

Created by T. Madas

RELATIVE MOTION

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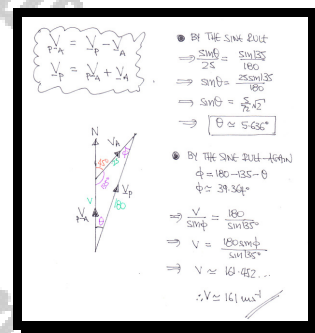
Question 1 ()**

An aeroplane is travelling at the same horizontal level. The speed of the aeroplane relative to the air is $v \text{ ms}^{-1}$ due north.

The air is blowing from south-west at 25 ms^{-1} .

Given the magnitude of the speed of the aeroplane relative to the ground is 180 ms^{-1} , determine the value of v .

$$v \approx 161 \text{ ms}^{-1}$$

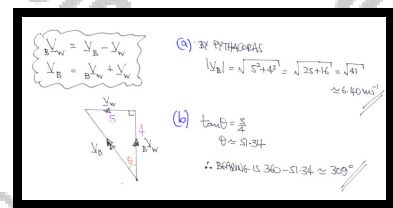


Question 2 ()**

As a boat moves, it travels at 4 ms^{-1} due north, relative to the water. The water is moving due west at 5 ms^{-1} .

- Find the magnitude of the velocity of the boat relative to the ground.
- Determine the bearing at which the boat is moving as viewed by a stationary observer outside the boat.

$$|\vec{v}_B| = \sqrt{41} \approx 6.40 \text{ ms}^{-1}, \quad \theta \approx 309^\circ$$



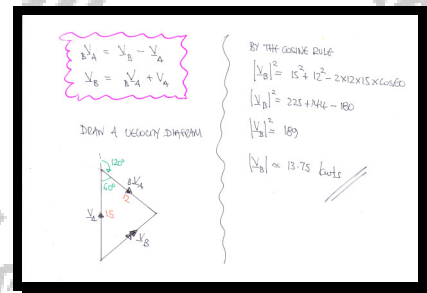
Question 3 ()**

A boat A is sailing at 15 knots due north.

To the captain of boat A another boat B is appearing to be sailing at 12 knots on a bearing of 120° .

Determine the actual speed of B.

speed ≈ 13.75 knots



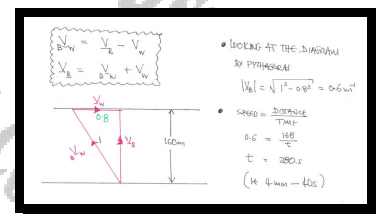
Question 4 ()**

A river which has parallel banks 168 m wide is flowing at constant speed 0.8 ms^{-1} .

A boy can capable of swimming at 1 ms^{-1} swims across at right angles to both banks.

Determine the time the boy takes to swim across the river.

280s



Question 5 (**)

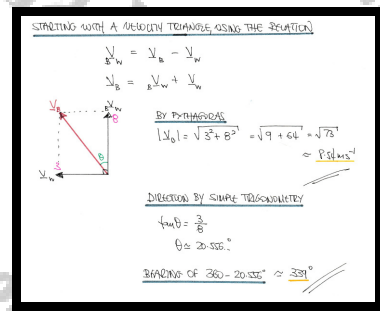
William is standing on the observation platform of a tall lighthouse.

He is observing a boat is sailing through water, which flowing due west at 3 ms^{-1} .

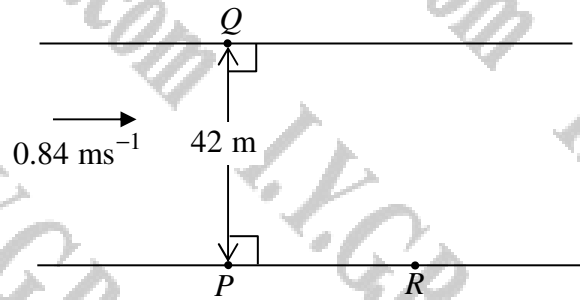
The velocity of the boat relative to the water is 8 ms^{-1} , due north.

Find the speed and direction, as a bearing, of the boat as it is observed by William.

$$\boxed{}, \quad |v| \approx 8.54 \text{ ms}^{-1}, \quad \theta \approx 339^\circ$$



Question 6 (**+)



The banks of a river are modelled as parallel lines of constant width 42 m. The river flows with constant speed of 0.84 ms^{-1} , throughout its width. The points P and Q are on opposite river banks so that $PQ = 42 \text{ m}$, as shown in the figure above.

