INTEGRATION
by substitution
(without answers)

Question 1

Carry out the following integrations **by substitution** only.

1. $\int 4x(2x-1)^4\,dx$

2. $\int \frac{2x}{2x+1}\,dx$

3. $\int x\left(4-x^2\right)^{-\frac{1}{2}}\,dx$

4. $\int \frac{4x}{6x^2-1}\,dx$

5. $\int x(3x-1)^4\,dx$

6. $\int \frac{8x}{\sqrt{4x-1}}\,dx$

7. $\int \frac{2x^2}{\sqrt{2x^3+1}}\,dx$

8. $\int \frac{4-3x}{x+2}\,dx$

9. $\int \frac{4x^2}{2x-1}\,dx$

10. $\int \frac{4x-3}{3x-4}\,dx$

Question 2

Carry out the following integrations **by substitution** only.

1. $\int 6x(3x-1)^3\,dx$

2. $\int \frac{5x}{5x-1}\,dx$

3. $\int 3x\left(x^2+1\right)^{\frac{1}{2}}\,dx$

4. $\int \frac{3x^2}{2x^3+1}\,dx$

5. $\int x(2x-1)^5\,dx$

6. $\int \frac{10x}{\sqrt{1-2x}}\,dx$

7. $\int \frac{3x^4}{\sqrt{2x^5+1}}\,dx$

8. $\int \frac{1-x}{1+2x}\,dx$

9. $\int \frac{6x^2}{2x+3}\,dx$

10. $\int \frac{1}{x^{\frac{1}{2}}\sqrt{x^{\frac{1}{2}}-1}}\,dx$

Question 3

Carry out the following integrations **by substitution** only.

1. $\int 10x(5x-3)^3\,dx$

2. $\int \frac{12x}{2x-1}\,dx$

3. $\int x\left(x^2-1\right)^{\frac{5}{2}}\,dx$

4. $\int \frac{5x^5}{2x^6+7}\,dx$

5. $\int 2x(1-5x)^4\,dx$

6. $\int \frac{9x}{\sqrt{4x^2+1}}\,dx$

7. $\int \frac{3x-1}{\sqrt{4x-1}}\,dx$

8. $\int \frac{1-2x}{1+3x}\,dx$

9. $\int \frac{6x^{\frac{1}{2}}}{2x^{\frac{3}{2}}+3}\,dx$

10. $\int \frac{x^{\frac{3}{2}}}{\sqrt{1-3x^{\frac{5}{2}}}}\,dx$

Question 4

Carry out the following integrations to the answers given, **by using substitution** only.

1. $\displaystyle\int_0^{\frac{1}{2}} 8x(2x-1)^4 \, dx = \frac{1}{15}$

2. $\displaystyle\int_2^3 \frac{3x}{3x-5} \, dx = 1+\frac{10}{3}\ln 2$

3. $\displaystyle\int_0^1 x\left(1-x^2\right)^{\frac{3}{2}} \, dx = \frac{1}{5}$

4. $\displaystyle\int_0^1 \frac{4x}{x^2+1} \, dx = 2\ln 2$

5. $\displaystyle\int_1^3 2x(3x-1)^4 \, dx = \frac{55808}{5}$

6. $\displaystyle\int_4^8 \frac{6x}{\sqrt{2x-7}} \, dx = 68$

7. $\displaystyle\int_0^1 \frac{x}{\sqrt{9-5x^2}} \, dx = \frac{1}{5}$

8. $\displaystyle\int_0^3 \frac{5-2x}{x+1} \, dx = 5\ln 3-6$

9. $\displaystyle\int_0^{\frac{1}{5}} \frac{10x^2}{5x+1} \, dx = \frac{\ln 4-1}{25}$

10. $\displaystyle\int_{-\frac{3}{2}}^{-\frac{1}{2}} \frac{5x-2}{2x-5} \, dx = \frac{10-21\ln 2}{4}$

Question 5

Carry out the following integrations to the answers given, **by using substitution** only.

