PENDULUM MOTION

SIMPLE PENDULUM

Question 1 (**)

A particle of mass m is attached to one end of a light inelastic string A of length l, and the other end of the string is attached to a fixed point O. The particle rests in equilibrium when it receives an impulse perpendicular to OA. Let θ be the angle that OA makes with the downward vertical.

a) Show that for small values of θ, the motion of the particle is approximately simple harmonic, stating its period in terms of l and g.

b) Given further that $l = 2.45$ m, calculate the frequency of the motion

$$\boxed{\nu = \frac{1}{\pi} \approx 0.318 \text{ s}^{-1}}$$

Question 2 (**+)

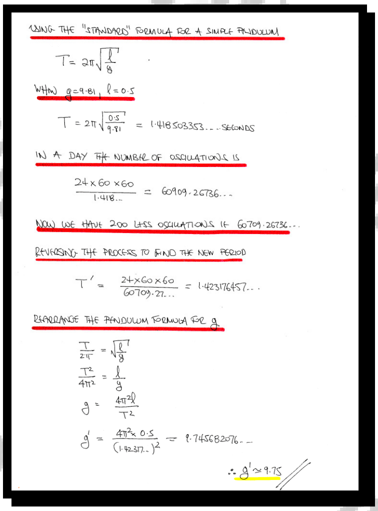
A simple pendulum consists of a heavy particle suspended from a fixed point by a light inextensible string of length 0.5 m.

We consider this pendulum performing small amplitude oscillations in the absence of any external forces, except its weight.

The pendulum is taken to a place where it performs in a day, 200 less small amplitude oscillations than to a place where $g = 9.81 \text{ ms}^{-2}$.

Determine the value of g in this place.

$\boxed{g' \approx 9.75 \text{ ms}^{-2}}$



USING THE "STANDARD" FORMULA FOR A SIMPLE PENDULUM

$$T = 2\pi\sqrt{\frac{l}{g}}$$

WHEN $g = 9.81$, $l = 0.5$

$$T = 2\pi\sqrt{\frac{0.5}{9.81}} = 1.418503353\ldots \text{ SECONDS}$$

IN A DAY THE NUMBER OF OSCILLATIONS IS

$$\frac{24 \times 60 \times 60}{1.418\ldots} = 60909.26736\ldots$$

NOW WE HAVE 200 LESS OSCILLATIONS IE $60709.26736\ldots$

REVERSING THE PROCESS TO FIND THE NEW PERIOD

$$T' = \frac{24 \times 60 \times 60}{60709.27\ldots} = 1.423176457\ldots$$

REARRANGE THE PENDULUM FORMULA FOR g

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

$$\frac{T^2}{4\pi^2} = \frac{l}{g}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$g' = \frac{4\pi^2 \times 0.5}{(1.42317\ldots)^2} = 9.745682076\ldots$$

$$\therefore g' \simeq 9.75$$

Question 3 (***)

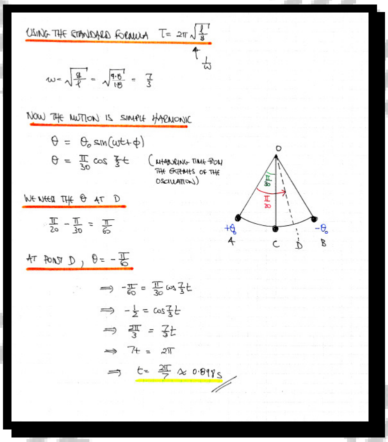
A simple pendulum consists of a heavy particle suspended from a fixed point O by a light inextensible string of length 1.8 m.

We consider this pendulum performing small amplitude oscillations in the absence of any external forces, except its weight.

The pendulum is released from rest with the string taut. On release the taut string forms an angle of $\dfrac{\pi}{30}$ radians with the downward vertical through O.

Calculate the time, from the instant of release, that the pendulum moves through an angle of $\dfrac{\pi}{20}$ radians.