Alex and Bradley are swimmers, both capable of swimming relative to the water with a speed of 1.4 ms^{-1} .

Alex sets from Q and decides to cross the river in the shortest possible time, and in doing so he reaches the opposite bank at the point R .

- a) Calculate the distance PR .

Bradley sets from P and decides to cross the river directly across reaching the opposite bank at the point Q .

- b) Calculate the time taken by Bradley to reach Q .

$$|PR| = 25.2 \text{ m}, \quad t = 37.5 \text{ s}$$

(a) $V_{A/W} = V_A - V_W$
 $V_A = V_{A/W} + V_W$
 To minimise time $V_{A/W}$ must be straight across.

 Thus $42 = 1.4 \times T$
 $T = 30 \text{ s}$
 Now to find how far Alex will be (downstream) stream by $0.84 \times 30 = 25.2 \text{ m}$

(b) To land across in a direct manner, then V_A must be straight across.

 By Pythagoras
 $|V_{A/W}| = \sqrt{1.4^2 - 0.84^2} = 1.12$
 so $42 = 1.12 \times T$
 $T = 37.5 \text{ s}$

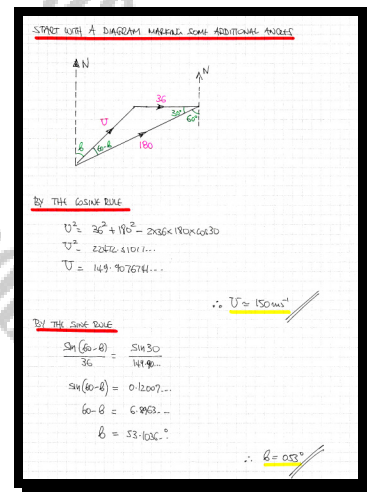
Question 7 (**+)

An aeroplane capable of speed $U \text{ ms}^{-1}$, is flying with this speed on a bearing of β .

As observed from the ground, due to the cross wind this plane is flying with a speed of 180 ms^{-1} , on a bearing of 60° .

If the cross wind is blowing from the west with a speed of 36 ms^{-1} , calculate the value of U and the value of β .

$$\boxed{50^\circ}, \quad \boxed{U \approx 149.91}, \quad \boxed{\beta \approx 053^\circ}$$

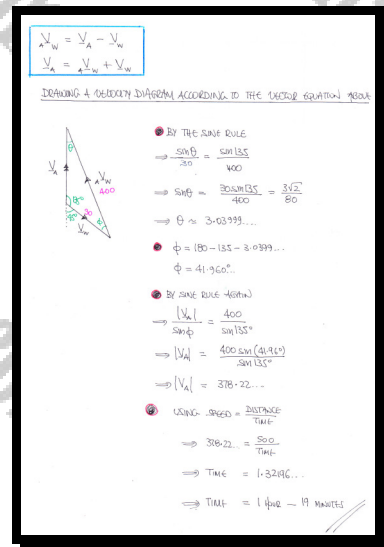


Question 8 (*)**

An aeroplane is travelling at the same horizontal level and in a northerly direction relative to the ground. The speed of the aeroplane relative to the air is 400 km h^{-1} . The air is blowing from north-west at 30 km h^{-1} .

Determine the time, in hours and minutes, it takes the aeroplane to cover 500 km .

$$t \approx 1 \text{ hour} - 19 \text{ minutes}$$



Question 9 (***)

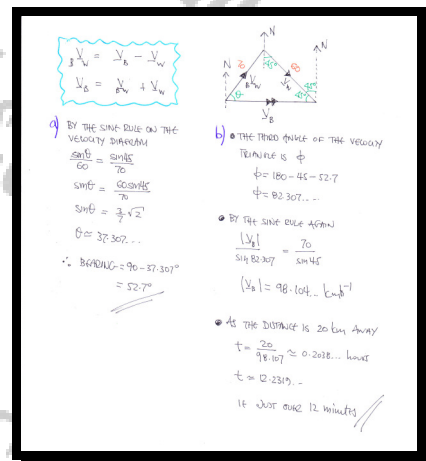
A bird is capable of flying at 70 km h^{-1} .