1. $\displaystyle\int_0^{\frac{3}{2}} 2x(2x-3)^4 \, dx = \frac{243}{20}$

2. $\displaystyle\int_0^{2} \frac{4x}{4x+1} \, dx = 2-\frac{1}{2}\ln 3$

3. $\displaystyle\int_0^{1} x^2\left(1-x^3\right)^{\frac{9}{2}} \, dx = \frac{2}{33}$

4. $\displaystyle\int_0^{4} \frac{12x}{x^2+9} \, dx = 12\ln\left(\frac{5}{3}\right)$

5. $\displaystyle\int_1^{3} 2x(3x-1)^4 \, dx = -\frac{64}{15}$

6. $\displaystyle\int_2^{6} \frac{6x}{\sqrt{3x-2}} \, dx = \frac{272}{9}$

7. $\displaystyle\int_0^{1} \frac{x}{\sqrt{16-7x^2}} \, dx = \frac{1}{7}$

8. $\displaystyle\int_5^{6} \frac{1-2x}{x-4} \, dx = -2-7\ln 2$

9. $\displaystyle\int_0^{\frac{1}{3}} \frac{9x^2}{3x+1} \, dx = \frac{\ln 4-1}{6}$

10. $\displaystyle\int_0^{\frac{3}{2}} \frac{2x-3}{2x+3} \, dx = \frac{3}{2}(1-\ln 4)$

Question 6

Carry out the following integrations.

1. $\int \frac{x}{\sqrt{x+1}}\,dx$

2. $\int \frac{2x}{(2x+1)^3}\,dx$

3. $\int \frac{x}{x+1}\,dx$

4. $\int \frac{x}{\sqrt{x-1}}\,dx$

5. $\int \frac{4x+1}{2x-5}\,dx$

6. $\int \frac{x^2}{2x-1}\,dx$

7. $\int \frac{2x+1}{2x-1}\,dx$

8. $\int \frac{6x}{\sqrt{2x+3}}\,dx$

9. $\int \frac{3x-1}{2x+3}\,dx$

10. $\int \frac{8x^2}{1-2x}\,dx$

Question 7

Carry out the following integrations **using the substitutions** given.

1. $\int x\sqrt{1-x}\,dx$ Use $u=1-x$, or $u=\sqrt{1-x}$

2. $\int \frac{6x}{\sqrt{2x+1}}\,dx$ Use $u=2x+1$, or $u=\sqrt{2x+1}$

3. $\int \cos^3 x\,dx$ Use $u=\sin x$

4. $\int \sec^4 x\,dx$ Use $u=\tan x$

5. $\int \frac{1}{\sqrt{x}(x-4)}\,dx$ Use $u=\sqrt{x}$

6. $\int \frac{\sqrt{x^2+9}}{x}\,dx$ Use $u=\sqrt{x^2+9}$

7. $\int \frac{1+\cos x}{\sin x}\,dx$ Use $u=\cos x$

8. $\int \frac{1}{1+\sqrt{x-2}}\,dx$ Use $u=\sqrt{x-2}$

9. $\int \sec^2 x\tan x\sqrt{1+\tan x}\,dx$ Use $u=\sqrt{1+\tan x}$

10. $\int \frac{9}{\sqrt{x}(9x-1)}\,dx$ Use $u=\sqrt{x}$

Question 8

Carry out the following integrations.

1. $\int \frac{1}{2+\sqrt{x}}\,dx$

2. $\int \frac{x^2}{1-2x}\,dx$

3. $\int \frac{3x^2+2}{4x+1}\,dx$

4. $\int \frac{4-3x}{2x+1}\,dx$

5. $\int \frac{x+1}{x-5}\,dx$

6. $\int \frac{x^2}{x-2}\,dx$

7. $\int \frac{1}{2+\sqrt{x-1}}\,dx$

8. $\int \frac{x+4}{x-4}\,dx$

9. $\int 3x^2\left(4-2x^3\right)^{\frac{3}{2}}\,dx$

Question 9

Carry out the following integrations.

1. $\int x\sqrt{x+1}\,dx$

2. $\int \frac{x+1}{\sqrt[3]{x^2+2x+3}}\,dx$

3. $\int \frac{3x^3+5x}{x^2+1}\,dx$

4. $\int \frac{2x+1}{3x-1}\,dx$

5. $\int \frac{1}{(x-1)\sqrt{x^2-1}}\,dx$, use $x-1=\frac{1}{u}$

6. $\int \frac{4x^3\sqrt{x^4+1}}{1+\sqrt{x^4+1}}\,dx$

Question 10

Carry out the following integrations to the answers given.

