TRIGONOMETRIC GRAPHS

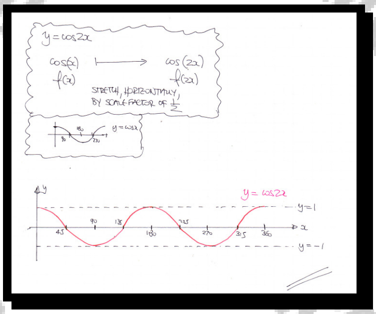
Question 1 (**)

Sketch the graph of

$$y = \cos 2x\,,\ 0° \le x \le 360°\,.$$

The sketch must include the coordinates of all the points where the graph meets the coordinate axes.

graph



Question 2 (+)**

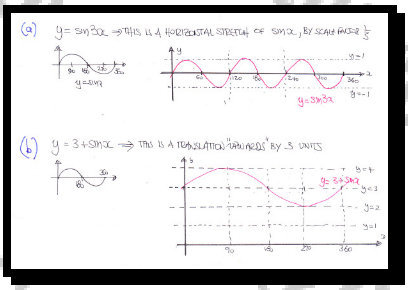
Sketch on separate diagrams the graph of

a) $y = \sin 3x°\,,\ 0 \le x \le 360\,.$

b) $y = 3 + \sin x°\,,\ 0 \le x \le 360\,.$

The sketches must include the coordinates of all the points where each of the graphs meets the coordinate axes.

graph



Question 3 (+)**

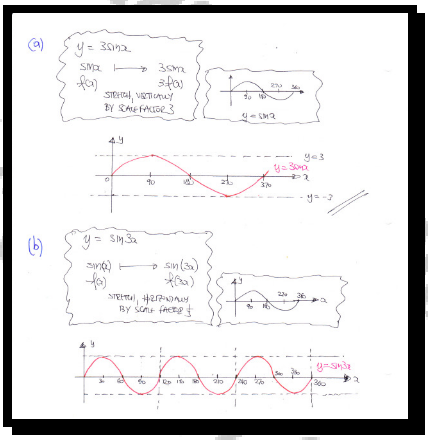
Sketch on separate diagrams the graph of

a) $y = 3\sin x°$, $0 \le x \le 360$.

b) $y = \sin 3x°$, $0 \le x \le 360$.

The sketches must include the coordinates of all the points where each of the graphs meets the coordinate axes.

graph

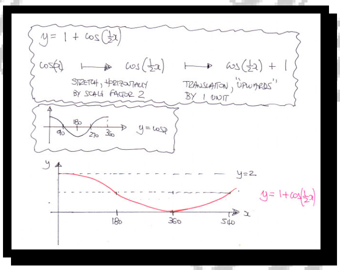


Question 4 (***)

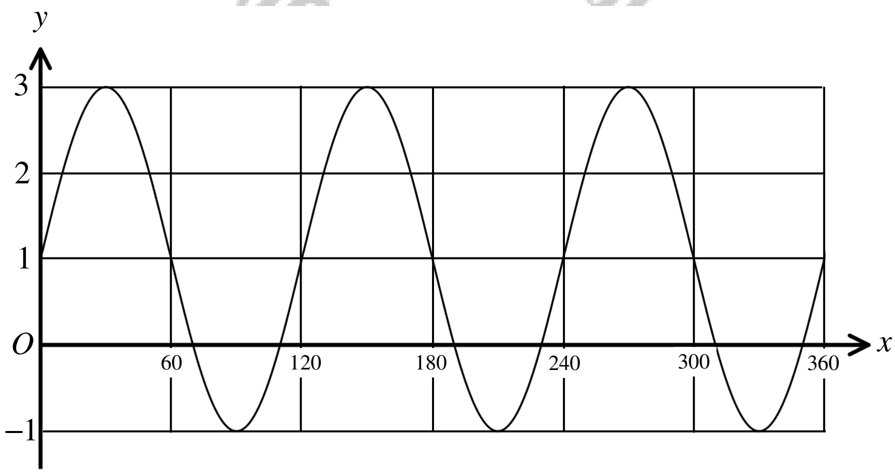
Sketch the graph of

$$y = 1 + \cos\frac{1}{2}x, \quad 0° \le x \le 360°.$$

graph



Question 5 (***)



The figure above shows an accurate graph of

$$y = A + B\sin Cx\,,$$

where x is measured in degrees and A, B and C are constants.

a) State the period of the graph.

b) Find the value of A, B and C.

$\boxed{}$, $\boxed{\tau = 120°}$, $\boxed{A = 1}$, $\boxed{B = 2}$, $\boxed{C = 3}$

(a) PERIOD IS 120°

(b) $y = A + B\sin Cx$

TRANSLATION UP BY 1

2

$y = \sin x$ EXISTS FOR $-1 \le y \le 1$ & THIS GRAPH HAS A VERTICAL "GAP" OF 4 i.e. FROM −1 to 3

3

"THREE FULL CYCLES WITHIN 0 to 360

$\therefore A = 1$, $B = 2$, $C = 3$

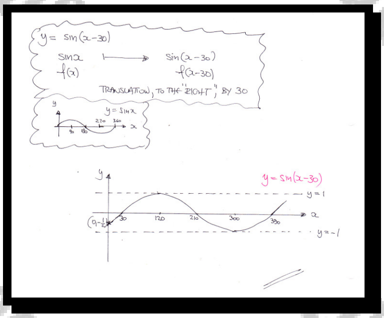
Question 6 (***)

Sketch the graph of

$$y = \sin(x - 30)°, \ 0 \le x \le 360 .$$

The sketch must include the coordinates of all the points where the graph meets the coordinate axes.

graph



Question 7 (***)

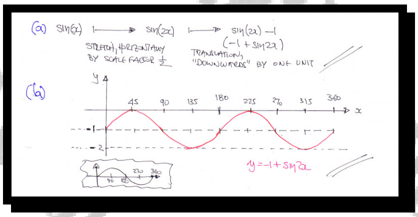
$$y = -1 + \sin 2x°, \ 0 \le x \le 360.$$

a) Describe geometrically the two transformations that map the graph of $y = \sin x°$ onto the graph of $y = -1 + \sin 2x°$.

b) Sketch the graph of

$$y = -1 + \sin 2x°, \ 0 \le x \le 360.$$

horizontal stretch by scale factor $\frac{1}{2}$,
followed by translation "downwards" by 1 unit



Question 8 (***)

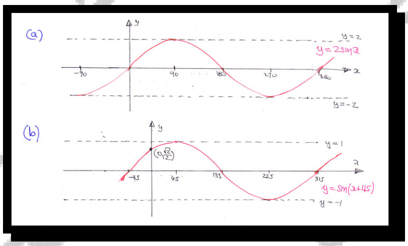
Sketch on separate diagrams the graph of

a) $y = 2\sin x°, \ 0 \le x \le 360.$

b) $y = \sin(x + 45)°, \ 0 \le x \le 360.$

The sketches must include the coordinates of all the points where each of the graphs meets the coordinate axes.

graph



Question 9 (***)

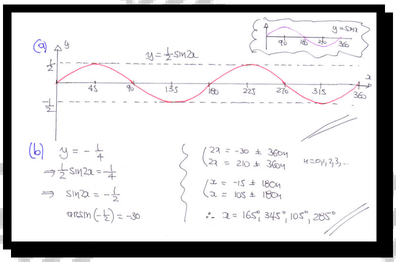
$$y = \frac{1}{2}\sin 2x\,,\ 0° \le x \le 360°.$$

a) Sketch the graph of y.

