INTEGRATION

BY TRIGONOMETRIC IDENTITIES

Question 1

Carry out the following integrations:

1. $\int 3\sin^2 x \, dx = \frac{3}{2}x - \frac{3}{4}\sin 2x + C$

2. $\int 4\cos^2 x \, dx = 2x + \sin 2x + C$

3. $\int 3\sin x \cos x \, dx = -\frac{3}{4}\cos 2x + C$

4. $\int (2 - 3\sin x)^2 \, dx = \frac{17}{2}x + 12\cos x - \frac{9}{4}\sin 2x + C$

5. $\int (1 - \cos 2x)^2 \, dx = \frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x + C$

6. $\int 2\tan^2 x \, dx = 2\tan x - 2x + C$

7. $\int 5\cot^2 x \, dx = -5\cot x - 5x + C$

8. $\int (2\tan x - \cot x)^2 \, dx = 4\tan x - \cot x - 9x + C$

9. $\int \frac{4\sin x}{\cos^2 x} \, dx = 4\sec x + C$

10. $\int \frac{\cos x}{3\sin^2 x} \, dx = -\frac{1}{3}\operatorname{cosec} x + C$

1. $\int 3\sin^2 x \, dx = \int 3\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx = \int \frac{3}{2} - \frac{3}{2}\cos 2x \, dx$
$= \frac{3}{2}x - \frac{3}{4}\sin 2x + C$

2. $\int 4\cos^2 x \, dx = \int 4\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx = \int 2 + 2\cos 2x \, dx$
$= 2x + \sin 2x + C$

3. $\int 3\sin x\cos x \, dx = \int \frac{3}{2}(2\sin x\cos x) \, dx = \int \frac{3}{2}\sin 2x \, dx = -\frac{3}{4}\cos 2x + C$

4. $\int (2 - 3\sin x)^2 \, dx = \int 4 - 12\sin x + 9\sin^2 x \, dx = \int 4 - 12\sin x + 9\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$
$= \int 4 - 12\sin x + \frac{9}{2} - \frac{9}{2}\cos 2x \, dx = \int \frac{17}{2} - 12\sin x + \frac{9}{2}\cos 2x \, dx$
$= \frac{17}{2}x + 12\cos x - \frac{9}{4}\sin 2x + C$

5. $\int (1 - \cos 2x)^2 \, dx = \int 1 - 2\cos 2x + \cos^2 2x \, dx$
$= \int 1 - 2\cos 2x + \left(\frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$
$= \int 1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x \, dx$
$= \int \frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x \, dx$
$= \frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x + C$

6. $\int 2\tan^2 x \, dx = \int 2(\sec^2 x - 1) \, dx = \int 2\sec^2 x - 2 \, dx = 2\tan x - x + C$

7. $\int 5\cot^2 x \, dx = \int 5(\operatorname{cosec}^2 x - 1) \, dx = \int 5\operatorname{cosec}^2 x - 5 \, dx = -5\cot x - 5x + C$

8. $\int (2\tan x - \cot x)^2 \, dx = \int 4\tan^2 x - 4\tan x\cot x + \cot^2 x \, dx$
$= \int 4(\sec^2 x - 1) - 4\tan x\left(\frac{1}{\tan x}\right) + (\operatorname{cosec}^2 x - 1) \, dx$
$= \int 4\sec^2 x - 4 - 4 + \operatorname{cosec}^2 x - 1 \, dx$
$= \int 4\sec^2 x + \operatorname{cosec}^2 x - 9 \, dx$
$= 4\tan x - \cot x - 9x + C$

9. $\int \frac{4\sin x}{\cos^2 x} \, dx = \int \frac{4\sin x}{\cos x} \times \frac{1}{\cos x} \, dx = \int 4\tan x\sec x \, dx = 4\sec x + C$

10. $\int \frac{\cos x}{3\sin^2 x} \, dx = \int \frac{1}{3} \times \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \, dx = \int \frac{1}{3}\cot x\operatorname{cosec} x \, dx = -\frac{1}{3}\operatorname{cosec} x + C$

Question 2

Carry out the following integrations:

1. $\int (2+\sin x)^2 \, dx = \frac{9}{2}x - 4\cos x - \frac{1}{4}\sin 2x + C$

2. $\int \sin x\left(1+\sec^2 x\right) \, dx = \sec x - \cos x + C$

3. $\int (1-2\cos x)^2 \, dx = 3x - 4\sin x + \sin 2x + C$

4. $\int \frac{1}{\cos^2 x \tan^2 x} \, dx = -\cot x + C$

5. $\int 2+2\tan^2 x \, dx = 2\tan x + C$

6. $\int \frac{1+\cos x}{\sin^2 x} \, dx = -\cot x - \operatorname{cosec} x + C$

7. $\int \frac{(1+\cos x)^2}{\sin^2 x} \, dx = -2\cot x - x - 2\operatorname{cosec} x + C$

8. $\int 4\cos^2 x \, dx = 2x + \sin 2x + C$

9. $\int 3\cot^2 x \, dx = -3\cot x - 3x + C$

10. $\int (2\cos x - 3\sin x)^2 \, dx = \frac{13}{2}x - \frac{5}{4}\sin 2x + 3\cos 2x + C$

1. $\int (2+\sin x)^2\,dx = \int 4 + 4\sin x + \sin^2 x\,dx =$

$= \int 4 + 4\sin x + \left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2x\right)dx = \int \tfrac{9}{2} + 4\sin x - \tfrac{1}{2}\cos 2x\,dx$

$= \tfrac{9}{2}x - 4\cos x - \tfrac{1}{4}\sin 2x + C$

2. $\int \sin x\,(1+\sec^2 x)\,dx = \int \sin x + \sin x\sec^2 x\,dx = \int \sin x + \frac{\sin x}{\cos^2 x}\,dx$

$\int \sin x + \frac{\sin x}{\cos x}\times\frac{1}{\cos x}\,dx = \int \sin x + \tan x\sec x\,dx = -\cos x + \sec x + C$

