TRIGONOMETRY EXAM QUESTIONS INTRODUCTION

Question 1 (**+)

Solve the following trigonometric equation in the range given.

$$\cos(2\theta + 25)^\circ = -0.454, \quad 0 \le \theta < 360.$$

$\boxed{}$, $\boxed{\theta \approx 46, 109, 226, 289}$

$\cos(2\theta + 25) = -0.454$

$\arccos(-0.454) = 117.0^\circ$

$\begin{cases} 2\theta + 25 = 117.0 \pm 360n \\ 2\theta + 25 = 243.0 \pm 360n \end{cases}$

$n = 0, 1, 2, 3, \ldots$

$\begin{cases} 2\theta = 92.0^\circ \pm 360n \\ 2\theta = 218.0^\circ \pm 360n \end{cases}$

$\begin{cases} \theta = 46.0^\circ \pm 180n \\ \theta = 109.0^\circ \pm 180n \end{cases}$

$\theta_1 = 46.0^\circ$

$\theta_2 = 226.0^\circ$

$\theta_3 = 109.0^\circ$

$\theta_4 = 289.0^\circ$

Question 2 (**+)

Solve the following trigonometric equation in the range given.

$$\cos(2y - 35)^\circ = 0.891, \quad 0 \le y < 360.$$

$\boxed{}$, $\boxed{y \approx 4, 31, 184, 211}$

$\cos(2y - 35) = 0.891 \quad 0 \le y < 360$

$\arccos(0.891) = 27.000823\ldots \simeq 27.0^\circ$

$\Rightarrow \begin{cases} 2y - 35 = 27.0^\circ \pm 360n \\ 2y - 35 = 333.0^\circ \pm 360n \end{cases} \quad n = 0, 1, 2, 3, \ldots$

$360 - 27$

$\Rightarrow \begin{cases} 2y = 62 \pm 360n \\ 2y = 368 \pm 360n \end{cases}$

$\Rightarrow \begin{cases} y = 31 \pm 180n \\ y = 184 \pm 180n \end{cases}$

LOOKING AT THE REQUIRED RANGE

$y_1 = 31^\circ$

$y_2 = 211^\circ$

$y_3 = 184^\circ$

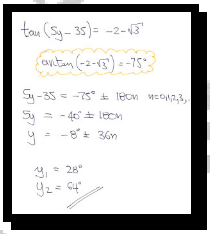
$y_4 = 4^\circ$

Question 3 (**+)

Solve the following trigonometric equation in the range given.

$$\tan(5y-35)^\circ = -2-\sqrt{3}, \quad 0 \le y < 90.$$

$\boxed{y \approx 28,\ 64}$


$\tan(5y-35) = -2-\sqrt{3}$

$\arctan(-2-\sqrt{3}) = -75^\circ$

$5y-35 = -75^\circ \pm 180n \quad n=0,1,2,3,\ldots$

$5y = -40^\circ \pm 180n$

$y = -8^\circ \pm 36n$

$y_1 = 28^\circ$

$y_2 = 64^\circ$

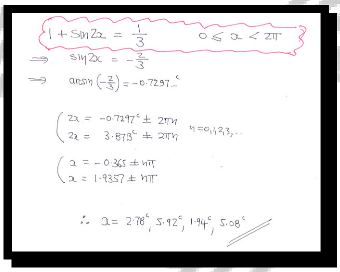
Question 4 (**+)

Solve, in **radians**, the following trigonometric equation

$$1+\sin 2x = \frac{1}{3}, \quad 0 \le x < 2\pi,$$

giving the answers correct to three significant figures.

$\boxed{x = 1.94^c,\ 2.78^c,\ 5.08^c,\ 5.92^c}$


$1+\sin 2x = \frac{1}{3} \quad 0 \le x < 2\pi$

$\Rightarrow \sin 2x = -\frac{2}{3}$

$\Rightarrow \arcsin\left(-\frac{2}{3}\right) = -0.7297\ldots^c$

$2x = -0.7297^c \pm 2\pi n$

$2x = 3.8713^c \pm 2\pi n \quad n=0,1,2,3,\ldots$

$x = -0.365 \pm n\pi$

$x = 1.9357 \pm n\pi$

$\therefore x = 2.78^c, 5.92^c, 1.94^c, 5.08^c$

Question 5 (**+)

Solve the following trigonometric equation in the range given.

$$2\cos\theta = \sin\theta, \quad 0° \le \theta < 360° .$$

$\boxed{\theta = 63.4°,\ 243.4°}$

$2\cos\theta = \sin\theta \qquad 0 \le \theta < 360$

$\Rightarrow 2\cos\theta = \sin\theta$

$\Rightarrow \frac{2\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow 2 = \tan\theta$

• $\arctan(2) = 63.4°$

$\theta = 63.4° \pm 180n \qquad n = 0,1,2,3,\ldots$

$\theta_1 = 63.4°$
$\theta_2 = 243.4°$

Question 6 (**+)

Solve the following trigonometric equation in the range given.

$$2\sin\theta = 5\cos\theta, \quad 0° \le \theta < 360° .$$

$\boxed{}$, $\boxed{x \approx 68.2°,\ 248.2°}$

$2\sin\theta = 5\cos\theta \qquad 0° \le \theta < 360°$

$\Rightarrow 2\sin\theta = 5\cos\theta$

$\Rightarrow \frac{2\sin\theta}{\cos\theta} = \frac{5\cos\theta}{\cos\theta}$

$\Rightarrow 2\tan\theta = 5$

$\Rightarrow \tan\theta = \frac{5}{2}$

• $\arctan\left(\frac{5}{2}\right) = 68.2°$

$\theta = 68.2° \pm 180n \qquad n = 0,1,2,3,4,\ldots$

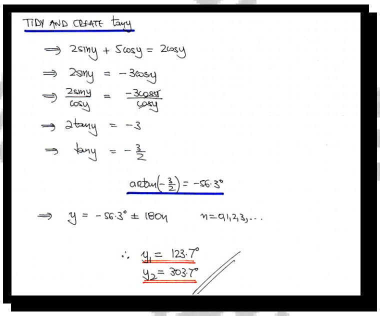
$\therefore \theta_1 = 68.2°$
$\theta_2 = 248.2°$

Question 7 (**+)

Solve the following trigonometric equation in the range given.

$$2\sin y + 5\cos y = 2\cos y, \quad 0° \le y < 360°.$$

$\boxed{}$, $\boxed{y \approx 123.7°,\ 303.7°}$



TIDY AND CREATE $\tan y$

$\Rightarrow 2\sin y + 5\cos y = 2\cos y$

$\Rightarrow 2\sin y = -3\cos y$

$\Rightarrow \dfrac{2\sin y}{\cos y} = \dfrac{-3\cos y}{\cos y}$

$\Rightarrow 2\tan y = -3$

$\Rightarrow \tan y = -\frac{3}{2}$

$\arctan\left(-\frac{3}{2}\right) = -56.3°$

$\Rightarrow y = -56.3° \pm 180n \qquad n = 0,1,2,3,\ldots$

$\therefore\ y_1 = 123.7°$

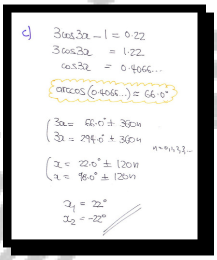
$y_2 = 303.7°$

Question 8 (***)

Solve the following trigonometric equation in the range given.

$$3\cos 3x - 1 = 0.22, \quad -90° \le x < 90°.$$

$\boxed{x \approx -22°, 22°}$



c) $3\cos 3x - 1 = 0.22$

$3\cos 3x = 1.22$

$\cos 3x = 0.4066\ldots$

$\arccos(0.4066\ldots) \approx 66.0°$

$\begin{cases} 3x = 66.0° \pm 360n \\ 3x = 294.0° \pm 360n \end{cases}$ $\quad n = 0,1,2,3,\ldots$

$\begin{cases} x = 22.0° \pm 120n \\ x = 98.0° \pm 120n \end{cases}$

$x_1 = 22°$

$x_2 = -22°$

Question 9 (***)

Solve the following trigonometric equation in the range given.

$$1 + 2\sin(\theta + 25)^\circ = 2.532, \quad 0 \le \theta < 360.$$

$$\boxed{\theta \approx 25,\ 105}$$

$$1 + 2\sin(\theta + 25^\circ) = 2.532$$
$$2\sin(\theta + 25^\circ) = 1.532$$
$$\sin(\theta + 25^\circ) = 0.766$$
$$\arcsin(0.766) = 50.0^\circ$$
$$\begin{cases} \theta + 25 = 50.0^\circ \pm 360n \\ \theta + 25 = 130.0^\circ \pm 360n \end{cases} \quad n = 0, 1, 2, 3 \ldots$$
$$\begin{cases} \theta = 25 \pm 360n \\ \theta = 105 \pm 360n \end{cases}$$
$$\theta_1 = 25^\circ$$
$$\theta_2 = 105^\circ$$

Question 10 (***)

Solve, in **radians**, the following trigonometric equation

$$4\sin^2\psi = 15\cos\psi, \quad 0 \le \psi < 2\pi,$$

giving the answers correct to three significant figures.

$$\boxed{\psi \approx 1.32^c, 4.97^c}$$

$$4\sin^2\psi = 15\cos\psi, \quad 0 \le \psi < 2\pi$$
$$4\sin^2\psi = 15\cos\psi$$
$$\Rightarrow 4(1 - \cos^2\psi) = 15\cos\psi$$
$$\Rightarrow 4 - 4\cos^2\psi = 15\cos\psi$$
$$\Rightarrow 0 = 4\cos^2\psi + 15\cos\psi - 4$$
$$\Rightarrow 0 = (4\cos\psi - 1)(\cos\psi + 4)$$
$$\Rightarrow \cos\psi = \cancel{-4},\ \frac{1}{4}$$
$$\arccos\left(\frac{1}{4}\right) = 1.318^c$$
$$\begin{cases} \psi = 1.318^c \pm 2\pi n \\ \psi = 4.965^c \pm 2\pi n \end{cases} \quad n = 0, 1, 2, 3, \ldots$$
$$\psi_1 = 1.32^c$$
$$\psi_2 = 4.97^c$$

Question 11 (***)

Solve the following trigonometric equation in the range given.

$$4\sin 2\theta + 3\cos 2\theta = 0\,, \quad 0° \le \theta < 360°\,.$$

$$\boxed{\theta = 71.6°, 161.6°, 251.6°, 341.6°}$$

$\boxed{4\sin 2\theta + 3\cos 2\theta = 0 \quad 0 \le \theta < 360}$

$\Rightarrow 4\sin 2\theta + 3\cos 2\theta = 0$

$\Rightarrow 4\sin 2\theta = -3\cos 2\theta$

$\Rightarrow \frac{4\sin 2\theta}{\cos 2\theta} = -\frac{3\cos 2\theta}{\cos 2\theta}$

$\Rightarrow 4\tan 2\theta = -3$

$\Rightarrow \tan 2\theta = -\frac{3}{4}$

- $\arctan\left(-\frac{3}{4}\right) = -36.87°$

$2\theta = -36.87° \pm 180n \quad n = 0,1,2,3\ldots$

$\theta = -18.4° \pm 90n$

$\theta = 71.6°, 161.6°, 251.6°, 341.6°$

Question 12 (***)

Solve the following trigonometric equation in the range given.

$$2 + 2\sin 3\varphi = 1\,,\ 0° \le \varphi < 180°\,.$$

$$\boxed{\varphi = 70°,\ 110°}$$

$\boxed{2 + 2\sin 3\phi = 1, \quad 0 \le \phi < 180}$

$\Rightarrow 2 + 2\sin 3\phi = 1$

$\Rightarrow 2\sin 3\phi = -1$

$\Rightarrow \sin 3\phi = -\frac{1}{2}$

$\arcsin\left(-\frac{1}{2}\right) = -30°$

THUS

$\begin{cases} 3\phi = -30° \pm 360n \\ 3\phi = 210° \pm 360n \end{cases} \quad n = 0,1,2,3,\ldots$

$\begin{cases} \phi = -10° \pm 120n \\ \phi = 70° \pm 120n \end{cases}$

THUS FOR THE RANGE GIVEN

$\phi_1 = 110°$

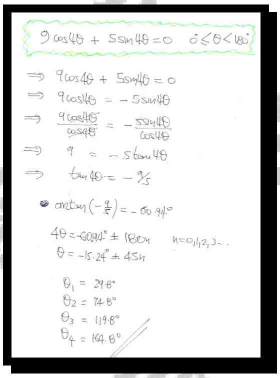
$\phi_2 = 70°$

Question 13 (***)

Solve the following trigonometric equation in the range given.

$$9\cos 4\theta + 5\sin 4\theta = 0, \quad 0° \le \theta < 180°.$$

$$\boxed{\theta \approx 29.8°,\ 74.8°,\ 119.8°,\ 164.8°}$$


$9\cos 4\theta + 5\sin 4\theta = 0 \quad 0° \le \theta < 180°$

$\Rightarrow 9\cos 4\theta + 5\sin 4\theta = 0$

$\Rightarrow 9\cos 4\theta = -5\sin 4\theta$

$\Rightarrow \frac{9\cos 4\theta}{\cos 4\theta} = -\frac{5\sin 4\theta}{\cos 4\theta}$

$\Rightarrow 9 = -5\tan 4\theta$

$\Rightarrow \tan 4\theta = -\frac{9}{5}$

• $\arctan\left(-\frac{9}{5}\right) = -60.94°$

$4\theta = -60.94° \pm 180n \quad n = 0, 1, 2, 3 \ldots$

$\theta = -15.24° \pm 45n$

$\theta_1 = 29.8°$

$\theta_2 = 74.8°$

$\theta_3 = 119.8°$

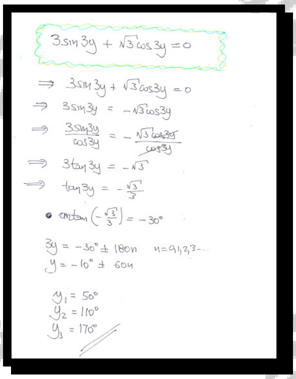
$\theta_4 = 164.8°$

Question 14 (***)

Solve the following trigonometric equation in the range given.

$$3\sin 3y + \sqrt{3}\cos 3y = 0, \quad 0° \le y < 180°.$$

$$\boxed{y = 50°,\ 110°,\ 170°}$$


$3\sin 3y + \sqrt{3}\cos 3y = 0$

$\Rightarrow 3\sin 3y + \sqrt{3}\cos 3y = 0$

$\Rightarrow 3\sin 3y = -\sqrt{3}\cos 3y$

$\Rightarrow \frac{3\sin 3y}{\cos 3y} = -\frac{\sqrt{3}\cos 3y}{\cos 3y}$

$\Rightarrow 3\tan 3y = -\sqrt{3}$

$\Rightarrow \tan 3y = -\frac{\sqrt{3}}{3}$

• $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -30°$

$3y = -30° \pm 180n \quad n = 0, 1, 2, 3 \ldots$

$y = -10° \pm 60n$

$y_1 = 50°$

$y_2 = 110°$

$y_3 = 170°$

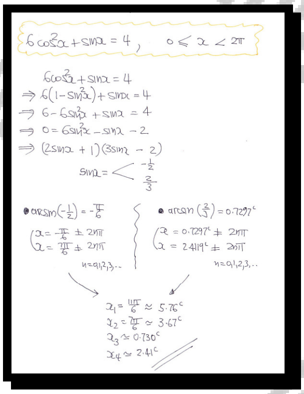
Question 15 (***)

