TRIGONOMETRY
EXAM QUESTIONS
INTRODUCTION
Question 1 (**+)**
Solve the following trigonometric equation in the range given.

\[
\cos(2\theta + 25)^\circ = -0.454, \quad 0 \leq \theta < 360.
\]

\[\theta \approx 46, 109, 226, 289\]

Question 2 (**+)**
Solve the following trigonometric equation in the range given.

\[
\cos(2y - 35)^\circ = 0.891, \quad 0 \leq y < 360.
\]

\[y \approx 4, 31, 184, 211\]
Question 3 (**+)**
Solve the following trigonometric equation in the range given.

\[ \tan(5y-35) \degree = -2 - \sqrt{3}, \quad 0 \leq y < 90. \]

\[ y \approx 28, 64 \]

Question 4 (**+)**
Solve, in radians, the following trigonometric equation

\[ 1 + \sin 2x = \frac{1}{3}, \quad 0 \leq x < 2\pi. \]

giving the answers correct to three significant figures.

\[ x = 1.94, 2.78, 5.08, 5.92 \]
Question 5 (**+)
Solve the following trigonometric equation in the range given.

\[ 2 \cos \theta = \sin \theta, \quad 0^\circ \leq \theta < 360^\circ. \]

\[ \theta = 63.4^\circ, 243.4^\circ \]

Question 6 (**+)
Solve the following trigonometric equation in the range given.

\[ 2 \sin \theta = 5 \cos \theta, \quad 0^\circ \leq \theta < 360^\circ. \]

\[ x = 68.2^\circ, 248.2^\circ \]
Question 7 (**+)**
Solve the following trigonometric equation in the range given.

\[ 2\sin y + 5\cos y = 2\cos y, \quad 0^\circ \leq y < 360^\circ. \]

\[ y \approx 123.7^\circ, \ 303.7^\circ \]

Question 8 (***)
Solve the following trigonometric equation in the range given.

\[ 3\cos 3x - 1 = 0.22, \quad -90^\circ \leq x < 90^\circ. \]

\[ x \approx -22^\circ, \ 22^\circ \]
Question 9 (***)
Solve the following trigonometric equation in the range given.

\[ 1 + 2\sin(\theta + 25)^\circ = 2.532, \quad 0 \leq \theta < 360. \]

\[ \theta \approx 25, 105 \]

Question 10 (***)
Solve, in radians, the following trigonometric equation

\[ 4\sin^2 \psi = 15 \cos \psi, \quad 0 \leq \psi < 2\pi, \]

giving the answers correct to three significant figures.

\[ \psi \approx 1.32\text{c}, 4.97\text{c} \]
Question 11  (***)
Solve the following trigonometric equation in the range given.

\[ 4 \sin 2\theta + 3 \cos 2\theta = 0, \quad 0^\circ \leq \theta < 360^\circ. \]

\[ \theta = 71.6^\circ, 161.6^\circ, 251.6^\circ, 341.6^\circ \]

Question 12  (***)
Solve the following trigonometric equation in the range given.

\[ 2 + 2 \sin 3\phi = 1, \quad 0^\circ \leq \phi < 180^\circ. \]

\[ \phi = 70^\circ, 110^\circ \]
Question 13 (★★★)
Solve the following trigonometric equation in the range given.

\[ 9\cos 4\theta + 5\sin 4\theta = 0, \quad 0^\circ \leq \theta < 180^\circ. \]

\[ \theta = 29.8^\circ, 74.8^\circ, 119.8^\circ, 164.8^\circ \]

Question 14 (★★★)
Solve the following trigonometric equation in the range given.

\[ 3\sin 3y + \sqrt{3}\cos 3y = 0, \quad 0^\circ \leq y < 180^\circ. \]

\[ y = 50^\circ, 110^\circ, 170^\circ \]
Question 15 (***)
Solve, in radians, the following trigonometric equation

$$6\cos^2 x + \sin x = 4, \quad 0 \leq x < 2\pi,$$

giving the answers correct to three significant figures.