The bird wishes to fly to its nest which 20 km due East from its current position.

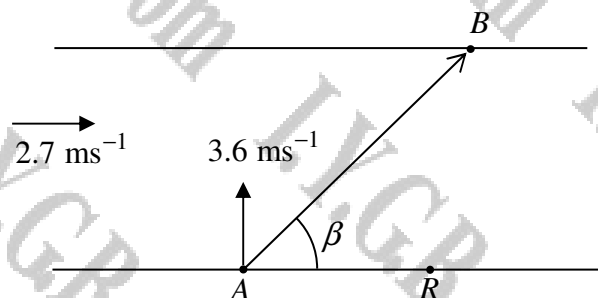
There is a wind blowing from North-West at 60 km h^{-1} .

- Find the direction, as a bearing, in which the bird must fly to reach its nest.
- Calculate the time, in minutes, for its journey.

$$\approx 52.7^\circ, \approx 12.23 \text{ min}$$



Question 10 (***)



The banks of a river are modelled as parallel lines of constant width. The river flows with constant speed of 2.7 ms^{-1} , throughout its width. A boat travels with constant velocity 3.6 ms^{-1} relative to the water, in a direction perpendicular to the river banks.

The boat starts at a point A on one river bank and ends up at a point B on the opposite river bank. The path AB forms an acute angle β with the river bank, as shown in the figure above.

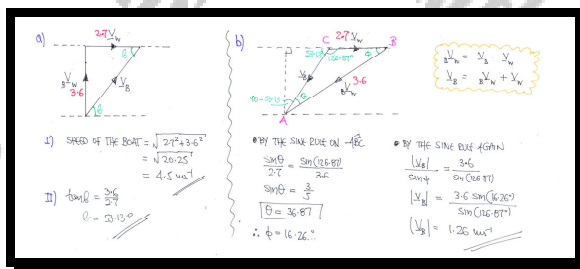
a) Calculate ...

- i. ... the speed of the boat as it travels from A to B .
- ii. ... the value of β .

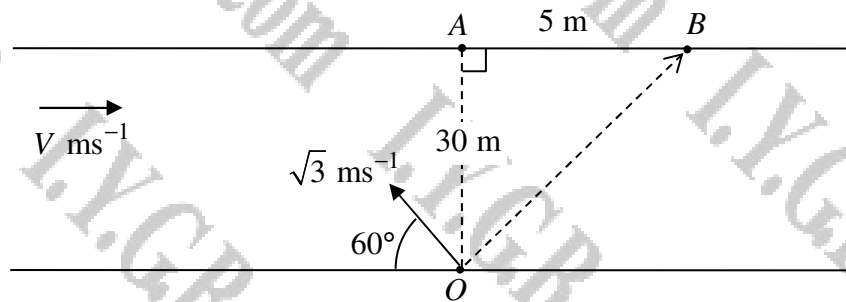
The boat returns from B to A , following exactly the same straight path.

- b) Determine the velocity of the boat when rowed back from B to A , given that it is still rowed with a constant velocity 3.6 ms^{-1} relative to the water.

$$v = 4.5 \text{ ms}^{-1}, \quad \beta = 53.13^\circ, \quad v = 1.26 \text{ ms}^{-1}$$



Question 11 (***)



A river flows with constant speed of $V \text{ ms}^{-1}$, throughout its width. The banks of the river are modelled as parallel lines of constant width of 30 m.

A boat starts at O and travels upstream with constant speed $\sqrt{3} \text{ ms}^{-1}$ relative to the water, in a direction of 60° to the river bank, as shown in the figure above. The point A is on the opposite river bank so that OA is perpendicular to both river banks.

The point B is on the same bank as A , so that $|AB| = 5 \text{ m}$, downstream. To an observer on one of the banks of the river the boat sails on a straight line from O to B .

- Calculate the time it takes the boat to travel from O to B .
- Determine the value of V , correct to two decimal places.

$$t = 20 \text{ s}, \quad V = 1.12 \text{ ms}^{-1}$$

Handwritten solution for Question 11:

Diagram: A river with parallel banks. A boat starts at point O on the bottom bank and travels upstream at a constant speed of $\sqrt{3} \text{ ms}^{-1}$ relative to the water, in a direction of 60° to the river bank. The boat reaches point A on the opposite bank. The distance OA is perpendicular to both river banks and is 30 m. Point B is on the same bank as A , so that $AB = 5 \text{ m}$, downstream. The river flows with a constant speed of $V \text{ ms}^{-1}$ to the right. A dashed line connects O to B .