Solve, in **radians**, the following trigonometric equation

$$6\cos^2 x + \sin x = 4, \quad 0 \le x < 2\pi,$$

giving the answers correct to three significant figures.

$$\boxed{x \approx 0.73^c, 2.41^c, 3.67^c, 5.76^c}$$



$6\cos^2 x + \sin x = 4, \quad 0 \leqslant x < 2\pi$

$6\cos^2 x + \sin x = 4$

$\Rightarrow 6(1-\sin^2 x) + \sin x = 4$

$\Rightarrow 6 - 6\sin^2 x + \sin x = 4$

$\Rightarrow 0 = 6\sin^2 x - \sin x - 2$

$\Rightarrow (2\sin x + 1)(3\sin x - 2)$

$\sin x = -\frac{1}{2}, \ \frac{2}{3}$

• $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$x = -\frac{\pi}{6} \pm 2n\pi$

$x = \frac{7\pi}{6} \pm 2n\pi$

$n = 0, 1, 2, 3, \ldots$

• $\arcsin\left(\frac{2}{3}\right) = 0.7297^c$

$x = 0.7297^c \pm 2n\pi$

$x = 2.4119^c \pm 2n\pi$

$n = 0, 1, 2, 3, \ldots$

$x_1 = \frac{11\pi}{6} \approx 5.76^c$

$x_2 = \frac{7\pi}{6} \approx 3.67^c$

$x_3 \approx 0.730^c$

$x_4 \approx 2.41^c$

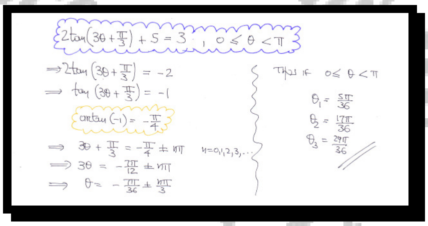
Question 16 (***)

Solve, in **radians**, the following trigonometric equation

$$5 + 2\tan\left(3\theta + \frac{\pi}{3}\right) = 3, \quad 0 \le \theta < \pi,$$

giving the answers in terms of π.

$$\boxed{\theta = \frac{5\pi}{36}, \frac{17\pi}{36}, \frac{29\pi}{36}}$$



$2\tan\left(3\theta + \frac{\pi}{3}\right) + 5 = 3, \quad 0 \leqslant \theta < \pi$

$\Rightarrow 2\tan\left(3\theta + \frac{\pi}{3}\right) = -2$

$\Rightarrow \tan\left(3\theta + \frac{\pi}{3}\right) = -1$

$\arctan(-1) = -\frac{\pi}{4}$

$\Rightarrow 3\theta + \frac{\pi}{3} = -\frac{\pi}{4} \pm n\pi \quad n = 0, 1, 2, 3, \ldots$

$\Rightarrow 3\theta = -\frac{7\pi}{12} \pm n\pi$

$\Rightarrow \theta = -\frac{7\pi}{36} \pm \frac{n\pi}{3}$

THUS IF $0 \leqslant \theta < \pi$

$\theta_1 = \frac{5\pi}{36}$

$\theta_2 = \frac{17\pi}{36}$

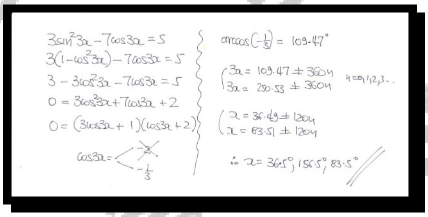
$\theta_3 = \frac{29\pi}{36}$

Question 17 (***)

Solve, in **degrees**, the following trigonometric equation

$$3\sin^2 3x - 7\cos 3x = 5, \quad 0° \le x < 180°.$$

$\boxed{}$, $\boxed{x \approx 36.5°,\ 83.5°,\ 156.5°}$


$3\sin^2 3x - 7\cos 3x = 5$
$3(1-\cos^2 3x) - 7\cos 3x = 5$
$3 - 3\cos^2 3x - 7\cos 3x = 5$
$0 = 3\cos^2 3x + 7\cos 3x + 2$
$0 = (3\cos 3x + 1)(\cos 3x + 2)$
$\cos 3x = \cancel{-2}, -\frac{1}{3}$
$\arccos\left(-\frac{1}{3}\right) = 109.47°$
$3x = 109.47 \pm 360n$, $3x = 250.53 \pm 360n$, $n = 0,1,2,3,\ldots$
$x = 36.49 \pm 120n$, $x = 83.51 \pm 120n$
$\therefore x = 36.5°, 156.5°, 83.5°$

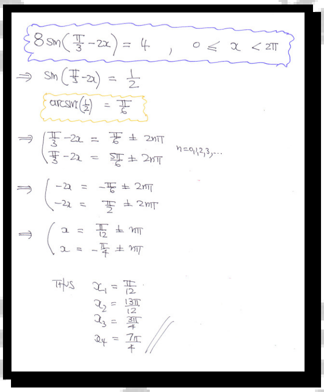
Question 18 (***)

Solve, in **radians**, the following trigonometric equation

$$8\sin\left(\frac{\pi}{3} - 2x\right) = 4, \quad 0 \le \theta < 2\pi,$$

giving the answers in terms of π.

$$\boxed{x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{7\pi}{4}}$$

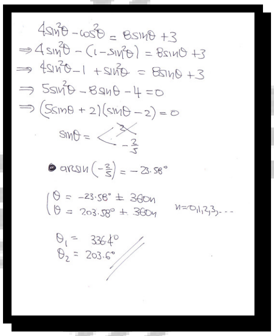

$8\sin\left(\frac{\pi}{3} - 2x\right) = 4, \quad 0 \le x < 2\pi$
$\Rightarrow \sin\left(\frac{\pi}{3} - 2x\right) = \frac{1}{2}$
$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$
$\Rightarrow \frac{\pi}{3} - 2x = \frac{\pi}{6} \pm 2n\pi$, $\frac{\pi}{3} - 2x = \frac{5\pi}{6} \pm 2n\pi$, $n = 0,1,2,3,\ldots$
$\Rightarrow -2x = -\frac{\pi}{6} \pm 2n\pi$, $-2x = \frac{\pi}{2} \pm 2n\pi$
$\Rightarrow x = \frac{\pi}{12} \pm n\pi$, $x = -\frac{\pi}{4} \pm n\pi$
THUS $x_1 = \frac{\pi}{12}$, $x_2 = \frac{13\pi}{12}$, $x_3 = \frac{3\pi}{4}$, $x_4 = \frac{7\pi}{4}$

Question 19 (***)

Solve the following trigonometric equation in the range given.

$$4\sin^2\theta - \cos^2\theta = 8\sin\theta + 3, \quad 0° \le \theta < 360°.$$

$\boxed{\theta \approx 203.6°, 336.4°}$



$4\sin^2\theta - \cos^2\theta = 8\sin\theta + 3$

$\Rightarrow 4\sin^2\theta - (1 - \sin^2\theta) = 8\sin\theta + 3$

$\Rightarrow 4\sin^2\theta - 1 + \sin^2\theta = 8\sin\theta + 3$

$\Rightarrow 5\sin^2\theta - 8\sin\theta - 4 = 0$

$\Rightarrow (5\sin\theta + 2)(\sin\theta - 2) = 0$

$\sin\theta = \cancel{2}, -\frac{2}{5}$

$\arcsin\left(-\frac{2}{5}\right) = -23.58°$

$\theta = -23.58° \pm 360n$
$\theta = 203.58° + 360n$ $\quad n = 0, 1, 2, 3, \ldots$

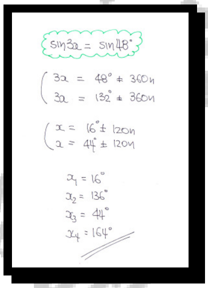
$\theta_1 = 336.4°$
$\theta_2 = 203.6°$

Question 20 (***)

Solve, in **degrees**, the following trigonometric equation

$$\sin 3x = \sin 48°, \quad 0° \le x < 180°.$$

$\boxed{}$, $\boxed{x = 16°, 44°, 136°, 164°}$



$\sin 3x = \sin 48°$

$3x = 48° \pm 360n$
$3x = 132° \pm 360n$

$x = 16° \pm 120n$
$x = 44° \pm 120n$

$x_1 = 16°$
$x_2 = 136°$
$x_3 = 44°$
$x_4 = 164°$

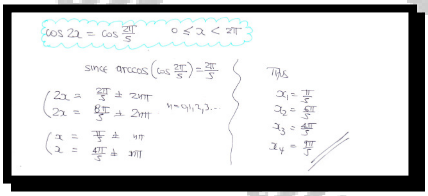
Question 21 (***)

Solve, in **radians**, the following trigonometric equation

$$\cos 2x = \cos \frac{2\pi}{5}, \quad 0 \le x < 2\pi,$$

giving the answers in terms of π.

$$\boxed{x = \frac{\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{9\pi}{5}}$$



$\cos 2x = \cos \frac{2\pi}{5} \qquad 0 \le x < 2\pi$

since $\arccos\left(\cos \frac{2\pi}{5}\right) = \frac{2\pi}{5}$

$2x = \frac{2\pi}{5} \pm 2n\pi$

$2x = \frac{8\pi}{5} \pm 2n\pi \qquad n = 0, 1, 2, 3 \ldots$

$x = \frac{\pi}{5} \pm n\pi$

$x = \frac{4\pi}{5} \pm n\pi$

Thus

$x_1 = \frac{\pi}{5}$

$x_2 = \frac{6\pi}{5}$

$x_3 = \frac{4\pi}{5}$

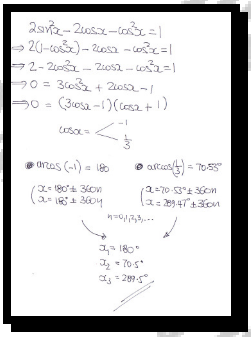
$x_4 = \frac{9\pi}{5}$

Question 22 (***)

Solve the following trigonometric equation in the range given.

$$2\sin^2 x - 2\cos x - \cos^2 x = 1, \quad 0° \le x < 360°.$$

$$\boxed{x \approx 70.5°, 289.5°, \quad x = 180°}$$



$2\sin^2 x - 2\cos x - \cos^2 x = 1$

$\Rightarrow 2(1-\cos^2 x) - 2\cos x - \cos^2 x = 1$

$\Rightarrow 2 - 2\cos^2 x - 2\cos x - \cos^2 x = 1$

$\Rightarrow 0 = 3\cos^2 x + 2\cos x - 1$

$\Rightarrow 0 = (3\cos x - 1)(\cos x + 1)$

$\cos x = -1, \ \frac{1}{3}$

$\arccos(-1) = 180$ $\qquad$ $\arccos\left(\frac{1}{3}\right) = 70.53°$

$x = 180° \pm 360n$

$x = 180° \pm 360n$

$x = 70.53° \pm 360n$

$x = 289.47° \pm 360n$

$n = 0, 1, 2, 3, \ldots$

$x_1 = 180°$

$x_2 = 70.5°$

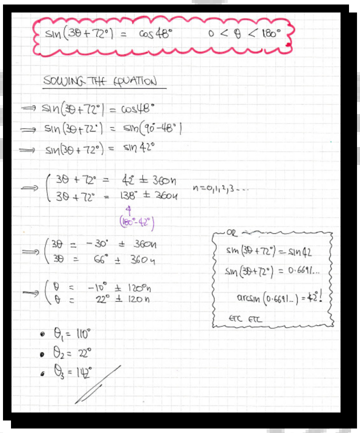
$x_3 = 289.5°$

Question 23 (***)

Solve the following trigonometric equation.

$$\sin(3\theta + 72)^\circ = \cos 48^\circ, \quad 0 \le \theta < 180.$$

$\boxed{}$, $\boxed{\theta = \{22, 110, 142\}}$



$\sin(3\theta + 72^\circ) = \cos 48^\circ \qquad 0 < \theta < 180^\circ$

SOLVING THE EQUATION

$\Rightarrow \sin(3\theta + 72^\circ) = \cos 48^\circ$

$\Rightarrow \sin(3\theta + 72^\circ) = \sin(90^\circ - 48^\circ)$

$\Rightarrow \sin(3\theta + 72^\circ) = \sin 42^\circ$

$\Rightarrow \begin{cases} 3\theta + 72^\circ = 42^\circ \pm 360n \\ 3\theta + 72^\circ = 138^\circ \pm 360n \end{cases}$ $n = 0, 1, 2, 3 \ldots$

$(180^\circ - 42^\circ)$

$\Rightarrow \begin{cases} 3\theta = -30^\circ \pm 360n \\ 3\theta = 66^\circ \pm 360n \end{cases}$

$\Rightarrow \begin{cases} \theta = -10^\circ \pm 120^\circ n \\ \theta = 22^\circ \pm 120n \end{cases}$

- $\theta_1 = 110^\circ$
- $\theta_2 = 22^\circ$
- $\theta_3 = 142^\circ$

OR

$\sin(3\theta + 72^\circ) = \sin 42$

$\sin(3\theta + 72^\circ) = 0.6691\ldots$

$\arcsin(0.6691\ldots) = 42^\circ!$

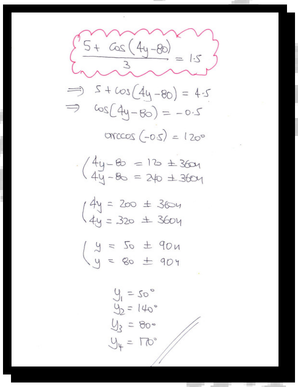
ETC ETC

Question 24 (***+)

Solve the following trigonometric equation in the range given.

$$\frac{5+\cos(4y-80)°}{3} = 1.5, \quad 0 \le y < 180.$$

$$\boxed{y = 50,\ 80,\ 140,\ 170}$$



$$\frac{5+\cos(4y-80)}{3} = 1.5$$

$\Rightarrow\ 5+\cos(4y-80) = 4.5$

$\Rightarrow\ \cos(4y-80) = -0.5$

$\arccos(-0.5) = 120°$

$\begin{cases} 4y-80 = 120 \pm 360n \\ 4y-80 = 240 \pm 360n \end{cases}$

$\begin{cases} 4y = 200 \pm 360n \\ 4y = 320 \pm 360n \end{cases}$

$\begin{cases} y = 50 \pm 90n \\ y = 80 \pm 90n \end{cases}$

$y_1 = 50°$

$y_2 = 140°$

$y_3 = 80°$

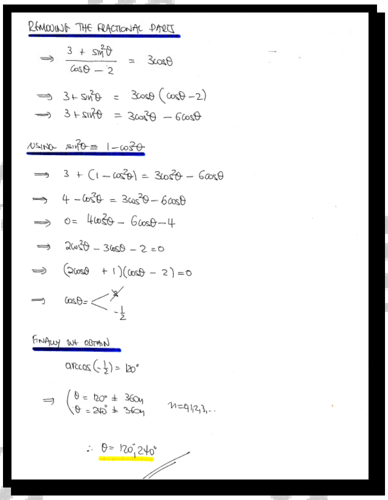
$y_4 = 170°$

Question 25 (***+)

Solve the following trigonometric equation in the range given.