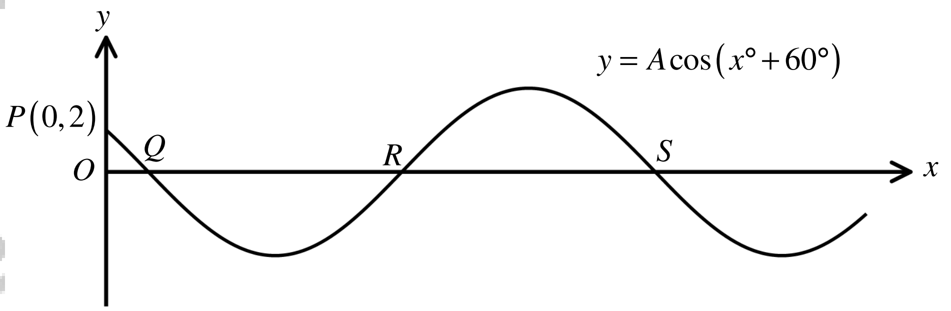
The sketch must include the coordinates of all the points where the graph meets the coordinate axes.

b) Solve the equation $y = -\frac{1}{4}$.

$\boxed{x = 105°, 165°, 285°, 345°}$



Question 10 (***)



The figure above shows part of the graph of the curve with equation

$$y = A\cos(x° + 60°),$$

where x is measured in degrees and A is a constant.

The point $P(0,2)$ lies on the curve.

a) Find the value of A.

The first three x intercepts of the curve, for which $x > 0$, are the points labelled as Q, R and S.

b) State the coordinates of Q, R and S.

$\boxed{}$, $\boxed{A = 4}$, $\boxed{B = 3}$, $\boxed{Q(30,0),\ R(210,0),\ S(390,0)}$

a) USING THE POINT (0,2)

$y = A\cos(x+60)$
$2 = A\cos 60$
$2 = \frac{1}{2}A$
$A = 4$ //

b) SOLVING $4\cos(x+60) = 0$

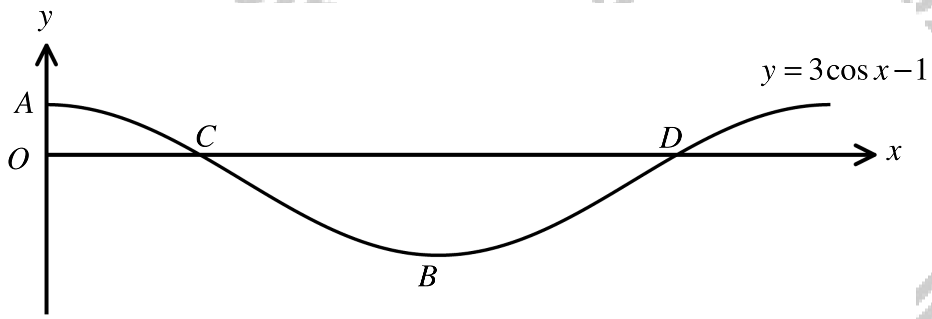
$\Rightarrow \cos(x+60) = 0$

$\Rightarrow \begin{cases} x+60 = 90 \pm 360n \\ x+60 = 270 \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$

$\Rightarrow \begin{cases} x = 30° \pm 360n \\ x = 210° \pm 360n \end{cases}$

$\therefore Q(30°,0)$, $R(210°,0)$, $S(390°,0)$ //

Question 11 (***+)



The figure above shows the graph of the curve with equation

$$y = 3\cos x - 1, \quad 0 \le x \le 2\pi .$$

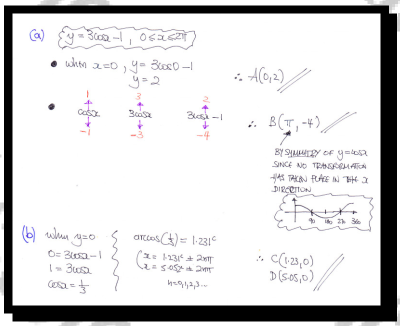
The graph meets the y axis at point A and the x axis at points C and D.

The point B is the first minimum of the graph for which $x > 0$.

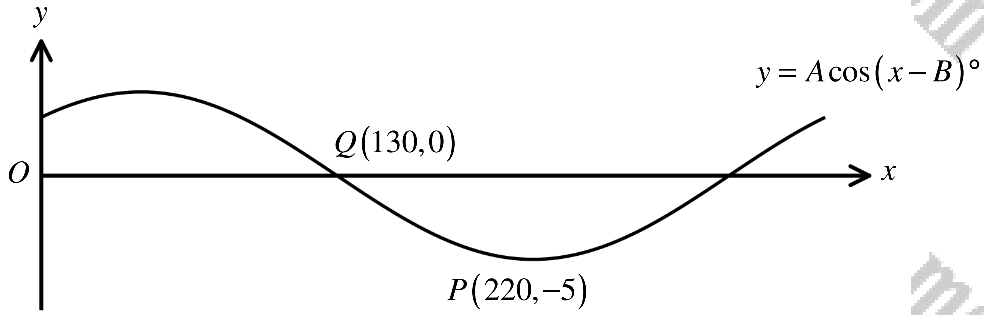
a) State the coordinates of A and B.

b) Determine the coordinates of C and D, correct to three significant figures.

$\boxed{}$, $\boxed{A(0,2)}$, $\boxed{B(\pi,-4)}$, $\boxed{C(1.23,0)}$, $\boxed{C(5.05,0)}$



Question 12 (*+)**



The figure above shows the graph of the curve with equation

$$y = A\cos(x - B)°,\ 0 \le x \le 360,$$

where A and B are positive constants with $0 < B < 90$.

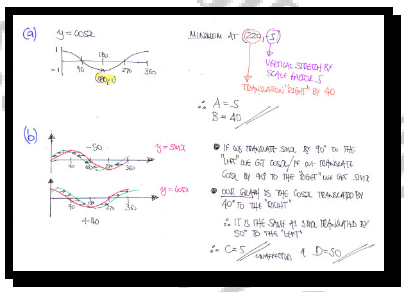
The graph meets the x axis at point $Q(130,0)$ and the point $P(220,-5)$ is the minimum point of the curve.

a) State the value of A and the value of B.

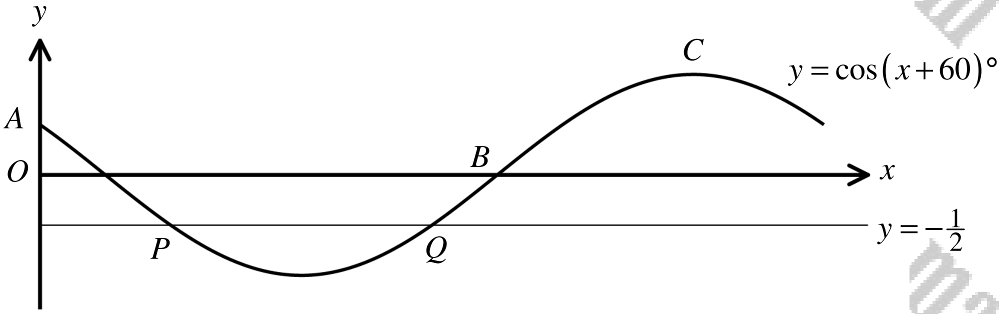
The graph of $y = A\cos(x - B)°$ can also be expressed in the form $y = C\sin(x + D)°$, where C and D are positive constants with $0 < D < 90°$.

b) State the value of C and the value of D.