$$x \approx 0.73^\circ, 2.41^\circ, 3.67^\circ, 5.76^\circ$$

Question 16 (***)
Solve, in radians, the following trigonometric equation

$$5 + 2\tan\left(3\theta + \frac{\pi}{3}\right) = 3, \quad 0 \leq \theta < \pi,$$

giving the answers in terms of \(\pi\).

$$\theta = \frac{5\pi}{36}, \frac{17\pi}{36}, \frac{29\pi}{36}$$
Question 17  (***)
Solve, in degrees, the following trigonometric equation

\[3 \sin^2 3x - 7 \cos 3x = 5, \quad 0^\circ \leq x < 180^\circ.\]

\[x \approx 36.5^\circ, 83.5^\circ, 156.5^\circ\]

Question 18  (***)
Solve, in radians, the following trigonometric equation

\[8 \sin \left(\frac{\pi}{3} - 2x\right) = 4, \quad 0 \leq \theta < 2\pi;\]

giving the answers in terms of \(\pi\).

\[x = \frac{3\pi}{4}, \frac{13\pi}{4}, \frac{7\pi}{4}\]
Question 19 (***)
Solve the following trigonometric equation in the range given.

\[ 4\sin^2 \theta - \cos^2 \theta = 8\sin \theta + 3, \quad 0^\circ \leq \theta < 360^\circ. \]

\[ \theta = 203.6^\circ, 336.4^\circ \]

Question 20 (***)
Solve, in degrees, the following trigonometric equation

\[ \sin 3x = \sin 48^\circ, \quad 0^\circ \leq x < 180^\circ. \]

\[ x = 16^\circ, 136^\circ, 164^\circ \]
Question 21 (***)
Solve, in radians, the following trigonometric equation

\[ \cos 2x = \cos \frac{2\pi}{5}, \quad 0 \leq x < 2\pi, \]

giving the answers in terms of \( \pi \).

\[ x = \frac{\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{9\pi}{5} \]

Question 22 (***)
Solve the following trigonometric equation in the range given.

\[ 2\sin^2 x - 2 \cos x - \cos^2 x = 1, \quad 0^\circ \leq x < 360^\circ. \]

\[ x = 70.5^\circ, 289.5^\circ, \quad x = 180^\circ \]
Question 23 (***)
Solve the following trigonometric equation.

\[ \sin(3\theta + 72)\degree = \cos 48\degree, \quad 0 \leq \theta < 180. \]

\[ \theta = \{22, 110, 142\} \]
Question 24 (***+)

Solve the following trigonometric equation in the range given.

\[
\frac{5 + \cos(4y - 80)°}{3} = 1.5, \quad 0 \leq y < 180°.
\]

\[y = 50, 80, 140, 170\]

Question 25 (***+)

Solve the following trigonometric equation in the range given.

\[
\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta, \quad 0° \leq \theta < 360°.
\]

\[\theta = 120°, 240°\]
Question 26  (***)

Solve, in radians, the following trigonometric equation

$$\sin^2 \left( \frac{3x}{2} \right) = \frac{1}{2}, \quad 0 \leq x < 2\pi,$$

giving the answers in terms of $\pi$.

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

Question 27  (***)

Solve the following trigonometric equation in the range given.

$$\sin x - \cos x = 2, \quad 0^\circ \leq x < 360^\circ.$$

$$x \approx 71.6^\circ, 251.6^\circ$$
Question 28  (***)
Solve, in radians, the following trigonometric equation
\[ \frac{1}{\tan^2 \varphi} = 3, \quad 0 \leq \varphi < 2\pi, \]
giving the answers in terms of \( \pi \).

\[ \varphi = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \]

Question 29  (***)
Solve, in radians, the following trigonometric equation
\[ 4\sin^2 2\varphi - \cos 2\varphi = 3 + 8\sin 2\varphi, \quad 0 \leq \varphi < 2\pi. \]
giving the answers correct to three significant figures.
\[ \varphi \approx 1.78^\circ, 2.94^\circ, 4.92^\circ, 6.08^\circ \]
Question 30 (***+)