$$\frac{3+\sin^2\theta}{\cos\theta - 2} = 3\cos\theta, \quad 0° \le \theta < 360°.$$

$$\boxed{\theta = 120°,\ 240°}$$



REMOVING THE FRACTIONAL PARTS

$\Rightarrow\ \dfrac{3+\sin^2\theta}{\cos\theta - 2} = 3\cos\theta$

$\Rightarrow\ 3+\sin^2\theta = 3\cos\theta(\cos\theta - 2)$

$\Rightarrow\ 3+\sin^2\theta = 3\cos^2\theta - 6\cos\theta$

USING $\sin^2\theta \equiv 1-\cos^2\theta$

$\Rightarrow\ 3+(1-\cos^2\theta) = 3\cos^2\theta - 6\cos\theta$

$\Rightarrow\ 4-\cos^2\theta = 3\cos^2\theta - 6\cos\theta$

$\Rightarrow\ 0 = 4\cos^2\theta - 6\cos\theta - 4$

$\Rightarrow\ 2\cos^2\theta - 3\cos\theta - 2 = 0$

$\Rightarrow\ (2\cos\theta + 1)(\cos\theta - 2) = 0$

$\Rightarrow\ \cos\theta = \cancel{2},\ -\frac{1}{2}$

FINALLY WE OBTAIN

$\arccos(-\frac{1}{2}) = 120°$

$\Rightarrow\ \begin{cases} \theta = 120° \pm 360n \\ \theta = 240° \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$

$\therefore\ \theta = 120°, 240°$

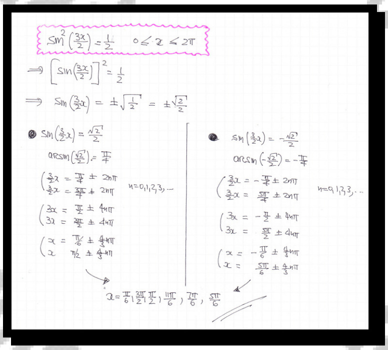
Question 26 (***+)

Solve, in **radians**, the following trigonometric equation

$$\sin^2\left(\frac{3x}{2}\right) = \frac{1}{2}, \quad 0 \le x < 2\pi,$$

giving the answers in terms of π.

$$\boxed{x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}}$$


$\sin^2\left(\frac{3x}{2}\right) = \frac{1}{2}$, $0 \le x < 2\pi$

$\Rightarrow \left[\sin\left(\frac{3x}{2}\right)\right]^2 = \frac{1}{2}$

$\Rightarrow \sin\left(\frac{3}{2}x\right) = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

• $\sin\left(\frac{3}{2}x\right) = \frac{\sqrt{2}}{2}$, $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\frac{3}{2}x = \frac{\pi}{4} \pm 2n\pi$, $\frac{3}{2}x = \frac{3\pi}{4} \pm 2n\pi$, $n = 0,1,2,3,\ldots$

$3x = \frac{\pi}{2} \pm 4n\pi$, $3x = \frac{3\pi}{2} \pm 4n\pi$

$x = \frac{\pi}{6} \pm \frac{4}{3}n\pi$, $x = \frac{\pi}{2} \pm \frac{4}{3}n\pi$

• $\sin\left(\frac{3}{2}x\right) = -\frac{\sqrt{2}}{2}$, $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

$\frac{3}{2}x = -\frac{\pi}{4} \pm 2n\pi$, $\frac{3}{2}x = \frac{5\pi}{4} \pm 2n\pi$, $n = 0,1,2,3,\ldots$

$3x = -\frac{\pi}{2} \pm 4n\pi$, $3x = \frac{5\pi}{2} \pm 4n\pi$

$x = -\frac{\pi}{6} \pm \frac{4}{3}n\pi$, $x = \frac{5\pi}{6} \pm \frac{4}{3}n\pi$

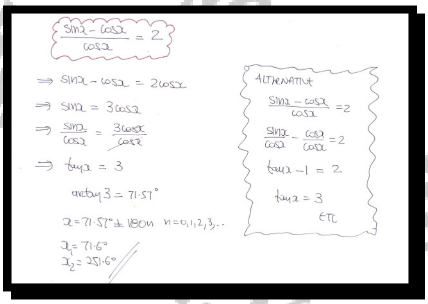
$x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{\pi}{2}, \frac{11\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}$

Question 27 (***+)

Solve the following trigonometric equation in the range given.

$$\frac{\sin x - \cos x}{\cos x} = 2, \quad 0° \le x < 360°.$$

$\boxed{2M}$, $\boxed{x \approx 71.6°, \ 251.6°}$


$\frac{\sin x - \cos x}{\cos x} = 2$

$\Rightarrow \sin x - \cos x = 2\cos x$

$\Rightarrow \sin x = 3\cos x$

$\Rightarrow \frac{\sin x}{\cos x} = \frac{3\cos x}{\cos x}$

$\Rightarrow \tan x = 3$

$\arctan 3 = 71.57°$

$x = 71.57° \pm 180n$, $n = 0,1,2,3,\ldots$

$x_1 = 71.6°$, $x_2 = 251.6°$

ALTERNATIVE

$\frac{\sin x - \cos x}{\cos x} = 2$

$\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} = 2$

$\tan x - 1 = 2$

$\tan x = 3$

ETC

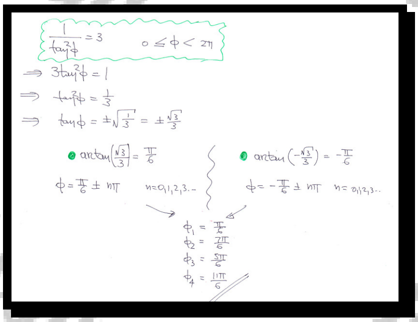
Question 28 (***+)

Solve, in **radians**, the following trigonometric equation

$$\frac{1}{\tan^2 \varphi} = 3, \quad 0 \le \varphi < 2\pi,$$

giving the answers in terms of π.

$$\boxed{\varphi = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$



$$\frac{1}{\tan^2\phi} = 3 \qquad 0 \le \phi < 2\pi$$

$\Rightarrow 3\tan^2\phi = 1$

$\Rightarrow \tan^2\phi = \frac{1}{3}$

$\Rightarrow \tan\phi = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

- $\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ $\quad \phi = \frac{\pi}{6} \pm n\pi \quad n = 0,1,2,3\ldots$
- $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$ $\quad \phi = -\frac{\pi}{6} \pm n\pi \quad n = 0,1,2,3\ldots$

$\phi_1 = \frac{\pi}{6}$

$\phi_2 = \frac{7\pi}{6}$

$\phi_3 = \frac{5\pi}{6}$

$\phi_4 = \frac{11\pi}{6}$

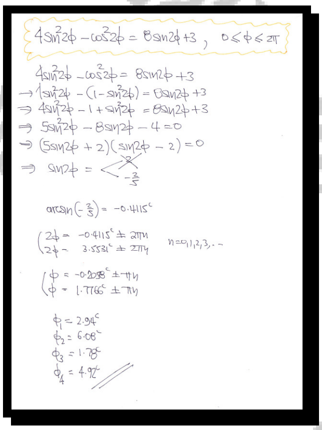
Question 29 (***+)

Solve, in **radians**, the following trigonometric equation

$$4\sin^2 2\varphi - \cos^2 2\varphi = 3 + 8\sin 2\varphi, \quad 0 \le \varphi < 2\pi,$$

giving the answers correct to three significant figures.

$$\boxed{\varphi \approx 1.78^c, 2.94^c, 4.92^c, 6.08^c}$$



$$4\sin^2 2\phi - \cos^2 2\phi = 8\sin 2\phi + 3, \quad 0 \le \phi \le 2\pi$$

$4\sin^2 2\phi - \cos^2 2\phi = 8\sin 2\phi + 3$

$\Rightarrow 4\sin^2 2\phi - (1 - \sin^2 2\phi) = 8\sin 2\phi + 3$

$\Rightarrow 4\sin^2 2\phi - 1 + \sin^2 2\phi = 8\sin 2\phi + 3$

$\Rightarrow 5\sin^2 2\phi - 8\sin 2\phi - 4 = 0$

$\Rightarrow (5\sin 2\phi + 2)(\sin 2\phi - 2) = 0$

$\Rightarrow \sin 2\phi = \cancel{2}, \ -\frac{2}{5}$

$\arcsin\left(-\frac{2}{5}\right) = -0.4115^c$

$2\phi = -0.4115^c \pm 2\pi n$

$2\phi = 3.5531^c \pm 2\pi n \quad n = 0,1,2,3,\ldots$

$\phi = -0.2058^c \pm \pi n$

$\phi = 1.7766^c \pm \pi n$

$\phi_1 = 2.94^c$

$\phi_2 = 6.08^c$

$\phi_3 = 1.78^c$

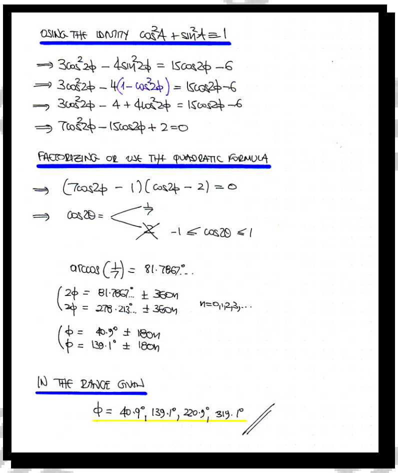
$\phi_4 = 4.92^c$

Question 30 (***+)

Solve the following trigonometric equation in the range given.

$$3\cos^2 2\varphi - 4\sin^2 2\varphi = 15\cos 2\varphi - 6, \quad 0° \le \varphi < 360°.$$

$\boxed{\varphi \approx 40.9°,\ 139.1°,\ 220.9°,\ 319.1°}$



USING THE IDENTITY $\cos^2 A + \sin^2 A \equiv 1$

$\Rightarrow 3\cos^2 2\phi - 4\sin^2 2\phi = 15\cos 2\phi - 6$

$\Rightarrow 3\cos^2 2\phi - 4(1 - \cos^2 2\phi) = 15\cos 2\phi - 6$

$\Rightarrow 3\cos^2 2\phi - 4 + 4\cos^2 2\phi = 15\cos 2\phi - 6$

$\Rightarrow 7\cos^2 2\phi - 15\cos 2\phi + 2 = 0$

FACTORIZING OR USE THE QUADRATIC FORMULA

$\Rightarrow (7\cos 2\phi - 1)(\cos 2\phi - 2) = 0$

$\Rightarrow \cos 2\theta = \frac{1}{7}$ or $\cancel{2}$ $\quad -1 \le \cos 2\theta \le 1$

$\arccos\left(\frac{1}{7}\right) = 81.7867...°$

$\begin{cases} 2\phi = 81.7867...° \pm 360n \\ 2\phi = 278.213...° \pm 360n \end{cases} \quad n = 0, 1, 2, 3, \dots$

$\begin{cases} \phi = 40.9° \pm 180n \\ \phi = 139.1° \pm 180n \end{cases}$

IN THE RANGE GIVEN

$\phi = 40.9°, 139.1°, 220.9°, 319.1°$

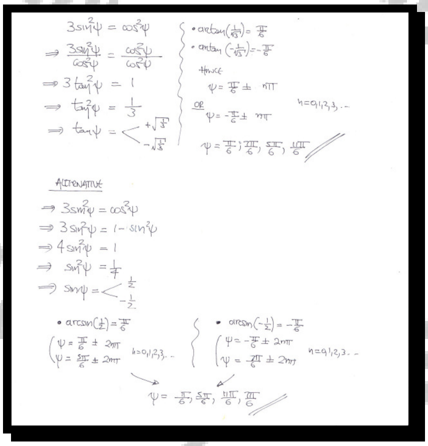
Question 31 (***+)

Solve, in **radians**, the following trigonometric equation

$$3\sin^2\psi = \cos^2\psi\,, \quad 0 \le \psi < 2\pi\,,$$

giving the answers in terms of π.

$$\boxed{\psi = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$



$3\sin^2\psi = \cos^2\psi$

$\Rightarrow \frac{3\sin^2\psi}{\cos^2\psi} = \frac{\cos^2\psi}{\cos^2\psi}$

$\Rightarrow 3\tan^2\psi = 1$

$\Rightarrow \tan^2\psi = \frac{1}{3}$

$\Rightarrow \tan\psi = \begin{cases} +\sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{3}} \end{cases}$

• $\arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

• $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

Hence

$\psi = \frac{\pi}{6} \pm n\pi$

OR $\quad n = 0, 1, 2, 3, \ldots$

$\psi = -\frac{\pi}{6} \pm n\pi$

$\psi = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

ALTERNATIVE

$\Rightarrow 3\sin^2\psi = \cos^2\psi$

$\Rightarrow 3\sin^2\psi = 1 - \sin^2\psi$

$\Rightarrow 4\sin^2\psi = 1$

$\Rightarrow \sin^2\psi = \frac{1}{4}$

$\Rightarrow \sin\psi = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases}$

• $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\begin{cases} \psi = \frac{\pi}{6} \pm 2n\pi \\ \psi = \frac{5\pi}{6} \pm 2n\pi \end{cases} \quad n = 0, 1, 2, 3, \ldots$

• $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\begin{cases} \psi = -\frac{\pi}{6} \pm 2n\pi \\ \psi = -\frac{7\pi}{6} \pm 2n\pi \end{cases} \quad n = 0, 1, 2, 3, \ldots$

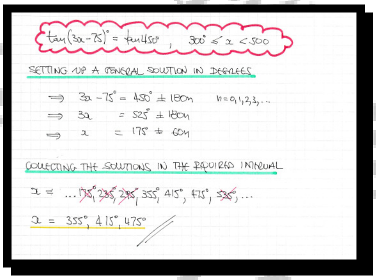
$\psi = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{7\pi}{6}$

Question 32 (***+)

Solve, in **degrees**, the following trigonometric equation

$$\tan(3x - 75)^\circ = \tan 450^\circ\,, \quad 300^\circ \le x < 500^\circ\,.$$

$$\boxed{x = 355^\circ,\ x = 415^\circ,\ x = 475^\circ}$$



$\tan(3x - 75)^\circ = \tan 450^\circ\,, \quad 300^\circ \le x < 500$

SETTING UP A GENERAL SOLUTION IN DEGREES

$\Rightarrow 3x - 75^\circ = 450^\circ \pm 180n \quad n = 0, 1, 2, 3, \ldots$

$\Rightarrow 3x = 525^\circ \pm 180n$

$\Rightarrow x = 175^\circ \pm 60n$

COLLECTING THE SOLUTIONS IN THE REQUIRED INTERVAL

$x = \ldots \cancel{175^\circ}, \cancel{235^\circ}, \cancel{295^\circ}, 355^\circ, 415^\circ, 475^\circ, \cancel{535^\circ}, \ldots$

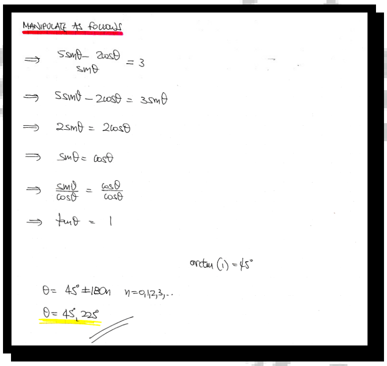
$x = 355^\circ, 415^\circ, 475^\circ$

Question 33 **(***+)**

Solve the following trigonometric equation in the range given.

$$\frac{5\sin\theta - 2\cos\theta}{\sin\theta} = 3, \quad 0° \le \theta < 360°.$$

$\boxed{}$, $\boxed{x = 45°,\ 225°}$


MANIPULATE AS FOLLOWS

$\Rightarrow \frac{5\sin\theta - 2\cos\theta}{\sin\theta} = 3$

$\Rightarrow 5\sin\theta - 2\cos\theta = 3\sin\theta$

$\Rightarrow 2\sin\theta = 2\cos\theta$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$

$\Rightarrow \tan\theta = 1$

$\arctan(1) = 45°$

$\theta = 45° \pm 180n \quad n = 0,1,2,3,\ldots$

$\theta = 45, 225$

Question 34 **(***+)**

$$2CT - 2C + T - 1$$

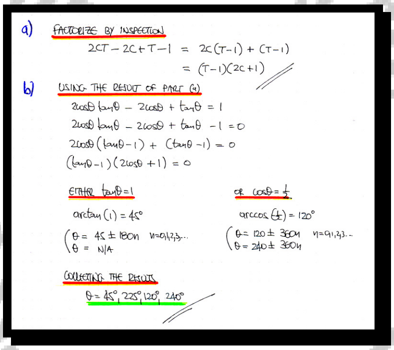
a) Write the above expression as a product of two linear factors.

b) Hence solve the trigonometric equation

$$2\cos\theta\tan\theta - 2\cos\theta + \tan\theta = 1,$$

for $0° \le \theta < 360°$.

