INTEGRATION
BY PARTS
Question 1

Carry out the following integrations:

1. \( \int x e^{2x} \, dx = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C \)

2. \( \int 3x \cos 2x \, dx = \frac{3}{2} x \sin 2x + \frac{3}{4} \cos 2x + C \)

3. \( \int x \sin 4x \, dx = -\frac{1}{4} x \cos 4x + \frac{1}{16} \sin 4x + C \)

4. \( \int -2x \sin 5x \, dx = \frac{2}{5} x \cos 5x - \frac{2}{25} \sin 5x + C \)

5. \( \int (1 - 2x) e^{-x} \, dx = (2x - 1) e^{-x} + 2e^{-x} + C \)

6. \( \int x^2 e^{-3x} \, dx = -\frac{1}{3} x^2 e^{-3x} - \frac{2}{9} x e^{-3x} - \frac{2}{27} e^{-3x} + C \)

7. \( \int 16x^3 \ln x \, dx = 4x^4 \ln |x| - x^4 + C \)

8. \( \int \ln x \, dx = x \ln x - x + C \)

9. \( \int x \cos \left( \frac{1}{2} x \right) \, dx = 2x \sin \left( \frac{1}{2} x \right) + 4 \cos \left( \frac{1}{2} x \right) + C \)

10. \( \int (3x - 1) \sin (3x - 1) \, dx = -\frac{1}{3} (3x - 1) \cos (3x - 1) + \frac{1}{3} \sin (3x - 1) + C \)
Question 2

Carry out the following integrations:

1. \[ \int 6xe^{3x} \, dx = 2xe^{3x} - \frac{2}{3}e^{3x} + C \]

2. \[ \int 12x \cos 3x \, dx = 4x \sin 3x + \frac{4}{3} \cos 3x + C \]

3. \[ \int x \sin 6x \, dx = -\frac{1}{6} x \cos 6x + \frac{1}{36} \sin 6x + C \]

4. \[ \int -x \sin 2x \, dx = \frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x + C \]

5. \[ \int (2-x)e^{-3x} \, dx = -\frac{1}{3}(2-x)e^{-3x} + \frac{1}{9}e^{-3x} + C \]

6. \[ \int x^2 e^{-4x} \, dx = -\frac{1}{4} x^2 e^{-4x} - \frac{1}{8} xe^{-4x} + \frac{1}{32} e^{-4x} + C \]

7. \[ \int x^2 e^{-\frac{x}{2}} \, dx = -2x^2 e^{-\frac{x}{2}} - 8xe^{-\frac{x}{2}} - 16e^{-\frac{x}{2}} + C \]

8. \[ \int 25x^4 \ln x \, dx = 5x^5 \ln x - x^5 + C \]

9. \[ \int 24x \cos \left( \frac{2}{3}x \right) \, dx = 36x \sin \left( \frac{2}{3}x \right) + 54 \cos \left( \frac{2}{3}x \right) + C \]

10. \[ \int x^2 \sin (1-x) \, dx = x^2 \cos (1-x) + 2x \sin (1-x) - 2 \cos (1-x) + C \]
1. \[ \int x^2 dx = \frac{x^3}{3} + C \]
2. \[ \int \frac{1}{x} dx = \ln|x| + C \]
3. \[ \int \cos(x) dx = \sin(x) + C \]
4. \[ \int e^x dx = e^x + C \]
5. \[ \int \sin(x) dx = -\cos(x) + C \]
6. \[ \int \frac{1}{x^2} dx = -\frac{1}{x} + C \]
7. \[ \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C \]
8. \[ \int \frac{1}{1+x^2} dx = \arctan(x) + C \]
9. \[ \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C \]
10. \[ \int \frac{1}{1+x^2} dx = \arctan(x) + C \]
Question 3

Carry out the following integrations:

1. \( \int \frac{1}{2} x e^{4x} \, dx = \frac{1}{8} x e^{4x} - \frac{1}{32} e^{4x} + C \)

2. \( \int 5x \sin 4x \, dx = \frac{5}{4} x \cos 4x + \frac{5}{16} \sin 4x + C \)

3. \( \int (2x + 1) \cos 2x \, dx = \frac{1}{2} (2x + 1) \sin 2x + \frac{1}{2} \cos 2x + C \)

4. \( \int -3x \cos 4x \, dx = -\frac{3}{4} x \sin 4x - \frac{3}{16} \cos 4x + C \)