Solve the following trigonometric equation in the range given.

\[ 3\cos^2 2\phi - 4\sin^2 2\phi = 15\cos 2\phi - 6, \quad 0^\circ \leq \phi < 360^\circ. \]

\[ \phi \approx 40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ. \]}
Question 31 (***)

Solve, in radians, the following trigonometric equation

\[ 3\sin^2 \psi = \cos^2 \psi, \quad 0 \leq \psi < 2\pi, \]

giving the answers in terms of \( \pi \).

\[ \psi = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \]

Question 32 (***)

Solve, in degrees, the following trigonometric equation

\[ \tan(3x - 75) = \tan 450^\circ, \quad 300^\circ \leq x < 500^\circ. \]

\[ x = 355^\circ, x = 415^\circ, x = 475^\circ \]
Question 33 (***+)
Solve the following trigonometric equation in the range given.
\[ \frac{5\sin \theta - 2\cos \theta}{\sin \theta} = 3, \quad 0^\circ \leq \theta < 360^\circ. \]
\[ x = 45^\circ, 225^\circ \]

Question 34 (***+)
\[ 2CT - 2C + T - 1 \]

a) Write the above expression as a product of two linear factors.

b) Hence solve the trigonometric equation
\[ 2\cos \theta \tan \theta - 2\cos \theta + \tan \theta = 1, \]
for \( 0^\circ \leq \theta < 360^\circ. \)
\[ (2C + 1)(T - 1), \quad \theta = 45^\circ, 120^\circ, 135^\circ, 240^\circ \]
Question 35  (***+)
Solve the following trigonometric equation in the range given.

\[ \cos(4\psi - 120^\circ) = \cos 200^\circ, \quad 0 \leq \psi < 180. \]

\[ \psi = 70, \ 80, \ 160, \ 170 \]

Question 36  (***+)
Solve, in radians, the following trigonometric equation

\[ 2 + 3\sin^2 4x = 4, \quad 0 \leq x < \frac{\pi}{2}, \]

giving the answers correct to three significant figures.

\[ x = 0.239^\circ, \ 0.547^\circ, \ 1.02^\circ, \ 1.33^\circ \]
Question 37 (***)

Solve the following trigonometric equation in the range given.

\[ \frac{5 \cos 2x + \sin 2x}{3 \sin 2x} = 7, \quad -90^\circ \leq x < 90^\circ. \]

\[ x \approx -83.0^\circ, \ 7.0^\circ \]
Question 38  (***+)***)
Solve each of the following trigonometric equations, in the range given.

a) \( \sin(2\theta + 30^\circ) = \frac{\sqrt{3}}{2}, \quad -180^\circ \leq \theta < 180^\circ \)

b) \( \sin x = 2\cos x, \quad 0 \leq x < 360^\circ \)

c) \( 2\sin^2 y - 5\cos y + 1 = 0, \quad 0 \leq y < 2\pi \)

\[ \theta = -165^\circ, -135^\circ, 15^\circ, 45^\circ, \quad x \approx 63.4^\circ, 243.4^\circ, \quad y = \frac{x}{3}, \frac{5\pi}{3} \]
Question 39 (***+)

A cubic curve is given by

\[ f(x) = 4x^3 - 8x^2 - x + k, \]

where \( k \) is a non zero constant.

a) Given that \((x - 2)\) is a factor of \(f(x)\), show that \((2x - 1)\) is also a factor of \(f(x)\).

b) Express \(f(x)\) as the product of three linear factors.

c) Hence solve the following trigonometric equation

\[ 4\sin^3 y - 8\sin^2 y - \sin y + k = 0, \]

for \(0^\circ \leq y < 360^\circ\).

\[ y = 30^\circ, \ 150^\circ, \ 210^\circ, \ 330^\circ \]
Question 40 (***+)