3. $\int (1-2\cos x)^2\,dx = \int 1 - 4\cos x + 4\cos^2 x\,dx = \int 1 - 4\cos x + 4\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right)dx$

$= \int 1 - 4\cos x + 2 + 2\cos 2x\,dx = \int 3 - 4\cos x + 2\cos 2x\,dx$

$= 3x - 4\sin x + \sin 2x + C$

4. $\int \frac{1}{\cos^2 x\tan^2 x}\,dx = \int \frac{1}{\cos^2 x \times \frac{\sin^2 x}{\cos^2 x}}\,dx = \int \operatorname{cosec}^2 x\,dx = -\cot x + C$

5. $\int 2 + 2\tan^2 x\,dx = \int 2 + 2(\sec^2 x - 1)\,dx = \int 2\sec^2 x\,dx = 2\tan x + C$

6. $\int \frac{1+\cos x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x} + \frac{\cos x}{\sin^2 x}\,dx = \int \operatorname{cosec}^2 x + \frac{\cos x}{\sin x}\times\frac{1}{\sin x}\,dx$

$= \int \operatorname{cosec}^2 x + \cot x\operatorname{cosec} x\,dx = -\cot x - \operatorname{cosec} x + C$

7. $\int \frac{(1+\cos x)^2}{\sin^2 x}\,dx = \int \frac{1 + 2\cos x + \cos^2 x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x} + \frac{2\cos x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}\,dx$

$= \int \operatorname{cosec}^2 x + \frac{2\cos x}{\sin x}\times\frac{1}{\sin x} + \cot^2 x\,dx = \int \operatorname{cosec}^2 x + 2\cot x\operatorname{cosec} x + (\operatorname{cosec}^2 x - 1)\,dx$

$= \int 2\operatorname{cosec}^2 x + 2\cot x\operatorname{cosec} x - 1\,dx = -2\cot - 2\operatorname{cosec} x - x + C$

8. $\int 4\cos^2 x\,dx = \int 4\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right)dx = \int 2 + 2\cos 2x\,dx = 2x + \sin 2x + C$

9. $\int 3\cot^2 x\,dx = \int 3(\operatorname{cosec}^2 x - 1)\,dx = \int 3\operatorname{cosec}^2 x - 3\,dx$

$= -3\cot x - 3x + C$

10. $\int (2\cos x - 3\sin x)^2\,dx = \int 4\cos^2 x - 12\cos x\sin x + 9\sin^2 x\,dx$

$= \int 4\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right) - 6\sin 2x + 9\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2x\right)dx$

$= \int 2 + 2\cos 2x - 6\sin 2x + \tfrac{9}{2} - \tfrac{9}{2}\cos 2x\,dx = \int \tfrac{13}{2} - \tfrac{5}{2}\cos 2x - 6\sin 2x\,dx$

$= \tfrac{13}{2}x - \tfrac{5}{4}\sin 2x + 3\cos 2x + C$

Question 3

Carry out the following integrations:

1. $\int \sin 2x \operatorname{cosec} x \, dx = 2\sin x + C$

2. $\int \frac{1+\sin x}{\cos^2 x} \, dx = \sec x + \tan x + C$

3. $\int \tan^2 x \, dx = \tan x - x + C$

4. $\int \frac{(1+\sin x)^2}{\cos^2 x} \, dx = 2\tan x + 2\sec x - x + C$

5. $\int \frac{\cos^2 x}{1+\sin x} \, dx = x + \cos x + C$

6. $\int \frac{1}{1+\cos x} \, dx = \operatorname{cosec} x - \cot x + C$

7. $\int \frac{(1+2\cos x)^2}{3\sin^2 x} \, dx = -\frac{5}{3}\cot x - \frac{4}{3}\operatorname{cosec} x - \frac{4}{3}x + C$

8. $\int \sin x \sin 3x \, dx = \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$

9. $\int \sin^2 2x \, dx = \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

10. $\int 2\cos 3x \sin x \, dx = \frac{1}{2}\cos 2x - \frac{1}{4}\cos 4x + C$

1. $\int \sin 2x \operatorname{cosec} x \, dx = \int 2\sin x \cos x \times \frac{1}{\sin x} \, dx = \int 2\cos x \, dx =$

$= 2\sin x + C$

2. $\int \frac{1+\sin x}{\cos^2 x} \, dx = \int \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \, dx = \int \sec^2 x + \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \, dx$

$= \int \sec^2 x + \tan x \sec x \, dx = \tan x + \sec x + C$

3. $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx = \tan x - x + C$

4. $\int \frac{(1+\sin x)^2}{\cos^2 x} \, dx = \int \frac{1 + 2\sin x + \sin^2 x}{\cos^2 x} \, dx = \int \frac{1}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \, dx$

$= \int \sec^2 x + \frac{2\sin x}{\cos x} \times \frac{1}{\cos x} + \tan^2 x \, dx$

$= \int \sec^2 x + 2\tan x \sec x + (\sec^2 x - 1) \, dx$

$= \int 2\sec^2 x + 2\tan x \sec x - 1 \, dx$

$= 2\tan x + 2\sec x - x + C$

5. $\int \frac{\cos^2 x}{1+\sin x} \, dx = \int \frac{1-\sin^2 x}{1+\sin x} \, dx = \int \frac{(1-\sin x)(1+\sin x)}{1+\sin x} \, dx$

$= \int 1 - \sin x \, dx = x + \cos x + C$

6. $\int \frac{1}{1+\cos x} \, dx = \int \frac{(1-\cos x)}{(1+\cos x)(1-\cos x)} \, dx = \int \frac{1-\cos x}{1-\cos^2 x} \, dx$

$= \int \frac{1-\cos x}{\sin^2 x} \, dx = \int \frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x} \, dx$

$= \int \operatorname{cosec}^2 x - \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \, dx = \int \operatorname{cosec}^2 x - \cot x \operatorname{cosec} x \, dx$