Vector diagram: A vector triangle showing the boat's velocity \vec{v}_b relative to the water, the river's velocity \vec{v}_r , and the boat's velocity \vec{v}_B relative to the ground. The angle between \vec{v}_b and the horizontal is 60° . The angle between \vec{v}_B and the horizontal is θ . The magnitude of \vec{v}_b is $\sqrt{3}$.

Calculations:

a) First, the component of the velocity used to cross the river is $|v_{b_y}| \sin 60^\circ = \sqrt{3} \cdot \frac{\sqrt{3}}{2} = 1.5 \text{ ms}^{-1}$

Speed = $\frac{\text{Distance}}{\text{Time}} \Rightarrow 1.5 = \frac{30}{\text{Time}} \Rightarrow \text{Time} = 20 \text{ s}$

b) Next $\tan \psi = \frac{5}{30} = \frac{1}{6}$
 $\psi = \arctan \frac{1}{6}$
 $\psi \approx 9.462^\circ$
 $\Rightarrow 30^\circ + \psi \approx 39.462^\circ$
 $\Rightarrow \theta = 180^\circ - 39.462^\circ - 60^\circ$
 $\Rightarrow \theta = 80.538^\circ$

By the sine rule on the velocity triangle

$$\frac{\sqrt{3}}{\sin \theta} = \frac{V}{\sin \psi}$$

$$V = \frac{\sqrt{3} \sin 80.538^\circ}{\sin (9.462^\circ)}$$

$$V \approx 1.12 \text{ ms}^{-1}$$

Question 12 (*)**

The vectors \mathbf{i} , \mathbf{j} and \mathbf{k} are unit vectors mutually perpendicular to one another.

At time $t = 0$ s, the respective position vectors of two particles P and Q , relative to a fixed origin O , are $(-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$ m and $(-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ m.

P has constant velocity $(3\mathbf{i} + \mathbf{j})$ ms⁻¹ and Q has constant velocity $(\mathbf{i} - \mathbf{k})$ ms⁻¹.

Find the cosine of the angle POQ when the distance between P and Q is least.

 $\frac{3}{5}$

$\mathbf{r}_P = (-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) + (3\mathbf{i} + \mathbf{j})t = (3t - 6)\mathbf{i} + (4 + t)\mathbf{j} - 3\mathbf{k}$
 $\mathbf{r}_Q = (-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + (\mathbf{i} - \mathbf{k})t = (-2 + t)\mathbf{i} + 2\mathbf{j} + (3 - t)\mathbf{k}$
 $\mathbf{r}_P - \mathbf{r}_Q = (2t - 4)\mathbf{i} + t\mathbf{j} - t\mathbf{k}$
 $|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{(2t - 4)^2 + t^2 + (-t)^2}$
 $|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{4t^2 - 16t + 16 + t^2 + t^2}$
 $|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{6t^2 - 16t + 16}$
 By calculus or completing the square
 $|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{6(t^2 - \frac{8}{3}t + \frac{16}{3})}$
 $|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{6((t - \frac{4}{3})^2 + \frac{32}{9})}$
 $|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{6(t - \frac{4}{3})^2 + 32}$
 \therefore Particles are closest when $t = \frac{4}{3}$
 At that time $\mathbf{r}_P = (0, \frac{16}{3}, -3)$
 $\mathbf{r}_Q = (-\frac{2}{3}, 2, 2)$
 By dot product involving $\hat{\mathbf{i}}$ component
 $(\mathbf{r}_P - \mathbf{r}_Q) \cdot (\mathbf{r}_P - \mathbf{r}_Q) = |\mathbf{r}_P - \mathbf{r}_Q|^2 = |\mathbf{r}_P|^2 |\mathbf{r}_Q|^2 \cos \theta$
 $12 - 3 = \sqrt{36 + 16} \sqrt{4 + 4} \cos \theta$
 $9 = \sqrt{52} \sqrt{8} \cos \theta$
 $\cos \theta = \frac{3}{5}$

Question 13 (***)

Two straight horizontal roads meet at right angles at a junction O .