$\boxed{}$, $\boxed{t = \frac{2}{7}\pi \approx 0.898 \text{ s}}$


USING THE STANDARD FORMULA $T = 2\pi\sqrt{\frac{l}{g}}$

$\uparrow \frac{1}{\omega}$

$\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{1.8}} = \frac{7}{3}$

NOW THE MOTION IS SIMPLE HARMONIC

$\theta = \theta_0 \sin(\omega t + \phi)$

$\theta = \frac{\pi}{30} \cos \frac{7}{3}t$ (MEASURING TIME FROM THE EXTREMES OF THE OSCILLATION)

WE NEED THE θ AT D

$\frac{\pi}{20} - \frac{\pi}{30} = \frac{\pi}{60}$

AT POINT D, $\theta = -\frac{\pi}{60}$

$\Rightarrow -\frac{\pi}{60} = \frac{\pi}{30}\cos\frac{7}{3}t$

$\Rightarrow -\frac{1}{2} = \cos\frac{7}{3}t$

$\Rightarrow \frac{2\pi}{3} = \frac{7}{3}t$

$\Rightarrow 7t = 2\pi$

$\Rightarrow t = \frac{2\pi}{7} \approx 0.898 \text{ s}$

Question 4 (***+)

A simple pendulum consists of a heavy particle suspended from a fixed point O by a light inextensible string of length 0.8 m.

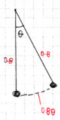
We consider this pendulum performing small amplitude oscillations in the absence of any external forces, except its weight.

The pendulum is released from rest with the string taut. On release the taut string forms an angle of $\frac{\pi}{20}$ radians with the downward vertical through O.

When the speed of the particle is 0.4 ms^{-1} the taut string forms an angle of θ radians with the downward vertical through O.

Calculate the value of θ.

$\boxed{}$, $\boxed{\theta \approx 0.0653^c}$



OBTAIN THE ANGULAR FREQUENCY FROM THE STANDARD FORMULA

$$T = 2\pi\sqrt{\frac{l}{g}} = \frac{2\pi}{\omega} \implies \omega = \sqrt{\frac{g}{l}} \implies \omega = \sqrt{\frac{9.8}{0.8}} \implies \omega = \frac{7}{2}$$

NOW USING THE S.H.M FORMULA

$$V^2 = \omega^2(a^2 - x^2)$$
$$0.4^2 = \left(\frac{7}{2}\right)^2\left[\left(0.8 \times \frac{\pi}{20}\right)^2 - (0.8\theta)^2\right]$$
$$0.4^2 = \left(\frac{7}{2}\right)^2 \times 0.8^2\left(\frac{\pi^2}{400} - \theta^2\right)$$
$$\frac{1}{49} = \frac{\pi^2}{400} - \theta^2$$
$$\theta^2 = \frac{\pi^2}{400} - \frac{1}{49}$$
$$\theta = 0.0653^c$$

Question 5 (***+)

A simple pendulum is modelled as a point mass attached at the end of a light inextensible string of length l.

a) Show that the period of the pendulum of small oscillations about its equilibrium position is given by

$$2\pi\sqrt{\frac{l}{g}},$$

stating clearly any approximations used.

A simple pendulum consists of a small heavy bob attached at the end of a light inextensible string of length 1.2 m.

The other end of the string is attached to a fixed point O so that the bob is hanging in equilibrium vertically below O.

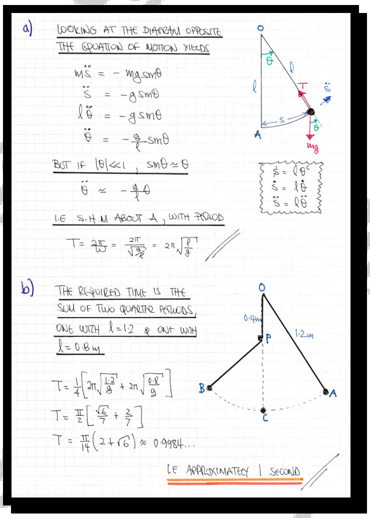
A small peg P is located at a distance of 0.4 m, vertically below O.

