TRIGONOMETRIC IDENTITIES

1. $(2\cos x + \sin x)^2 + (\cos x - 2\sin x)^2 \equiv 5$ **(**)**

$$\begin{aligned} \text{LHS} &= (2\cos x + \sin x)^2 + (\cos x - 2\sin x)^2 \\ &= 4\cos^2 x + \cancel{4\cos x \sin x} + \sin^2 x + \cos^2 x - \cancel{4\cos x \sin x} + 4\sin^2 x \\ &= 5\cos^2 x + 5\sin^2 x = 5(\cos^2 x + \sin^2 x) = 5 \times 1 = 5 = \text{RHS} \end{aligned}$$

2. $\sec\theta - \sec\theta\sin^2\theta \equiv \cos\theta$ **(**)**

$$\begin{aligned} \text{LHS} &= \sec\theta - \sec\theta\sin^2\theta = \sec\theta(1 - \sin^2\theta) \\ &= \sec\theta\cos^2\theta = \frac{1}{\cancel{\cos\theta}} \times \cos^{\cancel{2}}\theta = \cos\theta = \text{RHS} \end{aligned}$$

3. $\dfrac{\cos x}{\sin y} - \dfrac{\sin x}{\cos y} \equiv \dfrac{\cos(x+y)}{\sin y\cos y}$ **(**)**

$$\begin{aligned} \text{LHS} &= \frac{\cos x}{\sin y} - \frac{\sin x}{\cos y} = \frac{\cos x\cos y - \sin x\sin y}{\sin y\cos y} \\ &= \frac{\cos(x+y)}{\sin y\cos y} = \text{RHS} \end{aligned}$$

4. $\cos\left(2x + \dfrac{\pi}{3}\right) + \cos\left(2x - \dfrac{\pi}{3}\right) \equiv \cos 2x$ **(**)**

$$\begin{aligned} \text{LHS} &= \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \\ &= \cos 2x\cos\frac{\pi}{3} - \cancel{\sin 2x\sin\frac{\pi}{3}} + \cos 2x\cos\frac{\pi}{3} + \cancel{\sin 2x\sin\frac{\pi}{3}} \\ &= 2\cos 2x\cos\frac{\pi}{3} \\ &= 2\cos 2x \times \frac{1}{2} \\ &= \cos 2x \\ &= \text{RHS} \end{aligned}$$

5. $\left(\cos x + \sec x\right)^2 \equiv \tan^2 x + \cos^2 x + 3$ (**)

$$\text{LHS} = (\cos x + \sec x)^2 = \cos^2 x + 2\cos x \sec x + \sec^2 x = \cos^2 x + 2 + (1 + \tan^2 x)$$
$$= \cos^2 x + 3 + \tan^2 x = \text{RHS}$$

6. $\cos x \sin x \left(\cot x + \tan x\right) \equiv 1$ (**)

$$\text{LHS} = \cos x \sin x(\cot x + \tan x) = \cos x \sin x\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$$
$$= \cancel{\cos x \sin x}\left(\frac{\cos^2 x + \sin^2 x}{\cos x \sin x}\right) = \cos^2 x + \sin^2 x = 1 = \text{RHS}$$

7. $\cos x + \sin x \tan x \equiv \sec x$ (**)

$$\text{LHS} = \cos x + \sin x \tan x = \cos x + \sin x \times \frac{\sin x}{\cos x} = \cos x + \frac{\sin^2 x}{\cos x}$$
$$= \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \sec x = \text{RHS}$$

8. $\operatorname{cosec} x - \sin x \equiv \cos x \cot x$ (**)

$$\text{LHS} = \operatorname{cosec} x - \sin x = \frac{1}{\sin x} - \sin x = \frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$$
$$= \frac{\cos x}{\sin x} \times \cos x = \cot x \cos x = \text{RHS}$$

9. $\sin\left(x+\frac{\pi}{3}\right) - \sqrt{3}\cos\left(x+\frac{\pi}{3}\right) \equiv 2\sin x$ **(**)**

$$\begin{aligned}
\text{LHS} &= \sin\left(x+\tfrac{\pi}{3}\right) - \sqrt{3}\cos\left(x+\tfrac{\pi}{3}\right)\\
&= \sin x\cos\tfrac{\pi}{3} + \cos x\sin\tfrac{\pi}{3} - \sqrt{3}\left[\cos x\cos\tfrac{\pi}{3} - \sin x\sin\tfrac{\pi}{3}\right]\\
&= \tfrac{1}{2}\sin x + \tfrac{\sqrt{3}}{2}\cos x - \sqrt{3}\left[\tfrac{1}{2}\cos x - \tfrac{\sqrt{3}}{2}\sin x\right]\\
&= \tfrac{1}{2}\sin x + \cancel{\tfrac{\sqrt{3}}{2}\cos x} - \cancel{\tfrac{\sqrt{3}}{2}\cos x} + \tfrac{3}{2}\sin x\\
&= 2\sin x\\
&= \text{RHS}
\end{aligned}$$

10. $\cos\left(x+\frac{\pi}{3}\right) + \sqrt{3}\sin\left(x+\frac{\pi}{3}\right) \equiv 2\cos x$ **(**)**

$$\begin{aligned}
\text{LHS} &= \cos\left(x+\tfrac{\pi}{3}\right) + \sqrt{3}\sin\left(x+\tfrac{\pi}{3}\right)\\
&= \cos x\cos\tfrac{\pi}{3} - \sin x\sin\tfrac{\pi}{3} + \sqrt{3}\left[\sin x\cos\tfrac{\pi}{3} + \cos x\sin\tfrac{\pi}{3}\right]\\
&= \tfrac{1}{2}\cos x - \tfrac{\sqrt{3}}{2}\sin x + \sqrt{3}\left[\tfrac{1}{2}\sin x + \tfrac{\sqrt{3}}{2}\cos x\right]\\
&= \tfrac{1}{2}\cos x - \cancel{\tfrac{\sqrt{3}}{2}\sin x} + \cancel{\tfrac{\sqrt{3}}{2}\sin x} + \tfrac{3}{2}\cos x\\
&= 2\cos x\\
&= \text{RHS}
\end{aligned}$$

11. $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta$ **(**)**

$$\begin{aligned}
\text{LHS} &= \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\\
&= \frac{1}{\sin\theta\cos\theta} = \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta} = 2\operatorname{cosec} 2\theta = \text{RHS}
\end{aligned}$$

12. $\sec\theta\operatorname{cosec}\theta \equiv 2\operatorname{cosec} 2\theta$ **(**)**

$$\text{LHS} = \sec\theta\operatorname{cosec}\theta = \frac{1}{\cos\theta}\times\frac{1}{\sin\theta} = \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta} = \text{RHS}$$

13. $\dfrac{\cos 2\theta}{\cos\theta - \sin\theta} \equiv \cos\theta + \sin\theta$ **(**)**

$LHS = \dfrac{\cos 2\theta}{\cos\theta - \sin\theta} \quad \dfrac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \dfrac{\cancel{(\cos\theta - \sin\theta)}(\cos\theta + \sin\theta)}{\cancel{\cos\theta - \sin\theta}}$

$= \cos\theta + \sin\theta = RHS$

14. $\dfrac{1-\cos 2x}{\sin 2x} \equiv \tan x$ **(**)**

$LHS = \dfrac{1-\cos 2x}{\sin 2x} = \dfrac{1-(1-2\sin^2 x)}{2\sin x\cos x} = \dfrac{\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin x}\cos x} = \dfrac{\sin x}{\cos x}$

$= \tan x = RHS$

15. $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$ **(**)**

$LHS = \dfrac{\cot^2 x}{1+\cot^2 x} = \dfrac{\cot^2 x}{\operatorname{cosec}^2 x} = \cot^2 x \sin^2 x = \dfrac{\cos^2 x}{\cancel{\sin^2 x}} \times \cancel{\sin^2 x}$

$= \cos^2 x = RHS$

16. $\dfrac{1}{\sec x - \tan x} + \dfrac{1}{\sec x + \tan x} \equiv 2\sec x$ **(**)**

$LHS = \dfrac{1}{\sec x - \tan x} + \dfrac{1}{\sec x + \tan x} = \dfrac{(\sec x + \tan x) + (\sec x - \tan x)}{(\sec x - \tan x)(\sec x + \tan x)}$

$= \dfrac{2\sec x}{\sec^2 x - \tan^2 x} = \dfrac{2\sec x}{(1+\cancel{\tan^2 x}) - \cancel{\tan^2 x}} = 2\sec x = RHS$

17. $\tan\left(x+\frac{\pi}{4}\right)\tan\left(x-\frac{\pi}{4}\right) \equiv -1$ **(**)**

$$\text{LHS} = \tan\left(x+\frac{\pi}{4}\right)\tan\left(x-\frac{\pi}{4}\right) = \frac{\tan x + \tan\frac{\pi}{4}}{1-\tan x\tan\frac{\pi}{4}} \times \frac{\tan x - \tan\frac{\pi}{4}}{1+\tan x\tan\frac{\pi}{4}}$$

$$= \frac{\cancel{\tan x+1}}{1-\tan x} \times \frac{\tan x - 1}{\cancel{1+\tan x}} = \frac{\tan x - 1}{1-\tan x} = \frac{-(1-\cancel{\tan x})}{1-\cancel{\tan x}} = -1 = \text{RHS}$$

18. $\dfrac{1}{\cos\theta-\sin\theta}-\dfrac{1}{\cos\theta+\sin\theta} \equiv 2\sin\theta\sec 2\theta$ **(**)**

$$\text{LHS} = \frac{1}{\cos\theta-\sin\theta} - \frac{1}{\cos\theta+\sin\theta} = \frac{(\cos\theta+\sin\theta)-(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$$

$$= \frac{2\sin\theta}{\cos^2\theta-\sin^2\theta} = \frac{2\sin\theta}{\cos 2\theta} = 2\sin\theta\sec 2\theta = \text{RHS}$$

19. $\dfrac{\tan x\sec x}{1+\tan^2 x} \equiv \sin x$ **(**)**

$$\text{LHS} = \frac{\tan x\sec x}{1+\tan^2 x} = \frac{\tan x\sec x}{\sec^2 x} = \frac{\tan x}{\sec x} = \tan x\cos x$$

$$= \frac{\sin x}{\cancel{\cos x}}\cancel{\cos x} = \sin x = \text{RHS}$$

20. $\dfrac{\sin(x+y)}{\cos x\cos y} \equiv \tan x+\tan y$ **(**)**

$$\text{LHS} = \frac{\sin(x+y)}{\cos x\cos y} = \frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y}$$

$$= \frac{\sin x\cancel{\cos y}}{\cos x\cancel{\cos y}} + \frac{\cancel{\cos x}\sin y}{\cancel{\cos x}\cos y} = \tan x + \tan y = \text{RHS}$$

21. $\cot x + \tan x \equiv \sec x \operatorname{cosec} x$ **(**)**

$$\text{LHS} = \cot x + \tan x = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$
$$= \frac{1}{\sin x} \times \frac{1}{\cos x} = \operatorname{cosec} x \sec x = \text{RHS}$$

22. $\sec^2\theta\cos^5\theta + \cot\theta\operatorname{cosec}\theta\sin^4\theta \equiv \cos\theta$ **(**)**

$$\text{LHS} = \sec^2\theta\cos^5\theta + \cot\theta\operatorname{cosec}\theta\sin^4\theta$$
$$= \frac{1}{\cos^2\theta}\cos^5\theta + \frac{\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta} \times \sin^4\theta$$
$$= \cos^3\theta + \cos\theta\sin^2\theta$$
$$= \cos\theta(\cos^2\theta + \sin^2\theta)$$
$$= \cos\theta$$
$$= \text{RHS}$$

23. $\tan(x+60°)\tan(x-60°) \equiv \dfrac{\tan^2 x - 3}{1 - 3\tan^2 x}$ **(**)**

$$\text{LHS} = \tan(x+60)\tan(x-60) = \frac{\tan x + \tan 60}{1 - \tan x \tan 60} \times \frac{\tan x - \tan 60}{1 + \tan x \tan 60}$$
$$= \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} \times \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x} = \ldots \text{ Difference of squares}$$
$$= \frac{\tan^2 x - (\sqrt{3})^2}{1^2 - (\sqrt{3}\tan x)^2} = \frac{\tan^2 x - 3}{1 - 3\tan^2 x} = \text{RHS}$$

