = a + \frac{1}{x+b},$$

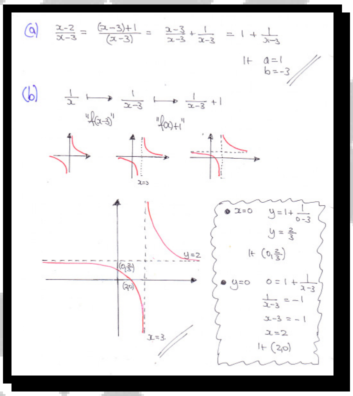
where a and b are integers.

c) By considering a series of transformations which map the graph of $\frac{1}{x}$ onto the graph of $f(x)$, sketch the graph of $f(x)$.

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

$\boxed{a=1}$, $\boxed{b=-3}$



Question 12 (****)

A cubic curve C has equation

$$y = (3-x)(4+x)^2.$$

a) Sketch the graph of C.

The sketch must include any points where the graph meets the coordinate axes.

b) Sketch in separate diagrams the graph of …

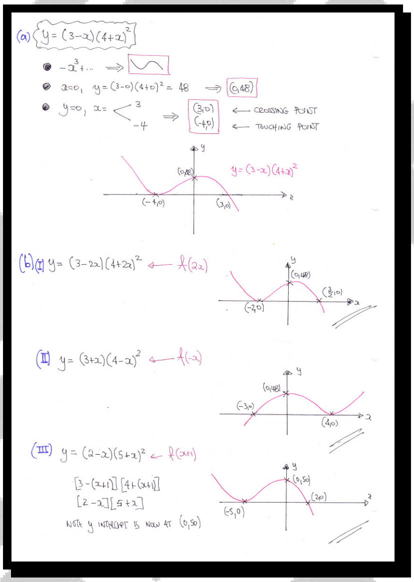
i. … $y = (3-2x)(4+2x)^2$.

ii. … $y = (3+x)(4-x)^2$.

iii. … $y = (2-x)(5+x)^2$.

The sketches must include any points where each of the graphs meets the coordinate axes.

graph



Question 13 (****)

The curve C has equation $y = f(x)$ given by

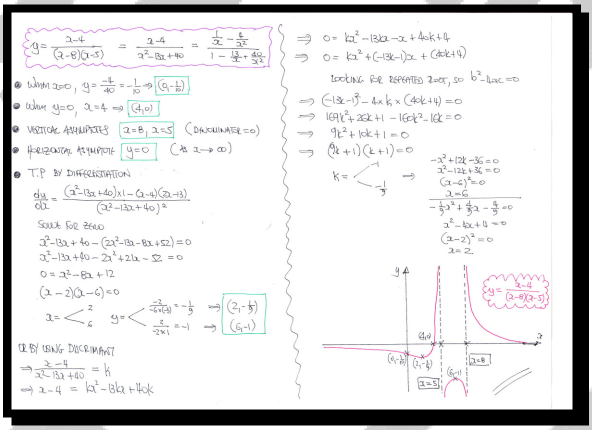
$$f(x) = \frac{x-4}{(x-5)(x-8)}, \quad x \in \mathbb{R}, \quad x \neq 5, \quad x \neq 8.$$

Sketch the graph of C.

Indicate clearly in the sketch …

- … the equations of the asymptotes
- … the coordinates of any intersections of C with the coordinate axes.
- … the coordinates of any turning points of C.

graph



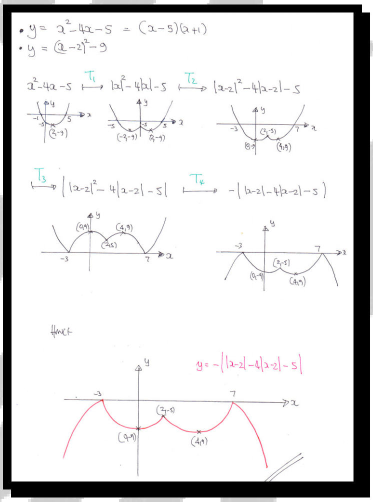
Question 14 (*****)

By considering a sequence of four transformations, or otherwise, sketch the graph of

$$y = -\left| |x-2|^2 - 4|x-2| - 5 \right|.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

$$\boxed{(-3,0),(7,0),(0,-9),(2,-5)}$$



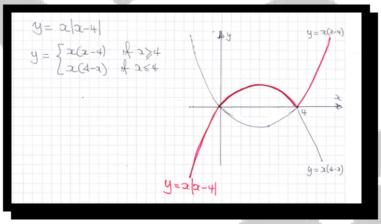
Question 15 (*****)

By considering the graphs of two separate curves, or otherwise, sketch the graph of

$$y = x|x-4|.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

$$\boxed{(0,0),(4,0)}$$



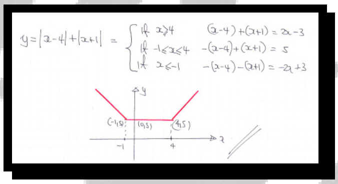
Question 16 **(*****)**

By considering the graphs of three separate lines, or otherwise, sketch the graph of

$$y = |x-4| + |x+1|$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

$\boxed{(-1,5),(0,5),(4,5)}$



Question 17 **(*****)**

The curve C has equation

$$y = \frac{x^2-4}{|x+5|}, \ x \in \mathbb{R}, \ x \neq -5.$$

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the asymptotes of the curve.

$\boxed{\text{graph}}$