$= -\cot x + \operatorname{cosec} x + C$

7. $\int \frac{(1+2\cos x)^2}{3\sin^2 x} \, dx = \int \frac{1 + 4\cos x + 4\cos^2 x}{3\sin^2 x} \, dx = \int \frac{1}{3\sin^2 x} + \frac{4\cos x}{3\sin^2 x} + \frac{4\cos^2 x}{3\sin^2 x} \, dx$

$= \int \frac{1}{3}\operatorname{cosec}^2 x + \frac{4\cos x}{3\sin x} \times \frac{1}{\sin x} + \frac{4}{3}\cot^2 x \, dx$

$= \int \frac{1}{3}\operatorname{cosec}^2 x + \frac{4}{3}\cot x \operatorname{cosec} x + \frac{4}{3}(\operatorname{cosec}^2 x - 1) \, dx$

$= \int \frac{5}{3}\operatorname{cosec}^2 x + \frac{4}{3}\cot x \operatorname{cosec} x - \frac{4}{3} \, dx$

$= -\frac{5}{3}\cot x - \frac{4}{3}\operatorname{cosec} x - \frac{4}{3}x + C$

8.

$\cos(3x+x) = \cos 3x \cos x - \sin 3x \sin x$

$\cos(3x-x) = \cos 3x \cos x + \sin 3x \sin x$ ← SUBTRACT "UPWARDS"

$\cos 2x - \cos 4x = 2\sin 3x \sin x$

$\sin 3x \sin x = \frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x$

$\therefore \int \sin x \sin 3x \, dx = \int \frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x \, dx = \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$

9. $\int \sin^2 2x \, dx = \int \frac{1}{2} - \frac{1}{2}\cos 4x \, dx = \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

10.

$\sin(3x+x) = \sin 3x \cos x + \cos 3x \sin x$

$\sin(3x-x) = \sin 3x \cos x - \cos 3x \sin x$

$\sin 4x - \sin 2x = 2\cos 3x \sin x$

If $2\cos 3x \sin x = \sin 4x - \sin 2x$

$\therefore \int 2\cos 3x \sin x \, dx = \int \sin 4x - \sin 2x \, dx$

$= -\frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x + C$

Question 4

Carry out the following integrations:

1. $\displaystyle\int \frac{\cos 2x}{1-\cos^2 2x}\, dx = -\frac{1}{2}\operatorname{cosec} 2x + C$

2. $\displaystyle\int \cot^2 3x \, dx = -x - \frac{1}{3}\cot 3x + C$

3. $\displaystyle\int \sin 2x \sec x \, dx = -2\cos x + C$

4. $\displaystyle\int \frac{1}{\sin x \cos^2 x}\, dx = \ln\left|\tan\left(\frac{x}{2}\right)\right| + \sec x + C$

5. $\displaystyle\int \frac{1}{\sec x - 1}\, dx = -x - \cot x - \operatorname{cosec} x + C$

6. $\displaystyle\int 1 - \cot^2 x \, dx = 2x + \cot x + C$

7. $\displaystyle\int (2\cos x - 3)^2 \, dx = 11x + \sin 2x - 12\sin x + C$

8. $\displaystyle\int (3\sin x - \cos x)^2 \, dx = 5x - 2\sin 2x + \frac{3}{2}\cos 2x + C = 5x - 2\sin 2x - 3\sin^2 x + C$

9. $\displaystyle\int \frac{1}{\cos x \sin^2 x}\, dx = \ln|\sec x + \tan x| - \operatorname{cosec} x + C$

10. $\displaystyle\int \sin^2 x \sec^2 x \, dx = \tan x - x + C$

1. $\int \frac{\cos 2x}{1-\cos^2 2x}\,dx = \int \frac{\cos 2x}{\sin^2 2x}\,dx = \int \frac{\cos 2x}{\sin 2x} \times \frac{1}{\sin 2x}\,dx$
$= \int \cot 2x \operatorname{cosec} 2x \,dx = -\frac{1}{2}\operatorname{cosec} 2x + C$

2. $\int \cot^2 3x \,dx = \int \operatorname{cosec}^2 3x - 1 \,dx = -\frac{1}{3}\cot 3x - x + C$

3. $\int \sin 2x \sec x \,dx = \int (2\sin x \cos x)\frac{1}{\cos x}\,dx = \int 2\sin x \,dx = -2\cos x + C$

4. $\int \frac{1}{\sin x \cos^2 x}\,dx = \int \frac{\sec^2 x}{\sin x}\,dx = \int \frac{1+\tan^2 x}{\sin x}\,dx = \int \frac{1}{\sin x} + \frac{\tan^2 x}{\sin x}\,dx$
$= \int \operatorname{cosec} x + \frac{\sin^2 x}{\cos^2 x} \times \frac{1}{\sin x}\,dx = \int \operatorname{cosec} x + \frac{\sin x}{\cos^2 x}\,dx$
$= \int \operatorname{cosec} x + \frac{\sin x}{\cos x} \times \frac{1}{\cos x}\,dx = \int \operatorname{cosec} x + \tan x \sec x \,dx$
$= \ln\left|\tan \frac{x}{2}\right| + \sec x + C$

5. $\int \frac{1}{\sec x - 1}\,dx = \int \frac{\sec x + 1}{(\sec x - 1)(\sec x + 1)}\,dx = \int \frac{\sec x + 1}{\sec^2 x - 1}\,dx$
$= \int \frac{\sec x + 1}{\tan^2 x}\,dx = \int \frac{\sec x}{\tan^2 x} + \frac{1}{\tan^2 x}\,dx$
$= \int \frac{1}{\cos x}\cot^2 x + \cot^2 x \,dx = \int \frac{1}{\cos x}\frac{\cos^2 x}{\sin^2 x} + (\operatorname{cosec}^2 x - 1)\,dx$
$= \int \frac{\cos x}{\sin^2 x} + \operatorname{cosec}^2 x - 1 \,dx = \int \frac{\cos x}{\sin x} \times \frac{1}{\sin x} + \operatorname{cosec}^2 x - 1 \,dx$
$= \int \cot x \operatorname{cosec} x + \operatorname{cosec}^2 x - 1 \,dx = -\operatorname{cosec} x - \cot x - x + C$