24. $\operatorname{cosec}^2 x\left(\tan^2 x - \sin^2 x\right) \equiv \tan^2 x$ **(**+)**

$$\text{LHS} = \operatorname{cosec}^2 x(\tan^2 x - \sin^2 x) = \operatorname{cosec}^2 x \tan^2 x - \operatorname{cosec}^2 x \sin^2 x$$
$$= \frac{1}{\sin^2 x}\frac{\sin^2 x}{\cos^2 x} - \frac{1}{\sin^2 x}\sin^2 x = \frac{1}{\cos^2 x} - 1 = \sec^2 x - 1$$
$$= (1 + \tan^2 x) - 1 = \tan^2 x = \text{RHS}$$

25. $\tan 2\theta \sec\theta \equiv 2\sin\theta \sec 2\theta$ **(**+)**

$$\text{LHS} = \tan 2\theta \sec\theta = \frac{\sin 2\theta}{\cos 2\theta}\sec\theta = \frac{2\sin\theta\cancel{\cos\theta}}{\cos 2\theta}\times\frac{1}{\cancel{\cos\theta}}$$
$$= \frac{2\sin\theta}{\cos 2\theta} = 2\sin\theta\sec 2\theta = \text{RHS}$$

26. $(1-\cos x)(1+\sec x) \equiv \sin x \tan x$ **(**+)**

$$\text{LHS} = (1-\cos x)(1+\sec x) = 1+\sec x-\cos x-\cos x\sec x$$
$$= \cancel{1}+\sec x-\cos x-\cancel{1} = \frac{1}{\cos x}-\cos x = \frac{1-\cos^2 x}{\cos x}$$
$$= \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x}\times\sin x = \tan x\sin x = \text{RHS}$$

27. $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\sin\theta} \equiv \sec^2\theta$ **(**+)**

$$\text{LHS} = \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\sin\theta} = \frac{\operatorname{cosec}\theta\sin\theta}{\operatorname{cosec}\theta\sin\theta-\sin\theta\sin\theta} \qquad \sin\theta\operatorname{cosec}\theta=1$$
$$= \frac{1}{1-\sin^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta = \text{RHS}$$

28. $\cos 2x+\tan x\sin 2x \equiv 1$ **(**+)**

$$\text{LHS} = \cos 2x+\tan x\sin 2x = (1-2\sin^2 x)+\frac{\sin x}{\cancel{\cos x}}(2\sin x\cancel{\cos x})$$
$$= 1-\cancel{2\sin^2 x}+\cancel{2\sin^2 x} = 1 = \text{RHS}$$

29. $\left(1-\sin\theta\right)\left(1+\operatorname{cosec}\theta\right) \equiv \cos\theta\cot\theta$ **(**+)**

$$\text{LHS} = (1-\sin\theta)(1+\operatorname{cosec}\theta) = 1+\operatorname{cosec}\theta - \sin\theta - \sin\theta\operatorname{cosec}\theta$$
$$= \cancel{1} + \frac{1}{\sin\theta} - \sin\theta - \cancel{1} = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$$
$$= \frac{\cos^2\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}\times\cos\theta = \cot\theta\cos\theta = \text{RHS}$$

30. $\dfrac{\cot x \operatorname{cosec} x}{1+\cot^2 x} \equiv \cos x$ **(**+)**

$$\text{LHS} = \frac{\cot x\operatorname{cosec}x}{1+\cot^2 x} = \frac{\cot x\operatorname{cosec}x}{\operatorname{cosec}^2 x} = \frac{\cot x}{\operatorname{cosec}x} = \cot x \times \frac{1}{\operatorname{cosec}x}$$
$$= \frac{\cos x}{\cancel{\sin x}}\times\cancel{\sin x} = \cos x = \text{RHS}$$

31. $\dfrac{1}{\sec x-\tan x} - \dfrac{1}{\sec x+\tan x} \equiv 2\tan x$ **(**+)**

$$\text{LHS} = \frac{1}{\sec x-\tan x} - \frac{1}{\sec x+\tan x} = \frac{(\cancel{\sec x}+\tan x)-(\cancel{\sec x}-\tan x)}{\sec^2 x-\tan^2 x}$$
$$= \frac{2\tan x}{(1+\cancel{\tan^2 x})-\cancel{\tan^2 x}} = \frac{2\tan x}{1} = 2\tan x = \text{RHS}$$

32. $\dfrac{\sin x}{1-\sin x} - \dfrac{\sin x}{1+\sin x} \equiv 2\tan^2 x$ **(**+)**

$$\text{LHS} = \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} = \frac{\sin x(1+\sin x)-\sin x(1-\sin x)}{(1-\sin x)(1+\sin x)}$$
$$= \frac{\cancel{\sin x}+\sin^2 x-\cancel{\sin x}+\sin^2 x}{1-\sin^2 x} = \frac{2\sin^2 x}{\cos^2 x} = \tan^2 x = \text{RHS}$$

33. $\sec^2\theta\left(\cot^2\theta - \cos^2\theta\right) \equiv \cot^2\theta$ **(**+)**

$$\text{LHS} = \sec^2\theta\left(\cot^2\theta - \cos^2\theta\right) = \sec^2\theta\cot^2\theta - \sec^2\theta\cos^2\theta = \frac{1}{\cancel{\cos^2\theta}}\frac{\cancel{\cos^2\theta}}{\sin^2\theta} - \frac{1}{\cos^2\theta}\cos^2\theta$$

$$= \frac{1}{\sin^2\theta} - 1 = \operatorname{cosec}^2\theta - 1 = \left(1 + \cot^2\theta\right) - 1 = \cot^2\theta = \text{RHS}$$

34. $\operatorname{cosec} x \sec^2 x \equiv \operatorname{cosec} x + \tan x \sec x$ **(**+)**

$$\text{RHS} = \operatorname{cosec}x + \tan x\sec x = \frac{1}{\operatorname{cosec}x} + \frac{\sin x}{\cos x}\times\frac{1}{\cos x} = \frac{1}{\operatorname{cosec}x} + \frac{\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x\cos^2 x} = \frac{1}{\sin x\cos^2 x} = \frac{1}{\sin x}\times\frac{1}{\cos^2 x} = \operatorname{cosec}x\sec^2 x = \text{RHS}$$

35. $\dfrac{\cos 2x}{\sin x} + \dfrac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x$ **(**+)**

$$\text{LHS} = \frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{1 - 2\sin^2 x}{\sin x} + \frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}$$

$$= \frac{1}{\sin x} - \cancel{2\sin x} + \cancel{2\sin x} = \frac{1}{\sin x} = \operatorname{cosec}x = \text{RHS}$$

ALTERNATIVE

$$\text{LHS} = \frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{\cos 2x\cos x + \sin 2x\sin x}{\sin x\cos x} = \frac{\cos(2x - x)}{\sin x\cos x}$$

$$= \frac{\cos x}{\sin x\cos x} = \frac{1}{\sin x} = \operatorname{cosec}x = \text{RHS}$$

36. $\sin\left(x + \dfrac{\pi}{4}\right) \equiv \cos\left(x - \dfrac{\pi}{4}\right)$ **(***)**

$$\text{LHS} = \sin\left(x + \frac{\pi}{4}\right) = \sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4} = \underbrace{\frac{\sqrt{2}}{2}}_{\cos\frac{\pi}{4}}\sin x + \underbrace{\frac{\sqrt{2}}{2}}_{\sin\frac{\pi}{4}}\cos x$$

$$= \cos x\cos\frac{\pi}{4} + \sin x\sin\frac{\pi}{4} = \cos\left(x - \frac{\pi}{4}\right) = \text{RHS}$$

ALTERNATIVE

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\cos(-x) = \cos x$

$$\text{LHS} = \sin\left(x + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2} - \left(x + \frac{\pi}{4}\right)\right) = \cos\left(\frac{\pi}{4} - x\right) = \cos\left(x - \frac{\pi}{4}\right) = \text{RHS}$$

37. $\left(\operatorname{cosec}\theta - \sin\theta\right)\sec^2\theta \equiv \operatorname{cosec}\theta$ (***)

$$\text{LHS} = (\operatorname{cosec}\theta - \sin\theta)\sec^2\theta = \left(\frac{1}{\sin\theta} - \sin\theta\right)\times\frac{1}{\cos^2\theta}$$
$$= \frac{1-\sin^2\theta}{\sin\theta}\times\frac{1}{\cos^2\theta} = \frac{\cancel{\cos^2\theta}}{\sin\theta}\times\frac{1}{\cancel{\cos^2\theta}} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta = \text{RHS}$$

38. $\dfrac{1}{1+\sin^2\theta} + \dfrac{1}{1+\operatorname{cosec}^2\theta} \equiv 1$ (***)

$$\text{LHS} = \frac{1}{1+\sin^2\theta} + \frac{1}{1+\operatorname{cosec}^2\theta} = \frac{1}{1+\sin^2\theta} + \frac{1}{1+\frac{1}{\sin^2\theta}}$$

MULTIPLY TOP & BOTTOM OF 2nd FRACTION BY $\sin^2\theta$

$$= \frac{1}{1+\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta+1} = \frac{\cancel{1+\sin^2\theta}}{\cancel{1+\sin^2\theta}} = 1 = \text{RHS}$$

$$\text{LHS} = \frac{1}{1+\sin^2\theta} + \frac{1}{1+\operatorname{cosec}^2\theta} = \frac{(1+\operatorname{cosec}^2\theta)+(1+\sin^2\theta)}{(1+\sin^2\theta)(1+\operatorname{cosec}^2\theta)}$$
$$= \frac{2+\sin^2\theta+\operatorname{cosec}^2\theta}{1+\operatorname{cosec}^2\theta+\sin^2\theta+1} = \frac{\cancel{2+\sin^2\theta+\operatorname{cosec}^2\theta}}{\cancel{2+\sin^2\theta+\operatorname{cosec}^2\theta}} = 1 = \text{RHS}$$

39. $\dfrac{1+\tan^2 x}{1-\tan^2 x} \equiv \sec 2x$ (***)

$$\text{LHS} = \frac{1+\tan^2 x}{1-\tan^2 x} = \frac{1+\frac{\sin^2 x}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}}$$

MULTIPLY TOP AND BOTTOM BY $\cos^2 x$

$$= \frac{\cos^2 x+\sin^2 x}{\cos^2 x-\sin^2 x} = \frac{1}{\cos 2x} = \sec 2x = \text{RHS}$$

40. $\cot 2x + \operatorname{cosec} 2x \equiv \cot x$ (***)

$$\text{LHS} = \cot 2x + \operatorname{cosec} 2x = \frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x} = \frac{\cos 2x + 1}{\sin 2x}$$
$$= \frac{(2\cos^2 x - 1)+1}{2\sin x\cos x} = \frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\cancel{\cos x}} = \frac{\cos x}{\sin x} = \cot x = \text{RHS}$$

41. $\sin 2x \equiv \dfrac{2\tan x}{1+\tan^2 x}$ (***)

$$\text{RHS} = \frac{2\tan x}{1+\tan^2 x} = \frac{2\tan x}{\sec^2 x} = 2\tan x\cos^2 x = \frac{2\sin x}{\cancel{\cos x}}\cos^{\cancel{2}} x$$
$$= 2\sin x\cos x = \sin 2x = \text{RHS}$$

42. $\dfrac{\sec^2\theta}{1-\tan^2\theta} \equiv \sec 2\theta$ (***)

$$\text{LHS} = \frac{\sec^2\theta}{1-\tan^2\theta} = \frac{\frac{1}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}} = \text{MULTIPLY TOP \& BOTTOM BY } \cos^2\theta$$
$$= \frac{1}{\cos^2\theta-\sin^2\theta} = \frac{1}{\cos 2\theta} = \sec 2\theta = \text{RHS}$$

43. $\dfrac{\tan 2\theta-\sin 2\theta}{\tan 2\theta} \equiv 2\sin^2\theta$ (***)

$$\text{LHS} = \frac{\tan 2\theta-\sin 2\theta}{\tan 2\theta} = \frac{\tan 2\theta}{\tan 2\theta} - \frac{\sin 2\theta}{\tan 2\theta}$$
$$= 1 - \sin 2\theta \times \cot 2\theta = 1 - \cancel{\sin 2\theta} \times \frac{\cos 2\theta}{\cancel{\sin 2\theta}}$$
$$= 1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta = \text{RHS}$$