$\boxed{C2M}$, $\boxed{A=5}$, $\boxed{B=40}$, $\boxed{C=5}$, $\boxed{D=50}$



Question 13 (***+)



The figure above shows the graph of

$$y = \cos(x+60)°,\ 0 \le x \le 360.$$

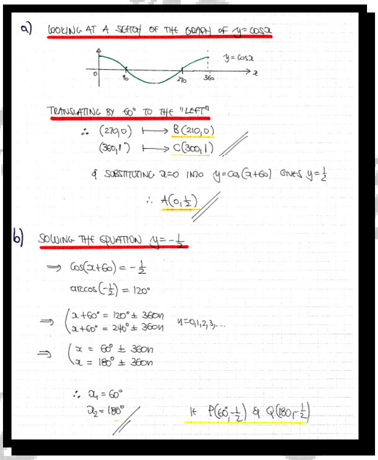
The graph meets the y axis at the point A and the point B is one of the two x intercepts of the curve. The point C is the maximum point of the curve.

a) State the coordinates of A, B and C.

The straight line with equation $y = -\frac{1}{2}$ meets the graph of $y = \cos(x+60)°$ at the points P and Q.

b) Determine the coordinates of P and Q.

$\boxed{A\left(0,\frac{1}{2}\right)}$, $\boxed{B(210,0)}$, $\boxed{C(300,1)}$, $\boxed{P\left(60,-\frac{1}{2}\right)}$, $\boxed{Q\left(180,-\frac{1}{2}\right)}$



a) LOOKING AT A SKETCH OF THE GRAPH OF $y = \cos x$

TRANSLATING BY 60° TO THE "LEFT"

$\therefore (270,0) \longmapsto B(210,0)$

$(360,1) \longmapsto C(300,1)$

& SUBSTITUTING $x=0$ INTO $y = \cos(x+60)$ GIVES $y = \frac{1}{2}$

$\therefore A(0,\frac{1}{2})$

b) SOLVING THE EQUATION $y = -\frac{1}{2}$

$\Rightarrow \cos(x+60) = -\frac{1}{2}$

$\arccos(-\frac{1}{2}) = 120°$

$\Rightarrow \begin{cases} x+60° = 120° \pm 360n \\ x+60° = 240° \pm 360n \end{cases}$ $n = 0,1,2,3,\ldots$

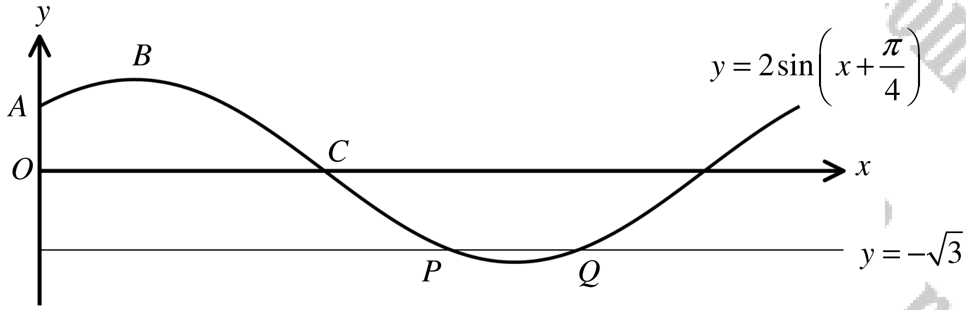
$\Rightarrow \begin{cases} x = 60° \pm 360n \\ x = 180° \pm 360n \end{cases}$

$\therefore x_1 = 60°$

$x_2 = 180°$

ie $P(60,-\frac{1}{2})$ & $Q(180,-\frac{1}{2})$

Question 14 (***+)



The figure above shows the graph of

$$y = 2\sin\left(x + \frac{\pi}{4}\right), \ 0 \le x \le 2\pi .$$

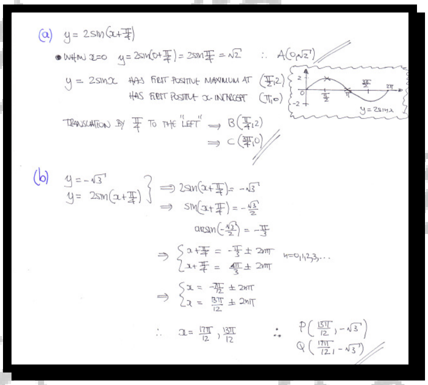
The graph meets the y axis at the point A and the point C is one of the two x intercepts of the curve. The point B is the maximum point of the curve.

a) State the coordinates of A, B and C.

The straight line with equation $y = -\sqrt{3}$ meets the graph of $y = 2\sin\left(x + \frac{\pi}{4}\right)$ at the points P and Q.

b) Determine the coordinates of P and Q.

$\boxed{A\left(0, \sqrt{2}\right)}$, $\boxed{B\left(\frac{\pi}{4}, 2\right)}$, $\boxed{C\left(\frac{3\pi}{4}, 0\right)}$, $\boxed{P\left(\frac{13\pi}{12}, -\sqrt{3}\right)}$, $\boxed{Q\left(\frac{17\pi}{12}, -\sqrt{3}\right)}$



Question 15 (***+)

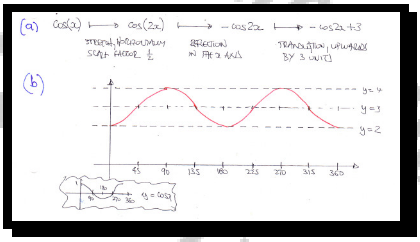
$$y = 3 - \cos 2x° \, , \; 0 \le x \le 360 \, .$$

a) Describe geometrically the three transformations that map the graph of $y = \cos x°$ onto the graph of $y = 3 - \cos 2x°$.

b) Sketch the graph of

$$y = 3 - \cos 2x° \, , \; 0 \le x \le 360 \, .$$

horizontal stretch by scale factor 2,   followed by reflection in the x axis,   followed by translation "upwards" by 3 units



Question 16 (***+)

$$y = 2\cos\left(x - \frac{\pi}{3}\right),\ 0 \le x \le 2\pi .$$

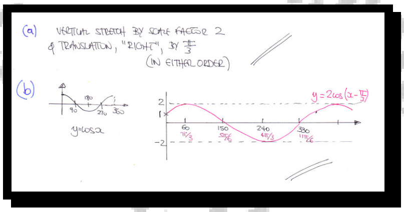
a) Describe geometrically the three transformations that map the graph of $y = \cos x$ onto the graph of $y = 2\cos\left(x - \frac{\pi}{3}\right)$.

b) Sketch the graph of

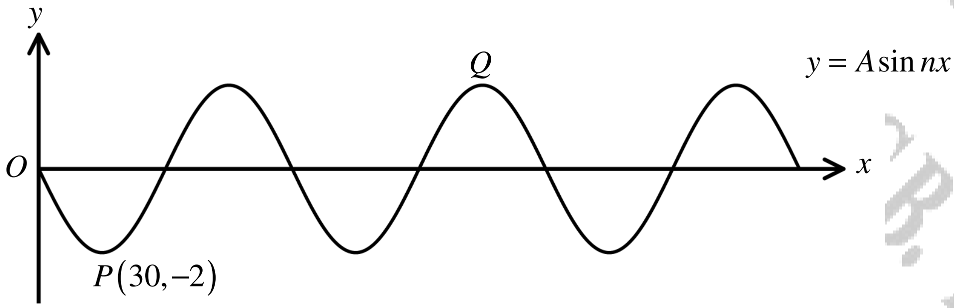
$$y = 2\cos\left(x - \frac{\pi}{3}\right),\ 0 \le x \le 2\pi .$$

vertical stretch by scale factor 2,

followed by translation "right" by $\dfrac{\pi}{3}$



Question 17 (***+)



The figure above shows part of the graph of

$$y = A\sin nx,$$

where x is measured in degrees and A and n are constants.