One of the roads is directed south to north and the other west to east.

A cyclist is travelling north on the first road at constant speed 6 ms^{-1} and at time $t = 0 \text{ s}$ is 200 m south of O .

A car is travelling west on the second road at constant speed 24 ms^{-1} and at time $t = 0 \text{ s}$ is 960 m east of O .

Determine the shortest distance between the cyclist and the car if they continue to move at the above described fashion.

$$d_{\min} = \frac{160}{\sqrt{17}} \approx 38.81...$$

	Position	Velocity
A	$-200\hat{j}$	$6\hat{j}$
B	$960\hat{i}$	$-24\hat{i}$

$\bullet \vec{r}_A = (0t - 200) + t(6\hat{j}) = (0, 6t - 200)$
 $\bullet \vec{r}_B = (960t) + t(-24\hat{i}) = (960 - 24t, 0)$
 $\Rightarrow \vec{r}_A - \vec{r}_B = (0, 6t - 200) - (960 - 24t, 0)$
 $\Rightarrow \vec{r}_A - \vec{r}_B = (24t - 960, 6t - 200)$
 $\Rightarrow |\vec{r}_A - \vec{r}_B| = \sqrt{(24t - 960)^2 + (6t - 200)^2}$
 $\bullet \text{Let } f(t) = (24t - 960)^2 + (6t - 200)^2$
 $\Rightarrow f'(t) = 48(24t - 960) + 12(6t - 200)$
 $\bullet \text{Solve for zero}$
 $\Rightarrow 48(24t - 960) + 12(6t - 200) = 0$
 $\Rightarrow 2(24t - 960) + (6t - 200) = 0$
 $\Rightarrow 48t - 1920 + 6t - 200 = 0$
 $\Rightarrow 54t = 2120$
 $\Rightarrow t = \frac{2120}{54}$ (which yields a min as this is a quadratic)

$\bullet \text{Hence } d_{\min} = \sqrt{(24 \times \frac{2120}{54} - 960)^2 + (6 \times \frac{2120}{54} - 200)^2}$
 $= \sqrt{(-\frac{480}{9})^2 + (-\frac{640}{9})^2}$
 $= \frac{48^2 200}{289} + \frac{25600}{17}$
 $\therefore d_{\min} = \frac{160}{\sqrt{17}} \approx 38.81$

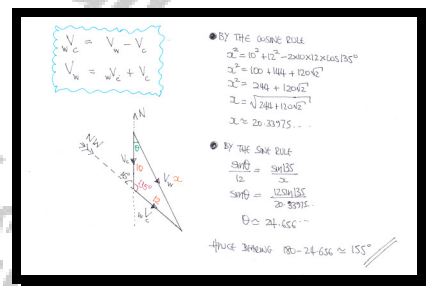
Question 14 (*)**

A cyclist is travelling in a southerly direction at a constant speed 10 km h^{-1} , on a straight horizontal road.

To the cyclist the wind appear to be blowing from the north-west with a constant horizontal speed of 12 km h^{-1} .

Determine, as a three figure bearing, the direction from which the wind is blowing.

155°



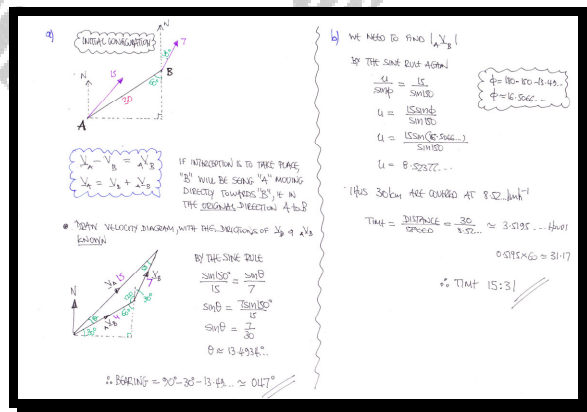
Question 15 (*)**

At noon, two ships A and B are 30 km apart, with A on a bearing of 240° from B .

Ship B is moving at 7 km h^{-1} on a bearing of 030° . The maximum speed of A is 15 km h^{-1} . Ship A sets a course to intercept B as soon as possible.