$\boxed{}$, $\boxed{(2C+1)(T-1)}$, $\boxed{\theta = 45°,\ 120°,\ 135°,\ 240°}$


a) FACTORIZE BY INSPECTION

$2CT - 2C + T - 1 = 2C(T-1) + (T-1)$

$= (T-1)(2C+1)$

b) USING THE RESULT OF PART (a)

$2\cos\theta\tan\theta - 2\cos\theta + \tan\theta = 1$

$2\cos\theta\tan\theta - 2\cos\theta + \tan\theta - 1 = 0$

$2\cos\theta(\tan\theta - 1) + (\tan\theta - 1) = 0$

$(\tan\theta - 1)(2\cos\theta + 1) = 0$

EITHER $\tan\theta = 1$

$\arctan(1) = 45°$

$\theta = 45 \pm 180n \quad n = 0,1,2,3\ldots$

$\theta = \text{N/A}$

OR $\cos\theta = -\frac{1}{2}$

$\arccos(-\frac{1}{2}) = 120°$

$\theta = 120 \pm 360n \quad n = 0,1,2,3\ldots$

$\theta = 240 \pm 360n$

COLLECTING THE RESULTS

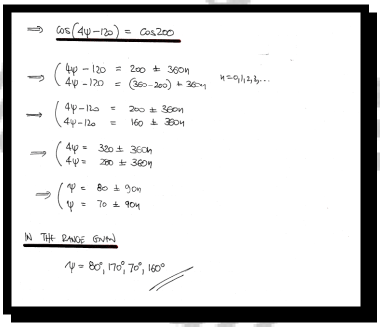
$\theta = 45°, 225°, 120°, 240°$

Question 35 (*+)**

Solve the following trigonometric equation in the range given.

$$\cos(4\psi - 120)° = \cos 200°, \ 0 \le \psi < 180.$$

$\boxed{}$, $\boxed{\psi = 70, \ 80, \ 160, 170}$



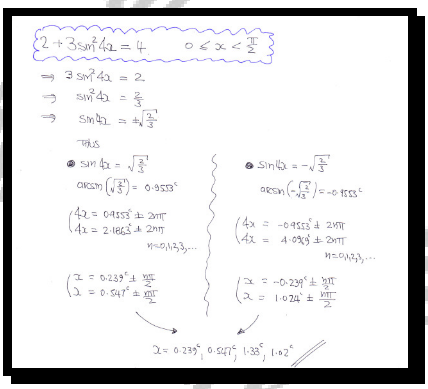
Question 36 (*+)**

Solve, in **radians**, the following trigonometric equation

$$2 + 3\sin^2 4x = 4, \ \ 0 \le x < \frac{\pi}{2},$$

giving the answers correct to three significant figures.

$$\boxed{x = 0.239^c, \ 0.547^c, 1.02^c, \ 1.33^c}$$

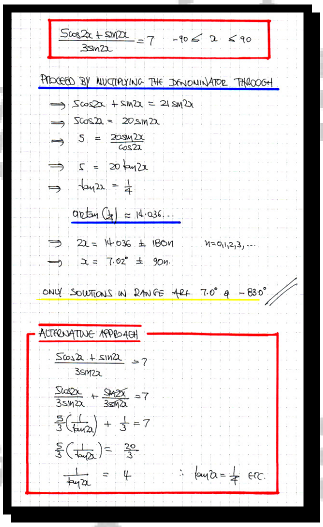


Question 37 (***+)

Solve the following trigonometric equation in the range given.

$$\frac{5\cos 2x + \sin 2x}{3\sin 2x} = 7, \quad -90° \le x < 90°.$$

$\boxed{}$, $\boxed{x \approx -83.0°,\ 7.0°}$



$$\boxed{\frac{5\cos 2x + \sin 2x}{3\sin 2x} = 7 \qquad -90 \le x \le 90}$$

PROCEED BY MULTIPLYING THE DENOMINATOR THROUGH

$\Rightarrow 5\cos 2x + \sin 2x = 21\sin 2x$

$\Rightarrow 5\cos 2x = 20\sin 2x$

$\Rightarrow 5 = \frac{20\sin 2x}{\cos 2x}$

$\Rightarrow 5 = 20\tan 2x$

$\Rightarrow \tan 2x = \frac{1}{4}$

$\arctan\left(\frac{1}{4}\right) \approx 14.036\ldots$

$\Rightarrow 2x = 14.036 \pm 180n \qquad n = 0, 1, 2, 3, \ldots$

$\Rightarrow x = 7.02° \pm 90n.$

ONLY SOLUTIONS IN RANGE ARE $7.0°$ & $-83.0°$

ALTERNATIVE APPROACH

$\frac{5\cos 2x + \sin 2x}{3\sin 2x} = 7$

$\frac{5\cos 2x}{3\sin 2x} + \frac{\cancel{\sin 2x}}{3\cancel{\sin 2x}} = 7$

$\frac{5}{3}\left(\frac{1}{\tan 2x}\right) + \frac{1}{3} = 7$

$\frac{5}{3}\left(\frac{1}{\tan 2x}\right) = \frac{20}{3}$

$\frac{1}{\tan 2x} = 4 \qquad \therefore \tan 2x = \frac{1}{4}$ ETC.

Question 38 (***+)

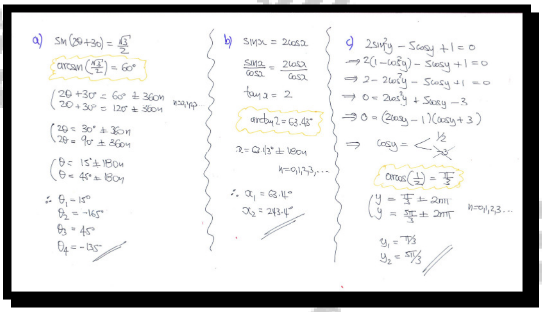
Solve each of the following trigonometric equations, in the range given.

a) $\sin(2\theta + 30°) = \dfrac{\sqrt{3}}{2}$, $\quad -180° \le \theta < 180°$

b) $\sin x = 2\cos x$, $\quad 0 \le x < 360°$

c) $2\sin^2 y - 5\cos y + 1 = 0$, $\quad 0 \le y < 2\pi$

$\boxed{}$, $\boxed{\theta = -165°, -135°, 15°, 45°}$, $\boxed{x \approx 63.4°,\ 243.4°}$, $\boxed{y = \dfrac{\pi}{3},\ \dfrac{5\pi}{3}}$



a) $\sin(2\theta+30) = \frac{\sqrt{3}}{2}$

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = 60°$

$2\theta + 30° = 60° \pm 360n$
$2\theta + 30° = 120° \pm 360n$ $\quad n = 0,1,2,3\ldots$

$2\theta = 30° \pm 360n$
$2\theta = 90° \pm 360n$

$\theta = 15° \pm 180n$
$\theta = 45° \pm 180n$

$\therefore \theta_1 = 15°$
$\theta_2 = -165°$
$\theta_3 = 45°$
$\theta_4 = -135°$

b) $\sin x = 2\cos x$

$\frac{\sin x}{\cos x} = \frac{2\cos x}{\cos x}$

$\tan x = 2$

$\arctan 2 = 63.43°$

$x = 63.43° \pm 180n$ $\quad n = 0,1,2,3,\ldots$

$\therefore x_1 = 63.4°$
$x_2 = 243.4°$

c) $2\sin^2 y - 5\cos y + 1 = 0$
$\Rightarrow 2(1-\cos^2 y) - 5\cos y + 1 = 0$
$\Rightarrow 2 - 2\cos^2 y - 5\cos y + 1 = 0$
$\Rightarrow 0 = 2\cos^2 y + 5\cos y - 3$
$\Rightarrow 0 = (2\cos y - 1)(\cos y + 3)$
$\Rightarrow \cos y = \frac{1}{2}$ or $\cancel{-3}$

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$y = \frac{\pi}{3} \pm 2n\pi$
$y = \frac{5\pi}{3} \pm 2n\pi$ $\quad n = 0,1,2,3\ldots$

$y_1 = \pi/3$
$y_2 = 5\pi/3$

Question 39 (***+)

A cubic curve is given by

$$f(x) \equiv 4x^3 - 8x^2 - x + k,$$

where k is a non zero constant.

a) Given that $(x-2)$ is a factor of $f(x)$, show that $(2x-1)$ is also a factor of $f(x)$.

b) Express $f(x)$ as the product of three linear factors.

c) Hence solve the following trigonometric equation

$$4\sin^3 y - 8\sin^2 y - \sin y + k = 0,$$

for $0° \le y < 360°$.

$\boxed{}$, $\boxed{y = 30°, \ 150°, \ 210°, \ 330°}$

a) $f(x) = 4x^3 - 8x^2 - x + k$

$x-2$ IS A FACTOR $\Rightarrow f(2) = 0$
$\Rightarrow 4 \times 2^3 - 8 \times 2^2 - 2 + k$
$\Rightarrow 32 - 32 - 2 + k$
$\Rightarrow k = 2$

$f(x) = 4x^3 - 8x^2 - x + 2$

$f(\tfrac{1}{2}) = 4(\tfrac{1}{2})^3 - 8(\tfrac{1}{2})^2 - \tfrac{1}{2} + 2$
$= \tfrac{1}{2} - 2 - \tfrac{1}{2} + 2$
$= 0$

$\therefore$ $(2x-1)$ IS INDEED ALSO A FACTOR

c) BY INSPECTION WE HAVE

$f(x) = 4x^3 - 8x^2 - x + 2 = (x-2)(2x-1)(2x+1)$

c) $4\sin^3 y - 8\sin^2 y - \sin y + k = 0$
$\Rightarrow (\sin y - 2)(2\sin y - 1)(2\sin y + 1) = 0$
(PART a)

$\Rightarrow \sin y = \cancel{2}, \ \tfrac{1}{2}, \ -\tfrac{1}{2}$

SOLVING SEPARATELY

• $\arcsin(\tfrac{1}{2}) = 30°$

$y = 30° \pm 360°n$
$y = 150° \pm 360°n$
$n = 0, 1, 2, 3, \ldots$

• $\arcsin(-\tfrac{1}{2}) = -30°$

$y = -30° \pm 360°n$
$y = 210° \pm 360°n$
$n = 0, 1, 2, 3, \ldots$

$y = 30°, 150°, 210°, 330°$

Question 40 (*+)**

Solve, in **radians**, the following trigonometric equation

$$7\cos(2x+3)^{c} = 5, \quad -\pi \le x < \pi,$$

giving the answers correct to three significant figures.

$$\boxed{x = -1.89^{c},\ -1.11^{c},\ 1.25^{c},\ 2.08^{c}}$$

$7\cos(2x+3)^{c} = 5 \qquad -\pi \le x < \pi$

$\Rightarrow \cos(2x+3) = \frac{5}{7}$

$\arccos\left(\frac{5}{7}\right) = 0.7752^{c}$

$\Rightarrow \begin{cases} 2x+3 = 0.7752^{c} \pm 2n\pi \\ 2x+3 = 5.5080^{c} \pm 2n\pi \end{cases} \quad n = 0,1,2,3,\ldots$

$\Rightarrow \begin{cases} 2x = -2.2248^{c} \pm 2n\pi \\ 2x = 2.5080^{c} \pm 2n\pi \end{cases}$

$\Rightarrow \begin{cases} x = -1.1124^{c} \pm n\pi \\ x = 1.2540^{c} \pm n\pi \end{cases}$

THUS FOR THE GIVEN INTERVAL

$x_1 = -1.11^{c}$

$x_2 = 2.03^{c}$

$x_3 = 1.25^{c}$

$x_4 = -1.89^{c}$

Question 41 (***+)

The graph of the curve with equation

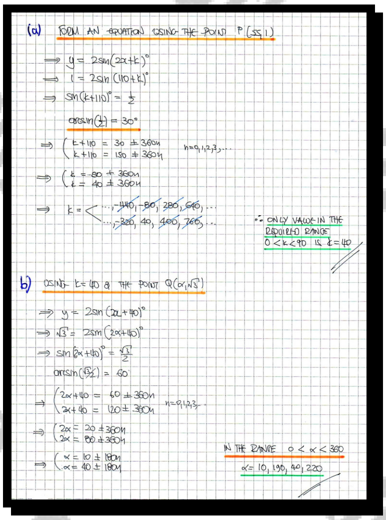
$$y = 2\sin(2x+k)°, \quad 0 \le x < 360,$$

where k is a constant so that $0 < k < 90$, passes through the points with coordinates $P(55,1)$ and $Q(\alpha, \sqrt{3})$.

a) Show, without verification, that $k = 40$.

b) Determine the possible values of α.