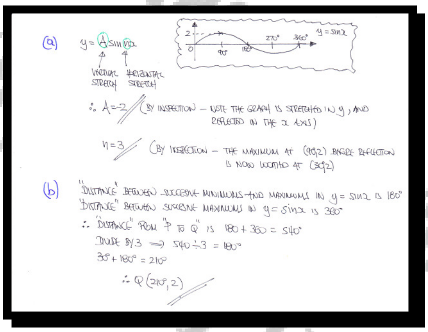
The first minimum of the curve for which $x > 0$ is the point $P(30,-2)$.

a) Find the value of A and the value of n.

The **second** maximum of the curve for which $x > 0$ is at the point Q.

b) Determine the coordinates of Q.

$\boxed{}$, $\boxed{A = -2}$, $\boxed{n = 3}$, $\boxed{Q(210,2)}$


(a) $y = A\sin nx$
VERTICAL STRETCH HORIZONTAL STRETCH
$\therefore A = -2$ (BY INSPECTION – NOTE THE GRAPH IS STRETCHED IN y, AND REFLECTED IN THE x AXIS)
$n = 3$ (BY INSPECTION – THE MAXIMUM AT $(90,2)$ BEFORE REFLECTION IS NOW LOCATED AT $(30,2)$)
(b) "DISTANCE" BETWEEN SUCCESSIVE MINIMUMS AND MAXIMUMS IN $y = \sin x$ IS $180°$
"DISTANCE" BETWEEN SUCCESSIVE MAXIMUMS IN $y = \sin x$ IS $360°$
$\therefore$ "DISTANCE" FROM "P TO Q" IS $180 + 360 = 540°$
DIVIDE BY 3 $\Longrightarrow 540 \div 3 = 180°$
$30° + 180° = 210°$
$\therefore Q(210°, 2)$

Question 18 (***+)

$$f(x) = 5\sin(3x)°, \quad 0 \le x \le 180.$$

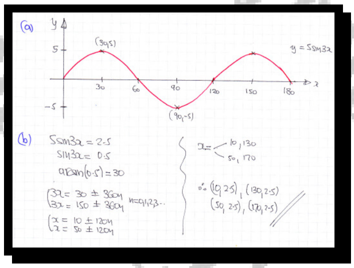
a) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes and the coordinates of any stationary points.

The line with equation $y = 2.5$ intersects the graph of $f(x)$ at four points.

b) Determine the coordinates of the points of intersections between the straight line with equation $y = 2.5$ and $f(x)$.

$\boxed{(10, 2.5)}$, $\boxed{(50, 2.5)}$, $\boxed{(130, 2.5)}$, $\boxed{(170, 2.5)}$



Question 19 **(***+)**

$$y = 1 + 2\cos x°, \ 0 \le x \le 360.$$

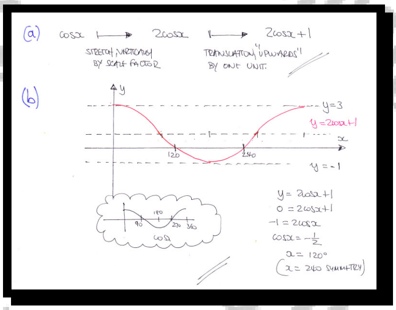
a) Describe geometrically the two transformations that map the graph of $y = \cos x°$ onto the graph of $y = 1 + 2\cos x°$.

b) Sketch the graph of

$$y = 1 + 2\cos x°, \ 0 \le x \le 360.$$

The sketch must include the coordinates of any points where the graph meets the coordinate axes.

vertical stretch by scale factor 2, followed by translation "upwards" by 1 unit



Question 20 (***+)

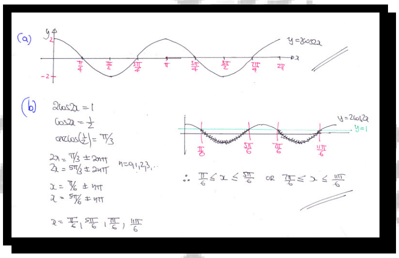
$$f(x) = 2\cos 2x, \quad 0 \le x \le 2\pi.$$

a) Sketch the graph of $f(x)$.

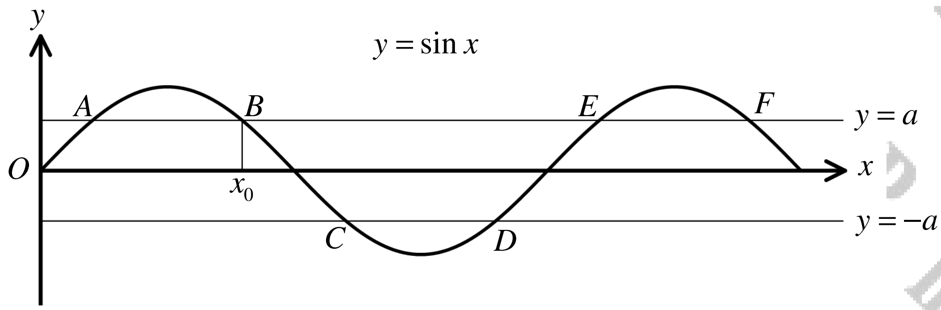
The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes.

b) Hence, or otherwise, solve the inequality $f(x) \le 1$.

$$\boxed{\frac{\pi}{6} \le x \le \frac{5\pi}{6} \text{ or } \frac{7\pi}{6} \le x \le \frac{11\pi}{6}}$$



Question 21 (*+)**



The figure above shows the graph of the curve with equation

$$y = \sin x, \; 0 \le x \le 3\pi .$$

The graph is intersected by the straight lines with equations

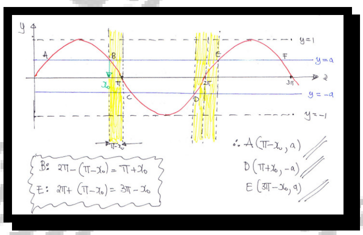
$$y = \pm a, \; 0 < a < 1 .$$

These intersections are labelled in the figure by the points A, B, C, D, E and F.

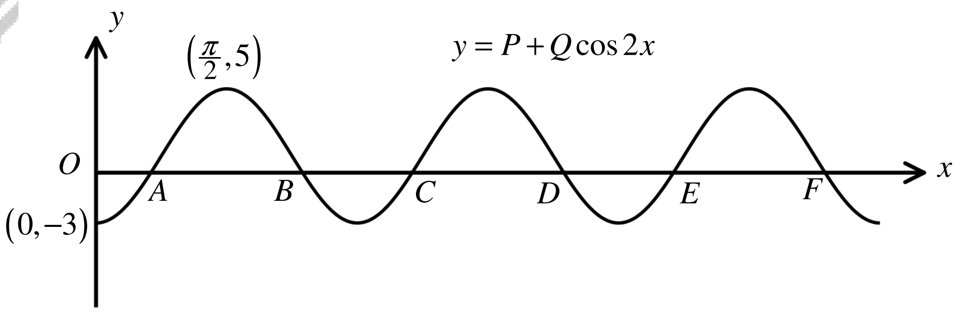
The x coordinate of the point B is x_0.