The bob is then displaced, so that the taut string makes a small angle with the downward vertical through O, and released from rest from the point A.

When in a vertical position, the string meets the peg and continues its small angle oscillations first coming to rest at the point B.

b) Show that the particle takes approximately 1 second to move from A to B.

$\boxed{}$, $\boxed{\text{proof}}$



Question 6 (***+)

A simple pendulum consists of a small heavy bob attached at the end of a light inextensible string of length 24.5 cm. The other end of the string is attached to a fixed point O so that the bob is hanging in equilibrium vertically below O.

The bob is then displaced, so that the taut string makes an angle of $\frac{\pi}{9}$ with the downward vertical through O, and released from rest.

In the subsequent motion, the angle the taut string makes with the downward vertical through O, is denoted by θ.

Calculate the time it takes for the pendulum to travel from $\theta = \frac{\pi}{18}$ to $\theta = \frac{\pi}{36}$.

$\boxed{}$, $\boxed{t \approx 0.0428 \text{ s}}$

LET THE MASS OF THE BOB BE m

THE EQUATION OF MOTION IS GIVEN BY

$$\Rightarrow \cancel{m}\ddot{s} = -\cancel{m}g\sin\theta$$
$$\Rightarrow l\ddot{\theta} = -g\sin\theta$$
$$\Rightarrow \ddot{\theta} = -\frac{g}{l}\sin\theta$$

FOR "SMALL" OSCILLATIONS, $\sin\theta \simeq \theta$

$$\Rightarrow \ddot{\theta} = -\frac{g}{l}\theta$$
$$\Rightarrow \ddot{\theta} = -\frac{9.8}{0.245}\theta$$
$$\Rightarrow \ddot{\theta} = -40\theta$$

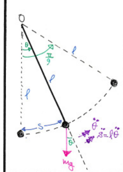
IE S.H.M WITH $\omega^2 = 40$, "ANGULAR AMPLITUDE" OF $\frac{\pi}{9}$

USING EQUATION WITH $t=0$ AT THE "POSITIVE" ENDPOINT

$$\Rightarrow x = a\cos\omega t$$
$$\Rightarrow \theta = \theta_0 \cos\omega t$$
$$\Rightarrow \theta = \frac{\pi}{9}\cos\left(\sqrt{40}\,t\right)$$

SOLVING FOR $\theta = \frac{\pi}{18}$

$$\Rightarrow \frac{\pi}{18} = \frac{\pi}{9}\cos\left(\sqrt{40}\,t\right)$$
$$\Rightarrow \frac{1}{2} = \cos\left(\sqrt{40}\,t\right)$$



$l = 24.5 \text{ cm}$
$l = 0.245 \text{ m}$

ALSO WE HAVE

$$s = l\theta$$
$$\dot{s} = l\dot{\theta}$$
$$\ddot{s} = l\ddot{\theta}$$

$$\Rightarrow \sqrt{40}\,t = \arccos\left(\tfrac{1}{2}\right) \quad \leftarrow \text{1ST POSITIVE SOLUTION}$$
$$\Rightarrow \sqrt{40}\,t = \frac{\pi}{3}$$
$$\Rightarrow t = \frac{\pi}{3\sqrt{40}} \approx 0.165576\ldots$$

SIMILARLY FOR $\theta = \pi/9$

$$\Rightarrow \frac{\pi}{36} = \frac{\pi}{9}\cos\left(\sqrt{40}\,t\right)$$
$$\Rightarrow \frac{1}{4} = \cos\left(\sqrt{40}\,t\right)$$
$$\Rightarrow \sqrt{40}\,t = \arccos\left(\tfrac{1}{4}\right) \quad \leftarrow \text{1ST POSITIVE SOLUTION}$$
$$\Rightarrow t = \frac{\arccos 1/4}{\sqrt{40}} \approx 0.208412\ldots$$

REQUIRED TIME IS

$$0.208412\ldots - 0.165576\ldots \simeq 0.0428 \text{ s}$$

COMPOUND PENDULUM