44. $\dfrac{\operatorname{cosec} x-\sin x}{\cos^2 x\cot x} \equiv \sec x$ (***)

$$\text{LHS} = \frac{\operatorname{cosec} x-\sin x}{\cos^2 x\cot x} = \frac{\frac{1}{\sin x}-\sin x}{\cos^2 x \times \frac{\cos x}{\sin x}} = \frac{\frac{1}{\sin x}-\sin x}{\frac{\cos^3 x}{\sin x}} =$$

MULTIPLY TOP/BOTTOM BY $\sin x$

$$= \frac{1-\sin^2 x}{\cos^3 x} = \frac{\cos^2 x}{\cos^3 x} = \frac{1}{\cos x} = \sec x = \text{RHS}$$

45. $\sin 2\theta \equiv \dfrac{2\cot\theta}{1+\cot^2\theta}$ **(***)**

$$\text{RHS} = \frac{2\cot\theta}{1+\cot^2\theta} = \frac{2\cot\theta}{\operatorname{cosec}^2\theta} = 2\cot\theta\sin^2\theta = 2\left(\frac{\cos\theta}{\sin\theta}\right)\sin^2\theta$$

$$= 2\cos\theta\sin\theta = \sin 2\theta = \text{LHS}$$

46. $\dfrac{\sin 2\theta}{\sin\theta} - \dfrac{\cos 2\theta}{\cos\theta} \equiv \sec\theta$ **(***)**

$$\text{LHS} = \frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \frac{\cos\theta\sin 2\theta - \cos 2\theta\sin\theta}{\sin\theta\cos\theta} = \frac{\sin(2\theta-\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\sin\theta\cos\theta} = \frac{1}{\cos\theta} = \sec\theta = \text{RHS}$$

ALTERNATIVE $\text{LHS} = \dfrac{\sin 2\theta}{\sin\theta} - \dfrac{\cos 2\theta}{\cos\theta} = \dfrac{2\sin\theta\cos\theta}{\sin\theta} - \dfrac{2\cos^2\theta - 1}{\cos\theta}$

$$= 2\cos\theta - \left(2\cos\theta - \frac{1}{\cos\theta}\right) = \frac{1}{\cos\theta} = \sec\theta = \text{RHS}$$

47. $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} \equiv 2\operatorname{cosec}\theta$ **(***)**

$$\text{LHS} = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta} = \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta = \text{RHS}$$

48. $\cos 2\theta \equiv \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ **(***)**

$$\text{RHS} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2\theta}{\sec^2\theta} = \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta} = \cos^2\theta - \tan^2\theta\cos^2\theta$$

$$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta}\times\cos^2\theta = \cos^2\theta - \sin^2\theta = \cos 2\theta = \text{LHS}$$

(OR SWITCH INTO SINES & COSINES & TIDY UP THE DOUBLE FRACTION)

49. $\sqrt{2+2\cos 2\theta} \equiv 2\cos\theta$ **(***)**

$$\text{LHS} = \sqrt{2+2\cos 2\theta} = \sqrt{2+2(2\cos^2\theta-1)} = \sqrt{2+4\cos^2\theta-2}$$
$$= \sqrt{4\cos^2\theta} = 2\cos\theta = \text{RHS}$$

50. $\dfrac{1}{\operatorname{cosec}\theta-1}+\dfrac{1}{\operatorname{cosec}\theta+1} \equiv 2\sec\theta\tan\theta$ **(***+)**

$$\text{LHS} = \frac{1}{\operatorname{cosec}\theta-1}+\frac{1}{\operatorname{cosec}\theta+1} = \frac{1(\operatorname{cosec}\theta+\cancel{1})+1(\operatorname{cosec}\theta-\cancel{1})}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}$$
$$= \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1} = \frac{2\operatorname{cosec}\theta}{(1+\cot^2\theta)-1} = \frac{2\operatorname{cosec}\theta}{\cot^2\theta}$$
$$= 2\operatorname{cosec}\theta\times\tan^2\theta = \frac{2}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta} = \frac{2\sin\theta}{\cos^2\theta}$$
$$= \frac{2\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta} = 2\tan\theta\sec\theta = \text{RHS}$$

51. $(3\sin\theta+5\cos\theta)^2 \equiv 17+8\cos 2\theta+15\sin 2\theta$ **(***+)**

$$\text{LHS} = (3\sin\theta+5\cos\theta)^2 = 9\sin^2\theta+25\cos^2\theta+30\sin\theta\cos\theta$$
$$= (9\sin^2\theta+9\cos^2\theta)+16\cos^2\theta+15(2\sin\theta\cos\theta)$$
$$= 9+16\left(\tfrac{1}{2}+\tfrac{1}{2}\cos 2\theta\right)^{*}+15\sin 2\theta$$
$$= 9+8+8\cos 2\theta+15\sin 2\theta$$
$$= 17+8\cos 2\theta+15\sin 2\theta$$
$$= \text{RHS}$$

$$\boxed{\begin{aligned} &{}^{*}\ \cos 2\theta = 2\cos^2\theta-1\\ &\cos 2\theta+1 = 2\cos^2\theta\\ &\cos^2\theta = \tfrac{1}{2}+\tfrac{1}{2}\cos 2\theta\end{aligned}}$$

52. $\left(\dfrac{1+\sin\theta}{\cos\theta}\right)^2+\left(\dfrac{1-\sin\theta}{\cos\theta}\right)^2 \equiv 2+4\tan^2\theta$ **(***+)**

$$\text{LHS} = \left(\frac{1+\sin\theta}{\cos\theta}\right)^2+\left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \frac{1+\cancel{2\sin\theta}+\sin^2\theta}{\cos^2\theta}+\frac{1-\cancel{2\sin\theta}+\sin^2\theta}{\cos^2\theta}$$
$$= \frac{2+2\sin^2\theta}{\cos^2\theta} = \frac{2}{\cos^2\theta}+\frac{2\sin^2\theta}{\cos^2\theta} = 2\sec^2\theta+2\tan^2\theta$$
$$= 2(\tan^2\theta+1)+2\tan^2\theta = 4\tan^2\theta+2 = \text{RHS}$$

53. $2\cot 2\theta + \tan\theta \equiv \cot\theta$ **(***+)**

$$\text{LHS} = 2\cot 2\theta + \tan\theta = \frac{2\cos 2\theta}{\sin 2\theta} + \tan\theta = \frac{\cancel{2}\cos 2\theta}{\cancel{2}\sin\theta\cos\theta} + \tan\theta$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} + \tan\theta = \frac{\cos^2\theta}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta} + \tan\theta$$

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} + \tan\theta = \cot\theta - \cancel{\tan\theta} + \cancel{\tan\theta} = \cot\theta = \text{RHS}$$

$$\text{LHS} = \cancel{2}\underbrace{\left(\frac{1-\tan^2\theta}{\cancel{2}\tan\theta}\right)}_{\cot 2\theta} + \tan\theta = \frac{1 - \cancel{\tan^2\theta} + \cancel{\tan^2\theta}}{\tan\theta} = \frac{1}{\tan\theta}$$

$$= \cot\theta = \text{RHS}$$

54. $\cos x + \sin x \tan 2x \equiv \dfrac{\cos x}{\cos 2x}$ **(***+)**

$$\text{LHS} = \cos x + \sin x \tan x = \cos x + \sin x \frac{\sin 2x}{\cos 2x} = \cos x + \sin x \frac{2\sin x\cos x}{\cos 2x}$$

$$= \cos x + \frac{2\sin^2 x\cos x}{\cos 2x} = \frac{\cos x\cos 2x + 2\sin^2 x\cos x}{\cos 2x}$$

$$= \frac{\cos x(\cos 2x + 2\sin^2 x)}{\cos 2x} = \frac{\cos x(1 - \cancel{2\sin^2 x} + \cancel{2\sin^2 x})}{\cos 2x}$$

$$= \frac{\cos x}{\cos 2x} = \text{RHS}$$

ALTERNATIVE

$$\text{LHS} = \cos x + \sin x \tan x = \cos x + \sin x \frac{\sin 2x}{\cos 2x} = \frac{\cos x\cos 2x + \sin x\sin 2x}{\cos 2x}$$

$$= \frac{\cos(2x - x)}{\cos 2x} = \frac{\cos x}{\cos 2x} = \text{RHS}$$

55. $4\text{cosec}^2 2\theta - \sec^2\theta \equiv \text{cosec}^2\theta$ (***+)

(a) $$\text{LHS} = 4\text{cosec}^2 2\theta - \sec^2\theta = \frac{4}{\sin^2 2\theta} - \frac{1}{\cos^2\theta} = \frac{4}{(\sin 2\theta)^2} - \frac{1}{\cos^2\theta}$$

$$= \frac{4}{(2\sin\theta\cos\theta)^2} - \frac{1}{\cos^2\theta} = \frac{4}{4\sin^2\theta\cos^2\theta} - \frac{1}{\cos^2\theta} = \frac{1}{\sin^2\theta\cos^2\theta} - \frac{1}{\cos^2\theta}$$

$$= \frac{1 - \sin^2\theta}{\sin^2\theta\cos^2\theta} = \frac{\cancel{\cos^2\theta}}{\sin^2\theta\cancel{\cos^2\theta}} = \frac{1}{\sin^2\theta} = \text{cosec}^2\theta = \text{RHS}$$

56. $\dfrac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} \equiv \tan x$ **(***+)**

$$\text{LHS} = \frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \frac{2\sin x\cos x + \sin x}{(2\cos^2 x \cancel{-1}) + \cos x \cancel{+1}} = \frac{\sin x(2\cos x + 1)}{2\cos^2 x + \cos x}$$

$$= \frac{\sin x\cancel{(2\cos x + 1)}}{\cos x\cancel{(2\cos x + 1)}} = \frac{\sin x}{\cos x} = \tan x = \text{RHS}$$

57. $\dfrac{\tan A - \cot B}{\tan B - \cot A} \equiv \tan A \cot B$ **(***+)**

$$\text{LHS} = \frac{\tan A - \cot B}{\tan B - \cot A} = \frac{\frac{\sin A}{\cos A} - \frac{\cos B}{\sin B}}{\frac{\sin B}{\cos B} - \frac{\cos A}{\sin A}} = \frac{\frac{\cancel{\sin A \sin B - \cos A \cos B}}{\cos A \sin B}}{\frac{\cancel{\sin B \sin A - \cos A \cos B}}{\cos B \sin A}}$$

$$= \frac{\cos B \sin A}{\cos A \sin B} = \frac{\sin A}{\cos A} \times \frac{\cos B}{\sin B} = \tan A \times \cot B = \text{RHS}$$

$$\text{LHS} = \frac{\tan A - \cot B}{\tan B - \cot A} = \frac{\tan A - \frac{1}{\tan B}}{\tan B - \frac{1}{\tan A}} = \frac{\frac{\cancel{\tan A \tan B - 1}}{\tan B}}{\frac{\cancel{\tan B \tan A - 1}}{\tan A}}$$

$$= \frac{\tan A}{\tan B} = \tan A \cot B = \text{RHS}$$

58. $\dfrac{(1+\sec x)(1-\cos x)}{\tan x} \equiv \sin x$ **(***+)**

$$\text{LHS} = \frac{(1+\sec x)(1-\cos x)}{\tan x} = \frac{1-\cos x+\sec x-1}{\tan x}$$

$$= \frac{\sec x - \cos x}{\tan x} = \frac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} = \dots$$

MULTIPLY TOP & BOTTOM BY $\cos x$ ...

$$= \frac{1-\cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x = \text{RHS}$$

59. $\cos 3x \equiv 4\cos^3 x - 3\cos x$ **(***+)**

$$\text{LHS} = \cos 3x = \cos(2x+x) = \cos 2x \cos x - \sin x \sin 2x$$

$$= (2\cos^2 x - 1)\cos x - \sin x(2\sin x \cos x)$$

$$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$$

$$= 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)$$

$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$

$$= 4\cos^3 x - 3\cos x$$

$$= \text{RHS}$$

60. $\dfrac{\tan x}{\sec x - 1} - \dfrac{\sec x - 1}{\tan x} \equiv 2\cot x$ **(***+)**

$$\text{LHS} = \frac{\tan x}{\sec x - 1} - \frac{\sec x - 1}{\tan x} = \frac{\tan^2 x - (\sec x - 1)^2}{(\sec x - 1)\tan x}$$

$$= \frac{\tan^2 x - (\sec^2 x - 2\sec x + 1)}{(\sec x - 1)\tan x} = \frac{\tan^2 x - \sec^2 x + 2\sec x - 1}{(\sec x - 1)\tan x}$$

$$= \frac{\cancel{\tan^2 x} - (1+\cancel{\tan^2 x}) + 2\sec x - 1}{(\sec x - 1)\tan x} = \frac{2\sec x - 2}{(\sec x - 1)\tan x}$$

$$= \frac{2\cancel{(\sec x - 1)}}{\cancel{(\sec x - 1)}\tan x} = \frac{2}{\tan x} = 2\cot x = \text{RHS}$$