6. $\int 1 - \cot^2 x \,dx = \int 1 - (\operatorname{cosec}^2 x - 1)\,dx = \int 2 - \operatorname{cosec}^2 x \,dx$
$= 2x + \cot x + C$

7. $\int (2\cos x - 3)^2\,dx = \int 4\cos^2 x - 12\cos x + 9 \,dx$
$= \int 4\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) - 12\cos x + 9 \,dx = \int 11 + 2\cos 2x - 12\cos x \,dx$
$= 11x + \sin 2x - 12\sin x + C$

8. $\int (3\sin x - \cos x)^2\,dx = \int 9\sin^2 x - 6\sin x\cos x + \cos^2 x \,dx$
$= \int 9\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) - 3(2\sin x\cos x) + \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx$
$= \int 5 - 4\cos 2x - 3\sin 2x \,dx$
$= 5x - 2\sin 2x + \frac{3}{2}\cos 2x + C$

9. $\int \frac{1}{\cos x \sin^2 x}\,dx = \int \frac{\operatorname{cosec}^2 x}{\cos x}\,dx = \int \frac{1+\cot^2 x}{\cos x}\,dx = \int \frac{1}{\cos x} + \frac{\cot^2 x}{\cos x}\,dx$
$= \int \sec x + \frac{\cos^2 x}{\sin^2 x} \times \frac{1}{\cos x}\,dx = \int \sec x + \frac{\cos x}{\sin^2 x}\,dx$
$= \int \sec x + \frac{\cos x}{\sin x} \times \frac{1}{\sin x}\,dx = \int \sec x + \cot x \operatorname{cosec} x \,dx$
$= \ln|\sec x + \tan x| - \operatorname{cosec} x + C$

10. $\int \sin^2 x \sec^2 x \,dx = \int \sin^2 x \times \frac{1}{\cos^2 x}\,dx = \int \tan^2 x \,dx$
$= \int \sec^2 x - 1 \,dx = \tan x - x + C$

Question 5

Carry out the following integrations:

1. $\displaystyle\int \sin 3x \cos 2x \, dx = -\frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C$

2. $\displaystyle\int \frac{1}{\sin x \cos x} \, dx = -\frac{1}{2}\ln\left|\text{cosec}\, 2x + \cot 2x\right| + C = \ln\left|\tan x\right| + C$

3. $\displaystyle\int \frac{1}{1-\sin x} \, dx = \sec x + \tan x + C$

4. $\displaystyle\int \sin^2 2x \, dx = \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

5. $\displaystyle\int \frac{\cos 2x}{\cos^2 x} \, dx = 2x - \tan x + C$

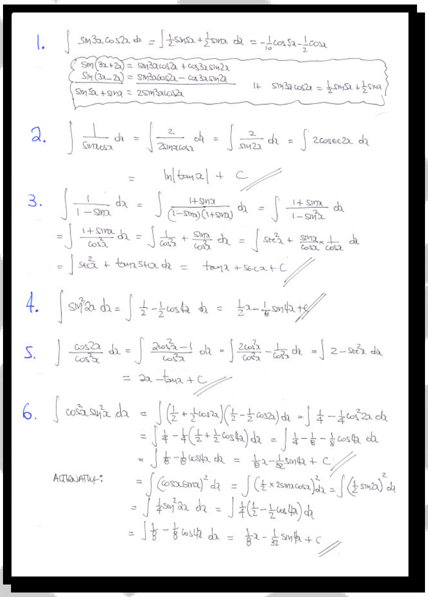
6. $\displaystyle\int \cos^2 x \sin^2 x \, dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

7. $\displaystyle\int (\sin x + 2\cos x)^2 \, dx = \frac{5}{2}x + 2\sin^2 x + \frac{3}{4}\sin 2x + C$

8. $\displaystyle\int \frac{1}{\sin^2 x \cos^2 x} \, dx = -2\cot 2x + C$

9. $\displaystyle\int \sqrt{\sin^2 x + (\cos x - 1)^2} \, dx = -4\cos\left(\frac{x}{2}\right) + C$

10. $\displaystyle\int \frac{1-\cos x}{1+\cos x} \, dx = 2\tan\left(\frac{x}{2}\right) - x + C = -2\cot x - x + 2\,\text{cosec}\, x + C$



1. $\int \sin 3x \cos 2x \, dx = \int \tfrac{1}{2}\sin 5x + \tfrac{1}{2}\sin x \, dx = -\tfrac{1}{10}\cos 5x - \tfrac{1}{2}\cos x$

$\sin(3x+2x) = \sin 3x \cos 2x + \cos 3x \sin 2x$
$\sin(3x-2x) = \sin 3x \cos 2x - \cos 3x \sin 2x$
$\sin 5x + \sin x = 2\sin 3x \cos 2x$ ∴ $\sin 3x \cos 2x = \tfrac{1}{2}\sin 5x + \tfrac{1}{2}\sin x$

2. $\int \frac{1}{\sin x \cos x} \, dx = \int \frac{2}{2\sin x \cos x} \, dx = \int \frac{2}{\sin 2x} \, dx = \int 2\operatorname{cosec} 2x \, dx$

$= \ln|\tan x| + C$

3. $\int \frac{1}{1-\sin x} \, dx = \int \frac{1+\sin x}{(1-\sin x)(1+\sin x)} \, dx = \int \frac{1+\sin x}{1-\sin^2 x} \, dx$

$= \int \frac{1+\sin x}{\cos^2 x} \, dx = \int \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \, dx = \int \sec^2 x + \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \, dx$