Express, in terms of x_0 and π, the x coordinates of the points A, D and E.

$\boxed{}$, $\boxed{\pi - x_0}$, $\boxed{\pi + x_0}$, $\boxed{3\pi - x_0}$



Question 22 (***+)



The figure above shows part of the graph of

$$y = P + Q\cos 2x, \ x \geq 0,$$

where P and Q are constants.

The points $(0,-3)$ and $\left(\frac{\pi}{2},5\right)$ lie on the graph of y.

a) Find the value of P and the value of Q.

The first six x intercepts of the graph are labelled A to F.

b) Determine to two decimal places the x coordinates of the six points, labelled as A to F.

$\boxed{}$, $\boxed{P=1}$, $\boxed{Q=-4}$, $\boxed{x \approx 0.66^c, 2.48^c, 3.80^c, 5.62^c, 6.94^c, 8.77^c}$

(a) $y = P + Q\cos 2x$

TRANSLATION, STRETCH VERTICALLY

$(0,-3) \Rightarrow -3 = P + Q\cos 0$
$\left(\frac{\pi}{2},5\right) \Rightarrow 5 = P + Q\cos\pi$
$\Rightarrow P + Q = -3$, $P - Q = 5$ $\Rightarrow$

$2P = 2$, $P = 1$

$2Q = -8$, $Q = -4$

(b) $y = 1 - 4\cos 2x$

$\Rightarrow 0 = 1 - 4\cos 2x$

$\Rightarrow 4\cos 2x = 1$

$\Rightarrow \cos 2x = \frac{1}{4}$

$\arccos\left(\frac{1}{4}\right) = 1.3181\ldots$

$2x = 1.3181 \pm 2n\pi$
$2x = 4.9651 \pm 2n\pi$ $\quad n = 0,1,2,3,\ldots$

$x = 0.659 \pm n\pi$
$x = 2.483 \pm n\pi$

$\therefore x = 0.66^c, 2.48^c, 3.80^c, 5.62^c, 6.94^c, 8.77^c$

Question 23 (***+)

$$y = \tan\left(\frac{1}{2}x - 45\right)°, \quad -90° \le x \le 630°.$$

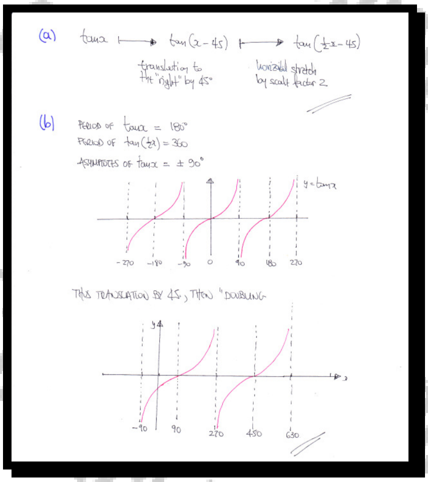
a) Describe geometrically the two transformations that map the graph of $y = \tan x°$ onto the graph of $y = \tan\left(\frac{1}{2}x - 45\right)°$.

b) Sketch the graph of

$$y = \tan\left(\frac{1}{2}x - 45\right)°, \quad -90° \le x \le 630°.$$

The sketch must include the coordinates of any points where the graph meets the coordinate axes and the equations of the vertical asymptotes of the graph.

translation "right" by 45 units, followed by horizontal stretch by scale factor 2



Question 24 (***+)

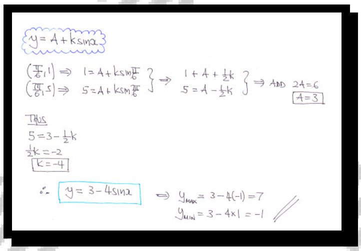
A trigonometric curve C has equation

$$y = A + k \sin x, \ 0 \le x < 2\pi,$$

where A and k are non zero constants.

Given that C passes through the points with coordinates $\left(\frac{\pi}{6}, 1\right)$ and $\left(\frac{7\pi}{6}, 5\right)$, determine the minimum and the maximum value of y.

$\boxed{y_{\text{min}} = -1}$, $\boxed{y_{\text{max}} = 7}$



$y = A + k\sin x$

$\left.\begin{array}{l}\left(\frac{\pi}{6}, 1\right) \Rightarrow 1 = A + k\sin\frac{\pi}{6} \\ \left(\frac{7\pi}{6}, 5\right) \Rightarrow 5 = A + k\sin\frac{7\pi}{6}\end{array}\right\} \Rightarrow \left.\begin{array}{l}1 + A + \frac{1}{2}k \\ 5 = A - \frac{1}{2}k\end{array}\right\} \Rightarrow$ ADD $2A = 6$ $\boxed{A = 3}$

Thus

$5 = 3 - \frac{1}{2}k$

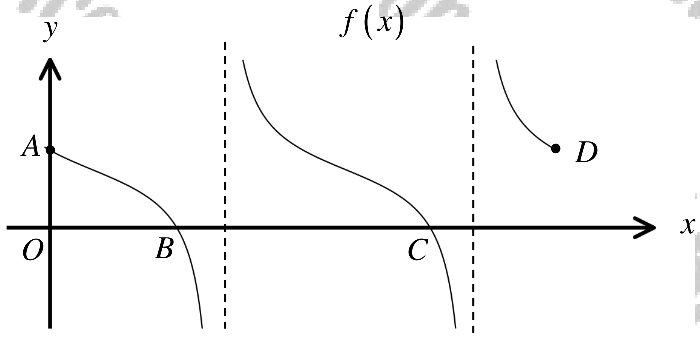
$\frac{1}{2}k = -2$

$\boxed{k = -4}$

$\therefore \boxed{y = 3 - 4\sin x} \Rightarrow y_{\text{MAX}} = 3 - 4(-1) = 7$

$y_{\text{MIN}} = 3 - 4 \times 1 = -1$

Question 25 (****)



The figure above shows the graph of the curve with equation

$$f(x)=\sqrt{3}-\tan\left(2x^\circ-\alpha^\circ\right),\quad 0\le x\le 180,\ 0<\alpha<90.$$

a) Given that the point $(52.5,-2)$ lies on the curve show that $\alpha=30$.

The curve crosses the x axis at the points B and C.

b) Determine the coordinates of B and C.

The points A and D are the endpoints of the curve.

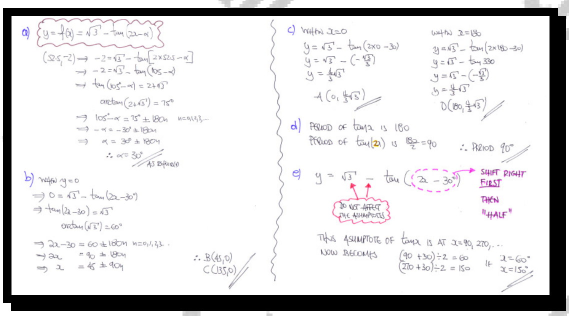
c) Find the exact coordinates of A and D.

The dotted lines represent the vertical asymptotes of the curve.