61. $\cot x - \tan x \equiv 2\cot 2x$ **(***+)**

$$\text{LHS} = \cot x - \tan x = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos 2x}{\sin x \cos x} = \frac{2\cos 2x}{2\sin x \cos x} = \frac{2\cos 2x}{\sin 2x} = 2\cot 2x = \text{RHS}$$

62. $\sin 3A \equiv 3\sin A - 4\sin^3 A$ **(***+)**

$$\begin{aligned} \text{LHS} = \sin 3A &= \sin(2A + A) \\ &= \sin 2A\cos A + \cos 2A\sin A \\ &= (2\sin A\cos A)\cos A + (1 - 2\sin^2 A)\sin A \\ &= 2\sin A\cos^2 A + \sin A - 2\sin^3 A \\ &= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A \\ &= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A \\ &= 3\sin A - 4\sin^3 A \\ &= \text{RHS} \end{aligned}$$

63. $\dfrac{\sec x - \cos x}{\operatorname{cosec} x - \sin x} \equiv \tan^3 x$ **(***+)**

$$\text{LHS} = \frac{\sec x - \cos x}{\operatorname{cosec} x - \sin x} \quad \frac{\frac{1}{\cos x} - \cos x}{\frac{1}{\sin x} - \sin x} = \frac{\frac{1 - \cos^2 x}{\cos x}}{\frac{1 - \sin^2 x}{\sin x}}$$

$$= \frac{\frac{\sin^2 x}{\cos x}}{\frac{\cos^2 x}{\sin x}} = \frac{\sin^3 x}{\cos^3 x} = \tan^3 x = \text{RHS}$$

64. $\operatorname{cosec} 2x \equiv \dfrac{\cot^2 x + 1}{2\cot x}$ **(***+)**

$$\text{RHS} = \frac{\cot^2 x + 1}{2\cot x} = \frac{\operatorname{cosec}^2 x}{2\cot x} = \frac{1}{2} \times \operatorname{cosec}^2 x \times \tan x = \frac{1}{2} \times \frac{1}{\sin^2 x} \times \frac{\sin x}{\cos x}$$

$$= \frac{1}{2\sin x\cos x} = \frac{1}{\sin 2x} = \operatorname{cosec} 2x = \text{LHS}$$

ALTERNATIVE

$$\text{LHS} = \operatorname{cosec} 2x = \frac{1}{\sin 2x} = \frac{\sin^2 x + \cos^2 x}{2\sin x\cos x} = \frac{\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}}{\frac{2\sin x\cos x}{\sin^2 x}}$$

$$= \frac{1 + \cot^2 x}{2\cot x} = \text{RHS}$$

65. $\dfrac{\sec 2x - 1}{\sec 2x + 1} \equiv \tan^2 x$ **(***+)**

$$\text{LHS} = \frac{\sec 2x - 1}{\sec 2x + 1} = \frac{\frac{1}{\cos 2x} - 1}{\frac{1}{\cos 2x} + 1} = \text{MULTIPLY TOP AND BOTTOM BY } \cos 2x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$= \frac{\cancel{1} - (\cancel{1} - 2\sin^2 x)}{\cancel{1} + (2\cos^2 x \cancel{-1})} = \frac{\cancel{2}\sin^2 x}{\cancel{2}\cos^2 x} = \tan^2 x = \text{RHS}$$

66. $4\text{cosec}^2 2\theta - \text{cosec}^2\theta \equiv \sec^2\theta$ (***+)

$$\text{LHS} = 4\text{cosec}^2 2\theta - \text{cosec}^2\theta = \frac{4}{\sin^2 2\theta} - \frac{1}{\sin^2\theta} = \frac{4}{(\sin 2\theta)^2} - \frac{1}{\sin^2\theta}$$

$$= \frac{4}{(2\sin\theta\cos\theta)^2} - \frac{1}{\sin^2\theta} = \frac{\cancel{4}}{\cancel{4}\sin^2\theta\cos^2\theta} - \frac{1}{\sin^2\theta} = \frac{1}{\sin^2\theta\cos^2\theta} - \frac{1}{\sin^2\theta}$$

$$= \frac{1 - \cos^2\theta}{\sin^2\theta\cos^2\theta} = \frac{\sin^2\theta}{\cancel{\sin^2\theta}\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta = \text{RHS}$$

67. $\dfrac{\cos 2x - \cos x + 1}{\sin 2x - \sin x} \equiv \cot x$ **(***+)**

$$\text{LHS} = \frac{\cos 2x - \cos x + 1}{\sin 2x - \sin x} = \frac{2\cos^2 x \cancel{-1} - \cos x \cancel{+1}}{2\sin x\cos x - \sin x}$$

$$= \frac{2\cos^2 x - \cos x}{\sin x(2\cos x - 1)} = \frac{\cos x\cancel{(2\cos x - 1)}}{\sin x\cancel{(2\cos x - 1)}} = \cot x = \text{RHS}$$

68. $(\tan\theta + \cot\theta)(\sin\theta + \cos\theta) \equiv \sec\theta + \text{cosec}\,\theta$ **(***+)**

$$\text{LHS} = (\tan\theta + \cot\theta)(\sin\theta + \cos\theta)$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)(\sin\theta + \cos\theta) = \frac{\cancel{\sin^2\theta + \cos^2\theta}}{\sin\theta\cos\theta} \times (\sin\theta + \cos\theta)$$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = \frac{\cancel{\sin\theta}}{\cancel{\sin\theta}\cos\theta} + \frac{\cancel{\cos\theta}}{\sin\theta\cancel{\cos\theta}}$$

$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \sec\theta + \text{cosec}\,\theta = \text{RHS}$$

69. $\sin^2\left(\theta+\frac{\pi}{4}\right)+\sin^2\left(\theta-\frac{\pi}{4}\right)\equiv 1$ **(***)**

$$\begin{aligned}\text{LHS} &= \sin^2\left(\theta+\tfrac{\pi}{4}\right)+\sin^2\left(\theta-\tfrac{\pi}{4}\right)=\left(\sin\left(\theta+\tfrac{\pi}{4}\right)\right)^2+\left(\sin\left(\theta-\tfrac{\pi}{4}\right)\right)^2\\ &= \left(\sin\theta\cos\tfrac{\pi}{4}+\cos\theta\sin\tfrac{\pi}{4}\right)^2+\left(\sin\theta\cos\tfrac{\pi}{4}-\cos\theta\sin\tfrac{\pi}{4}\right)^2\\ &= \left(\tfrac{\sqrt{2}}{2}\sin\theta+\tfrac{\sqrt{2}}{2}\cos\theta\right)^2+\left(\tfrac{\sqrt{2}}{2}\sin\theta-\tfrac{\sqrt{2}}{2}\cos\theta\right)^2\\ &= \left(\tfrac{1}{2}\sin^2\theta+\cancel{\sin\theta\cos\theta}+\tfrac{1}{2}\cos^2\theta\right)+\left(\tfrac{1}{2}\sin^2\theta-\cancel{\sin\theta\cos\theta}+\tfrac{1}{2}\cos^2\theta\right)\\ &= \sin^2\theta+\cos^2\theta\\ &= 1\\ &= \text{RHS}\end{aligned}$$

70. $\tan A\left(1+\sec 2A\right)\equiv\tan 2A$ **(****)**

$$\begin{aligned}\text{LHS} &= \tan A(1+\sec 2A)=\tan A\left(1+\frac{1}{\cos 2A}\right)\\ &= \tan A\left(\frac{\cos 2A+1}{\cos 2A}\right)=\tan A\times\frac{(2\cos^2 A-1)+1}{\cos 2A}\\ &= \tan A\ \frac{2\cos^2 A}{\cos 2A}=\frac{\sin A}{\cancel{\cos A}}\times\frac{2\cos^{\cancel{2}}A}{\cos 2A}=\frac{2\cos A\sin A}{\cos 2A}\\ &= \frac{\sin 2A}{\cos 2A}=\tan 2A=\text{RHS}\end{aligned}$$

71. $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}\equiv\tan x$.

$$\begin{aligned}\text{LHS} &= \frac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}=\frac{1-(1-2\sin^2 x)+\sin 2x}{1+(2\cos^2 x-1)+\sin 2x}=\frac{2\sin^2 x+\sin 2x}{2\cos^2 x+\sin 2x}\\ &= \frac{2\sin^2 x+2\sin x\cos x}{2\cos^2 x+2\sin x\cos x}=\frac{2\sin x\,\cancel{(\sin x+\cos x)}}{2\cos x\,\cancel{(\cos x+\sin x)}}=\tan x=\text{RHS}\end{aligned}$$

72. $\cos(x+y)\cos(x-y)\equiv\cos^2 x-\sin^2 y$ **(****)**

$$\begin{aligned}\text{LHS} &= \cos(x+y)\cos(x-y)=(\cos x\cos y-\sin x\sin y)(\cos x\cos y+\sin x\sin y)\\ &= \cos^2 x\cos^2 y-\sin^2 x\sin^2 y=\cos^2 x(1-\sin^2 y)-\sin^2 y(1-\cos^2 x)\\ &= \cos^2 x-\cancel{\cos^2 x\sin^2 y}-\sin^2 y+\cancel{\sin^2 y\cos^2 x}\\ &= \cos^2 x-\sin^2 y\\ &= \text{RHS}\end{aligned}$$

73. $\dfrac{\tan 2\theta + \sin 2\theta}{\tan 2\theta} \equiv 2\cos^2\theta$ (****)

$$\text{LHS} = \frac{\tan 2\theta + \sin 2\theta}{\tan 2\theta} = \frac{\frac{\sin 2\theta}{\cos 2\theta} + \sin 2\theta}{\frac{\sin 2\theta}{\cos 2\theta}}$$

MULTIPLY TOP/BOTTOM BY $\cos 2\theta$

$$= \frac{\cancel{\sin 2\theta} + \cancel{\sin 2\theta}\cos 2\theta}{\cancel{\sin 2\theta}} = \frac{1+\cos 2\theta}{1} = 1 + (2\cos^2\theta - 1)$$

$$= 2\cos^2\theta = \text{RHS}$$

74. $\dfrac{\sec x}{1+\sec x} - \dfrac{\sec x}{1-\sec x} \equiv 2\operatorname{cosec}^2 x$ (****)

$$\text{LHS} = \frac{\sec x}{1+\sec x} - \frac{\sec x}{1-\sec x} = \sec x\left[\frac{1}{1+\sec x} - \frac{1}{1-\sec x}\right] = \sec x\left[\frac{(\cancel{1} - \sec x) - (\cancel{1} + \sec x)}{(1+\sec x)(1-\sec x)}\right]$$

$$= \sec x \times \frac{-2\sec x}{1-\sec^2 x} = \sec x \times \frac{2\sec x}{\sec^2 x - 1} = \frac{2\sec^2 x}{(1+\tan^2 x) - 1} = \frac{2\sec^2 x}{\tan^2 x}$$

$$= 2\sec^2 x \times \frac{1}{\tan^2 x} = \frac{2}{\cancel{\cos^2 x}} \times \frac{\cancel{\cos^2 x}}{\sin^2 x} = \frac{2}{\sin^2 x} = 2\operatorname{cosec}^2 x = \text{RHS}$$