Solve, in radians, the following trigonometric equation

$$7 \cos(2x + 3)^\circ = 5, \quad -\pi \leq x < \pi,$$

giving the answers correct to three significant figures.

\[ x = -1.89^\circ, -1.11^\circ, 1.25^\circ, 2.08^\circ \]
Question 41  (***)

The graph of the curve with equation

\[ y = 2\sin(2x + k)^\circ, \ 0 \leq x < 360, \]

where \( k \) is a constant so that \( 0 < k < 90 \), passes through the points with coordinates \( P(55,1) \) and \( Q(\alpha, \sqrt{3}) \).

a) Show, without verification, that \( k = 40 \).

b) Determine the possible values of \( \alpha \).

\[ \alpha = 10, 40, 190, 220 \]
Question 42 (***+)
Solve, in radians, the following trigonometric equation
\[ \tan(3x - 5) = \tan 7, \quad 3 \leq x < 6, \]
giving the answers correct to three significant figures, where appropriate.
\[ x = 4, \ x = 5.05 \]

Question 43 (***+)
Solve, in radians, the following trigonometric equation
\[ \tan^4 y - \tan^2 y = 6, \quad 0 \leq y < 2\pi, \]
giving the answers in terms of \( \pi \).
\[ y = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \]
Question 44 (***+)
Solve the following trigonometric equation
\[
\frac{2 + \cos 2x}{3 + \sin^2 2x} = \frac{2}{5}, \text{ for } 0^\circ \leq x < 360^\circ.
\]
\[x = 60^\circ, 120^\circ, 240^\circ, 300^\circ\]

Question 45 (***+)
Solve, in degrees, the following trigonometric equation
\[
\tan^4 y = 6 + \tan^2 y, \text{ } 0^\circ \leq y < 360^\circ.
\]
\[y = 60^\circ, 120^\circ, 240^\circ, 300^\circ\]
A trigonometric curve is defined by the equation
\[ f(x) = 3 - 4\sin(2x + k) \, ^\circ, \quad 0 \leq x \leq 360 \]
where \( k \) is a constant such that \(-90 < k < 90\).

The curve passes through the point with coordinates \((15, 5)\) and further satisfies
\[ A \leq f(x) \leq B, \]
for some constants \( A \) and \( B \).

a) State the value of \( A \) and the value of \( B \).

b) Show that \( k = -60 \).

e) Solve the equation \( f(x) = -1 \).

\[ x = 75, 255, \quad A = -1, \quad B = 7, \quad x = 75, 255 \]
Question 47 (****)

Given that $\theta$ is measured in degrees, solve the following trigonometric equation

$$\frac{4}{\tan^2 3\theta} + 2 = \frac{7}{\sin 3\theta}, \quad 0 \leq \theta \leq 180.$$

$$\theta = 10^\circ, \ 50^\circ, \ 130^\circ, \ 170^\circ$$
Question 48  (***)

The depth of water in a harbour on a particular day can be modelled by the equation

\[ D = 12 + 3 \sin \left( \frac{\pi t}{6} \right), \]

where \( D \) is the depth of the water in metres, \( t \) hours after midnight.

Determine the times after noon, when the depth of water in the harbour is 10 metres.

\[ \boxed{19:24, 22:36} \]
Question 49  (***)

The height of tides in a harbour on a particular day can be modelled by the equation

\[ h = a + b \sin(30t)^\circ, \]

where \( h \) is the height of the water in metres, \( t \) hours after midnight, and \( a \) and \( b \) are constants.

At 02.00, \( h = 9.5 \) m and at 08.00, \( h = 3.5 \) m.