5. \( \int x^2 e^{-2x} \, dx = -\frac{1}{2} x^2 e^{-2x} - \frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C \)

6. \( \int x^2 \sin 5x \, dx = -\frac{1}{5} x^2 \cos 5x + \frac{2}{25} x \sin 5x + \frac{2}{125} \cos 5x + C \)

7. \( \int x^2 \cos \frac{1}{2} x \, dx = 3x^2 \sin \frac{1}{2} x + 18x \cos \frac{1}{2} x - 54 \sin \frac{1}{2} x + C \)

8. \( \int \frac{1}{2} x^3 \ln x \, dx = \frac{1}{8} x^4 \ln x - \frac{1}{32} x^4 + C \)

9. \( \int x \ln 3x \, dx = \frac{1}{2} x^2 \ln 3x - \frac{1}{4} x^2 + C \)

10. \( \int \frac{\ln x}{x^3} \, dx = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + C \)
Question 4

Carry out the following integrations:

1. \( \int x e^{5x} \, dx = \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} + C \)

2. \( \int 2x \cos 3x \, dx = \frac{2}{3} x \sin 3x + \frac{2}{9} \cos 3x + C \)

3. \( \int x \sin 3x \, dx = -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + C \)

4. \( \int x \sin 4x \, dx = \frac{1}{16} \sin 4x - \frac{1}{4} x \cos 4x + C \)

5. \( \int 2 \ln x \, dx = 2x \ln x - 2x + C \)

6. \( \int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln |x| - \frac{1}{9} x^3 + C \)

7. \( \int x \sin \left( \frac{1}{2} x \right) \, dx = 4 \sin \left( \frac{1}{2} x \right) - 2x \cos \left( \frac{1}{2} x \right) + C \)

8. \( \int x \sin (2x-1) \, dx = -\frac{1}{2} x \cos (2x-1) + \frac{1}{4} \sin (2x-1) + C \)

9. \( \int \frac{\ln x}{x^2} \, dx = -\frac{\ln x}{x} + \frac{1}{x} + C \)

10. \( \int x \sec^2 x \, dx = x \tan x - \ln |\sec x| + C \)
Question 5

Carry out the following integrations:

1. \[ \int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C \]
2. \[ \int x^3 \ln x \, dx = \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C \]
3. \[ \int \sin x \ln (\sec x) \, dx = -\cos x (1 + \ln |\sec x|) + C \]
4. \[ \int x \cos 5x \, dx = \frac{1}{5} x \sin 5x + \frac{1}{25} \cos 5x + C \]
5. \[ \int x^2 \sin 3x \, dx = -\frac{1}{3} x^2 \cos 3x + \frac{2}{9} x \sin 3x + \frac{2}{27} \cos 3x + C \]
6. \[ \int 4x e^{-3x} \, dx = -3(2x + 3)e^{-3x} + C \]
7. \[ \int x^2 \cos \left( \frac{1}{3} x \right) \, dx = 3x^2 \sin \left( \frac{1}{3} x \right) + 18x \cos \left( \frac{1}{3} x \right) - 54 \sin \left( \frac{1}{3} x \right) + C \]
8. \[ \int 2x^2 \sec^2 x \, dx = x^2 \sec^2 x - 2x \tan x + 2 \ln |\sec x| + C \]
9. \[ \int x^2 e^{4x} \, dx = \left( 2x^2 - 8x + 16 \right) e^{4x} + C \]
10. \[ \int x \sec x \tan x \, dx = x \sec x - \ln |\sec x + \tan x| + C \]
Question 6

Carry out the following integrations:

1. \[ \int x^2 e^{-x^2} \, dx = \frac{1}{2} e^{-x^2} (x^2 + 8x + 32) + C \]
2. \[ \int x^2 e^{-x^2} \, dx = -e^{-x^2} (x^2 + 2x + 2) + C \]
3. \[ \int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) + C \]
4. \[ \int (\ln x)^2 \, dx = x(\ln x)^2 - 2x \ln x + 2x + C \]
5. \[ \int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) + C \]
6. \[ \int (x^3 + 5x^2 - 2) e^{2x} \, dx = \frac{1}{8} e^{2x} (4x^3 + 14x^2 - 14x - 1) + C \]
7. \[ \int x \cos x \, dx = \frac{1}{8} x^2 + \frac{1}{4} x \sin 2x + \frac{1}{8} \cos 2x + C \]
8. \[ \int x \ln 2x^3 \, dx = \frac{3}{4} x^2 \ln (2\ln x - 1) + C \]
Question 7