$\boxed{}$, $\boxed{\alpha = 10, 40, 190, 220}$



(a) FORM AN EQUATION USING THE POINT $P(55,1)$

$\Rightarrow y = 2\sin(2x+k)°$

$\Rightarrow 1 = 2\sin(110+k)°$

$\Rightarrow \sin(k+110)° = \frac{1}{2}$

$\arcsin\left(\frac{1}{2}\right) = 30°$

$\Rightarrow \begin{cases} k+110 = 30 \pm 360n \\ k+110 = 150 \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$

$\Rightarrow \begin{cases} k = -80 \pm 360n \\ k = 40 \pm 360n \end{cases}$

$\Rightarrow k = \begin{cases} \ldots, \cancel{-440}, \cancel{-80}, \cancel{280}, \cancel{640}, \ldots \\ \ldots, \cancel{-320}, 40, \cancel{400}, \cancel{760}, \ldots \end{cases}$

$\therefore$ ONLY VALUE IN THE REQUIRED RANGE $0 < k < 90$ IS $k = 40$

b) USING $k = 40$ & THE POINT $Q(\alpha, \sqrt{3})$

$\Rightarrow y = 2\sin(2x+40)°$

$\Rightarrow \sqrt{3} = 2\sin(2\alpha+40)°$

$\Rightarrow \sin(2\alpha+40)° = \frac{\sqrt{3}}{2}$

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = 60$

$\Rightarrow \begin{cases} 2\alpha+40 = 60 \pm 360n \\ 2\alpha+40 = 120 \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$

$\Rightarrow \begin{cases} 2\alpha = 20 \pm 360n \\ 2\alpha = 80 \pm 360n \end{cases}$

$\Rightarrow \begin{cases} \alpha = 10 \pm 180n \\ \alpha = 40 \pm 180n \end{cases}$

IN THE RANGE $0 < \alpha < 360$

$\alpha = 10, 190, 40, 220$

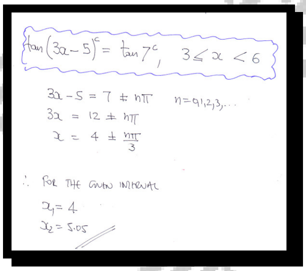
Question 42 **(***+)**

Solve, in **radians**, the following trigonometric equation

$$\tan(3x-5)^{\mathrm{c}} = \tan 7^{\mathrm{c}}, \quad 3 \le x < 6,$$

giving the answers correct to three significant figures, where appropriate.

$$\boxed{x = 4, \ x \approx 5.05}$$



$\tan(3x-5)^c = \tan 7^c, \quad 3 \le x < 6$

$3x - 5 = 7 \pm n\pi \qquad n = 0,1,2,3,\ldots$

$3x = 12 \pm n\pi$

$x = 4 \pm \frac{n\pi}{3}$

$\therefore$ FOR THE GIVEN INTERVAL

$x_1 = 4$

$x_2 = 5.05$

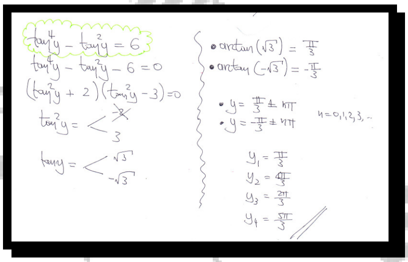
Question 43 **(***+)**

Solve, in **radians**, the following trigonometric equation

$$\tan^4 y - \tan^2 y = 6, \quad 0 \le y < 2\pi,$$

giving the answers in terms of π.

$$\boxed{y = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$



$\tan^4 y - \tan^2 y = 6$

$\tan^4 y - \tan^2 y - 6 = 0$

$(\tan^2 y + 2)(\tan^2 y - 3) = 0$

$\tan^2 y = \cancel{-2}, \ 3$

$\tan y = \sqrt{3}, \ -\sqrt{3}$

- $\arctan(\sqrt{3}) = \frac{\pi}{3}$
- $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$

- $y = \frac{\pi}{3} \pm n\pi \qquad n = 0,1,2,3,\ldots$
- $y = -\frac{\pi}{3} \pm n\pi$

$y_1 = \frac{\pi}{3}$

$y_2 = \frac{4\pi}{3}$

$y_3 = \frac{2\pi}{3}$

$y_4 = \frac{5\pi}{3}$

Question 44 **(***+)**

Solve the following trigonometric equation

$$\frac{2+\cos 2x}{3+\sin^2 2x}=\frac{2}{5}, \quad \text{for } 0° \le x < 360°.$$

$\boxed{}$, $\boxed{x = 60°, 120°, 240°, 300°}$

GETTING RID OF THE DENOMINATORS

$\Rightarrow \frac{2+\cos 2x}{3+\sin^2 2x} = \frac{2}{5}$

$\Rightarrow 10 + 5\cos 2x = 6 + 2\sin^2 2x$

USING $\cos^2 2x + \sin^2 2x \equiv 1$

$\Rightarrow 10 + 5\cos 2x = 6 + 2(1-\cos^2 2x)$

$\Rightarrow 10 + 5\cos 2x = 6 + 2 - 2\cos^2 2x$

$\Rightarrow 2\cos^2 2x + 5\cos 2x + 2 = 0$

$\Rightarrow (2\cos 2x + 1)(\cos 2x + 2) = 0$

$\Rightarrow \cos 2x = \cancel{-2}, \ -\frac{1}{2}$ $\quad -1 \le \cos 2x \le 1$

PROCEED WITH SOLUTION

$\arccos\left(-\frac{1}{2}\right) = 120°$

$\begin{cases} 2x = 120 \pm 360n \\ 2x = 240 \pm 360n \end{cases}$ $\quad n = 0,1,2,3,\dots$

$\begin{cases} x = 60 \pm 180n \\ x = 120 \pm 180n \end{cases}$

$x = 60°, 240°, 120°, 300°$

Question 45 **(***+)**

Solve, in **degrees**, the following trigonometric equation

$$\tan^4 y = 6 + \tan^2 y, \quad 0° \le y < 360°.$$

$\boxed{}$, $\boxed{y = 60°, \ 120°, \ 240°, \ 300°}$

$\tan^4 y = 6 + \tan^2 y \qquad 0 \le y < 360°$

THIS IS A QUADRATIC IN $\tan^2 y$ SO WE PROCEED BY FACTORIZATION

$\Rightarrow \tan^4 y - \tan^2 y - 6 = 0$

$\Rightarrow (\tan^2 y + 2)(\tan^2 y - 3) = 0$

$\Rightarrow \tan^2 y = \cancel{-2}, \ 3$

$\Rightarrow \tan y = \sqrt{3}, \ -\sqrt{3}$

SOLVING EACH OF THESE SEPARATELY

- $\tan y = \sqrt{3}$
  $\arctan\sqrt{3} = 60°$
  $y = 60° \pm 180n$
  $n = 0,1,2,3,\dots$

- $\tan y = -\sqrt{3}$
  $\arctan(-\sqrt{3}) = -60°$
  $y = -60° \pm 180n$
  $n = 0,1,2,3,\dots$

COLLECTING THE SOLUTIONS TOGETHER

$\Rightarrow y = 60°, 240°, 120°, 300°$

$\Rightarrow y = 60°, 120°, 240°, 300°$

Question 46 (***+)

A trigonometric curve is defined by the equation

$$f(x) = 3 - 4\sin(2x + k)°, \ 0 \le x \le 360$$

where k is a constant such that $-90 < k < 90$.

The curve passes through the point with coordinates $(15, 5)$ and further satisfies

$$A \le f(x) \le B,$$

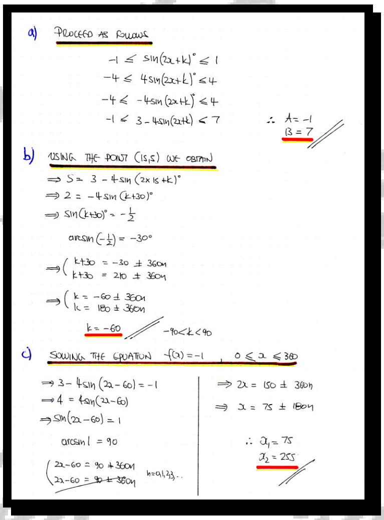
for some constants A and B.

a) State the value of A and the value of B.

b) Show that $k = -60$.

c) Solve the equation $f(x) = -1$.

$\boxed{}$, $\boxed{A = -1}$, $\boxed{B = 7}$, $\boxed{x = 75, \ 255}$



a) PROCEED AS FOLLOWS

$-1 \le \sin(2x+k)° \le 1$

$-4 \le 4\sin(2x+k)° \le 4$

$-4 \le -4\sin(2x+k)° \le 4$

$-1 \le 3 - 4\sin(2x+k) \le 7$

$\therefore A = -1$, $B = 7$

b) USING THE POINT (15,5) WE OBTAIN

$\Rightarrow 5 = 3 - 4\sin(2 \times 15 + k)°$

$\Rightarrow 2 = -4\sin(k+30)°$

$\Rightarrow \sin(k+30)° = -\frac{1}{2}$

$\arcsin\left(-\frac{1}{2}\right) = -30°$

$\Rightarrow \begin{cases} k+30 = -30 \pm 360n \\ k+30 = 210 \pm 360n \end{cases}$

$\Rightarrow \begin{cases} k = -60 \pm 360n \\ k = 180 \pm 360n \end{cases}$

$k = -60$ $\quad -90 < k < 90$

c) SOLVING THE EQUATION $f(x) = -1$, $0 \le x \le 360$

$\Rightarrow 3 - 4\sin(2x - 60) = -1$

$\Rightarrow 4 = 4\sin(2x - 60)$

$\Rightarrow \sin(2x - 60) = 1$

$\arcsin 1 = 90$

$\begin{cases} 2x - 60 = 90 \pm 360n \\ \cancel{2x - 60 = 90 \pm 360n} \end{cases}$ $\quad n = 0, 1, 2, 3, \dots$

$\Rightarrow 2x = 150 \pm 360n$

$\Rightarrow x = 75 \pm 180n$

$\therefore x_1 = 75$, $x_2 = 255$

Question 47 (****)

Given that θ is measured in degrees, solve the following trigonometric equation

$$\frac{4}{\tan^2 3\theta} + 2 = \frac{7}{\sin 3\theta}, \quad 0 \le \theta \le 180 .$$

$$\boxed{\theta = 10°,\ 50°,\ 130°,\ 170°}$$

MANIPULATE AS FOLLOWS

$$\Rightarrow \frac{4}{\tan^2 3\theta} + 2 = \frac{7}{\sin 3\theta}$$

$$\Rightarrow \frac{4}{\frac{\sin^2 3\theta}{\cos^2 3\theta}} + 2 = \frac{7}{\sin 3\theta}$$

$$\Rightarrow \frac{4\cos^2 3\theta}{\sin^2 3\theta} + 2 = \frac{7}{\sin 3\theta}$$

MULTIPLY THROUGH BY $\sin^2 3\theta$

$$\Rightarrow 4\cos^2 3\theta + 2\sin^2 3\theta = 7\sin 3\theta$$

$$\Rightarrow 4(1-\sin^2 3\theta) + 2\sin^2 3\theta = 7\sin 3\theta \qquad \{\cos^2 A + \sin^2 A \equiv 1\}$$

$$\Rightarrow 4 - 4\sin^2 3\theta + 2\sin^2 3\theta = 7\sin 3\theta$$

$$\Rightarrow 0 = 2\sin^2 3\theta + 7\sin 3\theta - 4$$

FACTORIZING YIELDS

$$\Rightarrow (2\sin 3\theta - 1)(\sin 3\theta + 4)$$

$$\Rightarrow \sin 3\theta = \begin{cases} \frac{1}{2} \\ \cancel{-4} \end{cases}$$

$$\Rightarrow \arcsin\left(\tfrac{1}{2}\right) = 30°$$

$$\begin{cases} 3\theta = 30° \pm 360n° \\ 3\theta = 150° \pm 360n° \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$$\begin{cases} \theta = 10° \pm 120n° \\ \theta = 50° \pm 120n° \end{cases}$$

$$\theta = 10°, 130°, 50°, 170°$$

Question 48 **(****)**

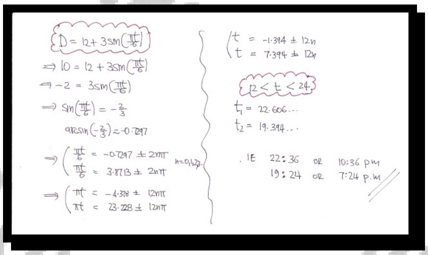
The depth of water in a harbour on a particular day can be modelled by the equation

$$D = 12 + 3\sin\left(\frac{\pi t}{6}\right),$$

where D is the depth of the water in metres, t hours after midnight.

Determine the times after noon, when the depth of water in the harbour is 10 metres.

$\boxed{}$, $\boxed{19:24}$, $\boxed{22:36}$



Question 49 (****)

The height of tides in a harbour on a particular day can be modelled by the equation

$$h = a + b\sin(30t)°,$$

where h is the height of the water in metres, t hours after midnight, and a and b are constants.

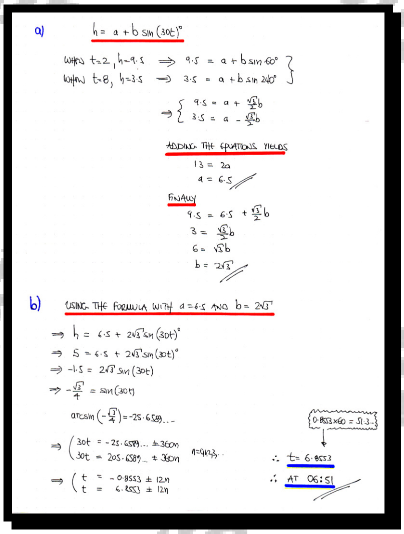
At 02.00, $h = 9.5$ m and at 08.00, $h = 3.5$ m.

Determine …

a) … the value of a and the exact value of b.

b) … the first time after midnight when the height of the tide is 5 metres.

$\boxed{\text{MP1-A}}$, $\boxed{a = 6.5}$, $\boxed{b = 2\sqrt{3}}$, $\boxed{06:51}$



a) $h = a + b\sin(30t)°$

WHEN $t=2$, $h=9.5$ $\Rightarrow$ $9.5 = a + b\sin 60°$

WHEN $t=8$, $h=3.5$ $\Rightarrow$ $3.5 = a + b\sin 240°$

$\Rightarrow \begin{cases} 9.5 = a + \frac{\sqrt{3}}{2}b \\ 3.5 = a - \frac{\sqrt{3}}{2}b \end{cases}$

ADDING THE EQUATIONS YIELDS

$13 = 2a$

$a = 6.5$

FINALLY

$9.5 = 6.5 + \frac{\sqrt{3}}{2}b$

$3 = \frac{\sqrt{3}}{2}b$

$6 = \sqrt{3}b$

$b = 2\sqrt{3}$

b) USING THE FORMULA WITH $a = 6.5$ AND $b = 2\sqrt{3}$

$\Rightarrow h = 6.5 + 2\sqrt{3}\sin(30t)°$

$\Rightarrow 5 = 6.5 + 2\sqrt{3}\sin(30t)°$

$\Rightarrow -1.5 = 2\sqrt{3}\sin(30t)$

$\Rightarrow -\frac{\sqrt{3}}{4} = \sin(30t)$

$\arcsin\left(-\frac{\sqrt{3}}{4}\right) = -25.6589\ldots$

$\Rightarrow \begin{cases} 30t = -25.6589\ldots \pm 360n \\ 30t = 205.6589\ldots \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$

$\Rightarrow \begin{cases} t = -0.8553 \pm 12n \\ t = 6.8553 \pm 12n \end{cases}$

$0.8553 \times 60 = 51.3\ldots$

$\therefore t = 6.8553$

$\therefore$ AT 06:51

Question 50 (****)

Solve the following trigonometric equation, in the range given.

$$\sqrt{3}+2\sin\left(3x+\frac{\pi}{4}\right)=0, \quad 0 \le x < \frac{\pi}{2}.$$

Give the answers in terms of π.