$= \int \sec^2 x + \tan x \sec x \, dx = \tan x + \sec x + C$

4. $\int \sin^2 2x \, dx = \int \tfrac{1}{2} - \tfrac{1}{2}\cos 4x \, dx = \tfrac{1}{2}x - \tfrac{1}{8}\sin 4x + C$

5. $\int \frac{\cos 2x}{\cos^2 x} \, dx = \int \frac{2\cos^2 x - 1}{\cos^2 x} \, dx = \int \frac{2\cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} \, dx = \int 2 - \sec^2 x \, dx$

$= 2x - \tan x + C$

6. $\int \cos^2 x \sin^2 x \, dx = \int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right)\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2x\right) dx = \int \tfrac{1}{4} - \tfrac{1}{4}\cos^2 2x \, dx$

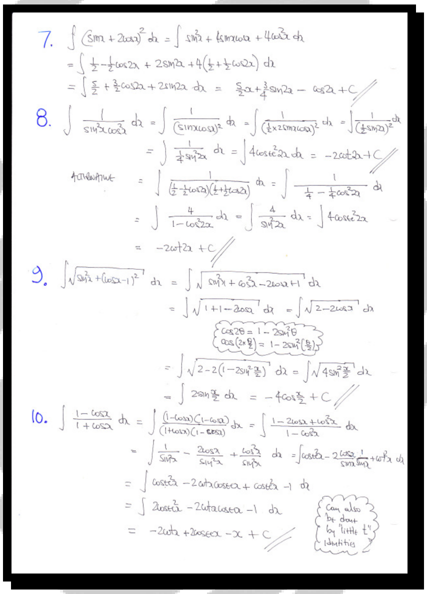
$= \int \tfrac{1}{4} - \tfrac{1}{4}\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 4x\right) dx = \int \tfrac{1}{4} - \tfrac{1}{8} - \tfrac{1}{8}\cos 4x \, dx$

$= \int \tfrac{1}{8} - \tfrac{1}{8}\cos 4x \, dx = \tfrac{1}{8}x - \tfrac{1}{32}\sin 4x + C$

ALTERNATIVE: $= \int (\cos x \sin x)^2 \, dx = \int \left(\tfrac{1}{2} \times 2\sin x \cos x\right)^2 dx = \int \left(\tfrac{1}{2}\sin 2x\right)^2 dx$

$= \int \tfrac{1}{4}\sin^2 2x \, dx = \int \tfrac{1}{4}\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 4x\right) dx$

$= \int \tfrac{1}{8} - \tfrac{1}{8}\cos 4x \, dx = \tfrac{1}{8}x - \tfrac{1}{32}\sin 4x + C$



7. $\int (\sin x + 2\cos x)^2 \, dx = \int \sin^2 x + 4\sin x \cos x + 4\cos^2 x \, dx$

$= \int \tfrac{1}{2} - \tfrac{1}{2}\cos 2x + 2\sin 2x + 4\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right) dx$

$= \int \tfrac{5}{2} + \tfrac{3}{2}\cos 2x + 2\sin 2x \, dx = \tfrac{5}{2}x + \tfrac{3}{4}\sin 2x - \cos 2x + C$

8. $\int \frac{1}{\sin^2 x \cos^2 x} \, dx = \int \frac{1}{(\sin x \cos x)^2} \, dx = \int \frac{1}{\left(\tfrac{1}{2} \times 2\sin x \cos x\right)^2} \, dx = \int \frac{1}{\left(\tfrac{1}{2}\sin 2x\right)^2} \, dx$

$= \int \frac{1}{\tfrac{1}{4}\sin^2 2x} \, dx = \int 4\operatorname{cosec}^2 2x \, dx = -2\cot 2x + C$

ALTERNATIVE $= \int \frac{1}{\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2x\right)\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right)} \, dx = \int \frac{1}{\tfrac{1}{4} - \tfrac{1}{4}\cos^2 2x} \, dx$

$= \int \frac{4}{1 - \cos^2 2x} \, dx = \int \frac{4}{\sin^2 2x} \, dx = \int 4\operatorname{cosec}^2 2x$

$= -2\cot 2x + C$

9. $\int \sqrt{\sin^2 x + (\cos x - 1)^2} \, dx = \int \sqrt{\sin^2 x + \cos^2 x - 2\cos x + 1} \, dx$

$= \int \sqrt{1 + 1 - 2\cos x} \, dx = \int \sqrt{2 - 2\cos x} \, dx$

$\cos 2\theta = 1 - 2\sin^2\theta$
$\cos\left(2 \times \tfrac{\theta}{2}\right) = 1 - 2\sin^2\left(\tfrac{\theta}{2}\right)$

$= \int \sqrt{2 - 2\left(1 - 2\sin^2 \tfrac{x}{2}\right)} \, dx = \int \sqrt{4\sin^2 \tfrac{x}{2}} \, dx$

$= \int 2\sin \tfrac{x}{2} \, dx = -4\cos \tfrac{x}{2} + C$

10. $\int \frac{1 - \cos x}{1 + \cos x} \, dx = \int \frac{(1-\cos x)(1-\cos x)}{(1+\cos x)(1-\cos x)} \, dx = \int \frac{1 - 2\cos x + \cos^2 x}{1 - \cos^2 x} \, dx$

$= \int \frac{1}{\sin^2 x} - \frac{2\cos x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \, dx = \int \operatorname{cosec}^2 x - 2\frac{\cos x}{\sin x}\frac{1}{\sin x} + \cot^2 x \, dx$

$= \int \operatorname{cosec}^2 x - 2\cot x \operatorname{cosec} x + \operatorname{cosec}^2 x - 1 \, dx$

$= \int 2\operatorname{cosec}^2 x - 2\cot x \operatorname{cosec} x - 1 \, dx$

$= -2\cot x + 2\operatorname{cosec} x - x + C$

Can also be done by "little t" identities

Question 6

Carry out the following integrations:

1. $\displaystyle\int \frac{1+\sin x}{1-\sin x}\,dx = 2\tan x - x + 2\sec x + C$

2.