- Find the course set by A , giving the answer as a bearing to the nearest degree.
- Determine the time at which A intercepts B .

$$\approx 047^\circ, 15:31$$



Question 16 (***)

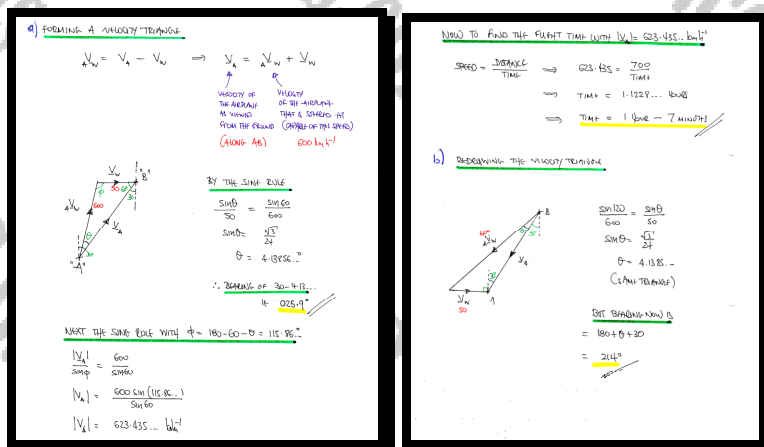
An airplane flying at 600 km h^{-1} in still air travels directly from A to B .

The point B is 700 km away from A , on a bearing 030° from A .

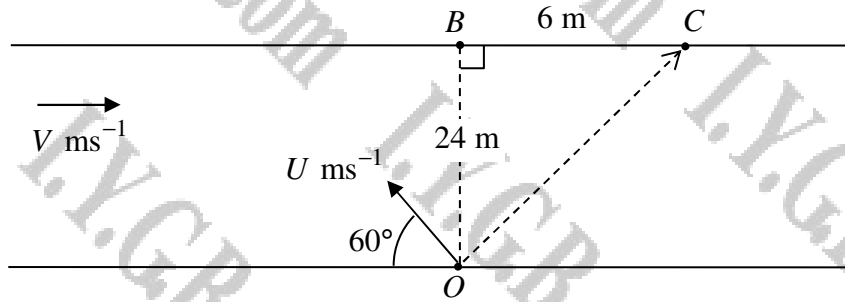
There is a steady wind, blowing from the west with a speed of 50 km h^{-1} , throughout the flight.

- Determine, as a bearing, the course the pilot should steer the airplane in order to travel directly from A to B , and hence calculate the flight time.
- Find, as a bearing, the course the pilot should steer the airplane in order to travel directly from B to A , assuming the wind is blowing steadily from the west with the same speed.

60, 025.9° , 1 hour, 7 minutes, 234°



Question 17 (*)**



A river flows with constant speed of $V \text{ ms}^{-1}$, throughout its width. The banks of the river are modelled as parallel lines of constant width of 24 m.

A boat starts at O and travels upstream with constant speed $U \text{ ms}^{-1}$ relative to the water, in a direction of 60° to the river bank, as shown in the figure above. The point B is on the opposite river bank so that OB is perpendicular to both river banks.

The point C is on the same bank as B , so that $|BC| = 6$ m, downstream.

To a stationary observer on one of the banks of the river the boat sails on a straight line from O to C .

Show that $V = \frac{1}{9}(3 + 4\sqrt{3})$.

, proof

● SPINDLING WITH A DISK

● CONSTANT ANGULAR VELOCITY

⇒ $\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$

⇒ $\omega_{\text{spool}} = \frac{2\pi}{18}$

⇒ $\frac{V_1}{r_1} = \frac{V_2}{r_2}$

⇒ $\omega = \frac{8}{3\sqrt{3}}$

● BY SIMILAR GEOMETRY ON CIRC

$\tan \theta = \frac{3}{4}$

$\sin \theta = \frac{3}{5}$

$\cos \theta = \frac{4}{5}$

● FINALLY WE HAVE BY THE SAME RULE ON THE TOP TRIANGLE BELOW

$\frac{V}{\sin(\theta + \phi)} = \frac{V'}{\sin \theta}$

$\Rightarrow V' = \frac{V \sin(\theta + \phi)}{\sin \theta}$

$\Rightarrow V' = V \left[\frac{\sin(\theta + \phi)}{\sin \theta} \right]$

$\Rightarrow V' = \frac{8}{3\sqrt{3}} \left[\frac{\sin \theta \cos \phi + \cos \theta \sin \phi}{\sin \theta} \right]$