1. $\int_0^{\frac{1}{2}} \frac{x}{(2-x)^2}\, dx = \frac{1}{3} + \ln\frac{3}{4}$

2. $\int_1^2 \frac{x}{(2x-1)^2}\, dx = \frac{2+\ln 27}{12}$

3. $\int_0^2 \frac{x+2}{\sqrt{4x+1}}\, dx = \frac{17}{6}$

4. $\int_0^{36} \frac{1}{\sqrt{x}\left(\sqrt{x}+2\right)}\, dx = \ln 16$

5. $\int_{-6}^{\frac{3}{2}} \frac{x}{\sqrt{4-2x}}\, dx = -\frac{9}{2}$

6. $\int_1^5 \frac{x+1}{(2x-1)^{\frac{3}{2}}}\, dx = 2$

7. $\int_{\frac{2}{3}}^1 \frac{x}{2x-1}\, dx = \frac{1}{6} + \frac{1}{4}\ln 3$

8. $\int_{-1}^7 \frac{x^2}{\sqrt{x+2}}\, dx = \frac{652}{15}$

9. $\int_1^{\frac{5}{2}} \frac{4x}{\sqrt{2x-1}}\, dx = \frac{20}{3}$

10. $\int_0^1 \frac{x}{(1+x)^2}\, dx = \ln 2 - \frac{1}{2}$

Question 11

Carry out the following integrations to the answers given.

1. $\displaystyle\int_0^3 x\sqrt{x+1}\ dx = \frac{116}{15}$

2. $\displaystyle\int_0^2 \frac{6x^3}{\sqrt{x^2+1}}\ dx = 4\left(1+\sqrt{5}\right)$

3. $\displaystyle\int_{-1}^0 \frac{x^2}{1-x}\ dx = -\frac{1}{2}+\ln 2$

4. $\displaystyle\int_0^{100} \frac{1}{20-\sqrt{x}}\ dx = 40\ln 2 - 20$

5. $\displaystyle\int_0^{\frac{1}{4}} 2x\sqrt{1-4x}\ dx = \frac{1}{30}$

6. $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\cos x\left(1+\sin x\right)^5\ dx = \frac{107}{14}$

7. $\displaystyle\int_2^5 \frac{x^2}{\sqrt{x-1}}\ dx = \frac{356}{15}$

Question 12

Carry out the following integrations to the answers given.

1. $\displaystyle\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx = \frac{\pi}{2}-1$, use $x = 2\sin\theta$

2. $\displaystyle\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,dx = \frac{1}{4}\left(\sqrt{3}-1\right)$, use $x = 2\cos\theta$

3. $\displaystyle\int_0^{1} \frac{1}{\left(1+x^2\right)^2}\,dx = \frac{1}{8}\left(\pi+2\right)$, use $x = \tan\theta$

4. $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,dx = \frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$, use $x = \sec\theta$

5. $\displaystyle\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,dx = \frac{\pi}{6}$, use $x = \frac{\sqrt{3}}{2}\sin\theta$

6. $\displaystyle\int_0^{1} \frac{1}{\left(1+3x^2\right)^{\frac{3}{2}}}\,dx = \frac{1}{2}$, use $x = \frac{1}{\sqrt{3}}\tan\theta$

7. $\displaystyle\int_0^{1} \frac{1}{\sqrt{2-x^2}}\,dx = \frac{\pi}{4}$, use $x = \sqrt{2}\sin\theta$

8. $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{4x^2+3}\,dx = \frac{\pi\sqrt{3}}{36}$, use $x = \frac{\sqrt{3}}{2}\tan\theta$

9. $\displaystyle\int_0^{1} \frac{1}{\left(4-x^2\right)^{\frac{3}{2}}}\,dx = \frac{\sqrt{3}}{12}$, use $x = 2\sin\theta$

10. $\displaystyle\int_{\sqrt{2}}^{2} \frac{\sqrt{x^2-1}}{x}\,dx = \sqrt{3}-1-\frac{\pi}{12}$, use $x=\operatorname{cosec}\theta$

11. $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{4-3x^2}}\,dx = \frac{\pi\sqrt{3}}{9}$, use $x=\frac{2}{\sqrt{3}}\sin\theta$

12. $\displaystyle\int_{1}^{\sqrt{3}} \frac{x^2}{x^2+1}\,dx = \sqrt{3}-1-\frac{\pi}{12}$, use $x=\tan\theta$

13. $\displaystyle\int_{0}^{2} \sqrt{16-x^2}\,dx = \frac{1}{3}\left(4\pi+6\sqrt{3}\right)$, use $x=4\sin\theta$

14. $\displaystyle\int_{0}^{2} \frac{1}{\left(3x^2+4\right)^{\frac{3}{2}}}\,dx = \frac{1}{8}$, use $x=\frac{2}{\sqrt{3}}\tan\theta$

15. $\displaystyle\int_{0}^{2} \sqrt{16-3x^2}\,dx = \frac{8\pi\sqrt{3}}{9}+2$, use $x=\frac{4}{\sqrt{3}}\sin\theta$

16. $\displaystyle\int_{0}^{3} \frac{27}{\left(9+x^2\right)^2}\,dx = \frac{\pi}{8}+\frac{1}{4}$, use $x=3\tan\theta$