Question 7

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{2}} 4\sin^2 x \ \ dx = \pi$

2. $\displaystyle\int_0^{\frac{\pi}{6}} 24\cos^2 x \ \ dx = \pi + 3$

3. $\displaystyle\int_0^{\frac{\pi}{6}} 8\sin x\cos x \ \ dx = 1$

4. $\displaystyle\int_0^{\frac{\pi}{2}} (1-\sin x)^2 \ \ dx = \frac{3\pi}{2} - 4$

5. $\displaystyle\int_0^{\frac{\pi}{6}} (1-\cos 3x)^2 \ \ dx = \frac{\pi}{4} - \frac{2}{3}$

6. $\displaystyle\int_0^{\frac{\pi}{4}} 4\tan^2 x \ \ dx = 4 - \pi$

7. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (3\cot x + \tan x)^2 \ \ dx = \frac{2}{3}\left(10\sqrt{3} - \pi\right)$

8. $\displaystyle\int_0^{\frac{\pi}{4}} (\sec x + 4\cos x)^2 \ \ dx = 4\pi + 5$

9. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x} \ \ dx = 1$

10. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} \ \ dx = \sqrt{2} - 1$

1) $\displaystyle\int_0^{\frac{\pi}{2}} 4\sin^2 x\,dx = \int_0^{\frac{\pi}{2}} 4\left(\tfrac{1}{2}-\tfrac{1}{2}\cos 2x\right)dx = \int_0^{\frac{\pi}{2}} 2-2\cos 2x\,dx$

$= \left[2x - \sin 2x\right]_0^{\frac{\pi}{2}} = (\pi - \sin\pi) - (0 - \sin 0) = \pi$

2) $\displaystyle\int_0^{\frac{\pi}{12}} 24\cos^2 x\,dx = \int_0^{\frac{\pi}{12}} 24\left(\tfrac{1}{2}+\tfrac{1}{2}\cos 2x\right)dx = \int_0^{\frac{\pi}{12}} 12 + 12\cos 2x\,dx$

$= \left[12x + 6\sin 2x\right]_0^{\frac{\pi}{12}} = \left(\pi + 6\sin\tfrac{\pi}{6}\right) - (0 + 6\sin 0)$

$= \pi + 3$

3) $\displaystyle\int_0^{\frac{\pi}{6}} 8\sin x\cos x\,dx = \int_0^{\frac{\pi}{6}} 4(2\sin x\cos x)\,dx = \int_0^{\frac{\pi}{6}} 4\sin 2x\,dx = \left[-2\cos 2x\right]_0^{\frac{\pi}{6}}$

$= \left[2\cos 2x\right]_{\frac{\pi}{6}}^{0} = 2\cos 0 - 2\cos\tfrac{\pi}{3} = 2 - 1 = 1$

4) $\displaystyle\int_0^{\frac{\pi}{2}} (1-2\sin x)^2\,dx = \int_0^{\frac{\pi}{2}} 1 - 4\sin x + 4\sin^2 x\,dx = \int_0^{\frac{\pi}{2}} 1 - 4\sin x + 4\left(\tfrac{1}{2}-\tfrac{1}{2}\cos 2x\right)dx$

$\displaystyle = \int_0^{\frac{\pi}{2}} 1 - 4\sin x + 2 - 2\cos 2x\,dx = \int_0^{\frac{\pi}{2}} 3 - 4\sin x - 2\cos 2x\,dx$

$= \left[3x + 4\cos x - \sin 2x\right]_0^{\frac{\pi}{2}}$

$= \left(\tfrac{3\pi}{2} + 4\cos\tfrac{\pi}{2} - \sin\pi\right) - (0 + 4\cos 0 - \sin 0) = \tfrac{3\pi}{2} - 4$

5) $\displaystyle\int_0^{\frac{\pi}{6}} (1-\cos 3x)^2\,dx = \int_0^{\frac{\pi}{6}} 1 - 2\cos 3x + \cos^2 3x\,dx$

$\displaystyle = \int_0^{\frac{\pi}{6}} 1 - 2\cos 3x + \left(\tfrac{1}{2}+\tfrac{1}{2}\cos 6x\right)dx$

$\displaystyle = \int_0^{\frac{\pi}{6}} \tfrac{3}{2} - 2\cos 3x + \tfrac{1}{2}\cos 6x\,dx = \left[\tfrac{3}{2}x - \tfrac{2}{3}\sin 3x + \tfrac{1}{12}\sin 6x\right]_0^{\frac{\pi}{6}}$

$= \left(\tfrac{\pi}{4} - \tfrac{2}{3}\sin\tfrac{\pi}{2} + \tfrac{1}{12}\sin\pi\right) - \left(0 - \tfrac{2}{3}\sin 0 + \tfrac{1}{12}\sin 0\right)$

$= \tfrac{\pi}{4} - \tfrac{2}{3}$

6) $\displaystyle\int_0^{\frac{\pi}{4}} 4\tan^2 x\,dx = \int_0^{\frac{\pi}{4}} 4(\sec^2 x - 1)\,dx = \int_0^{\frac{\pi}{4}} 4\sec^2 x - 4\,dx$

$= \left[4\tan x - 4x\right]_0^{\frac{\pi}{4}} = \left(4\tan\tfrac{\pi}{4} - \pi\right) - (4\tan 0 - 0)$

$= 4 - \pi$

7) $\displaystyle\int_{\pi/6}^{\pi/3} (3\cot x + \tan x)^2\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 9\cot^2 x + 6\cot x\tan x + \tan^2 x\,dx$

$\displaystyle = \int_{\pi/6}^{\frac{\pi}{3}} 9(\operatorname{cosec}^2 x - 1) + 6 + (\sec^2 x - 1)\,dx$

$\displaystyle = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 9\operatorname{cosec}^2 x + \sec^2 x - 4\,dx$

