

Created by T. Madas

TRIGONOMETRIC EQUATIONS GENERAL SOLUTIONS

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Question 1 ()**

Find the general solution of the trigonometric equation

$$\cos(4x - 40)^\circ = -0.5.$$

$$x = \begin{cases} (4 \pm 9n)10^\circ \\ (7 \pm 9n)10^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 2 ()**

Find the general solution of the trigonometric equation

$$\sin\left(x + \frac{\pi}{3}\right) = \sqrt{3}.$$

$$x = \begin{cases} (11 \pm 12n)\frac{\pi}{6} \\ (1 \pm 4n)\frac{\pi}{2} \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 3 ()**

Find the general solution of the trigonometric equation

$$\sqrt{2} \cos\left(2x + \frac{\pi}{6}\right) = 1.$$

$$x = \begin{cases} (1 \pm 24n) \frac{\pi}{24} \\ (19 \pm 24n) \frac{19\pi}{24} \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 4 ()**

Find the general solution of the trigonometric equation

$$\sin(4x + 10)^\circ = \sin 50^\circ.$$

$$x = \begin{cases} (1 \pm 9n) 10^\circ \\ (1 \pm 3n) 30^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 5 ()**

Find the general solution of the trigonometric equation

$$\cos(3x)^\circ = \cos 30^\circ.$$

$$x = (\pm 1 \pm 12n)10^\circ, \quad n = 0, 1, 2, 3, \dots$$

Question 6 ()**

Find the general solution of the trigonometric equation

$$\tan\left(x + \frac{\pi}{3}\right) = 1.$$

$$x = (11 \pm 12n)\frac{\pi}{12}, \quad n = 0, 1, 2, 3, \dots$$

Question 7 ()**

Find the general solution of the trigonometric equation

$$\tan\left(2x - \frac{\pi}{4}\right) = \sqrt{3}.$$

$$x = (7 \pm 12n) \frac{\pi}{24}, \quad n = 0, 1, 2, 3, \dots$$

Question 8 ()**

Find the general solution of the trigonometric equation

$$2 \sin\left(2x - \frac{\pi}{2}\right) = \sqrt{3}.$$

$$x = \begin{cases} (5 \pm 6n) \frac{\pi}{12} \\ (7 \pm 6n) \frac{\pi}{12} \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 9 ()**

Find the general solution of the trigonometric equation

$$\sqrt{2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) = 1.$$

$$x = \begin{cases} (0 \pm 4n)\pi \\ (1 \pm 4n)\pi \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 10 ()**

Find the general solution of the trigonometric equation

$$\tan\left(\frac{\pi}{2} - 3x\right) = \sqrt{3}.$$

$$x = (1 \pm 6n)\frac{\pi}{18}, \quad n = 0, 1, 2, 3, \dots$$

Question 11 ()**

Find the general solution of the trigonometric equation

$$\tan^2(30-x)^\circ = \sqrt{3}.$$

$$x = \begin{cases} (5 \pm 6n)30^\circ \\ (1 \pm 2n)90^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 12 ()**

Find in degrees the general solution of the trigonometric equation

$$2\cos\theta \tan\theta = \sqrt{3}.$$

$$\theta = \begin{cases} (1 \pm 6n)60^\circ \\ (1 \pm 3n)120^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 13 (**)

Find **in radians** the general solution of the trigonometric equation

$$\cos 3\theta = \cos 2\theta .$$

$$\theta = \pm \frac{2n\pi}{5}, \quad n = 0, 1, 2, 3, \dots$$

Question 14 (**)

Find **in radians** the general solution of the trigonometric equation

$$\sin 2x = \cos x .$$

$$x = \begin{cases} (1 \pm 4n) \frac{\pi}{2} \\ (1 \pm 4n) \frac{\pi}{6} \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 15 ()**

Find in degrees the general solution of the trigonometric equation

$$\sin 2\theta + \cos \theta = 0.$$

$$\theta = \begin{cases} (3 \pm 4n)30^\circ \\ (3 \pm 4n)90^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 16 ()**

Find in degrees the general solution of the trigonometric equation

$$\cos 2\theta = \cos(\theta + 60).$$

$$\theta = \begin{cases} (1 \pm 6n)60^\circ \\ (5 \pm 6n)20^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 17 ()**

Solve in degrees the trigonometric equation

$$\tan 4x - \tan 2x = 0, \quad 0 \leq x < 360.$$

$$x = 0^\circ, 90^\circ, 180^\circ, 270^\circ$$

Handwritten solution for Question 17:

Method 1:

$$\tan 4x - \tan 2x = 0$$

$$\tan 4x = \tan 2x$$

$$4x = 2x + 180n$$

$$2x = 180n$$

$$x = 90n$$

$$x = 0, 90, 180, 270$$

Method 2:

$$\tan 4x = \tan 2x$$

$$\frac{2 \tan 2x}{1 - \tan^2 2x} = \tan 2x$$

$$2 \tan 2x = \tan 2x (1 - \tan^2 2x)$$

$$2 = 1 - \tan^2 2x$$

$$\tan^2 2x = -1$$

$$\tan 2x = \pm i$$

$$2x = 90^\circ, 270^\circ$$

$$x = 45^\circ, 135^\circ$$

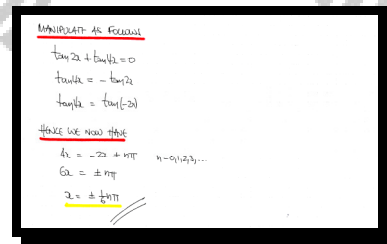
Question 18 (**)

Find the general solution of the following trigonometric equation

$$\tan 2x + \tan 4x = 0,$$

where x is measured in radians.

$$\boxed{}, \quad x = \pm \frac{1}{6}n\pi, \quad n \in \mathbb{N}$$



tan 2x + tan 4x = 0
 tan 4x = -tan 2x
 tan 4x = tan(-2x)
SINCE WE WANT THEM
 4x = -2x + n\pi \quad n = 0, 1, 2, \dots
 6x = \pm n\pi
x = \pm \frac{1}{6}n\pi

Question 19 (***)

Show that if x is measured in radians, the general solution of

$$6 \tan^2 x = 1 + 4 \sin^2 x,$$

is given by

$$x = \frac{1}{6} \pi f(n),$$

where $f(n)$ is an integer function to be found.

$$\boxed{}, \quad \boxed{f(n) = 6n \pm (-1)^n}$$

SWITCHING INTO SIN & COS

$$6 \tan^2 x = 1 + 4 \sin^2 x$$

$$\frac{6 \sin^2 x}{\cos^2 x} = 1 + 4 \sin^2 x$$

$$6 \sin^2 x = \cos^2 x (1 + 4 \sin^2 x)$$

$$6 \sin^2 x = (1 - \sin^2 x)(1 + 4 \sin^2 x)$$

$$6 \sin^2 x = 1 + 4 \sin^2 x - \sin^2 x - 4 \sin^4 x$$

$$4 \sin^2 x + 3 \sin^4 x - 1 = 0$$

PREPARE AS A QUADRATIC IN $\sin^2 x$

$$(4 \sin^2 x - 1)(\sin^2 x + 1) = 0$$

$$\sin^2 x = \frac{1}{4} \quad \text{or} \quad \sin^2 x = -1$$

$$\sin x = \pm \frac{1}{2} \quad \text{or} \quad \sin x = \pm i$$

SETTING UP THE GENERAL SOLUTION

$$\cos(2x) = \frac{1}{2} \quad \text{or} \quad \cos(2x) = -\frac{1}{2}$$

$$2x = \pi + (-1)^n \frac{\pi}{2} \quad \text{or} \quad 2x = \pi + (-1)^n \frac{\pi}{2}$$

$$2x = \frac{\pi}{3} + (-1)^n \frac{\pi}{3} \quad \text{or} \quad 2x = \frac{2\pi}{3} + (-1)^n \frac{\pi}{3}$$

$$\therefore x = \frac{\pi}{6} + (-1)^n \frac{\pi}{6} \quad \text{or} \quad x = \frac{\pi}{3} + (-1)^n \frac{\pi}{6}$$

* $f(n) = 6n \pm (-1)^n$

Question 20 (***)

Show that if x is measured in radians, the general solution of

$$\sin 2x = 1 + \cos 2x,$$

is given by

$$x = \frac{1}{8}\pi f(n),$$

where $f(n)$ is an integer function to be found.

$$\boxed{}, \quad \boxed{f(n) = 4n + 1 + (-1)^n}$$

NOTE: $\sin 2x = \cos 2x = \frac{1}{\sqrt{2}}$
 $\Rightarrow \sin 2x = 1 + \cos 2x$
 $\Rightarrow \sin 2x - \cos 2x = 1$
 $\Rightarrow \frac{1}{\sqrt{2}} \sin 2x - \frac{1}{\sqrt{2}} \cos 2x = \frac{1}{\sqrt{2}}$
 $\Rightarrow \cos \frac{\pi}{4} \sin 2x - \sin \frac{\pi}{4} \cos 2x = \frac{1}{\sqrt{2}}$ (SIN(A-B) = SIN A COS B - COS A SIN B)
 $\Rightarrow \sin(2x - \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$
 SETTING UP THE SOLUTION
 $2x - \frac{\pi}{4} = \arcsin(\frac{1}{\sqrt{2}})$ (SIN(A) = SIN B)
 $2x \sim \frac{\pi}{4} + \arcsin(\frac{1}{\sqrt{2}})$
 $2x = \frac{\pi}{4} [1 + 4n + (-1)^n]$
 $x = \frac{\pi}{8} [4n + 1 + (-1)^n]$ (1) $f(n) = 4n + 1 + (-1)^n$