75. $\cot 2x \equiv \dfrac{\cot^2 x - 1}{2\cot x}$ (****)

$$\text{LHS} = \cot 2x = \frac{\cos 2x}{\sin 2x} = \frac{\cos^2 x - \sin^2 x}{2\sin x\cos x}$$

$$= \frac{\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}}{\frac{2\sin x\cos x}{\sin^2 x}} = \frac{\cot^2 x - 1}{2\frac{\cos x}{\sin x}} = \frac{\cot^2 x - 1}{2\cot x} = \text{RHS}$$

$$\text{LHS} = \cot 2x = \frac{1}{\tan 2x} = \frac{1-\tan^2 x}{2\tan x}$$

$$= \frac{\frac{1}{\tan^2 x} - \frac{\tan^2 x}{\tan^2 x}}{\frac{2\tan x}{\tan^2 x}} = \frac{\cot^2 x - 1}{\frac{2}{\tan x}} = \frac{\cot^2 x - 1}{2\cot x} = \text{RHS}$$

76. $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} \equiv 2$ (****)

$$\text{LHS} = \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta} = \frac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin 2\theta}{\sin\theta\cos\theta} = \frac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}} = 2 = \text{RHS}$$

77. $2-2\tan x-\dfrac{2\tan x}{\tan 2x}\equiv\left(1-\tan x\right)^2$ **(****)**

$$\text{LHS} = 2-2\tan x-\frac{2\tan x}{\tan 2x} = 2-2\tan x-\frac{2\tan x\,(1-\tan^2 x)}{\dfrac{2\tan x}{\cancel{1-\tan^2 x}}\,\cancel{(1-\tan^2 x)}}$$

MULTIPLY TOP & BOTTOM OF THE FRACTION BY $(1-\tan^2 x)$

$$= 2-2\tan x-\frac{\cancel{2\tan x}(1-\tan^2 x)}{\cancel{2\tan x}} = 2-2\tan x-1+\tan^2 x$$

$$= \tan^2 x-2\tan x+1 = (\tan x-1)^2 = \text{RHS}$$

SAME AS $(1-\tan x)^2$

78. $\dfrac{1+\cos x}{1-\cos x}\equiv\cot^2\dfrac{1}{2}x$ **(****)**

$\cos 2\theta = 2\cos^2\theta-1$

$\cos(2\times\frac{1}{2}\theta) = 2\cos^2\frac{\theta}{2}-1$

$\cos\theta = 2\cos^2\frac{\theta}{2}-1$

$\cos 2\theta = 1-2\sin^2\theta$

$\cos(2\times\frac{1}{2}\theta) = 1-2\sin^2\frac{\theta}{2}$

$\cos\theta = 1-2\sin^2\frac{\theta}{2}$

$$\text{LHS} = \frac{1+\cos x}{1-\cos x} = \frac{1+\left(2\cos^2\frac{x}{2}-1\right)}{1-\left(1-2\sin^2\frac{x}{2}\right)}$$

$$= \frac{\cancel{2}\cos^2\frac{x}{2}}{\cancel{2}\sin^2\frac{x}{2}} = \cot^2\frac{x}{2} = \text{RHS}$$

79. $2\left(\sec\theta-\cos\theta\right)\left(\operatorname{cosec}\theta-\sin\theta\right)\equiv\sin 2\theta$ **(****)**

$$\text{LHS} = 2(\sec\theta-\cos\theta)(\operatorname{cosec}\theta-\sin\theta) = 2\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{1}{\sin\theta}-\sin\theta\right)$$

$$= 2\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\times\left(\frac{1-\sin^2\theta}{\sin\theta}\right) = 2\left(\frac{\sin^{\cancel{2}}\theta}{\cancel{\cos\theta}}\right)\times\left(\frac{\cos^{\cancel{2}}\theta}{\cancel{\sin\theta}}\right)$$

$$= 2\sin\theta\cos\theta = \sin 2\theta = \text{RHS}$$

80. $\cot x-2\cot 2x\equiv\tan x$ (****)

$$\text{LHS} = \cot x-2\cot 2x = \cot x-\frac{2}{\tan 2x} = \cot x-\frac{2(1-\tan^2 x)}{2\tan x}$$

$$= \cot x-\frac{1-\tan^2 x}{\tan x} = \frac{\cot x\tan x-(1-\tan^2 x)}{\tan x}$$

$$= \frac{\cancel{1-1}+\tan^2 x}{\tan x} = \frac{\tan^2 x}{\tan x} = \tan x = \text{RHS}$$

81. $\dfrac{1}{\cos\theta - \sin\theta} + \dfrac{1}{\cos\theta + \sin\theta} \equiv \dfrac{2\sec\theta}{1 - \tan^2\theta}$ (****)

$$\text{LHS} = \frac{1}{\cos\theta - \sin\theta} + \frac{1}{\cos\theta + \sin\theta} = \frac{\cancel{\cos\theta + \sin\theta} + \cos\theta \cancel{- \sin\theta}}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$= \frac{2\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{\frac{2\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}} = \frac{\frac{2}{\cos\theta}}{1 - \tan^2\theta}$$

$$= \frac{2\sec\theta}{1 - \tan^2\theta} = \text{RHS}$$

82. $\dfrac{1 + \sin x}{\cos x} \equiv \dfrac{\cos x}{1 - \sin x}$ (****)

$$\text{LHS} = \frac{1 + \sin x}{\cos x} = \frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)} = \frac{1 - \sin^2 x}{\cos x(1 - \sin x)} = \frac{\cos^2 x}{\cos x(1 - \sin x)}$$

$$= \frac{\cos x}{1 - \sin x} = \text{RHS}$$

83. $\cos^3\theta + \sin^3\theta \equiv (\cos\theta + \sin\theta)(1 - \sin\theta\cos\theta)$ (****)

$$\text{RHS} = (\cos\theta + \sin\theta)(1 - \sin\theta\cos\theta) = (\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)$$

$$= \cos^3\theta + \cancel{\cos\theta\sin^2\theta} - \cancel{\sin\theta\cos^2\theta} + \cancel{\sin\theta\cos^2\theta} + \sin^3\theta - \cancel{\sin^2\theta\cos\theta}$$

$$= \cos^3\theta + \sin^3\theta = \text{LHS}$$

ALTERNATIVE USING $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$ AND START FROM THE LEFT

84. $\dfrac{2\tan 2x}{\tan 2x - \sin 2x} \equiv \operatorname{cosec}^2 x$ (****)

$$\text{LHS} = \frac{2\tan 2x}{\tan 2x - \sin 2x} = \frac{\frac{2\sin 2x}{\cos 2x}}{\frac{\sin 2x}{\cos 2x} - \sin 2x} =$$

MULTIPLY TOP & BOTTOM BY $\cos 2x$

$$= \frac{2\sin 2x}{\sin 2x - \sin 2x\cos 2x} =$$

DIVIDE TOP & BOTTOM BY $\sin 2x$

$$= \frac{2}{1 - \cos 2x} = \frac{2}{1 - (1 - 2\sin^2 x)} = \frac{\cancel{2}}{\cancel{2}\sin^2 x} = \operatorname{cosec}^2 x = \text{RHS}$$

85. $\operatorname{cosec}2\theta - \cot 2\theta \equiv \tan\theta$ (****)

$$\text{LHS} = \operatorname{cosec}2\theta - \cot 2\theta = \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1-\cos 2\theta}{\sin 2\theta}$$
$$= \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta} = \frac{\cancel{2\sin^2\theta}}{\cancel{2\sin\theta}\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$$

86. $\dfrac{\sin x}{1-\cos x} \equiv \cot\dfrac{1}{2}x$ (****)

$$\text{LHS} = \frac{\sin x}{1-\cos x} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1-\left(1-2\sin^2\frac{x}{2}\right)}$$
$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}} = \frac{\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}}$$
$$= \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \cot\frac{x}{2} = \text{RHS}$$

$\sin 2A = 2\sin A\cos A$
$\sin A = \sin\left(2\times\frac{A}{2}\right) = 2\sin\frac{A}{2}\cos\frac{A}{2}$

$\cos 2A = 1-2\sin^2 A$
$\cos A = \cos\left(2\times\frac{A}{2}\right) = 1-2\sin^2\frac{A}{2}$

87. $\dfrac{2\tan x}{\tan x+\sin x} \equiv \sec^2\left(\dfrac{x}{2}\right)$ (****)

$$\text{LHS} = \frac{2\tan x}{\tan x+\sin x} = \frac{\frac{2\sin x}{\cos x}}{\frac{\sin x}{\cos x}+\sin x} = \frac{2\sin x}{\sin x+\sin x\cos x}$$

MULTIPLY TOP & BOTTOM BY $\cos x$

$$= \frac{2}{1+\cos x} = \frac{2}{1+\left(2\cos^2\frac{x}{2}-1\right)}$$

$\cos 2x = 2\cos^2 x - 1$
$\cos x = 2\cos^2\left(\frac{x}{2}\right)-1$

$$= \frac{\cancel{2}}{\cancel{2}\cos^2\left(\frac{x}{2}\right)} = \frac{1}{\cos^2\left(\frac{x}{2}\right)} = \sec^2\frac{x}{2} = \text{RHS}$$

88. $\dfrac{\operatorname{cosec}x}{1+\operatorname{cosec}x} - \dfrac{\operatorname{cosec}x}{1-\operatorname{cosec}x} \equiv 2\sec^2 x$ (****)

$$\text{LHS} = \frac{\operatorname{cosec}x}{1+\operatorname{cosec}x} - \frac{\operatorname{cosec}x}{1-\operatorname{cosec}x} = \frac{\frac{1}{\sin x}}{1+\frac{1}{\sin x}} - \frac{\frac{1}{\sin x}}{1-\frac{1}{\sin x}}$$
$$= \frac{1}{\sin x+1} - \frac{1}{\sin x-1} = \frac{(\sin x-1)-(\sin x+1)}{(\sin x+1)(\sin x-1)}$$
$$= \frac{-2}{\sin^2 x-1} = \frac{2}{1-\sin^2 x} = \frac{2}{\cos^2 x} = 2\sec^2 x$$

89. $\sin(x+y)\sin(x-y) \equiv \cos^2 y - \cos^2 x$ **(****)**

$$
\begin{aligned}
\text{LHS} &= \sin(x+y)\sin(x-y)\\
&= [\sin x\cos y + \cos x\sin y][\sin x\cos y - \cos x\sin y]\\
&= \text{Difference of squares} \ldots = (\sin x\cos y)^2 - (\cos x\sin y)^2\\
&= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y = (1-\cos^2 x)\cos^2 y - \cos^2 x\sin^2 y\\
&= \cos^2 y - \cos^2 x\cos^2 y - \cos^2 x\sin^2 y\\
&= \cos^2 y - \cos^2 x(\cancel{\cos^2 y + \sin^2 y})\\
&= \cos^2 y - \cos^2 x
\end{aligned}
$$

90. $\cot(x+y) \equiv \dfrac{\cot x\cot y - 1}{\cot x + \cot y}$ **(****)**

$$
\text{LHS} = \cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)} = \frac{\cos x\cos y - \sin x\sin y}{\sin x\cos y + \cos x\sin y}
$$

$$
= \frac{\frac{\cos x\cos y}{\sin x\sin y} - \cancel{\frac{\sin x\sin y}{\sin x\sin y}}}{\frac{\cancel{\sin x}\cos y}{\cancel{\sin x}\sin y} + \frac{\cos x\cancel{\sin y}}{\sin x\cancel{\sin y}}} = \frac{\cot x\cot y - 1}{\cot y + \cot x} = \text{RHS}
$$

$$
\text{LHS} = \cot(x+y) = \frac{1}{\tan(x+y)} = \frac{1}{\frac{\tan x + \tan y}{1-\tan x\tan y}} = \frac{1-\tan x\tan y}{\tan x + \tan y}
$$

$$
= \frac{\frac{1}{\tan x\tan y} - \cancel{\frac{\tan x\tan y}{\tan x\tan y}}}{\frac{\cancel{\tan x}}{\cancel{\tan x}\tan y} + \frac{\cancel{\tan y}}{\tan x\cancel{\tan y}}} = \frac{\cot x\cot y - 1}{\cot y + \cot x} = \text{RHS}
$$

91. $\dfrac{\cos x}{1-\sin x} + \dfrac{1-\sin x}{\cos x} \equiv 2\sec x$ **(****)**

$$
\text{LHS} = \frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x} = \frac{\cos^2 x + (1-\sin x)^2}{(1-\sin x)\cos x}
$$

$$
= \frac{\cos^2 x + 1 - 2\sin x + \sin^2 x}{(1-\sin x)\cos x} = \frac{2-2\sin x}{(1-\sin x)\cos x}
$$

$$
= \frac{2\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}\cos x} = \frac{2}{\cos x} = 2\sec x = \text{RHS}
$$