Determine …

a) … the value of \( a \) and the exact value of \( b \).

b) … the first time after midnight when the height of the tide is 5 metres.

\[ a = 6.5, \, b = 2\sqrt{3}, \, 06:51 \]
Question 50  (***)

Solve the following trigonometric equation, in the range given.

\[
\sqrt{3} + 2\sin \left(3x + \frac{\pi}{4}\right) = 0, \quad 0 \leq x < \frac{\pi}{2}.
\]

Give the answers in terms of \( \pi \).

\[x = \frac{13\pi}{36}, \frac{17\pi}{36}\]

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Question 51  (***)

Solve the following trigonometric equation in the range given.

\[
4\tan^2 \theta \cos \theta = 15, \quad 0 \leq \theta < 360^\circ.
\]

\[\theta \approx 75.5^\circ, 284.5^\circ\]

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Question 52  (****)
Solve the following trigonometric equation in the range given.

\[ 2 \tan \phi \sin \phi = 3, \quad 0 \leq \phi < 2\pi. \]

Give the answers in terms of \( \pi \).

\[ \phi = \frac{\pi}{3}, \frac{5\pi}{3} \]

Question 53 (****)
Solve the following trigonometric equation in the range given.

\[ 2 \cos x = 3 \tan x, \quad 0^\circ \leq x < 360^\circ. \]

\[ x = 30^\circ, 150^\circ \]
Question 54  (****)

\[ f(x) = x^3 - 4x^2 - \frac{1}{2}x + 2, \ x \in \mathbb{R}. \]

a) Show that \((x - 4)\) is a factor of \(f(x)\).

b) Express \(f(x)\) as the product of a linear and one quadratic factor.

c) Hence solve the trigonometric equation

\[ \cos^3 \theta - 4 \cos^2 \theta - \frac{1}{2} \cos \theta + 2 = 0, \]

for \(0^\circ \leq \theta < 360^\circ\).

\[ \boxed{\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ} \]
Question 55  (****)
Solve the following trigonometric equation in the range given.

\[2 \cos x - 3 \tan x = 0, \quad 0 \leq x < 2\pi.\]

Give the answers in terms of \(\pi\).

\[x = \frac{\pi}{6}, \frac{5\pi}{6}\]

Question 56  (****)
Solve the following trigonometric equation in the range given.

\[3 \tan \phi \sin \phi = 8, \quad 0 \leq \phi < 2\pi.\]

Give the answers in radians correct to two decimal places.

\[\phi = 1.23^\circ, 5.05^\circ\]
Question 57  (****)
Solve the following trigonometric equation in the range given.

\[ 4 \tan \psi \sin \psi \cos \psi + 4 \tan \psi \cos \psi + 1 = 0, \quad 0^\circ \leq \psi < 360^\circ. \]

\[ \psi = 210^\circ, 330^\circ \]

Question 58  (****)
Solve the following trigonometric equation in the range given.

\[ \frac{1}{2} \tan x - \sin x = 0, \quad 0^\circ \leq x < 360^\circ. \]

\[ x = 0^\circ, 60^\circ, 180^\circ, 300^\circ \]
Question 59 (****)
Solve the following trigonometric equation in the range given.

\[ 3\tan \theta \sin \theta = \cos \theta + 1, \quad 0 \leq \theta < 2\pi. \]

Give the answers in radians correct to two decimal places.

\[ \theta \approx 0.72, 3.14, 5.56 \]

Question 60 (****)
Solve the following trigonometric equation in the range given.

\[ (\sqrt{3} + 2\sin y)(\sqrt{3} + \tan 2y) = 0, \quad 0 \leq y < \pi. \]

Give the answers in terms of \( \pi \).

\[ y = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6} \]
Question 61 (****)

Solve the following trigonometric equation in the range given.

\[ 6 \cos \psi = 5 \tan \psi , \ 0 \leq \psi < 2\pi . \]

Give the answers in radians, correct to two decimal places.

\[ \psi = 0.73^\circ , 2.41^\circ \]
Question 62     (****)

\[ f(x) = x^3 - x^2 - 3x + 3. \]

a) Show that \((x - 1)\) is a factor of \(f(x)\).

b) Express \(f(x)\) as the product of three linear factors.

c) Hence solve the trigonometric equation

\[ \tan^3 \theta - \tan^2 \theta - 3 \tan \theta + 3 = 0, \]

for \(0^\circ \leq \theta < 360^\circ\).