$= \left[-9\cot x + \tan x - 4x\right]_{\pi/6}^{\pi/3}$

$= \left[9\cot x + 4x - \tan x\right]_{\pi/3}^{\pi/6}$

$= \left(9\cot\tfrac{\pi}{6} + \tfrac{2\pi}{3} - \tan\tfrac{\pi}{6}\right) - \left(9\cot\tfrac{\pi}{3} + \tfrac{4\pi}{3} - \tan\tfrac{\pi}{3}\right)$

$= \left(9\sqrt{3} + \tfrac{2\pi}{3} - \tfrac{\sqrt{3}}{3}\right) - \left(3\sqrt{3} + \tfrac{4\pi}{3} - \sqrt{3}\right)$

$= 7\sqrt{3} - \tfrac{1}{3}\sqrt{3} - \tfrac{2\pi}{3} = \tfrac{20}{3}\sqrt{3} - \tfrac{2\pi}{3} = \tfrac{2}{3}(10\sqrt{3} - \pi)$

8) $\displaystyle\int_0^{\frac{\pi}{4}} (\sec x + 4\cos x)^2\,dx = \int_0^{\frac{\pi}{4}} \sec^2 x + 8\cos x\sec x + 16\cos^2 x\,dx$

$\displaystyle = \int_0^{\frac{\pi}{4}} \sec^2 x + 8 + 16\left(\tfrac{1}{2}+\tfrac{1}{2}\cos 2x\right)dx$

$\displaystyle = \int_0^{\frac{\pi}{4}} \sec^2 x + 16 + 8\cos 2x\,dx$

$= \left[\tan x + 16x + 4\sin 2x\right]_0^{\frac{\pi}{4}}$

$= \left(\tan\tfrac{\pi}{4} + 4\pi + 4\sin\tfrac{\pi}{2}\right) - (\tan 0 + 4\sin 0)$

$= 1 + 4\pi + 4$

$= 4\pi + 5$

9. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x}\,dx = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x}\times\frac{1}{\cos x}\,dx = \int_0^{\frac{\pi}{3}} \tan x\sec x\,dx$

$= \left[\sec x\right]_0^{\pi/3} = \sec\tfrac{\pi}{3} - \sec 0$

$= 2 - 1 = 1$

10. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x}\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\times\frac{1}{\sin x}\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\operatorname{cosec} x\,dx$

$= \left[-\operatorname{cosec} x\right]_{\pi/4}^{\pi/2} = \left[\operatorname{cosec} x\right]_{\pi/2}^{\pi/4}$

$= \operatorname{cosec}\tfrac{\pi}{4} - \operatorname{cosec}\tfrac{\pi}{2} = \sqrt{2} - 1$

Question 8

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2 x \ dx = \frac{1}{8}(\pi + 2)$

2. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \ dx = \frac{\pi}{4}$

3. $\displaystyle\int_0^{\frac{\pi}{2}} (2\sin x - 3\cos x)^2 \ dx = \frac{1}{4}(13\pi - 24)$

4. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2 \ dx = 4\pi + 3\sqrt{3}$

5. $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2 x \ dx = \frac{1}{4}(4 - \pi)$

6. $\displaystyle\int_0^{\frac{\pi}{6}} \sin x \sin 3x \ dx = \frac{\sqrt{3}}{16}$

7. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{1}{1 - \sin x} \ dx = 1 + \sqrt{3}$

8. $\displaystyle\int_0^{\frac{\pi}{2}} \left(1 + \tan \tfrac{x}{2}\right)^2 \ dx = 2 + \ln 4$

9. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x \ dx = \frac{2}{3}$

10. $\displaystyle\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cot^2 2x \ dx = \frac{1}{2} - \frac{\sqrt{3}}{6} - \frac{\pi}{24}$

1. $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2 x \, dx = \int_0^{\frac{\pi}{4}} \frac{1}{2} + \frac{1}{2}\cos 2x \, dx = \left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{4}}$

$= \left[\frac{1}{2}\left(\frac{\pi}{4}\right) + \frac{1}{4}\sin\frac{\pi}{2}\right] - [0] = \frac{\pi}{8} + \frac{1}{4} = \frac{1}{8}(\pi + 2)$ //

2. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \, dx = \int_0^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2}\cos 2x \, dx = \left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$

$= \left(\frac{1}{2} \times \frac{\pi}{2} - \frac{1}{4}\sin\pi\right) - (0) = \frac{\pi}{4}$ //

3. $\displaystyle\int_0^{\frac{\pi}{2}} (2\sin x - 3\cos x)^2 \, dx = \int_0^{\frac{\pi}{2}} 4\sin^2 x - 12\sin x\cos x + 9\cos^2 x \, dx$

$\displaystyle = \int_0^{\frac{\pi}{2}} 4\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) - 6(2\sin x\cos x) + 9\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$

$\displaystyle = \int_0^{\frac{\pi}{2}} \frac{13}{2} + \frac{5}{2}\cos 2x - 6\sin 2x \, dx$

$= \left[\frac{13}{2}x + \frac{5}{4}\sin 2x + 3\cos 2x\right]_0^{\frac{\pi}{2}}$

$= \left(\frac{13}{2} \times \frac{\pi}{2} + \frac{5}{4}\sin\pi + 3\cos\pi\right) - (0 + 0 + 3)$

$= \frac{13\pi}{4} - 3 - 3 = \frac{13\pi}{4} - 6 = \frac{1}{4}(13\pi - 24)$ //

4. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2 \, dx = \int_{\pi/3}^{\frac{5\pi}{3}} 1 - 4\cos x + 4\cos^2 x \, dx$

$\displaystyle = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 1 - 4\cos x + 4\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$

$\displaystyle = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 3 - 4\cos x + 2\cos 2x \, dx$

$= \left[3x - 4\sin x + \sin 2x\right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$