92. $\sin^2\left(\theta+\dfrac{\pi}{4}\right)-\sin^2\left(\theta-\dfrac{\pi}{4}\right)\equiv\sin 2\theta$ **(****)**

$$\text{LHS} = \sin^2\left(\theta+\tfrac{\pi}{4}\right)-\sin^2\left(\theta-\tfrac{\pi}{4}\right) = \left[\sin\left(\theta+\tfrac{\pi}{4}\right)\right]^2-\left[\sin\left(\theta-\tfrac{\pi}{4}\right)\right]^2$$
$$= \left(\sin\theta\cos\tfrac{\pi}{4}+\cos\theta\sin\tfrac{\pi}{4}\right)^2-\left(\sin\theta\cos\tfrac{\pi}{4}+\cos\theta\sin\tfrac{\pi}{4}\right)^2$$
$$= \left(\tfrac{\sqrt{2}}{2}\sin\theta+\tfrac{\sqrt{2}}{2}\cos\theta\right)^2-\left(\tfrac{\sqrt{2}}{2}\sin\theta-\tfrac{\sqrt{2}}{2}\cos\theta\right)^2$$
$$= \tfrac{1}{2}(\sin\theta+\cos\theta)^2-\tfrac{1}{2}(\sin\theta-\cos\theta)^2$$
$$= \tfrac{1}{2}(\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta)-\tfrac{1}{2}(\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta)$$
$$= \tfrac{1}{2}(1+2\sin\theta\cos\theta)-\tfrac{1}{2}(1-2\sin\theta\cos\theta)$$
$$= \tfrac{1}{2}+\sin\theta\cos\theta-\tfrac{1}{2}+\sin\theta\cos\theta$$
$$= 2\sin\theta\cos\theta = \sin 2\theta = \text{RHS}$$

93. $\tan\left(\theta+\dfrac{\pi}{4}\right)+\tan\left(\theta-\dfrac{\pi}{4}\right)\equiv 2\tan 2\theta$ **(****)**

$$\text{LHS} = \tan\left(\theta+\tfrac{\pi}{4}\right)+\tan\left(\theta-\tfrac{\pi}{4}\right) = \frac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}+\frac{\tan\theta-\tan\frac{\pi}{4}}{1+\tan\theta\tan\frac{\pi}{4}}$$
$$= \frac{\tan\theta+1}{1-\tan\theta}+\frac{\tan\theta-1}{1+\tan\theta} = \frac{(\tan\theta+1)^2+(\tan\theta-1)(1-\tan\theta)}{(1-\tan\theta)(1+\tan\theta)}$$
$$= \frac{\cancel{\tan^2\theta}+2\tan\theta\cancel{+1}+\tan\theta-\cancel{\tan^2\theta}\cancel{-1}+\tan\theta}{1-\tan^2\theta}$$
$$= \frac{4\tan\theta}{1-\tan^2\theta} = 2\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = 2\tan 2\theta = \text{RHS}$$

94. $(\cos x+\sin x)(\operatorname{cosec} x-\sec x)\equiv 2\cot 2x$ **(****)**

$$\text{LHS} = (\cos x+\sin x)(\operatorname{cosec} x-\sec x)$$
$$= \cos x\operatorname{cosec} x-\cos x\sec x+\sin x\operatorname{cosec} x-\sin x\sec x$$
$$= \frac{\cos x}{\sin x}-1+1-\frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}$$
$$= \frac{\cos^2 x-\sin^2 x}{\cos x\sin x} = \frac{\cos 2x}{\sin x\cos x} = \frac{2\cos 2x}{2\sin x\cos x} = \frac{2\cos 2x}{\sin 2x}$$
$$= 2\cot 2x = \text{RHS}$$

95. $\sin P - \sin Q \equiv 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$ **(****)**

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$ SUBTRACT EQUATIONS

$\sin(A+B) - \sin(A-B) = 2\cos A\sin B$

- LET $\left\{\begin{array}{l} P = A+B \\ Q = A-B \end{array}\right.$ $\Rightarrow$ ADD $P+Q = 2A$, $A = \frac{P+Q}{2}$

$\Rightarrow$ SUBTRACT $P-Q = 2B$, $B = \frac{P-Q}{2}$

HENCE $\sin(A+B) - \sin(A-B) = 2\cos A\sin B$ NOW BECOMES

$\sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$

96. $\frac{\cot\theta}{\operatorname{cosec}\theta - 1} - \frac{\cos\theta}{1+\sin\theta} \equiv 2\tan\theta$ **(****)**

$\text{LHS} = \frac{\cot\theta}{\operatorname{cosec}\theta - 1} - \frac{\cos\theta}{1+\sin\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} - 1} - \frac{\cos\theta}{1+\sin\theta}$ MULTIPLY TOP/BOTTOM OF FIRST FRACTION BY $\sin\theta$

$= \frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta(1+\sin\theta) - \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{\cancel{\cos\theta} + \cos\theta\sin\theta - \cancel{\cos\theta} + \cos\theta\sin\theta}{1-\sin^2\theta} = \frac{2\cos\theta\sin\theta}{\cos^2\theta} = \frac{2\sin\theta}{\cos\theta}$

$= 2\tan\theta = \text{RHS}$

97. $\operatorname{cosec}\theta - \cot\theta \equiv \tan\frac{1}{2}\theta$ **(****)**

$\text{LHS} = \operatorname{cosec}\theta - \cot\theta = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1 - \left(1 - 2\sin^2\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$\cos 2A \equiv 1 - 2\sin^2 A$

$\cos A \equiv 1 - 2\sin^2\frac{A}{2}$

$\sin 2A = 2\sin A\cos A$

$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

$= \frac{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}{\cancel{2}\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}$

$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$= \tan\frac{\theta}{2}$

$= \text{RHS}$

98. $\dfrac{\sin 3\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}\equiv 2\cot 2\theta$ **(****)**

$$\text{LHS} = \frac{\sin 3\theta}{\cos\theta}+\frac{\cos 3\theta}{\sin\theta} = \frac{\sin 3\theta\sin\theta+\cos\theta\cos 3\theta}{\cos\theta\sin\theta} \leftarrow \text{COMPOUND ANGLE } \cos(A-B)$$

$$= \frac{\cos(3\theta-\theta)}{\cos\theta\sin\theta} = \frac{\cos 2\theta}{\cos\theta\sin\theta} = \frac{2\cos 2\theta}{2\cos\theta\sin\theta} = \frac{2\cos 2\theta}{\sin 2\theta} = 2\cot 2\theta = \text{RHS}$$

99. $\dfrac{\cot x}{\operatorname{cosec} x-1}-\dfrac{\operatorname{cosec} x-1}{\cot x}\equiv 2\tan x$ **(****)**

$$\text{LHS} = \frac{\cot x}{\operatorname{cosec}x-1}-\frac{\operatorname{cosec}x-1}{\cot x} = \frac{\cot^2 x-(\operatorname{cosec}x-1)^2}{\cot x(\operatorname{cosec}x-1)} = \frac{\cot^2 x-(\operatorname{cosec}^2 x-2\operatorname{cosec}x+1)}{\cot x(\operatorname{cosec}x-1)}$$

$$= \frac{\cot^2 x-\operatorname{cosec}^2 x+2\operatorname{cosec}x-1}{\cot x(\operatorname{cosec}x-1)} = \frac{(\cancel{\operatorname{cosec}^2 x}-1)-\cancel{\operatorname{cosec}^2 x}+2\operatorname{cosec}x-1}{\cot x(\operatorname{cosec}x-1)} = \frac{2\operatorname{cosec}x-2}{\cot x(\operatorname{cosec}x-1)}$$

$$= \frac{2\cancel{(\operatorname{cosec}x-1)}}{\cot x\cancel{(\operatorname{cosec}x-1)}} = \frac{2}{\cot x} = 2\tan x = \text{RHS}$$

100. $\sin^2(x+y)-\sin^2(x-y)\equiv \sin 2x\sin 2y$ **(****)**

$$\begin{aligned}
\text{LHS} &= \sin^2(x+y)-\sin^2(x-y)\\
&= [\sin(x+y)-\sin(x-y)][\sin(x+y)+\sin(x-y)]\\
&= [\cancel{\sin x\cos y}+\cos x\sin y-(\cancel{\sin x\cos y}+\cos x\sin y)]\\
&\quad\times[\sin x\cos y+\cancel{\cos x\sin y}+\sin x\cos y-\cancel{\cos x\sin y}]\\
&= (2\cos x\sin y)(2\sin x\cos y)\\
&= (2\cos x\sin x)(2\sin y\sin y)\\
&= \sin 2x\sin 2y\\
&= \text{RHS}
\end{aligned}$$

101. $\cos P+\cos Q\equiv 2\cos\left(\dfrac{P+Q}{2}\right)\cos\left(\dfrac{P-Q}{2}\right)$ **(****)**

$$\left.\begin{aligned}\cos(A+B) &= \cos A\cos B-\sin A\sin B\\ \cos(A-B) &= \cos A\cos B+\sin A\sin B\end{aligned}\right\}\ \text{Add equations}$$

$$\boxed{\cos(A+B)+\cos(A-B) = 2\cos A\cos B}\ (*)$$

LET $\left.\begin{aligned}P&=A+B\\ Q&=A-B\end{aligned}\right\}$ add equations $P+Q=2A$, $\boxed{\frac{P+Q}{2}=A}$; subtract equations $P-Q=2B$, $\boxed{\frac{P-Q}{2}=B}$

HENCE $(*)$ BECOMES

$$\cos P+\cos Q = 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

102. $\dfrac{\cos 4\theta + \cos 2\theta}{\sin 4\theta - \sin 2\theta} \equiv \cot\theta$ **(****+)**

USING $\cos A + \cos B \equiv 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$\sin A - \sin B \equiv 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$$\text{LHS} = \frac{\cos 4\theta + \cos 2\theta}{\sin 4\theta - \sin 2\theta} = \frac{\cancel{2\cos\frac{4\theta+2\theta}{2}}\cos\frac{4\theta-2\theta}{2}}{\cancel{2\cos\frac{4\theta+2\theta}{2}}\sin\frac{4\theta-2\theta}{2}} = \frac{\cos\theta}{\sin\theta} = \cot\theta = \text{RHS}$$

(ALTERNATIVE)

- $\sin 2A = 2\sin A\cos A$

$\sin 4\theta = \sin(2\times 2\theta) = 2\sin 2\theta\cos 2\theta$

- $\cos 2A = 2\cos^2 A - 1$

$\cos 4\theta = \cos(2\times 2\theta) = 2\cos^2 2\theta - 1$

$$\text{LHS} = \frac{\cos 4\theta + \cos 2\theta}{\sin 4\theta - \sin 2\theta} = \frac{(2\cos^2 2\theta - 1) + \cos 2\theta}{2\sin 2\theta\cos 2\theta - \sin 2\theta} = \frac{2\cos^2 2\theta + \cos 2\theta - 1}{\sin 2\theta(2\cos 2\theta - 1)}$$

$$= \frac{\cancel{(2\cos 2\theta - 1)}(\cos 2\theta + 1)}{\sin 2\theta\cancel{(2\cos 2\theta - 1)}} = \frac{(2\cos^2\theta - 1) + 1}{2\sin\theta\cos\theta} = \frac{2\cos^2\theta}{2\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta = \text{RHS}$$

103. $\dfrac{\sin x}{1+\tan x} \equiv \dfrac{\cos x}{1+\cot x}$ **(****+)**

$$\text{LHS} = \frac{\sin x}{1+\tan x} = \frac{\sin x}{1+\frac{\sin x}{\cos x}} = \text{MULTIPLY TOP/BOTTOM BY } \cos x = \frac{\sin x\cos x}{\cos x + \sin x}$$

$$= \frac{\frac{\cancel{\sin x}\cos x}{\cancel{\sin x}}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}} = \frac{\cos x}{\cot x + 1} = \text{RHS}$$

ALTERNATIVE

$$\text{LHS} = \frac{\sin x}{1+\tan x} = \frac{\sin x}{1+\frac{1}{\cot x}} = \text{MULTIPLY TOP/BOTTOM BY } \cot x = \frac{\cot x\sin x}{\cot x + 1}$$

$$= \frac{\frac{\cos x}{\cancel{\sin x}}\times\cancel{\sin x}}{\cot x + 1} = \frac{\cos x}{\cot x + 1} = \text{RHS}$$