$\boxed{}$, $\boxed{x=\frac{13\pi}{36},\ \frac{17\pi}{36}}$

$\sqrt{3}+2\sin\left(3x+\frac{\pi}{4}\right)=0$

$\Rightarrow 2\sin\left(3x+\frac{\pi}{4}\right)=-\sqrt{3}$

$\Rightarrow \sin\left(3x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$

$\arcsin\left(-\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}$

$\Rightarrow \begin{cases} 3x+\frac{\pi}{4} = -\frac{\pi}{3} \pm 2n\pi \\ 3x+\frac{\pi}{4} = \frac{4\pi}{3} \pm 2n\pi \end{cases}$ $n=0,1,2,3,\ldots$

$\Rightarrow \begin{cases} 3x = -\frac{7\pi}{12} \pm 2n\pi \\ 3x = \frac{13\pi}{12} \pm 2n\pi \end{cases}$

$\Rightarrow \begin{cases} x = -\frac{7\pi}{36} \pm \frac{2n\pi}{3} \\ x = \frac{13\pi}{36} \pm \frac{2}{3}n\pi \end{cases}$

$x_1 = \frac{17\pi}{36}$

$x_2 = \frac{13\pi}{36}$

Question 51 (****)

Solve the following trigonometric equation in the range given.

$$4\tan^2\theta\cos\theta = 15, \ 0 \le \theta < 360°.$$

$\boxed{}$, $\boxed{\theta \approx 75.5°,\ 284.5°}$

$4\tan^2\theta\cos\theta = 15$

$\Rightarrow 4\left(\frac{\sin\theta}{\cos\theta}\right)^2\cos\theta = 15$

$\Rightarrow \frac{4\sin^2\theta}{\cos^2\theta} \times \cos\theta = 15$

$\Rightarrow \frac{4\sin^2\theta}{\cos\theta} = 15$

$\Rightarrow 4\sin^2\theta = 15\cos\theta$

$\Rightarrow 4(1-\cos^2\theta) = 15\cos\theta$

$\Rightarrow 4-4\cos^2\theta = 15\cos\theta$

$\Rightarrow 0 = 4\cos^2\theta + 15\cos\theta - 4$

$\Rightarrow (4\cos\theta - 1)(\cos\theta + 4) = 0$

$\Rightarrow \cos\theta = \frac{1}{4}$, $\cancel{-4}$

$\arccos\left(\frac{1}{4}\right) = 75.5°$

$\begin{cases} \theta = 75.5° \pm 360n \\ \theta = 284.5 \pm 360n \end{cases}$ $n=0,1,2,3,\ldots$

$\theta_1 = 75.5°$

$\theta_2 = 284.5°$

Question 52 (****)

Solve the following trigonometric equation in the range given.

$$2\tan\varphi\sin\varphi = 3, \quad 0 \le \varphi < 2\pi .$$

Give the answers in terms of π.

$$\boxed{\varphi = \frac{\pi}{3}, \frac{5\pi}{3}}$$

$2\tan\phi\sin\phi = 3$

$\Rightarrow 2\left(\frac{\sin\phi}{\cos\phi}\right)\sin\phi = 3$

$\Rightarrow \frac{2\sin^2\phi}{\cos\phi} = 3$

$\Rightarrow 2\sin^2\phi = 3\cos\phi$

$\Rightarrow 2(1-\cos^2\phi) = 3\cos\phi$

$\Rightarrow 2 - 2\cos^2\phi = 3\cos\phi$

$\Rightarrow 0 = 2\cos^2\phi + 3\cos\phi - 2$

$\Rightarrow (2\cos\phi - 1)(\cos\phi + 2)$

$\cos\phi = \frac{1}{2}$ or $\cancel{-2}$

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\phi = \frac{\pi}{3} \pm 2n\pi$, $\phi = \frac{5\pi}{3} \pm 2n\pi$, $n = 0, 1, 2, 3, \ldots$

$\phi_1 = \frac{\pi}{3}$

$\phi_2 = \frac{5\pi}{3}$

Question 53 (****)

Solve the following trigonometric equation in the range given.

$$2\cos x = 3\tan x, \quad 0° \le x < 360° .$$

$\boxed{}$, $\boxed{x = 30°, 150°}$

START BY WRITING THE $\tan x$ IN TERMS OF SINES & COSINES

$\Rightarrow 2\cos x = 3\tan x$

$\Rightarrow 2\cos x = \frac{3\sin x}{\cos x}$

$\Rightarrow 2\cos^2 x = 3\sin x$

USING THE IDENTITY $\cos^2 x + \sin^2 x \equiv 1$

$\Rightarrow 2(1 - \sin^2 x) = 3\sin x$

$\Rightarrow 2 - 2\sin^2 x = 3\sin x$

$\Rightarrow 0 = 2\sin^2 x + 3\sin x - 2$

$\Rightarrow (2\sin x - 1)(\sin x + 2) = 0$

$\Rightarrow \sin x = \cancel{-2}$ or $\frac{1}{2}$

$\arcsin\frac{1}{2} = 30°$

$x = 30° \pm 360n$, $x = 150° \pm 360n$, $n = 0, 1, 2, 3, \ldots$

THUS IN THE RANGE GIVEN

$x_1 = 30°$

$x_2 = 150°$

Question 54 (****)

$$f(x) = x^3 - 4x^2 - \tfrac{1}{2}x + 2, \; x \in \mathbb{R}.$$

a) Show that $(x-4)$ is a factor of $f(x)$.

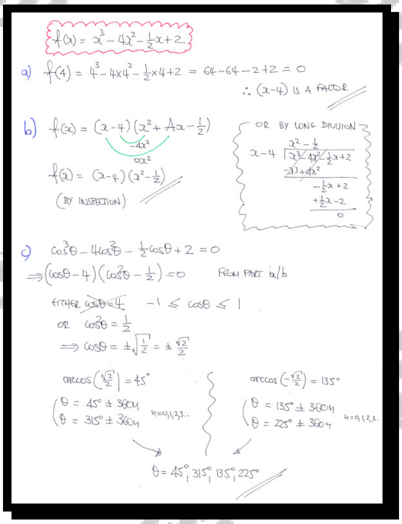
b) Express $f(x)$ as the product of a linear and one quadratic factor.

c) Hence solve the trigonometric equation

$$\cos^3\theta - 4\cos^2\theta - \tfrac{1}{2}\cos\theta + 2 = 0,$$

for $0° \le \theta < 360°$.

$\boxed{}$, $\boxed{f(x) \equiv (x-4)\left(x^2 - \tfrac{1}{2}\right)}$, $\boxed{\theta = 45°, 135°, 225°, 315°}$


$f(x) = x^3 - 4x^2 - \frac{1}{2}x + 2$

a) $f(4) = 4^3 - 4\times4^2 - \frac{1}{2}\times4 + 2 = 64 - 64 - 2 + 2 = 0$

$\therefore (x-4)$ IS A FACTOR

b) $f(x) = (x-4)(x^2 + Ax - \frac{1}{2})$

$-4x^2$

$0x^2$

$f(x) = (x-4)(x^2 - \frac{1}{2})$

(BY INSPECTION)

OR BY LONG DIVISION

$x - 4 \,\overline{)\, x^3 - 4x^2 - \frac{1}{2}x + 2}$ quotient $x^2 - \frac{1}{2}$

$-x^3 + 4x^2$

$-\frac{1}{2}x + 2$

$+\frac{1}{2}x - 2$

0

c) $\cos^3\theta - 4\cos^2\theta - \frac{1}{2}\cos\theta + 2 = 0$

$\Rightarrow (\cos\theta - 4)(\cos^2\theta - \frac{1}{2}) = 0$ FROM PART (a)/(b)

EITHER $\cancel{\cos\theta = 4}$ $\quad -1 \le \cos\theta \le 1$

OR $\cos^2\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$\arccos\left(\frac{\sqrt{2}}{2}\right) = 45°$

$\theta = 45° \pm 360n$

$\theta = 315° \pm 360n$ $\quad n = 0, 1, 2, 3 \ldots$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = 135°$

$\theta = 135° \pm 360n$

$\theta = 225° \pm 360n$ $\quad n = 0, 1, 2, 3 \ldots$

$\theta = 45°, 315°, 135°, 225°$

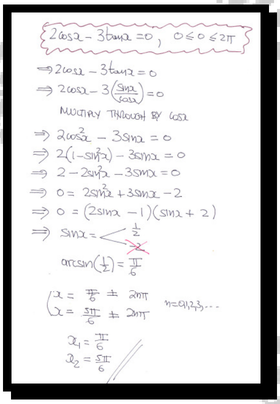
Question 55 (****)

Solve the following trigonometric equation in the range given.

$$2\cos x - 3\tan x = 0, \quad 0 \le x < 2\pi .$$

Give the answers in terms of π.

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$



$2\cos x - 3\tan x = 0, \quad 0 \le \theta \le 2\pi$

$\Rightarrow 2\cos x - 3\tan x = 0$

$\Rightarrow 2\cos x - 3\left(\frac{\sin x}{\cos x}\right) = 0$

MULTIPLY THROUGH BY $\cos x$

$\Rightarrow 2\cos^2 x - 3\sin x = 0$

$\Rightarrow 2(1-\sin^2 x) - 3\sin x = 0$

$\Rightarrow 2 - 2\sin^2 x - 3\sin x = 0$

$\Rightarrow 0 = 2\sin^2 x + 3\sin x - 2$

$\Rightarrow 0 = (2\sin x - 1)(\sin x + 2)$

$\Rightarrow \sin x = \frac{1}{2}, \cancel{-2}$

$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$x = \frac{\pi}{6} \pm 2n\pi$
$x = \frac{5\pi}{6} \pm 2n\pi$ $\quad n = 0, 1, 2, 3, \ldots$

$x_1 = \frac{\pi}{6}$
$x_2 = \frac{5\pi}{6}$

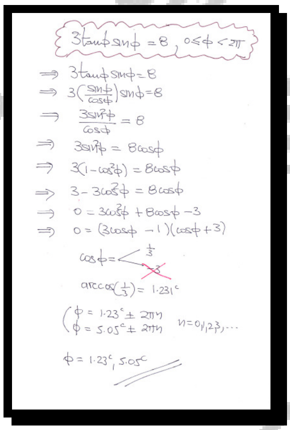
Question 56 (****)

Solve the following trigonometric equation in the range given.

$$3\tan\varphi\sin\varphi = 8, \quad 0 \le \varphi < 2\pi .$$

Give the answers in radians correct to two decimal places.

$$\boxed{\varphi \approx 1.23^{c}, 5.05^{c}}$$



$3\tan\phi\sin\phi = 8, \quad 0 \le \phi < 2\pi$

$\Rightarrow 3\tan\phi\sin\phi = 8$

$\Rightarrow 3\left(\frac{\sin\phi}{\cos\phi}\right)\sin\phi = 8$

$\Rightarrow \frac{3\sin^2\phi}{\cos\phi} = 8$

$\Rightarrow 3\sin^2\phi = 8\cos\phi$

$\Rightarrow 3(1-\cos^2\phi) = 8\cos\phi$

$\Rightarrow 3 - 3\cos^2\phi = 8\cos\phi$

$\Rightarrow 0 = 3\cos^2\phi + 8\cos\phi - 3$

$\Rightarrow 0 = (3\cos\phi - 1)(\cos\phi + 3)$

$\cos\phi = \frac{1}{3}, \cancel{-3}$

$\arccos\left(\frac{1}{3}\right) = 1.231^{c}$

$\phi = 1.23^{c} \pm 2\pi n$
$\phi = 5.05^{c} \pm 2\pi n$ $\quad n = 0, 1, 2, 3, \ldots$

$\phi = 1.23^{c}, 5.05^{c}$

Question 57 (****)

Solve the following trigonometric equation in the range given.

$$4\tan\psi\sin\psi\cos\psi + 4\tan\psi\cos\psi + 1 = 0\,, \quad 0° \le \psi < 360°\,.$$

$\boxed{}$, $\boxed{\psi = 210°,\ 330°}$

$$4\tan\psi\sin\psi\cos\psi + 4\tan\psi\cos\psi + 1 = 0$$
$$\Rightarrow 4\frac{\sin\psi}{\cancel{\cos\psi}}\sin\psi\cancel{\cos\psi} + 4\frac{\sin\psi}{\cancel{\cos\psi}}\cancel{\cos\psi} + 1 = 0$$
$$\Rightarrow 4\sin^2\psi + 4\sin\psi + 1 = 0$$
$$\Rightarrow (2\sin\psi + 1)^2 = 0$$
$$\Rightarrow \sin\psi = -\tfrac{1}{2}$$
$$\arcsin\left(-\tfrac{1}{2}\right) = -30°$$
$$\begin{cases}\psi = -30° \pm 360n \\ \psi = 210° \pm 360n\end{cases} \qquad \therefore \begin{array}{l}\psi_1 = 210° \\ \psi_2 = 330°\end{array}$$

Question 58 (****)

Solve the following trigonometric equation in the range given.

$$\frac{1}{2}\tan x - \sin x = 0\,, \quad 0° \le x < 360°\,.$$

$\boxed{}$, $\boxed{x = 0°,\ 60°,\ 180°,\ 300°}$

$$\tfrac{1}{2}\tan x - \sin x = 0$$
$$\Rightarrow \tfrac{1}{2}\left(\frac{\sin x}{\cos x}\right) - \sin x = 0$$
$$\Rightarrow \frac{\sin x}{2\cos x} - \sin x = 0$$
$$\Rightarrow \sin x - 2\sin x\cos x = 0$$
$$\Rightarrow \sin x(1 - 2\cos x) = 0$$
$$\sin x = 0 \quad \underline{\underline{\text{OR}}} \quad \cos x = \tfrac{1}{2}$$

- $\arcsin(0) = 0°$

$$x = 0° \pm 360n, \quad x = 180° \pm 360n \qquad n = 0, 1, 2, 3, \ldots$$

- $\arccos\left(\tfrac{1}{2}\right) = 60°$

$$x = 60° \pm 360n, \quad x = 300° \pm 360n \qquad n = 0, 1, 2, 3$$

$$x_1 = 0°, \quad x_2 = 180°, \quad x_3 = 60°, \quad x_4 = 300°$$

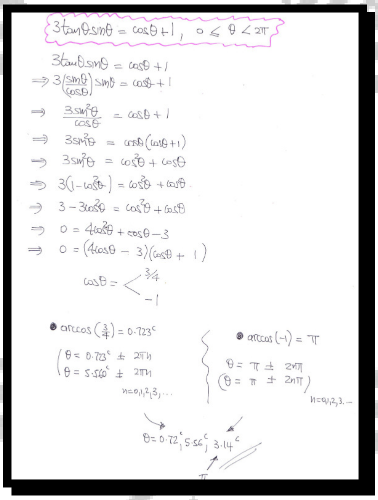
Question 59 (****)

Solve the following trigonometric equation in the range given.

$$3\tan\theta\sin\theta = \cos\theta + 1, \quad 0 \le \theta < 2\pi .$$

Give the answers in radians correct to two decimal places.

$\boxed{}$, $\boxed{\theta \approx 0.72^{c}, 3.14^{c}, 5.56^{c}}$


$3\tan\theta\sin\theta = \cos\theta+1, \quad 0 \le \theta < 2\pi$

$3\tan\theta\sin\theta = \cos\theta+1$

$\Rightarrow 3\left(\frac{\sin\theta}{\cos\theta}\right)\sin\theta = \cos\theta+1$

$\Rightarrow \frac{3\sin^2\theta}{\cos\theta} = \cos\theta+1$

$\Rightarrow 3\sin^2\theta = \cos\theta(\cos\theta+1)$

$\Rightarrow 3\sin^2\theta = \cos^2\theta+\cos\theta$

$\Rightarrow 3(1-\cos^2\theta) = \cos^2\theta+\cos\theta$

$\Rightarrow 3-3\cos^2\theta = \cos^2\theta+\cos\theta$

$\Rightarrow 0 = 4\cos^2\theta+\cos\theta-3$

$\Rightarrow 0 = (4\cos\theta-3)(\cos\theta+1)$

$\cos\theta = \frac{3}{4}$ or -1

• $\arccos\left(\frac{3}{4}\right) = 0.723^{c}$

$\theta = 0.723^{c} \pm 2\pi n$

$\theta = 5.560^{c} \pm 2\pi n$

$n = 0,1,2,3,\ldots$

• $\arccos(-1) = \pi$

$\theta = \pi \pm 2n\pi$

$(\theta = \pi \pm 2n\pi)$

$n = 0,1,2,3,\ldots$

$\theta = 0.72^{c}, 5.56^{c}, 3.14^{c}$

π

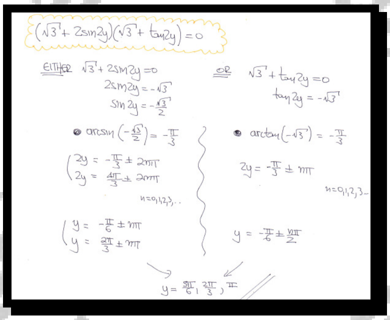
Question 60 (****)

Solve the following trigonometric equation in the range given.

$$\left(\sqrt{3} + 2\sin 2y\right)\left(\sqrt{3} + \tan 2y\right) = 0, \quad 0 \le y < \pi .$$

Give the answers in terms of π.