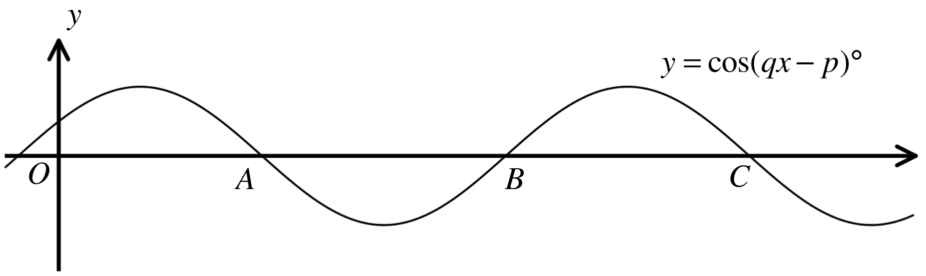
d) Write down the period of $f(x)$.

e) Determine the equations of the two vertical asymptotes of the curve.

$\boxed{}$, $\boxed{B(45,0),\ C(135,0)}$, $\boxed{A\left(0,\frac{4}{3}\sqrt{3}\right),\ D\left(180,\frac{4}{3}\sqrt{3}\right)}$, $\boxed{x=60,\ 150}$, $\boxed{\text{period}=90}$



Question 26 (****)



The figure below shows part of the graph of

$$y = \cos(qx - p)°, \ x \in \mathbb{R},$$

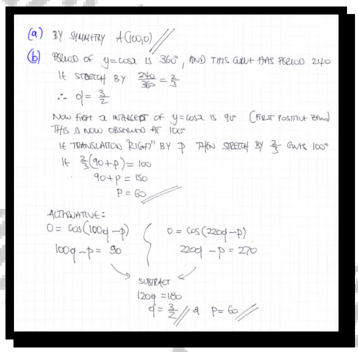
where q and p are positive constants.

The graph crosses the x axis at the points A, $B(220,0)$ and $C(340,0)$.

a) State the coordinates of A.

b) Determine the value of q and the value of p.

$\boxed{A(100,0)}$, $\boxed{q = \frac{3}{2}, \ p = 60}$


(a) BY SYMMETRY $A(100,0)$

(b) PERIOD OF $y = \cos x$ IS $360°$, AND THIS CURVE HAS PERIOD 240

∴ STRETCH BY $\frac{240}{360} = \frac{2}{3}$

$\therefore q = \frac{3}{2}$

NOW FIRST x INTERCEPT OF $y = \cos x$ IS $90°$ (FIRST POSITIVE ZERO)

THIS IS NOW OBSERVED AT $100°$

∴ TRANSLATION "RIGHT" BY p THEN STRETCH BY $\frac{2}{3}$ GIVES $100°$

∴ $\frac{2}{3}(90 + p) = 100$

$90 + p = 150$

$p = 60$

ALTERNATIVE:

$0 = \cos(100q - p)$ $\quad$ $0 = \cos(220q - p)$

$100q - p = 90$ $\quad$ $220q - p = 270$

SUBTRACT

$120q = 180$

$q = \frac{3}{2}$ & $p = 60$

Question 27 **(****)**

The graph of $f(x) = \sin x$ is subjected to a sequence of transformations consisting of

- a horizontal stretch by scale factor 2,
- followed by a vertical stretch by scale factor 2,
- followed by a translation in the positive y direction by 1 unit.

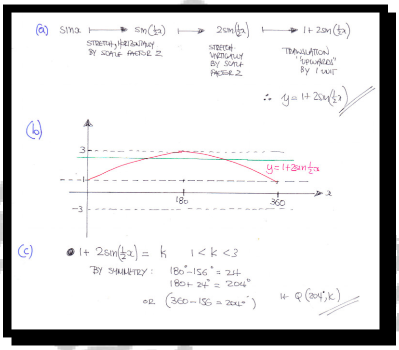
a) Write an equation of the transformed graph, in the form $y = g(x)$.

b) Sketch $y = g(x)$, for $0 \le x° \le 360$.

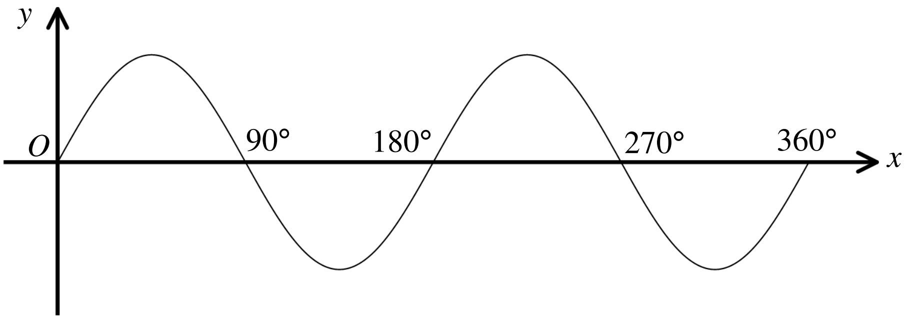
The horizontal line with equation $y = k$, $1 < k < 3$, meets $y = g(x)$ at two distinct points P and Q.

c) Given that the coordinates of P are $(24°, k)$, find the coordinates of Q.

$\boxed{y = 1 + 2\sin\left(\frac{1}{2}x\right)}$, $\boxed{Q(204°, k)}$



Question 28 (****)



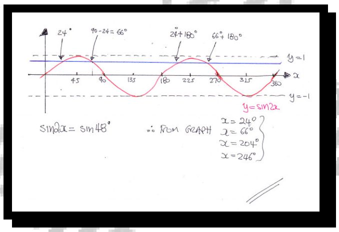
The figure above shows the graph of the curve with equation

$$y = \sin 2x,\ 0 \le x° \le 360.$$

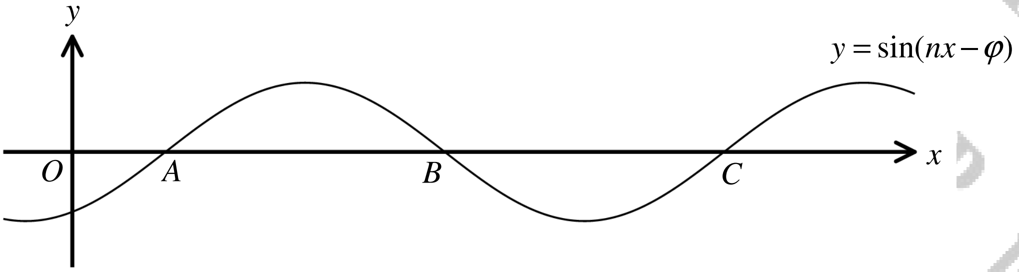
By drawing a suitable horizontal line on a copy of this graph and by fully communicating your method, solve the equation

$$\sin 2x° = \sin 48°,\ 0 \le x° \le 360.$$

$$\boxed{x = 24°,\ 66°,\ 204°,\ 246°}$$



Question 29 (****)



The figure above shows part of the graph of

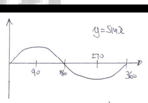
$$y = \sin(nx - \varphi),$$

where n and φ are positive constants, with $0 \le \varphi < \frac{\pi}{2}$.

The graph of $y = \sin(nx - \varphi)$ crosses the x axis at the points A, B and C with respective coordinates $\left(\frac{\pi}{9}, 0\right)$, $\left(\frac{4\pi}{9}, 0\right)$ and $\left(\frac{7\pi}{9}, 0\right)$,.