104. $\dfrac{1+\sin\theta}{1-\sin\theta} \equiv (\sec\theta + \tan\theta)^2$ **(****+)**

$$\text{LHS} = \frac{1+\sin\theta}{1-\sin\theta} = \frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = \frac{1+2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$$

$$= \frac{1+2\sin\theta+\sin^2\theta}{\cos^2\theta} = \sec^2\theta + \frac{2\sin\theta}{\cos^2\theta} + \tan^2\theta$$

$$= \sec^2\theta + 2\frac{\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta} + \tan^2\theta$$

$$= \sec^2\theta + 2\tan\theta\sec\theta + \tan^2\theta$$

$$= (\sec\theta + \tan\theta)^2 = \text{RHS}$$

105. $\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x$ **(****+)**

$$\text{LHS} = \cot^2 x - \tan^2 x = \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$$
$$= \frac{(\cos^2 x)^2 - (\sin^2 x)^2}{\frac{1}{4} \times 4\sin^2 x \cos^2 x} = \frac{(\cos^2 x - \sin^2 x)\cancel{(\cos^2 x + \sin^2 x)}}{\frac{1}{4}(2\sin x \cos x)^2}$$
$$= \frac{\cos 2x}{\frac{1}{4}(\sin 2x)^2} = \frac{4\cos 2x}{\sin^2 2x} = \frac{4\cos 2x}{\sin 2x} \times \frac{1}{\sin 2x}$$
$$= 4\cot 2x \operatorname{cosec} 2x = \text{RHS}$$

OR

$$\text{RHS} = 4\cot 2x \operatorname{cosec} 2x = 4\frac{\cos 2x}{\sin 2x} \times \frac{1}{\sin 2x} = \frac{4\cos 2x}{\sin^2 2x}$$
$$= \frac{4(\cos^2 x - \sin^2 x)}{(2\sin x \cos x)^2} = \frac{4\cos^2 x - 4\sin^2 x}{4\sin^2 x \cos^2 x} = \frac{\cancel{4\cos^2 x}}{\cancel{4}\sin^2 x \cancel{\cos^2 x}} - \frac{\cancel{4\sin^2 x}}{4\cancel{\sin^2 x}\cos^2 x}$$
$$= \operatorname{cosec}^2 x - \sec^2 x = (1 + \cot^2 x) - (1 + \tan^2 x)$$
$$= \cot^2 x - \tan^2 x = \text{LHS}$$

106. $\tan\left(\theta - \frac{\pi}{4}\right) \equiv \frac{\sin 2\theta - 1}{\cos 2\theta}$ **(****+)**

$$\text{LHS} = \tan\left(\theta - \frac{\pi}{4}\right) = \frac{\tan\theta - \tan\frac{\pi}{4}}{1 + \tan\theta \tan\frac{\pi}{4}} = \frac{\tan\theta - 1}{1 + \tan\theta} = \frac{\frac{\sin\theta}{\cos\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1}$$

MULTIPLY TOP/BOTTOM BY $\cos\theta$

$$= \frac{\sin\theta - \cos\theta}{\cos\theta + \sin\theta} = \frac{(\sin\theta - \cos\theta)(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$$
$$= \frac{-(\sin\theta - \cos\theta)^2}{\cos^2\theta - \sin^2\theta} = -\frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{\cos 2\theta} = -\frac{1 - \sin 2\theta}{\cos 2\theta}$$
$$= \frac{\sin 2\theta - 1}{\cos 2\theta} = \text{RHS}$$

107. $\frac{1 + \cos\theta}{1 - \cos\theta} \equiv (\operatorname{cosec}\theta + \cot\theta)^2$ **(****+)**

$$\text{LHS} = \frac{1 + \cos\theta}{1 - \cos\theta} = \frac{(1 + \cos\theta)(1 + \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)} = \frac{1 + 2\cos\theta + \cos^2\theta}{1 - \cos^2\theta}$$
$$= \frac{1 + 2\cos\theta + \cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} + \frac{2\cos\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} =$$
$$= \operatorname{cosec}^2\theta + 2\frac{\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta} + \cot^2\theta = \operatorname{cosec}^2\theta + 2\cot\theta\operatorname{cosec}\theta + \cot^2\theta$$
$$= (\operatorname{cosec}\theta + \cot\theta)^2 = \text{RHS}$$

108. $\dfrac{2\sin x\cos x-\cos x}{1-\sin x+\sin^2 x-\cos^2 x}\equiv\cot x$ **(****+)**

$$\text{LHS}=\frac{2\sin x\cos x-\cos x}{1-\sin x+\sin^2 x-\cos^2 x}=\frac{\cos x(2\sin x-1)}{1-\sin x+\sin^2 x-\cos^2 x}$$
$$=\frac{\cos x(2\sin x-1)}{1-\sin x+\sin^2 x-(1-\sin^2 x)}=\frac{\cos x(2\sin x-1)}{2\sin^2 x-\sin x}$$
$$=\frac{\cos x\cancel{(2\sin x-1)}}{\sin x\cancel{(2\sin x-1)}}=\cot x=\text{RHS}$$

109. $\tan\left(\theta+\dfrac{\pi}{4}\right)\equiv\sec 2\theta+\tan 2\theta$ **(****+)**

$$\text{LHS}=\tan\left(\theta+\frac{\pi}{4}\right)=\frac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}=\frac{\tan\theta+1}{1-\tan\theta}$$
$$=\frac{(\tan\theta+1)(1+\tan\theta)}{(1-\tan\theta)(1+\tan\theta)}=\frac{\tan^2\theta+1+2\tan\theta}{1-\tan^2\theta}$$
$$=\frac{\sec^2\theta}{1-\tan^2\theta}+\frac{2\tan\theta}{1-\tan^2\theta}=\frac{\frac{1}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}+\tan 2\theta$$

MULTIPLY TOP/BOTTOM BY $\cos^2\theta$

$$=\frac{1}{\cos^2\theta-\sin^2\theta}+\tan 2\theta$$
$$=\frac{1}{\cos 2\theta}+\tan 2\theta=\sec 2\theta+\tan 2\theta=\text{RHS}$$

110. $2\cos^4\theta+\dfrac{1}{2}\sin^2 2\theta-1\equiv\cos 2\theta$ **(****+)**

$\cos 2\theta = 2\cos^2\theta-1$
$1+\cos 2\theta = 2\cos^2\theta$
$(1+\cos 2\theta)^2 = 4\cos^4\theta$
$1+2\cos 2\theta+\cos^2 2\theta = 4\cos^4\theta$
$\frac{1}{2}+\cos 2\theta+\frac{1}{2}\cos^2 2\theta = 2\cos^4\theta$

$$\text{LHS}=2\cos^4\theta+\tfrac{1}{2}\sin^2 2\theta-1$$
$$=\left(\tfrac{1}{2}+\cos 2\theta+\tfrac{1}{2}\cos^2 2\theta\right)+\tfrac{1}{2}\sin^2 2\theta-1$$
$$=-\tfrac{1}{2}+\tfrac{1}{2}\left(\cos^2 2\theta+\sin^2 2\theta\right)+\cos 2\theta$$
$$=-\tfrac{1}{2}+\tfrac{1}{2}+\cos 2\theta$$
$$=\cos 2\theta$$
$$=\text{RHS}$$

111. $\sqrt{1+\sin 2\theta}\equiv\sin\theta+\cos\theta$ **(****+)**

$$\text{LHS}=\sqrt{1+\sin 2\theta}=\sqrt{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta}$$
$$=\sqrt{(\cos\theta+\sin\theta)^2}=\cos\theta+\sin\theta=\text{RHS}$$

112. $8\cos^4\left(\frac{1}{2}\theta\right) \equiv \cos 2\theta + 4\cos\theta + 3$ **(****+)**

NOTE $\cos 2A = 2\cos^2 A - 1$

$2\cos^2 A = 1 + \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$

HENCE $\cos^2\left(\frac{1}{2}\theta\right) = \frac{1}{2} + \frac{1}{2}\cos\theta$

$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

$$\text{LHS} = 8\cos^4\left(\tfrac{1}{2}\theta\right) = 8\left(\cos^2\tfrac{1}{2}\theta\right)^2 = 8\left(\tfrac{1}{2} + \tfrac{1}{2}\cos\theta\right)^2$$

$$= 8\times\left(\tfrac{1}{2}\right)^2(1+\cos\theta)^2 = 2(1+\cos\theta)^2 = 2(1 + 2\cos\theta + \cos^2\theta)$$

$$= 2 + 4\cos\theta + 2\cos^2\theta = 2 + 4\cos\theta + 2\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta\right)$$

$$= 2 + 4\cos\theta + 1 + \cos 2\theta = \cos 2\theta + 4\cos\theta + 3 = \text{RHS}$$

113. $\tan(A+B+C) \equiv \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$ **(****+)**

$$\text{LHS} = \tan(A+B+C) = \tan\left((A+B)+C\right) = \frac{\frac{\tan A + \tan B}{1 - \tan A \tan B} + \tan C}{1 - \left(\frac{\tan A + \tan B}{1 - \tan A \tan B}\right)\tan C}$$

MULTIPLY TOP & BOTTOM BY $1 - \tan A \tan B$

$$= \frac{(\tan A + \tan B) + \tan C(1 - \tan A \tan B)}{1 - \tan A \tan B - (\tan A + \tan B)\tan C}$$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan A \tan C - \tan B \tan C}$$

$= \text{RHS}$

114. $\dfrac{2\sec^2\theta - \cos 2\theta - 1}{2\tan\theta + \sin 2\theta} \equiv \tan\theta$ **(****+)**

$$\text{LHS} = \frac{2\sec^2\theta - \cos 2\theta - 1}{2\tan\theta + \sin 2\theta} = \frac{2\sec^2\theta - (2\cos^2\theta - 1) - 1}{2\tan\theta + 2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}\sec^2\theta - \cancel{2}\cos^2\theta}{\cancel{2}\tan\theta + \cancel{2}\sin\theta\cos\theta} = \frac{\frac{1}{\cos^2\theta} - \cos^2\theta}{\frac{\sin\theta}{\cos\theta} + \sin\theta\cos\theta} =$$

MULTIPLY TOP & BOTTOM BY $\cos^2\theta$, OR SIMPLIFY DOUBLE FRACTION "SLOWER"

$$= \frac{1 - \cos^4\theta}{\sin\theta\cos\theta + \sin\theta\cos^3\theta} = \frac{(1-\cos^2\theta)\cancel{(1+\cos^2\theta)}}{\sin\theta\cos\theta\cancel{(1+\cos^2\theta)}}$$

DIFFERENCE OF SQUARES

$$= \frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$$

115. $\tan 3x \equiv \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$ **(****+)**

$$\text{LHS} = \tan 3x = \tan(2x+x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$= \frac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \frac{2\tan x}{1-\tan^2 x}\tan x}$$

MULTIPLY TOP & BOTTOM OF THE FRACTION BY $(1-\tan^2 x)$

$$= \frac{2\tan x + \tan x(1-\tan^2 x)}{(1-\tan^2 x) - 2\tan^2 x} = \frac{2\tan x + \tan x - \tan^3 x}{1 - 3\tan^2 x}$$

$$= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = \text{RHS}$$

116. $\dfrac{\sqrt{2-2\cos x}}{\sin x} \equiv \sec\dfrac{x}{2}$ **(*****)**

113 $\quad \text{LHS} = \dfrac{\sqrt{2-2\cos x}}{\sin x}$

$$= \frac{\sqrt{2-2\left(1-2\sin^2\frac{x}{2}\right)}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \frac{\sqrt{4\sin^2\frac{x}{2}}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{\cancel{2\sin\frac{x}{2}}}{\cancel{2\sin\frac{x}{2}}\cos\frac{x}{2}}$$

$$= \sec\frac{x}{2} = \text{RHS}$$

$\cos 2A = 1 - 2\sin^2 A$
$\cos A = 1 - 2\sin^2\frac{A}{2}$

$\sin 2A = 2\sin A\cos A$
$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

117. $\sin^2\theta\tan\theta + \cos^2\theta\cot\theta + 2\sin\theta\cos\theta \equiv \tan\theta + \cot\theta$ **(*****)**