Carry out the following integrations, to the answer given:

1. \[ \int_{\ln 2}^{2} x e^{2x} \, dx = \ln 4 - \frac{3}{4} \]

2. \[ \int_{0}^{\frac{\pi}{2}} 6x \sin 3x \, dx = \frac{2\pi}{3} \]

3. \[ \int_{0}^{\frac{\pi}{2}} x^2 \cos x \, dx = \frac{1}{4}(\pi^2 - 8) \]

4. \[ \int_{0}^{e} x \ln x \, dx = \frac{1}{4}(e^2 + 1) \]

5. \[ \int_{0}^{1} 4x e^{3x} \, dx = \frac{4}{9}(2e^3 + 1) \]

6. \[ \int_{0}^{\frac{\pi}{2}} x \sin 4x \, dx = \frac{\pi}{16} \]

7. \[ \int_{0}^{2} x^3 \ln x \, dx = 4\ln 2 - \frac{15}{16} \]

8. \[ \int_{0}^{1} x e^{-2x} \, dx = \frac{1}{4}(1 - 3e^{-2}) \]

9. \[ \int_{0}^{\frac{\pi}{4}} 12x \cos 2x \, dx = \frac{3}{2}(\pi - 2) \]

10. \[ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4x \sin 2x \, dx = \pi - 1 \]
\[
1. \int \left( x^2 - 4x + 3 \right) dx = \left( \frac{1}{3}x^3 - 2x^2 + 3x \right) + C
\]
\[
2. \int \left( \cos x - \sin x \right) dx = \sin x + \cos x + C
\]
\[
3. \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + C
\]
\[
4. \int \left( \frac{1}{\sqrt{1-x^2}} \right) dx = \sin^{-1} x + C
\]
\[
5. \int \left( \frac{1}{2}x^2 + 3x + 1 \right) dx = \left( \frac{1}{6}x^3 + \frac{3}{2}x^2 + x \right) + C
\]
\[
6. \int \left( \frac{1}{x} - \frac{1}{x^2} \right) dx = \ln |x| + x^{-1} + C
\]
Question 8

Carry out the following integrations, to the answer given:

1. \( \int_{0}^{\pi/3} x \sin 3x \, dx = \frac{\pi}{9} \)

2. \( \int_{0}^{\pi/4} 2x \cos 4x \, dx = -\frac{1}{4} \)

3. \( \int_{0}^{\ln 2} 4x e^{-x} \, dx = 2 - \ln 4 \)

4. \( \int_{1}^{e} \ln x \, dx = 1 \)

5. \( \int_{0}^{\pi/2} x \sin 2x \, dx = \frac{\pi}{4} \)

6. \( \int_{0}^{\ln 4} x e^{x^2} \, dx = 8\ln 2 - 4 \)

7. \( \int_{0}^{\pi} x \cos \left(\frac{1}{4} x\right) \, dx = 2\sqrt{2}(\pi + 4) - 16 \)

8. \( \int_{0}^{1} (2x + 1)e^{2x} \, dx = e^2 \)

9. \( \int_{\frac{3}{4}}^{1} x \ln x \, dx = \frac{1}{4} \left(\frac{3}{e^2} - 1\right) \)

10. \( \int_{-1}^{0} 3\ln(2x + 3) \, dx = \frac{3}{2}(\ln 27 - 2) \) **REQUIRES ADDITIONAL TECHNIQUES**
1. \[
\int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C
\]
2. \[
\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \left( \frac{x}{a} \right) + C
\]
3. \[
\int \frac{1}{x^2 + b^2} \, dx = \frac{1}{b} \arctan \left( \frac{x}{b} \right) + C
\]
4. \[
\int \frac{1}{x^2 + c^2} \, dx = \frac{1}{c} \arctan \left( \frac{x}{c} \right) + C
\]
5. \[
\int \frac{1}{x^2 + d^2} \, dx = \frac{1}{d} \arctan \left( \frac{x}{d} \right) + C
\]
Question 9

Carry out the following integrations, to the answer given:

1. \[ \int_{0}^{\frac{\pi}{4}} x \sec^2 x \, dx = \frac{1}{4} (\pi - \ln 4) \]

2. \[ \int_{1}^{2} \frac{\ln x}{x} \, dx = \frac{1}{2} (\ln 2)^2 \]

3. \[ \int_{0}^{\frac{\pi}{2}} x \sin^2 x \, dx = \frac{1}{16} (\pi^2 + 4) \]