Determine the value of n and the value of φ.

$\boxed{}$, $\boxed{n = 3}$, $\boxed{\varphi = \frac{\pi}{3}}$



- $y = \sin x$ INTERCEPTS THE x AXIS EVERY 180° (π)
- THIS GRAPH INTERCEPTS EVERY $\frac{3\pi}{9} = \frac{\pi}{3} = 60$, BY LOOKING AT THE DIFFERENCES OF A, B, C
- OUR GRAPH HAS BEEN "STRETCHED" HORIZONTALLY BY SCALE FACTOR OF $\frac{1}{3}$

$\therefore n = 3$

- NEXT DEAL WITH THE TRANSLATION

TRANSLATION TAKES PLACE FIRST THEN STRETCH BY $\frac{1}{3}$

TAKE THE "ORIGIN" OF $y = \sin x$

$\frac{0 + \phi}{3} = \frac{\pi}{9}$

$\phi = \frac{\pi}{3}$

ALTERNATIVE

$y = \sin(nx - \phi)$

$n\left(\frac{\pi}{9}\right) - \phi = 0$ ← FIRST x-INTERCEPT OF $y = \sin x$

$n\left(\frac{4\pi}{9}\right) - \phi = \pi$ ← 2ND x-INTERCEPT OF $y = \sin x$

$\left.\begin{array}{l}\phi = \frac{n\pi}{9} \\ \phi = \frac{4\pi n}{9} - \pi\end{array}\right\} \Rightarrow$

$\frac{n\pi}{9} = \frac{4\pi n}{9} - \pi$

$\frac{n}{9} = \frac{4}{9}n - 1$

$1 = \frac{3}{9}n$

$n = 3$

& $\phi = \frac{n\pi}{9}$

$\phi = \frac{\pi}{3}$

Question 30 (****)

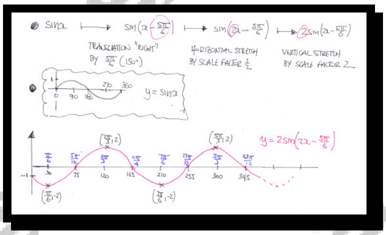
Sketch the graph of

$$y = 2\sin\left(2x - \frac{5\pi}{6}\right), \quad 0 \le x \le 2\pi .$$

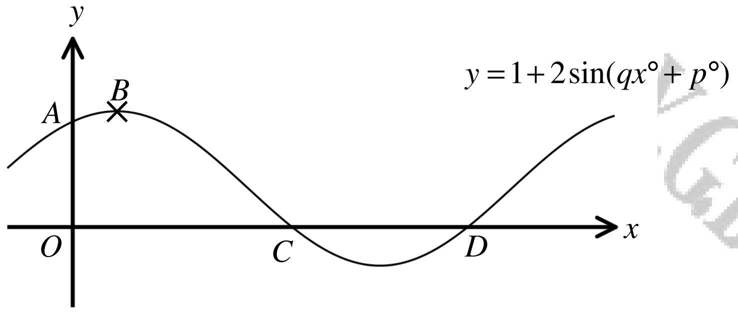
The sketch must include the exact coordinates …

- … of any stationary points.
- … of any points where the graph meets the coordinate axes.

graph



Question 31 (****)



The figure above shows part of the graph of

$$y = 1 + 2\sin(qx° + p°),\ x \in \mathbb{R},$$

where q and p are positive constants with $0° < p < 90°$, $0 < q < 5$.

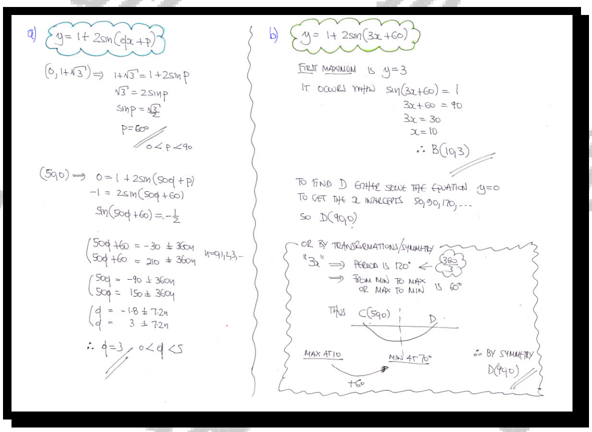
The graph crosses the y axis at the point $A\left(0, 1+\sqrt{3}\right)$, and the x axis at the points $C(50,0)$ and D.

The point B is a maximum point on the curve.

a) Determine the value of q and the value of p.

b) Find the coordinates of B and D.

$\boxed{}$, $\boxed{q = 3,\ \ p = 60}$, $\boxed{A(90,0)}$


a) $y = 1 + 2\sin(qx + p)$

$(0, 1+\sqrt{3}) \Rightarrow 1+\sqrt{3} = 1 + 2\sin p$

$\sqrt{3} = 2\sin p$

$\sin p = \frac{\sqrt{3}}{2}$

$p = 60°$ // $0 < p < 90$

$(50,0) \Longrightarrow 0 = 1 + 2\sin(50q + p)$

$-1 = 2\sin(50q + 60)$

$\sin(50q + 60) = -\frac{1}{2}$

$50q + 60 = -30 \pm 360n$, $50q + 60 = 210 \pm 360n$, $n = 0, 1, 2, 3, \ldots$

$50q = -90 \pm 360n$, $50q = 150 \pm 360n$

$q = -1.8 \pm 7.2n$, $q = 3 \pm 7.2n$

$\therefore q = 3$, $0 < q < 5$

b) $y = 1 + 2\sin(3x + 60)$

FIRST MAXIMUM IS $y = 3$

IT OCCURS WHEN $\sin(3x + 60) = 1$

$3x + 60 = 90$

$3x = 30$

$x = 10$

$\therefore B(10, 3)$

TO FIND D EITHER SOLVE THE EQUATION $y = 0$ TO GET THE x INTERCEPTS 50, 90, 170, ...

SO $D(90,0)$

OR BY TRANSFORMATIONS/SYMMETRY

"$3x$" $\Longrightarrow$ PERIOD IS $120°$ $\leftarrow \frac{360}{3}$

$\Longrightarrow$ FROM MIN TO MAX OR MAX TO MIN IS $60°$

THUS $C(50,0)$ D

MAX AT 10 $\xrightarrow{+60}$ MIN AT 70°

$\therefore$ BY SYMMETRY $D(90,0)$

Question 32 (****)

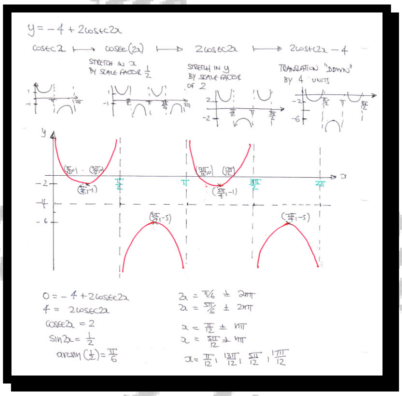
Sketch the graph of

$$y = -4 + 2\operatorname{cosec} 2x, \quad 0 \le x \le 2\pi$$

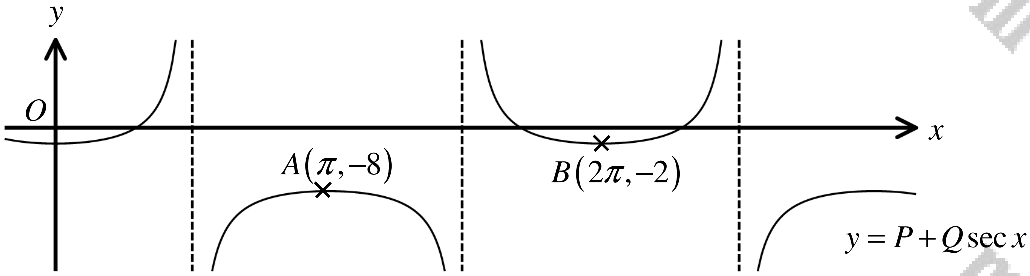
The sketch must include

- the equations of any asymptotes to the curve
- the exact coordinates of any stationary points.
- the exact coordinates of any points where the curve meets the coordinate axes.

graph



Question 33 (****)



The figure above shows part of the curve with equation

$$y = P + Q\sec x,$$

where P and Q are non zero constants.