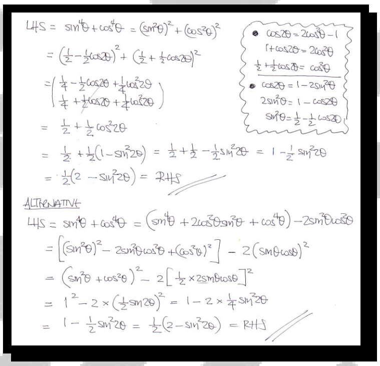
$$\text{LHS} = \sin^2\theta\tan\theta + \cos^2\theta\cot\theta + 2\sin\theta\cos\theta = \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\cos\theta} + 2\sin\theta\cos\theta$$

$$= \frac{\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} = \frac{(\sin^2\theta)^2 + 2\sin^2\theta\cos^2\theta + (\cos^2\theta)^2}{\sin\theta\cos\theta}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2}{\sin\theta\cos\theta} = \frac{1^2}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \cot\theta + \tan\theta = \text{RHS}$$

118. $\sin^4\theta + \cos^4\theta \equiv \frac{1}{2}\left(2 - \sin^2 2\theta\right)$ (*****)



LHS $= \sin^4\theta + \cos^4\theta = (\sin^2\theta)^2 + (\cos^2\theta)^2$

$= \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)^2 + \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)^2$

$= \left(\frac{1}{4} - \cancel{\frac{1}{2}\cos 2\theta} + \frac{1}{4}\cos^2 2\theta \quad \frac{1}{4} + \cancel{\frac{1}{2}\cos 2\theta} + \frac{1}{4}\cos^2 2\theta \right)$

$= \frac{1}{2} + \frac{1}{2}\cos^2 2\theta$

$= \frac{1}{2} + \frac{1}{2}(1 - \sin^2 2\theta) = \frac{1}{2} + \frac{1}{2} - \frac{1}{2}\sin^2 2\theta = 1 - \frac{1}{2}\sin^2 2\theta$

$= \frac{1}{2}(2 - \sin^2 2\theta) =$ RHS

- $\cos 2\theta = 2\cos^2\theta - 1$
  $1 + \cos 2\theta = 2\cos^2\theta$
  $\frac{1}{2} + \frac{1}{2}\cos 2\theta = \cos^2\theta$
- $\cos 2\theta = 1 - 2\sin^2\theta$
  $2\sin^2\theta = 1 - \cos 2\theta$
  $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

ALTERNATIVE

LHS $= \sin^4\theta + \cos^4\theta = \left(\sin^4\theta + 2\cos^2\theta\sin^2\theta + \cos^4\theta\right) - 2\sin^2\theta\cos^2\theta$

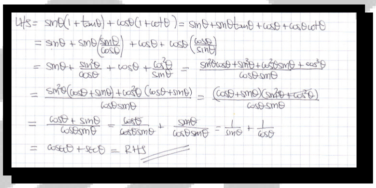
$= \left[(\sin^2\theta)^2 - 2\sin^2\theta\cos^2\theta + (\cos^2\theta)^2\right] - 2(\sin\theta\cos\theta)^2$

$= \left(\sin^2\theta + \cos^2\theta\right)^2 - 2\left[\frac{1}{2} \times 2\sin\theta\cos\theta\right]^2$

$= 1^2 - 2 \times \left(\frac{1}{2}\sin 2\theta\right)^2 = 1 - 2 \times \frac{1}{4}\sin^2 2\theta$

$= 1 - \frac{1}{2}\sin^2 2\theta = \frac{1}{2}(2 - \sin^2 2\theta) =$ RHS

119. $\sin\theta\left(1 + \tan\theta\right) + \cos\theta\left(1 + \cot\theta\right) \equiv \sec\theta + \operatorname{cosec}\theta$ (*****)



LHS $= \sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta) = \sin\theta + \sin\theta\tan\theta + \cos\theta + \cos\theta\cot\theta$

$= \sin\theta + \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \cos\theta + \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$

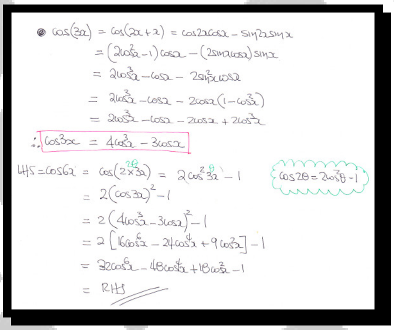
$= \sin\theta + \frac{\sin^2\theta}{\cos\theta} + \cos\theta + \frac{\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta\cos\theta + \sin^3\theta + \cos^2\theta\sin\theta + \cos^3\theta}{\cos\theta\sin\theta}$

$= \frac{\sin^2\theta(\cos\theta + \sin\theta) + \cos^2\theta(\cos\theta + \sin\theta)}{\cos\theta\sin\theta} = \frac{(\cos\theta + \sin\theta)\cancel{(\sin^2\theta + \cos^2\theta)}}{\cos\theta\sin\theta}$

$= \frac{\cos\theta + \sin\theta}{\cos\theta\sin\theta} = \frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta} = \frac{1}{\sin\theta} + \frac{1}{\cos\theta}$

$= \operatorname{cosec}\theta + \sec\theta =$ RHS

120. $\cos 6x \equiv 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ (*****)



- $\cos(3x) = \cos(2x + x) = \cos 2x\cos x - \sin 2x\sin x$

$= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x$

$= 2\cos^3 x - \cos x - 2\sin^2 x\cos x$

$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$\therefore \boxed{\cos 3x = 4\cos^3 x - 3\cos x}$

LHS $= \cos 6x = \cos(2 \times 3x) = 2\cos^2 3x - 1$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2(\cos 3x)^2 - 1$

$= 2\left(4\cos^3 x - 3\cos x\right)^2 - 1$

$= 2\left[16\cos^6 x - 24\cos^4 x + 9\cos^2 x\right] - 1$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

$=$ RHS

121. $32\sin^2 x\cos^4 x \equiv 2+\cos 2x - 2\cos 4x - \cos 6x$ (*****)

$$\text{LHS} = 32\sin^2 x\cos^4 x = 8\left(4\sin^2 x\cos^2 x\right)\cos^2 x = 8\left(2\sin x\cos x\right)^2\cos^2 x$$
$$= 8\left(\sin 2x\right)^2\cos^2 x = 8\sin^2 2x\cos^2 x$$

$\cos 2A = 2\cos^2 A - 1$; $1+\cos 2A = 2\cos^2 A$; $\cos^2 A = \frac{1}{2}+\frac{1}{2}\cos 2A$

$\cos 2A = 1 - 2\sin^2 A$; $2\sin^2 A = 1-\cos 2A$; $\sin^2 A = \frac{1}{2}-\frac{1}{2}\cos 2A$

$$= 8\times\left(\tfrac{1}{2}-\tfrac{1}{2}\cos 4x\right)\left(\tfrac{1}{2}+\tfrac{1}{2}\cos 2x\right)$$
$$= 2\left(1-\cos 4x\right)\left(1+\cos 2x\right)$$
$$= 2+2\cos 2x-2\cos 4x-\boxed{2\cos 2x\cos 4x}$$

$\cos(4x+2x) = \cos 4x\cos 2x - \sin 4x\sin 2x$

$\cos(4x-2x) = \cos 4x\cos 2x + \sin 4x\sin 2x$

ADD $\cos 6x + \cos 2x = 2\cos 4x\cos 2x$

$$= 2+2\cos 2x-2\cos 4x-\left(\cos 6x+\cos 2x\right)$$
$$= 2+\cos 2x-2\cos 4x-\cos 6x$$
$$= \text{RHS}$$

As required

122. $\sin^4\theta+\cos^4\theta \equiv \frac{1}{4}\left(3+\cos 4\theta\right)$ (*****)

$$\text{RHS} = \tfrac{1}{4}\left(3+\cos 4\theta\right) = \tfrac{1}{4}\left(3+2\cos^2 2\theta-1\right)$$
$$= \tfrac{1}{4}\left(2+2\cos^2 2\theta\right) = \tfrac{1}{2}\left(1+\cos^2 2\theta\right)$$
$$= \tfrac{1}{2}\left(1+\left(2\cos^2\theta-1\right)^2\right) = \tfrac{1}{2}\left[1+4\cos^4\theta-4\cos^2\theta+1\right]$$
$$= \tfrac{1}{2}\left[4\cos^4\theta-4\cos^2\theta+2\right] = 2\cos^4\theta-2\cos^2\theta+1$$
$$= \cos^4\theta+\left(\cos^4\theta-2\cos^2\theta+1\right) = \cos^4\theta+\left(1-\cos^2\theta\right)^2$$
$$= \cos^4\theta+\left(\sin^2\theta\right)^2 = \cos^4\theta+\sin^4\theta = \text{LHS}$$

$\cos 2A = 2\cos^2 A - 1$

$\cos 4A = 2\cos^2 2A - 1$

$$\text{LHS} = \cos^4\theta+\sin^4\theta = \left(\cos^4\theta+2\sin^2\theta\cos^2\theta+\sin^4\theta\right)-2\sin^2\theta\cos^2\theta$$
$$= \left(\cos^2\theta+\sin^2\theta\right)^2-2\sin^2\theta\cos^2\theta = 1^2-\tfrac{1}{2}\left(2\sin\theta\cos\theta\right)^2$$
$$= 1-\tfrac{1}{2}\sin^2 2\theta$$
$$= 1-\tfrac{1}{2}\left(\tfrac{1}{2}-\tfrac{1}{2}\cos 4\theta\right)$$
$$= 1-\tfrac{1}{4}+\tfrac{1}{4}\cos 4\theta$$
$$= \tfrac{3}{4}+\tfrac{1}{4}\cos 4\theta$$
$$= \tfrac{1}{4}\left(3+\cos 4\theta\right) = \text{RHS}$$

$\cos 2A = 1-2\sin^2 A$

$2\sin^2 A = 1-\cos 2A$

$\sin^2 A = \frac{1}{2}-\frac{1}{2}\cos 2A$

123. $\dfrac{\cos 2x}{\sqrt{1+\sin 2x}} \equiv \cos x - \sin x$ (*****)

$$\text{LHS} = \frac{\cos 2x}{\sqrt{1+\sin 2x}} = \frac{\cos^2 x-\sin^2 x}{\sqrt{1+2\sin x\cos x}} = \frac{(\cos x-\sin x)(\cos x+\sin x)}{\sqrt{\cos^2 x+2\sin x\cos x+\sin^2 x}}$$
$$= \frac{(\cos x-\sin x)(\cos x+\sin x)}{\sqrt{(\cos x+\sin x)^2}} = \frac{(\cos x-\sin x)\cancel{(\cos x+\sin x)}}{\cancel{\cos x+\sin x}}$$
$$= \cos x-\sin x = \text{RHS}$$

124. $\dfrac{\sin 4\theta - 8\sin^3\theta\cos\theta}{\sin\theta} \equiv 4\cos 3\theta$ **(*****)**

$$\text{LHS} = \frac{\sin 4\theta - 8\sin^3\theta\cos\theta}{\sin\theta} = \frac{2\sin 2\theta\cos 2\theta - 8\sin^3\theta\cos\theta}{\sin\theta}$$

$$= \frac{2\cos 2\theta(2\cancel{\sin\theta}\cos\theta) - 8\sin^{\cancel{3}\,2}\theta\cos\theta}{\cancel{\sin\theta}}$$

$$= 4\cos\theta\cos 2\theta - 8\sin^2\theta\cos\theta = 4\cos\theta\cos 2\theta - 4\sin\theta(2\cos\theta\sin\theta)$$

$$= 4\cos\theta\cos 2\theta - 4\sin\theta\sin 2\theta = 4[\cos 2\theta\cos\theta - \sin 2\theta\sin\theta]$$

$$= 4\cos(2\theta + \theta) = 4\cos 3\theta = \text{RHS}$$

125. $\cos^2 x + \sin^2 x \equiv 1$ **(*****)**

Let $f(x) = \cos^2 x + \sin^2 x$

$f'(x) = -2\cos x\sin x + 2\sin x\cos x$

$f'(x) = 0$ REGARDLESS OF x

$\therefore f(x) = k$ = CONSTANT, SO IT DIFFERENTIATES TO ZERO

AS FUNCTION IS CONSTANT EVALUATE BY ANY x, SAY $x = 0$

$f(0) = \cos^2 0 + \sin^2 0 = 1^2 + 0^2 = 1$

$\therefore f(x) = 1$

$\cos^2 x + \sin^2 x \equiv 1$