$\boxed{}$, $\boxed{y = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}}$


$(\sqrt{3}+2\sin 2y)(\sqrt{3}+\tan 2y) = 0$

EITHER $\sqrt{3}+2\sin 2y = 0$

$2\sin 2y = -\sqrt{3}$

$\sin 2y = -\frac{\sqrt{3}}{2}$

• $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

$2y = -\frac{\pi}{3} \pm 2n\pi$

$2y = \frac{4\pi}{3} \pm 2n\pi$

$n = 0,1,2,3,\ldots$

$y = -\frac{\pi}{6} \pm n\pi$

$y = \frac{2\pi}{3} \pm n\pi$

OR $\sqrt{3}+\tan 2y = 0$

$\tan 2y = -\sqrt{3}$

• $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$

$2y = -\frac{\pi}{3} \pm n\pi$

$n = 0,1,2,3,\ldots$

$y = -\frac{\pi}{6} \pm \frac{n\pi}{2}$

$y = \frac{5\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{3}$

Question 61 (****)

Solve the following trigonometric equation in the range given.

$$6\cos\psi = 5\tan\psi \ , \ 0 \le \psi < 2\pi \ .$$

Give the answers in radians, correct to two decimal places.

$\boxed{}$, $\boxed{\psi \approx 0.73^{c}, 2.41^{c}}$

$$6\cos\psi = 5\tan\psi$$

$$\Rightarrow 6\cos\psi = \frac{5\sin\psi}{\cos\psi}$$

$$\Rightarrow 6\cos^2\psi = 5\sin\psi$$

$$\Rightarrow 6(1-\sin^2\psi) = 5\sin\psi$$

$$\Rightarrow 6 - 6\sin^2\psi = 5\sin\psi$$

$$\Rightarrow 0 = 6\sin^2\psi + 5\sin\psi - 6$$

$$\Rightarrow (3\sin\psi - 2)(2\sin\psi + 3) = 0$$

$$\Rightarrow \sin\psi = \begin{cases} 2/3 \\ \cancel{-3/2} \end{cases}$$

$$\arcsin\left(\tfrac{2}{3}\right) \simeq 0.7297^{c}$$

$$\begin{cases} \psi = 0.7297^{c} \pm 2\pi n \\ \psi = 2.4119^{c} \pm 2\pi n \end{cases} \quad n = 0, 1, 2, 3$$

$$\psi_1 = 0.73^{c}$$

$$\psi_2 = 2.41^{c}$$

Question 62 (****)

$$f(x) = x^3 - x^2 - 3x + 3.$$

a) Show that $(x-1)$ is a factor of $f(x)$.

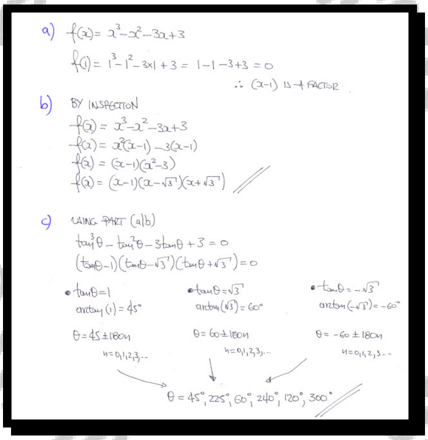
b) Express $f(x)$ as the product of **three linear factors**.

c) Hence solve the trigonometric equation

$$\tan^3\theta - \tan^2\theta - 3\tan\theta + 3 = 0,$$

for $0° \le \theta < 360°$.

$\boxed{\theta = 45°, 60°, 120°, 225°, 240°, 300°}$


a) $f(x) = x^3 - x^2 - 3x + 3$

$f(1) = 1^3 - 1^2 - 3\times1 + 3 = 1 - 1 - 3 + 3 = 0$

$\therefore (x-1)$ IS A FACTOR

b) BY INSPECTION

$f(x) = x^3 - x^2 - 3x + 3$

$f(x) = x^2(x-1) - 3(x-1)$

$f(x) = (x-1)(x^2-3)$

$f(x) = (x-1)(x-\sqrt{3})(x+\sqrt{3})$

c) USING PART (a|b)

$\tan^3\theta - \tan^2\theta - 3\tan\theta + 3 = 0$

$(\tan\theta - 1)(\tan\theta - \sqrt{3})(\tan\theta + \sqrt{3}) = 0$

- $\tan\theta = 1$ — $\arctan(1) = 45°$ — $\theta = 45 \pm 180n$, $n = 0,1,2,3,\ldots$
- $\tan\theta = \sqrt{3}$ — $\arctan(\sqrt{3}) = 60°$ — $\theta = 60 \pm 180n$, $n = 0,1,2,3,\ldots$
- $\tan\theta = -\sqrt{3}$ — $\arctan(-\sqrt{3}) = -60°$ — $\theta = -60 \pm 180n$, $n = 0,1,2,3,\ldots$

$\theta = 45°, 225°, 60°, 240°, 120°, 300°$

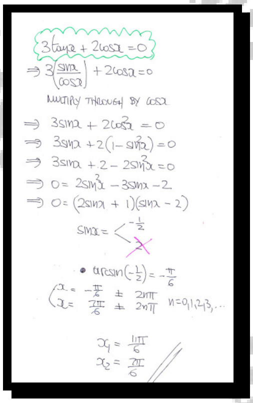
Question 63 (****)

Solve the following trigonometric equation in the range given.

$$3\tan x + 2\cos x = 0, \quad 0 \le x < 2\pi .$$

Give the answers in terms of π.

$$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$$



$3\tan x + 2\cos x = 0$

$\Rightarrow 3\left(\frac{\sin x}{\cos x}\right) + 2\cos x = 0$

MULTIPLY THROUGH BY $\cos x$

$\Rightarrow 3\sin x + 2\cos^2 x = 0$

$\Rightarrow 3\sin x + 2(1 - \sin^2 x) = 0$

$\Rightarrow 3\sin x + 2 - 2\sin^2 x = 0$

$\Rightarrow 0 = 2\sin^2 x - 3\sin x - 2$

$\Rightarrow 0 = (2\sin x + 1)(\sin x - 2)$

$\sin x = \begin{cases} -\frac{1}{2} \\ \cancel{2} \end{cases}$

- $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\begin{cases} x = -\frac{\pi}{6} \pm 2n\pi \\ x = \frac{7\pi}{6} \pm 2n\pi \end{cases} \quad n = 0, 1, 2, 3, \ldots$

$x_1 = \frac{11\pi}{6}$

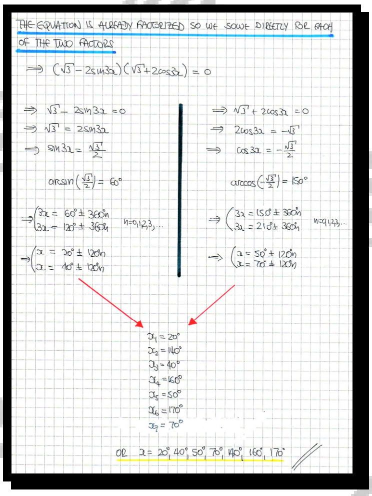
$x_2 = \frac{7\pi}{6}$

Question 64 (****)

Solve the following trigonometric equation in the range given.

$$\left(\sqrt{3} - 2\sin 3x\right)\left(\sqrt{3} + 2\cos 3x\right) = 0, \quad 0° \le x < 180° .$$

$$\boxed{x = 20°, 50°, 70°, 140°, 160°, 170°}$$



THE EQUATION IS ALREADY FACTORIZED SO WE SOLVE DIRECTLY FOR EACH OF THE TWO FACTORS

$\Rightarrow (\sqrt{3} - 2\sin 3x)(\sqrt{3} + 2\cos 3x) = 0$

$\Rightarrow \sqrt{3} - 2\sin 3x = 0$

$\Rightarrow \sqrt{3} = 2\sin 3x$

$\Rightarrow \sin 3x = \frac{\sqrt{3}}{2}$

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = 60°$

$\Rightarrow \begin{cases} 3x = 60° \pm 360n \\ 3x = 120° \pm 360n \end{cases} \quad n = 0, 1, 2, 3, \ldots$

$\Rightarrow \begin{cases} x = 20° \pm 120n \\ x = 40° \pm 120n \end{cases}$

$\Rightarrow \sqrt{3} + 2\cos 3x = 0$

$\Rightarrow 2\cos 3x = -\sqrt{3}$

$\Rightarrow \cos 3x = -\frac{\sqrt{3}}{2}$

$\arccos\left(-\frac{\sqrt{3}}{2}\right) = 150°$

$\Rightarrow \begin{cases} 3x = 150° \pm 360n \\ 3x = 210° \pm 360n \end{cases} \quad n = 0, 1, 2, 3, \ldots$

$\Rightarrow \begin{cases} x = 50° \pm 120n \\ x = 70° \pm 120n \end{cases}$

$x_1 = 20°$

$x_2 = 140°$

$x_3 = 40°$

$x_4 = 160°$

$x_5 = 50°$

$x_6 = 170°$

$x_7 = 70°$

OR $x = 20°, 40°, 50°, 70°, 140°, 160°, 170°$

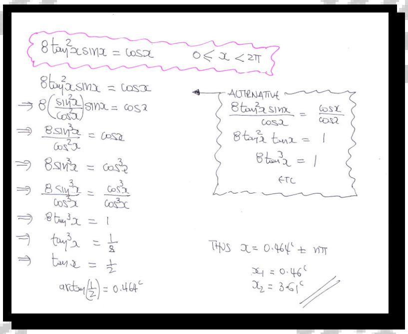
Question 65 (****)

Solve the following trigonometric equation in the range given.

$$8\tan^2 x \sin x = \cos x, \quad 0 \le x < 2\pi .$$

Give the answers in radians correct to two decimal places.

$\boxed{x \approx 0.46^c,\ 3.61^c}$

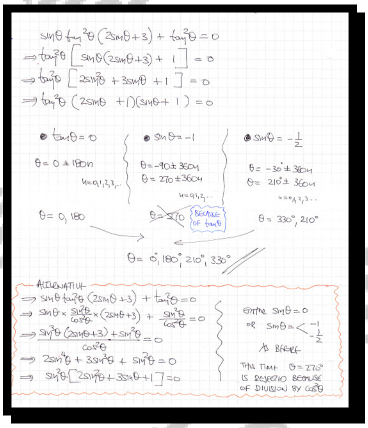


Question 66 (****+)

Solve the following trigonometric equation for $0 \le \theta < 360°$

$$\sin\theta \tan^2\theta\left(2\sin\theta + 3\right) + \tan^2\theta = 0 .$$

$\boxed{\theta = 0°,\ 180°,\ 210°,\ 330°}$



Question 67 (****+)

Calculate in **degrees**, correct to one decimal place, the solution of the following trigonometric equation

$$\frac{1-\cos\theta}{\sin\theta} = \sqrt{3}\sin\theta, \quad 0 < \theta < \pi .$$

$\boxed{}$, $\boxed{\theta \approx 2.01^{c}}$

MULTIPLY ACROSS & TIDY USING $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \frac{1-\cos\theta}{\sin\theta} = \sqrt{3}\sin\theta$$

$$\Rightarrow 1-\cos\theta = \sqrt{3}\sin^2\theta$$

$$\Rightarrow 1-\cos\theta = \sqrt{3}(1-\cos^2\theta)$$

$$\Rightarrow 1-\cos\theta = \sqrt{3}-\sqrt{3}\cos^2\theta$$

$$\Rightarrow \sqrt{3}\cos^2\theta - \cos\theta + 1 - \sqrt{3} = 0$$

BY THE QUADRATIC FORMULA

$$\Rightarrow \cos\theta = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\times\sqrt{3}\times(1-\sqrt{3})}}{2\times\sqrt{3}}$$

$$\Rightarrow \cos\theta = \frac{1 \pm \sqrt{1-4\sqrt{3}+12}}{2\sqrt{3}} = \begin{cases} 1 \\ -0.4226\ldots \end{cases}$$

SOLVING EACH CASE SEPARATELY

- $\cos\theta = 1$

  $\arccos(1) = 0$

  $\left(\begin{array}{l} \theta = 0 \pm 2n\pi \\ \theta = 2\pi \pm 2n\pi \end{array}\right. \quad n = 0,1,2,3,\ldots$

- $\cos\theta = -0.4226\ldots$

  $\arccos(-0.4226\ldots) = 2.0072\ldots^{c}$

  $\left(\begin{array}{l} \theta = 2.01 \pm 2n\pi \\ \theta = 4.28 \pm 2n\pi \end{array}\right. \quad n = 0,1,2,3,\ldots$

$\theta = 2.01^{c}$ IS THE ONLY SOLUTION IN RANGE

Question 68 (****+)

The three angles in a triangle are denoted as α, β and γ.

It is further given that

$$\tan\alpha = -4.705 \qquad \text{and} \qquad \tan(\beta-\gamma) = 0.404$$

Determine, in degrees, the size of each of the angles α, β and γ.

$\boxed{}$, $\boxed{\alpha \approx 102°}$, $\boxed{\beta \approx 50°}$, $\boxed{\gamma \approx 28°}$

START BY SOLVING EACH EQUATION NOTING THAT $0° < \alpha, \beta, \gamma < 180°$

- $\tan\alpha = -4.705$

  $\arctan(-4.705) = -78.00°$

  $\alpha = -78 \pm 180n \quad n = 0,1,2,3,\dots$

  $\alpha = 102°$

- $\tan(\beta-\gamma) = 0.404$

  $\arctan(0.404) = 21.9987\dots$

  $\beta - \gamma = 22.0 \pm 180n \quad n = 0,1,2,3,\dots$

  $\boxed{\beta - \gamma = 22°}$

BUT AS α, β, γ ARE ANGLES OF A TRIANGLE

$\Rightarrow \alpha + \beta + \gamma = 180°$

$\Rightarrow 102° + \beta + \gamma = 180°$

$\Rightarrow \boxed{\beta + \gamma = 78°}$

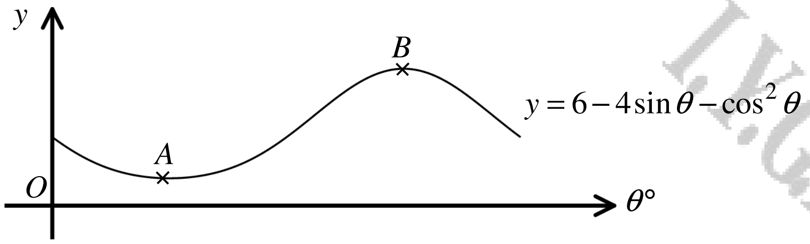
ADDING THE FOLLOWING EQUATIONS

$$\left.\begin{array}{l} \beta - \gamma = 22° \\ \beta + \gamma = 78° \end{array}\right) \Rightarrow 2\beta = 100°$$

$\Rightarrow \beta = 50°$

$\Rightarrow \gamma = 28°$

Question 69 (****+)



The figure above shows the graph of the curve with equation

$$y = 6 - 4\sin\theta - \cos^2\theta, \quad 0° \le \theta \le 360° .$$

The curve has a minimum at the point A and a maximum at the point B.