The curve has turning points at $A(\pi,-8)$ and $B(2\pi,-2)$.

Determine the value of P and the value of Q.

$\boxed{P=-5}$, $\boxed{Q=3}$

$y = P + Q\sec x$

$\left.\begin{array}{l}(\pi,-8) \Rightarrow -8 = P + Q\sec\pi \\ (2\pi,-2) \Rightarrow -2 = P + Q\sec 2\pi\end{array}\right\} \Rightarrow \left.\begin{array}{l}-8 = P - Q \\ -2 = P + Q\end{array}\right\}$

ADD $\Rightarrow$ $2P = -10$, $P = -5$ SUBTRACT $-6 = -2Q$, $Q = 3$

ALTERNATIVE

- $\sec x$ exist between -1 & 1, ie A "GAP" OF 2
- THIS GRAPH HAS A GAP OF 6 (FROM -2 to -8), SO IT MUST HAVE BEEN STRETCHED BY FACTOR OF 3 IN THE y DIRECTION
- BUT THIS MEANS IT SHOULD HAVE A GAP BETWEEN -3 & 3 BUT IT HAS A GAP BETWEEN -8 & -2, SO IT MUST HAVE BEEN TRANSLATED BY 5 UNITS DOWN

$\therefore y = -5 + 3\sec x$

Question 34 (****)

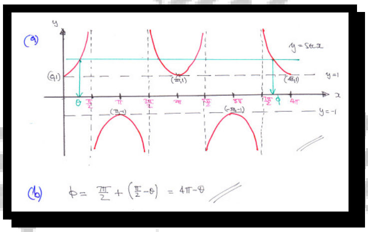
$$f(x) = \sec x, \ x \in \mathbb{R}, \ 0 \le x \le 4\pi$$

a) Sketch the graph of $f(x)$, showing clearly the coordinates of any stationary points and equations of asymptotes.

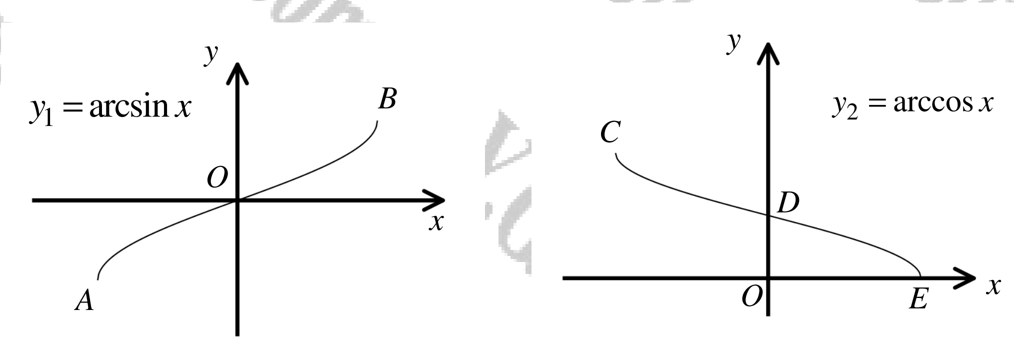
It is given that $\sec\theta = \sec\varphi$, where $0 < \theta < \frac{\pi}{2}$ and $\frac{7\pi}{2} < \varphi < 4\pi$.

b) Express φ in terms of θ.

$$\boxed{\varphi = 4\pi - \theta}$$



Question 35 (****)



The figures above show the graph of $y_1 = \arcsin x$ and the graph of $y_2 = \arccos x$.

The graph of y_1 has endpoints at A and B.

The graph of y_2 has endpoints at C and E, and D is the point where the graph of y_2 crosses the y axis.

a) State the coordinates of A, B, C, D and E.

The graph of y_2 can be obtained from the graph of y_1 by a series of two geometric transformations which can be carried out in a specific order.

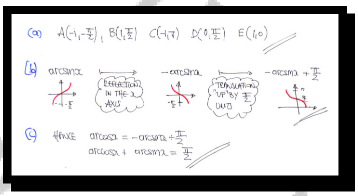
b) Describe these two geometric transformations.

c) Deduce using valid arguments that

$$\arcsin x + \arccos x = \text{constant},$$

stating the exact value of this constant.

$\boxed{A\left(-1, -\frac{\pi}{2}\right)}$, $\boxed{B\left(1, \frac{\pi}{2}\right)}$, $\boxed{C(-1, \pi)}$, $\boxed{D\left(0, \frac{\pi}{2}\right)}$, $\boxed{E(1,0)}$, $\boxed{\text{constant} = \frac{\pi}{2}}$



Question 36 (****+)

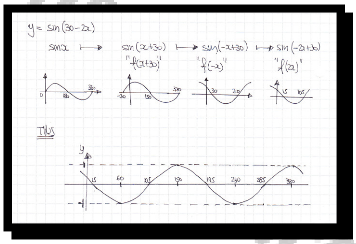
Sketch the graph of

$$y = \sin(30 - 2x)°, \quad 0 \le x \le 180.$$

The sketch must include the coordinates …

- … of any stationary points.
- … of any points where the graph meets the x axis.

$\boxed{}$, $\boxed{\text{graph}}$



Question 37 **(****+)**

$$y_1 = 2\cos 2x\,, \quad y_2 = 3\tan 2x\,, \quad 0 \le x \le 2\pi\,.$$

a) Sketch in a single set of axes the graph of y_1 and the graph of y_2.

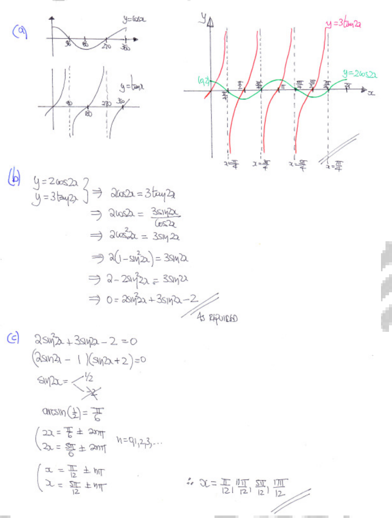
The sketch must include the coordinates of any points where the two graphs meet the coordinate axes and the equations of any asymptotes.

b) Show that the coordinates of the points of intersection between the graphs of y_1 and y_2 are solutions of the equation

$$2\sin^2 2x + 3\sin 2x - 2 = 0\,.$$

c) Hence find the x coordinates of the points of intersection between the graphs of y_1 and y_2.

$$\boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}}$$



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