Question 21 (***)

Find in radians the general solution of the trigonometric equation

$$\cos 3x + \cos x = 0.$$

$$x = \begin{cases} (1 \pm 2n)\frac{\pi}{4} \\ (1 \pm 2n)\frac{\pi}{2} \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 22 (***)

Find in degrees the general solution of the trigonometric equation

$$\sqrt{3} \sin \theta - \cos \theta = \sqrt{2}.$$

$$\theta = \begin{cases} (5 \pm 24n)15^\circ \\ (11 \pm 24n)15^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 23 (***)

Find **in radians** the general solution of the trigonometric equation

$$\sqrt{3} \sin\left(x - \frac{\pi}{6}\right) = \sin x.$$

$$x = (1 \pm 3n) \frac{\pi}{3}, \quad n = 0, 1, 2, 3, \dots$$

Question 24 (***)

Find **in degrees** the general solution of the trigonometric equation

$$\sin 2\theta - \tan \theta = 0.$$

$$\theta = \begin{cases} (1 \pm 4n) 45^\circ \\ (3 \pm 4n) 45^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 25 (*)**

Find in degrees the general solution of the trigonometric equation

$$\sin \theta + \cos \theta = \frac{1}{\sqrt{2}}.$$

$$\theta = \begin{cases} (7 \pm 24n)15^\circ \\ (13 \pm 24n)15^\circ \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 26 (*)**

Find in radians the general solution of the trigonometric equation

$$\cos 2x + 1 = \sin 2x.$$

$$x = \begin{cases} (1 \pm 4n)\frac{\pi}{4} \\ (1 \pm 2n)\frac{\pi}{2} \end{cases} \quad n = 0, 1, 2, 3, \dots$$

Question 27

Find, in radians, the general solution of the trigonometric equation

$$\sin 5x + \sin 3x = 0.$$

$$x = \pm \frac{n\pi}{4}, \quad n = 0, 1, 2, 3, \dots$$

Handwritten solution for Question 27:

$$\begin{aligned} \sin 5x + \sin 3x &= 0 \\ \sin 5x &= -\sin 3x \\ \sin 5x &= \sin(-3x) \\ 5x &= -3x + 2n\pi \\ 5x &= (\pi + 3x) + 2n\pi \quad n=0,1,2,3 \\ 5x &= 0 + 2n\pi \\ 5x &= \pi + 2n\pi \end{aligned}$$

Second solution is included in the first solution.

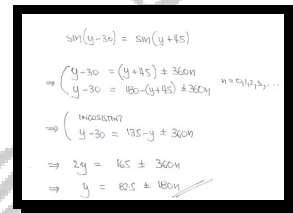
$$\therefore x = \pm \frac{n\pi}{4}$$

Question 28 (*) (non calculator)**

Find the general solution of the trigonometric equation

$$\sin(y - 30) = \sin(y - 45).$$

$$y = 82.5^\circ \pm 180^\circ n, \quad n = 0, 1, 2, 3, \dots$$



Handwritten solution for Question 28:

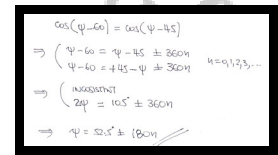
$$\begin{aligned} \sin(y - 30) &= \sin(y - 45) \\ \Rightarrow \begin{cases} y - 30 = (y - 45) \pm 360n \\ y - 30 = 180 - (y - 45) \pm 360n \end{cases} & n = 0, 1, 2, 3, \dots \\ \Rightarrow \begin{cases} \text{no solution} \\ y - 30 = 135 - y \pm 360n \end{cases} \\ \Rightarrow 2y &= 165 \pm 360n \\ \Rightarrow y &= 82.5 \pm 180n \end{aligned}$$

Question 29 (*) (non calculator)**

Find the general solution of the trigonometric equation

$$\cos(\psi - 60) = \cos(\psi - 45), \quad 0 \leq \psi < 360^\circ.$$

$$\psi = 52.5^\circ \pm 180^\circ n, \quad n = 0, 1, 2, 3, \dots$$



Handwritten solution for Question 29:

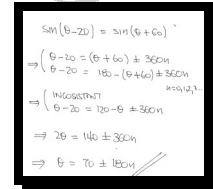
$$\begin{aligned} \cos(\psi - 60) &= \cos(\psi - 45) \\ \Rightarrow \begin{cases} \psi - 60 = \psi - 45 \pm 360n \\ \psi - 60 = 360 - (\psi - 45) \pm 360n \end{cases} & n = 0, 1, 2, 3, \dots \\ \Rightarrow \begin{cases} \text{no solution} \\ 2\psi = 105 \pm 360n \end{cases} \\ \Rightarrow \psi &= 52.5^\circ \pm 180n \end{aligned}$$

Question 30 (*) (non calculator)**

Find the general solution of the trigonometric equation

$$\sin(\theta - 20) = \sin(\theta + 60), \quad 0 \leq \theta < 360^\circ.$$

$$\theta = 70^\circ \pm 180^\circ n, \quad n = 0, 1, 2, 3, \dots$$



Handwritten solution for Question 30:

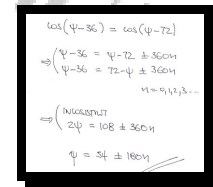
$$\begin{aligned} \sin(\theta - 20) &= \sin(\theta + 60) \\ \Rightarrow \theta - 20 &= (\theta + 60) \pm 360n \\ \theta - 20 &= 180 - (\theta + 60) \pm 360n \\ \Rightarrow \theta - 20 &= 120 - \theta \pm 360n \\ \Rightarrow 2\theta &= 140 \pm 360n \\ \Rightarrow \theta &= 70 \pm 180n \end{aligned}$$

Question 31 (*) (non calculator)**

Find the general solution of the trigonometric equation

$$\cos(\psi - 36) = \cos(\psi - 72), \quad 0 \leq \psi < 360^\circ.$$

$$\psi = 54^\circ \pm 180^\circ n, \quad n = 0, 1, 2, 3, \dots$$



Handwritten solution for Question 31:

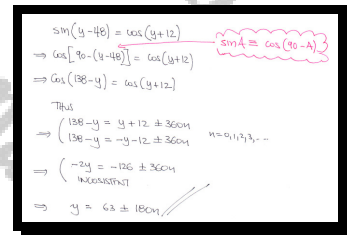
$$\begin{aligned} \cos(\psi - 36) &= \cos(\psi - 72) \\ \Rightarrow \psi - 36 &= \psi - 72 \pm 360n \\ \psi - 36 &= 72 - \psi \pm 360n \\ \Rightarrow 2\psi &= 108 \pm 360n \\ \psi &= 54 \pm 180n \end{aligned}$$

Question 32 (***) (non calculator)

Find the general solution of the trigonometric equation

$$\sin(y - 48) = \cos(y + 12), \quad 0 \leq y < 360^\circ.$$

$$y = 63^\circ \pm 180^\circ n, \quad n = 0, 1, 2, 3, \dots$$



Handwritten solution for Question 32:

$$\begin{aligned} \sin(y - 48) &= \cos(y + 12) \\ \Rightarrow \cos[90 - (y - 48)] &= \cos(y + 12) \\ \Rightarrow \cos(138 - y) &= \cos(y + 12) \end{aligned}$$

Thus

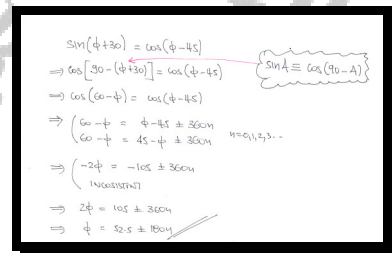
$$\begin{aligned} 138 - y &= y + 12 \pm 360n \\ 138 - y &= -y - 12 \pm 360n \quad n = 0, 1, 2, 3, \dots \\ \Rightarrow -2y &= -126 \pm 360n \\ \Rightarrow y &= 63 \pm 180n \end{aligned}$$

Question 33 (***) (non calculator)

Find the general solution of the trigonometric equation

$$\sin(\phi + 30) = \cos(\phi - 45), \quad 0 \leq \phi < 360^\circ.$$

$$\phi = 52.5^\circ \pm 180^\circ n, \quad n = 0, 1, 2, 3, \dots$$



Handwritten solution for Question 33:

$$\begin{aligned} \sin(\phi + 30) &= \cos(\phi - 45) \\ \Rightarrow \cos[90 - (\phi + 30)] &= \cos(\phi - 45) \\ \Rightarrow \cos(60 - \phi) &= \cos(\phi - 45) \end{aligned}$$

Thus

$$\begin{aligned} 60 - \phi &= \phi - 45 \pm 360n \\ 60 - \phi &= 45 - \phi \pm 360n \quad n = 0, 1, 2, 3, \dots \\ \Rightarrow -2\phi &= -105 \pm 360n \\ \Rightarrow 2\phi &= 105 \pm 360n \\ \Rightarrow \phi &= 52.5 \pm 180n \end{aligned}$$