Determine the coordinates of A and B.

$\boxed{A(90°,2)}$, $\boxed{B(270°,10)}$

FIRST WRITE THE EQUATION IN $\sin\theta$

$y = 6 - 4\sin\theta - \cos^2\theta$
$y = 6 - 4\sin\theta - (1 - \sin^2\theta)$
$y = 5 - 4\sin\theta + \sin^2\theta$

BY INSPECTION, LOOKING AT THE SINE GRAPH, AND NOTING THAT $\sin 270° = -1$

$y_{MAX} = 5 - 4(-1) + (-1)^2 = 10$

$\therefore$ $B(270°, 10)$

COMPLETING THE SQUARE IN $\sin\theta$

$y = \sin^2\theta - 4\sin\theta + 5$
$y = (\sin\theta - 2)^2 + 1$

BUT $\sin\theta \neq 2$, SO MINIMUM WILL BE ACHIEVED WHEN $\sin\theta = +1$

$y_{MIN} = (+1-2)^2 + 1 = 1 + 1 = 2$

$\therefore$ $A(90°, 2)$

Question 70 (*****)

Solve the following trigonometric equation for $0 \le \theta < 360°$

$$2 + 4\cos^2\theta = 7\cos\theta\sin\theta .$$

$\boxed{}$, $\boxed{\theta \approx 56.3°,\ \theta \approx 63.4°,\ \theta \approx 236.3°,\ \theta \approx 243.4°}$

PROCEED AS FOLLOWS

$\Rightarrow 2 + 4\cos^2\theta = 7\cos\theta\sin\theta$

$\Rightarrow 2(\cos^2\theta + \sin^2\theta) + 4\cos^2\theta = 7\cos\theta\sin\theta$

$\Rightarrow 6\cos^2\theta - 7\cos\theta\sin\theta + 2\sin^2\theta = 0$

FACTORIZE THE QUADRATIC EXPRESSION

$\Rightarrow (3\cos\theta - 2\sin\theta)(2\cos\theta - \sin\theta) = 0$

HENCE WE OBTAIN TWO EQUATIONS

$\Rightarrow 3\cos\theta - 2\sin\theta = 0$

$\Rightarrow 3\cos\theta = 2\sin\theta$

$\Rightarrow \frac{3\cos\theta}{\cos\theta} = \frac{2\sin\theta}{\cos\theta}$

$\Rightarrow 3 = 2\tan\theta$

$\Rightarrow \tan\theta = \frac{3}{2}$

$\arctan\left(\frac{3}{2}\right) = 56.3°$

$\Rightarrow \theta = 56.3 \pm 180n \quad n = 0,1,2,3,\ldots$

$\Rightarrow 2\cos\theta - \sin\theta = 0$

$\Rightarrow 2\cos\theta = \sin\theta$

$\Rightarrow \frac{2\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow 2 = \tan\theta$

$\arctan(2) = 63.4°$

$\Rightarrow \theta = 63.4 \pm 180n \quad n = 0,1,2,3,\ldots$

$\theta = 56.3°, 63.4°, 236.3°, 243.4°$

ALTERNATIVE APPROACH USING MINOR TRIG RATIOS AS FOLLOWS

$\Rightarrow 2 + 4\cos^2\theta = 7\cos\theta\sin\theta$

$\Rightarrow \frac{2}{\cos^2\theta} + \frac{4\cos^2\theta}{\cos^2\theta} = \frac{7\cos\theta\sin\theta}{\cos^2\theta}$

$\Rightarrow 2\sec^2\theta + 4 = 7\tan\theta$

$\Rightarrow 2(1 + \tan^2\theta) + 4 = 7\tan\theta$

$\Rightarrow 2 + 2\tan^2\theta + 4 = 7\tan\theta$

$\Rightarrow 2\tan^2\theta - 7\tan\theta + 6 = 0$

$\Rightarrow (2\tan\theta - 3)(\tan\theta - 2) = 0$

$\Rightarrow \tan\theta = \frac{3}{2}$ or 2

ETC. ETC. AS WITH THE PREVIOUS METHOD

Question 71 (*****)

It is given that

$$4\sin x - \frac{\cos x}{2} = \frac{4}{\sin x} - \frac{1}{2\cos x}.$$

Show clearly that the above equation is equivalent to

$$\tan x = 2.$$

$\boxed{}$, $\boxed{\text{proof}}$

$$4\sin x - \frac{\cos x}{2} = \frac{4}{\sin x} - \frac{1}{2\cos x}$$

- MULTIPLY BY 2

$$8\sin x - \cos x = \frac{8}{\sin x} - \frac{1}{\cos x}$$

- MULTIPLY BY $\sin x$, THEN BY $\cos x$

$$\Rightarrow 8\sin^2 x - \sin x\cos x = 8 - \frac{\sin x}{\cos x}$$

$$\Rightarrow 8\sin^2 x\cos x - \sin x\cos^2 x = 8\cos x - \sin x$$

$$\Rightarrow 8\sin^2 x\cos x - 8\cos x = \sin x\cos^2 x - \sin x$$

$$\Rightarrow 8\cos x(\sin^2 x - 1) = \sin x(\cos^2 x - 1)$$

$$\Rightarrow 8\cos x(1 - \sin^2 x) = \sin x(1 - \cos^2 x)$$

$$\Rightarrow 8\cos^3 x = \sin^3 x$$

$$\Rightarrow \frac{8\cancel{\cos^3 x}}{\cancel{\cos^3 x}} = \frac{\sin^3 x}{\cos^3 x}$$

$$\Rightarrow 8 = \tan^3 x$$

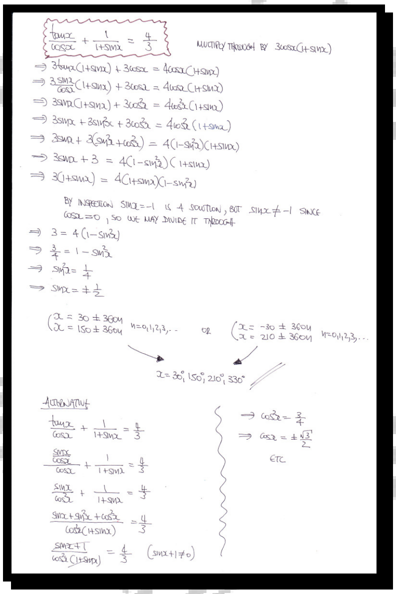
$$\Rightarrow \tan x = 2$$

Question 72 (*****)

Solve the following trigonometric equation for $0 \le x < 360°$

$$\frac{\tan x}{\cos x} + \frac{1}{1+\sin x} = \frac{4}{3}.$$

$\boxed{}$, $\boxed{x = 30°,\ 150°,\ 210°,\ 330°}$



$$\boxed{\frac{\tan x}{\cos x} + \frac{1}{1+\sin x} = \frac{4}{3}}$$

MULTIPLY THROUGH BY $3\cos x(1+\sin x)$

$\Rightarrow 3\tan x(1+\sin x) + 3\cos x = 4\cos x(1+\sin x)$

$\Rightarrow \frac{3\sin x}{\cos x}(1+\sin x) + 3\cos x = 4\cos x(1+\sin x)$

$\Rightarrow 3\sin x(1+\sin x) + 3\cos^2 x = 4\cos^2 x(1+\sin x)$

$\Rightarrow 3\sin x + 3\sin^2 x + 3\cos^2 x = 4\cos^2 x(1+\sin x)$

$\Rightarrow 3\sin x + 3(\sin^2 x + \cos^2 x) = 4(1-\sin^2 x)(1+\sin x)$

$\Rightarrow 3\sin x + 3 = 4(1-\sin^2 x)(1+\sin x)$

$\Rightarrow 3(1+\sin x) = 4(1+\sin x)(1-\sin^2 x)$

BY INSPECTION $\sin x = -1$ IS A SOLUTION, BUT $\sin x \neq -1$ SINCE $\cos x = 0$, SO WE MAY DIVIDE IT THROUGH

$\Rightarrow 3 = 4(1-\sin^2 x)$

$\Rightarrow \frac{3}{4} = 1 - \sin^2 x$

$\Rightarrow \sin^2 x = \frac{1}{4}$

$\Rightarrow \sin x = \pm\frac{1}{2}$

$\begin{cases} x = 30 \pm 360n \\ x = 150 \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$ OR $\begin{cases} x = -30 \pm 360n \\ x = 210 \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$

$x = 30°, 150°, 210°, 330°$

ALTERNATIVE

$\frac{\tan x}{\cos x} + \frac{1}{1+\sin x} = \frac{4}{3}$

$\frac{\frac{\sin x}{\cos x}}{\cos x} + \frac{1}{1+\sin x} = \frac{4}{3}$

$\frac{\sin x}{\cos^2 x} + \frac{1}{1+\sin x} = \frac{4}{3}$

$\frac{\sin x + \sin^2 x + \cos^2 x}{\cos^2 x(1+\sin x)} = \frac{4}{3}$

$\frac{\cancel{\sin x + 1}}{\cos^2 x\cancel{(1+\sin x)}} = \frac{4}{3}$ $(\sin x + 1 \neq 0)$

$\Rightarrow \cos^2 x = \frac{3}{4}$

$\Rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$

ETC

Question 73 (*****)

Solve the following trigonometric equation

$$\left(19+2\sin^2 2\theta\right)\tan 2\theta=\frac{3}{\cos 2\theta}-17\cos 2\theta\,,\qquad 0°\le\theta<360°\,.$$

$\boxed{}$, $\boxed{\theta=105°,\quad \theta=165°,\quad \theta=285°,\quad \theta=345°}$

$$\boxed{(19+2\sin^2 2\theta)\tan 2\theta=\frac{3}{\cos 2\theta}-17\cos 2\theta}$$

- START MANIPULATING THE EQUATION BY SWITCHING THE $\tan 2\theta$ INTO SINES AND COSINES AND THEN GETTING RID OF THE DENOMINATORS

$$\Rightarrow (19+2\sin^2 2\theta)\times\frac{\sin 2\theta}{\cos 2\theta}=\frac{3}{\cos 2\theta}-17\cos 2\theta \qquad \times\cos 2\theta$$

$$\Rightarrow (19+2\sin^2 2\theta)\times\sin 2\theta=3-17\cos^2 2\theta$$

$$\Rightarrow (19+2\sin^2 2\theta)\sin 2\theta=3-17(1-\sin^2 2\theta)$$

$$\Rightarrow 19\sin 2\theta+2\sin^3 2\theta=-14+17\sin^2 2\theta$$

$$\Rightarrow 2\sin^3 2\theta-17\sin^2 2\theta+19\sin 2\theta+14=0$$

- THIS IS A CUBIC IN $\sin 2\theta$, SO LOOK FOR SOME SENSIBLE LINEAR FACTORS

$$f(x)=2x^3-17x^2+19x+14$$

- $f(1)=2-17+19+14\neq 0$
- $f(-1)=-2-17-19+14\neq 0$
- $f(2)=16-68+38+14=0$

$\therefore\ (x-2)$ IS A FACTOR

- FACTORIZE THE CUBIC BY USING LONG DIVISION, INSPECTION OR ANY OTHER SENSIBLE METHOD

$$\begin{aligned}f(x)&=2x^3-17x^2+19x+14\\&=2x^2(x-2)-13x(x-2)-7(x-2)\\&=(x-2)(2x^2-13x-7)\\&=(x-2)(2x+1)(x-7)\end{aligned}$$

$$\therefore\ f(x)=0\Rightarrow x=\begin{cases}2\\7\\-\frac{1}{2}\end{cases}$$

$$f(\sin 2\theta)=0\Rightarrow \sin 2\theta=\begin{cases}\cancel{2}\\\cancel{7}\\-\frac{1}{2}\end{cases}$$

- SOLVING THE EQUATION $\sin 2\theta=-\frac{1}{2}$ FOR $0\le\theta<360°$

$$\Rightarrow \arcsin\left(-\tfrac{1}{2}\right)=-30°$$

$$\Rightarrow \begin{cases}2\theta=-30°\pm 360n\\2\theta=210°\pm 360n\end{cases}\qquad n=0,1,2,3,\ldots$$

$$\Rightarrow \begin{cases}\theta=-15°\pm 180n\\\theta=105°\pm 180n\end{cases}$$

$$\Rightarrow \theta=165°,\ 345°,\ 105°,\ 285°$$

$$\Rightarrow \theta=105°,\ 165°,\ 285°,\ 345°$$

Question 74 (*****)

$$4\cos^2\theta + \tan^4\theta = 10, \quad 0 \le \theta < 2\pi.$$

Show that $\theta = \frac{1}{3}\pi$ is a solution of the above trigonometric equation and use a non verification method to find the other solutions.

$\boxed{}$, $\boxed{\theta = \frac{2}{3}\pi, \ \frac{4}{3}\pi, \ \frac{5}{3}\pi}$

- START BY ATTEMPTING TO CREATE AN EQUATION IN A SINGLE TRIGONOMETRIC FUNCTION, AS FOLLOWS

$\Rightarrow 4\cos^2\theta + \tan^4\theta = 10$

$\Rightarrow 4\cos^2\theta + \frac{\sin^4\theta}{\cos^4\theta} = 10$

$\Rightarrow 4\cos^6\theta + \sin^4\theta = 10\cos^4\theta$

$\Rightarrow 4\cos^6\theta + (1-\cos^2\theta)^2 = 10\cos^4\theta$

$\Rightarrow 4\cos^6\theta + 1 - 2\cos^2\theta + \cos^4\theta = 10\cos^4\theta$

$\Rightarrow 4\cos^6\theta - 9\cos^4\theta - 2\cos^2\theta + 1 = 0$

THIS IS A CUBIC IN $\cos^2\theta$ — LET $a = \cos^2\theta$

$\Rightarrow 4a^3 - 9a^2 - 2a + 1 = 0$

- WE ARE GIVEN THAT $\theta = \frac{\pi}{3}$ IS A SOLUTION

$\Rightarrow \cos\frac{\pi}{3} = \frac{1}{2}$

$\Rightarrow \cos^2\frac{\pi}{3} = \frac{1}{4}$

$\Rightarrow a = \frac{1}{4}$

$\Rightarrow 4a - 1 = 0$

$\therefore (4a-1)$ IS A FACTOR OF THE CUBIC

- BY LONG DIVISION OR MANIPULATIONS

$\Rightarrow a^2(4a-1) - 2a(4a-1) - (4a-1) = 0$

$\Rightarrow (4a-1)(a^2-2a-1) = 0$

- NOW LOOKING AT THE QUADRATIC

$\Rightarrow a^2 - 2a - 1 = 0$

$\Rightarrow (a-1)^2 - 2 = 0$

$\Rightarrow (a-1)^2 = 2$

$\Rightarrow a - 1 = \pm\sqrt{2}$

$\Rightarrow a = 1 \pm \sqrt{2}$

$\Rightarrow \cos^2\theta = \begin{cases} \cancel{1+\sqrt{2}} > 1 \\ \cancel{1-\sqrt{2}} < 0 \end{cases}$

$\left[\cos\theta = \begin{cases} \sqrt{1+\sqrt{2}} > 1 \\ -\sqrt{1+\sqrt{2}} < -1 \end{cases} \right]$

- THUS ONLY SOLUTION IS $a = \frac{1}{4}$

$\Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases}$

- THUS $\cos\theta = \frac{1}{2}$ & $\cos\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ & $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$