\[ \{ \theta = 45^\circ, 60^\circ, 120^\circ, 225^\circ, 240^\circ, 300^\circ \} \]
Question 63 (****)
Solve the following trigonometric equation in the range given.

\[3\tan x + 2\cos x = 0, \quad 0 \leq x < 2\pi.\]

Give the answers in terms of \(\pi\).

\[x = \frac{7\pi}{6}, \quad \frac{11\pi}{6}\]

Question 64 (****)
Solve the following trigonometric equation in the range given.

\[\left(\sqrt{3} - 2\sin 3x\right)\left(\sqrt{3} + 2\cos 3x\right) = 0, \quad 0^\circ \leq x < 180^\circ.\]

\[x = 20^\circ, 50^\circ, 70^\circ, 140^\circ, 160^\circ, 170^\circ\]
Question 65 (****)
Solve the following trigonometric equation in the range given.

\[ 8 \tan^2 x \sin x = \cos x, \quad 0 \leq x < 2\pi. \]

Give the answers in radians correct to two decimal places.

\[ x \approx 0.46, 3.61 \]

Question 66 (****+)
Solve the following trigonometric equation for \( 0 \leq \theta < 360^\circ \)

\[ \sin \theta \tan^2 \theta (2 \sin \theta + 3) + \tan^2 \theta = 0. \]

\[ \theta = 0^\circ, 180^\circ, 210^\circ, 330^\circ \]
Question 67  (****+)

Calculate in degrees, correct to one decimal place, the solution of the following trigonometric equation

\[
\frac{1 - \cos \theta}{\sin \theta} = \sqrt{3} \sin \theta, \quad 0 < \theta < \pi.
\]

\[
\theta \approx 2.01^\circ
\]
Question 68  (**++)

The three angles in a triangle are denoted as $\alpha$, $\beta$ and $\gamma$.

It is further given that

$$\tan \alpha = -4.705 \quad \text{and} \quad \tan (\beta - \gamma) = 0.404$$

Determine, in degrees, the size of each of the angles $\alpha$, $\beta$ and $\gamma$.

$$\alpha \approx 102^\circ, \quad \beta \approx 50^\circ, \quad \gamma \approx 28^\circ$$
The figure above shows the graph of the curve with equation
\[ y = 6 - 4\sin \theta - \cos^2 \theta, \quad 0^\circ \leq \theta \leq 360^\circ. \]

The curve has a minimum at the point \( A \) and a maximum at the point \( B \).

Determine the coordinates of \( A \) and \( B \).

\[ A(90^\circ,2), \quad B(270^\circ,10) \]
Question 70  (***)
Solve the following trigonometric equation for $0 \leq \theta < 360^\circ$

$$2 + 4 \cos^2 \theta = 7 \cos \theta \sin \theta.$$ 

\[ \theta \approx 56.3^\circ, \theta \approx 63.4^\circ, \theta \approx 236.3^\circ, \theta \approx 243.4^\circ \]
Question 71  (*****)

It is given that

$$4\sin x - \frac{\cos x}{2} = \frac{4}{\sin x} - \frac{1}{2\cos x}.$$

Show clearly that the above equation is equivalent to

$$\tan x = 2.$$
Question 72  (*****)

Solve the following trigonometric equation for $0 \leq x < 360^\circ$

\[
\frac{\tan x}{\cos x} + \frac{1}{1 + \sin x} = \frac{4}{3}.
\]

$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$
Question 73  (*****)

Solve the following trigonometric equation

\[
(19 + 2\sin^2 2\theta)\tan 2\theta = \frac{3}{\cos 2\theta} - 17\cos 2\theta, \quad 0^\circ \leq \theta < 360^\circ.
\]

\[
\theta = 105^\circ, \quad \theta = 165^\circ, \quad \theta = 285^\circ, \quad \theta = 345^\circ
\]
Show that $\theta = \frac{1}{3}\pi$ is a solution of the above trigonometric equation and use a non-verification method to find the other solutions.

$\theta = \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$