$= \left[3\left(\frac{5\pi}{3}\right) - 4\sin\left(\frac{5\pi}{3}\right) + \sin\left(\frac{10\pi}{3}\right)\right] - \left[3\left(\frac{\pi}{3}\right) - 4\sin\frac{\pi}{3} + \sin\frac{2\pi}{3}\right]$

$= 5\pi - 4\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right) - \pi + 2\sqrt{3} - \frac{\sqrt{3}}{2}$

$= 4\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2} + 2\sqrt{3} - \frac{\sqrt{3}}{2} = 4\pi + 3\sqrt{3}$

5. $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = \int_0^{\frac{\pi}{4}} \sec^2 x - 1 \, dx = \left[\tan x - x\right]_0^{\frac{\pi}{4}}$

$= \left[\tan\frac{\pi}{4} - \frac{\pi}{4}\right] - [0] = 1 - \frac{\pi}{4} = \frac{1}{4}(4 - \pi)$ //

6. $\cos(3x + x) = \cos 3x\cos x - \sin 3x\sin x$

$\cos(3x - x) = \cos 3x\cos x + \sin 3x\sin x$

SUBTRACT EQUATIONS "UPWARDS"

$\boxed{\cos 2x - \cos 4x = 2\sin 3x\sin x}$

$\displaystyle\int_0^{\frac{\pi}{6}} \sin x\sin 3x \, dx = \int_0^{\frac{\pi}{6}} \frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x \, dx$

$= \left[\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x\right]_0^{\frac{\pi}{6}}$

$= \left(\frac{1}{4}\sin\frac{\pi}{3} - \frac{1}{8}\sin\frac{2\pi}{3}\right) - (0)$

$= \frac{1}{4} \times \frac{\sqrt{3}}{2} - \frac{1}{8} \times \frac{\sqrt{3}}{2} = \frac{1}{16}\sqrt{3}$ //

7. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{1}{1 - \sin x} \, dx = \int_0^{\frac{\pi}{3}} \frac{1 + \sin x}{(1 - \sin x)(1 + \sin x)} \, dx = \int_0^{\frac{\pi}{3}} \frac{1 + \sin x}{1 - \sin^2 x} \, dx$

$\displaystyle = \int_0^{\frac{\pi}{3}} \frac{1 + \sin x}{\cos^2 x} \, dx = \int_0^{\frac{\pi}{3}} \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \, dx$

$\displaystyle = \int_0^{\frac{\pi}{3}} \sec^2 x + \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \, dx = \int_0^{\pi/3} \sec^2 x + \tan x\sec x \, dx$

$= \left[\tan x + \sec x\right]_0^{\frac{\pi}{3}} = \left(\tan\frac{\pi}{3} + \sec\frac{\pi}{3}\right) - (0 + 1)$

$= \sqrt{3} + 2 - 1 = \sqrt{3} + 1$ //

8. $\displaystyle\int_0^{\frac{\pi}{2}} \left(1 + \tan\frac{x}{2}\right)^2 dx = \int_0^{\frac{\pi}{2}} 1 + 2\tan\frac{x}{2} + \tan^2\frac{x}{2} \, dx$

$\displaystyle = \int_0^{\frac{\pi}{2}} 1 + 2\tan\frac{x}{2} + \sec^2\frac{x}{2} - 1 \, dx$

$= \left[4\ln\left|\sec\frac{x}{2}\right| + 2\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$\left[4\ln\left|\sec\frac{\pi}{4}\right| + 2\tan\left(\frac{\pi}{4}\right)\right] - \left[4\ln|\sec 0| + 0\right]$

$= 4\ln\sqrt{2} + 2 = 4\ln 2^{\frac{1}{2}} + 2$

$= \ln\left(2^{\frac{1}{2}}\right)^4 + 2 = \ln 4 + 2$ //

9. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x \, dx = \int_0^{\frac{\pi}{2}} \cos^2 x\cos x \, dx = \int_0^{\frac{\pi}{2}} (1 - \sin^2 x)\cos x \, dx$

$\displaystyle = \int_0^{\frac{\pi}{2}} \cos x - \sin^2 x\cos x \, dx = \left[\sin x - \frac{1}{3}\sin^3 x\right]_0^{\frac{\pi}{2}}$

$= \left[\sin\frac{\pi}{2} - \frac{1}{3}\sin^3\frac{\pi}{2}\right] - [0] = 1 - \frac{1}{3} = \frac{2}{3}$ //

10. $\displaystyle\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cot^2 2x \, dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \operatorname{cosec}^2 2x - 1 \, dx = \left[-\frac{1}{2}\cot 2x - x\right]_{\frac{\pi}{8}}^{\frac{\pi}{6}}$

$= \left[\frac{1}{2}\cot 2x + x\right]_{\frac{\pi}{6}}^{\frac{\pi}{8}}$

$= \left(\frac{1}{2}\cot\frac{\pi}{4} + \frac{\pi}{8}\right) - \left(\frac{1}{2}\cot\frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \frac{1}{2} + \frac{\pi}{8} - \frac{1}{2}\frac{\sqrt{3}}{3} - \frac{\pi}{6}$

$= \frac{1}{2} - \frac{1}{6}\sqrt{3} - \frac{\pi}{24}$ //

Question 9

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{12}} 6\sin^2\theta \; d\theta = \frac{1}{4}(\pi - 3)$

2. $\displaystyle\int_0^{\frac{\pi}{6}} \sin^3\theta \; d\theta = \frac{5}{24}$

3. $\displaystyle\int_0^{\frac{\pi}{12}} 10\sin 8\theta \cos 2\theta \; d\theta = \frac{1}{12}\left(16 + 3\sqrt{3}\right)$

4. $\displaystyle\int_0^{\frac{\pi}{4}} (\cos x + \sec x)^2 \; dx = \frac{5}{8}(\pi + 2)$

5. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x + \cot x)^2 \; dx = \frac{1}{8}\left(26 - \pi - 4\sqrt{2}\right)$