$\Rightarrow V' = \frac{8}{3\sqrt{3}} \left[\frac{16}{5} + \frac{1}{2} \right]$

$\Rightarrow V' = \frac{1}{2} + \frac{4}{3\sqrt{3}} = \frac{3 + 4\sqrt{3}}{3}$

Question 18 (***)

At 14.00 hours a coastguard patrol ship first sights an unidentified boat, 12 km away on a bearing of 210° .

The unidentified boat is sailing at 18 km h^{-1} on a bearing of 290° .

The coastguard patrol ship is sailing at 36 km h^{-1} .

- Find, as a bearing, the course at which the coastguard patrol ship should steer in order to intercept the unidentified boat.
- Calculate the time at which the interception will take place.

, $\approx 236^\circ$, 14:31

a) SPEEDING WITH A VECTOR - P = PATROLSHIP A = UNIDENTIFIED BOAT

$V_P = V_A - V_B$
 $V_C = V_B + V_A$

THE V_C MUST BE ALONG PB, SO THAT AN COURSE ON B WISE P WOULD INTERCEPT BOAT AT 14:00

SEEK A UNUSUAL TRICK

$\sin \theta = \frac{18 \sin 80}{36}$
 $\sin \theta = \frac{1}{2}$
 $\theta = 25.69^\circ$
 \therefore BEARING OF $210 + 25.69^\circ$
BEARING OF 236°

b) FIND THE RELATIVE VELOCITY, LABELED α , BY THE SINE RULE

$\phi = 180 - 9 - 120$
 $\phi = 60 - 9$
 $\phi = 51.41^\circ$

SAME RULE AGAIN

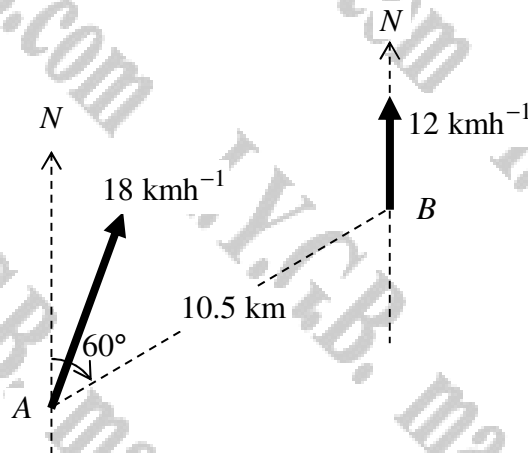
$\frac{\alpha}{\sin \theta} = \frac{36}{\sin 20}$
 $\alpha = \frac{36 \sin \theta}{\sin 20}$
 $\alpha = \frac{36 \sin (25.69)}{\sin (20)}$
 $\alpha = 23.44 \text{ km h}^{-1}$

THAT THE PATROL SHIP HAS TO COVER 12 km AT THIS SPEED

$T = \frac{12}{23.44}$
 $T = 0.517 \dots$ HOURS
 $T = 30.7 \text{ MINUTES}$
 $T \approx 31 \text{ MINUTES}$

\therefore INTERCEPTION TIME AT 14:31

Question 19 (***)



At noon two boats A and B are 10.5 km apart with B on a bearing 060° from A .

Boat B is travelling due North with a constant speed of 12 kmh^{-1} . Boat A is capable of a maximum speed of 18 kmh^{-1} and sets on a course to intercept A .

- Calculate the bearing at which A must travel in order to intercept A in the least possible time.
- Given instead that A travels on a bearing of 060° determine ...
 - ... the closest distance between A and B .
 - ... the time it takes for the two boats to get closest together.

$$\approx 025^\circ, \quad d_{\min} = \frac{3}{2}\sqrt{21} \approx 6.87 \text{ km}, \quad 12:30$$

(a)

• FOR INTERCEPTION THE RELATIVE VELOCITY $V_{A/B}$ MUST ALWAYS BE IN THE DIRECTION AB THEREFORE A & B SHARE THE INITIAL (CONFIGURATION) POSITION

• $V_A = V_B + V_{A/B}$

• BY SINE RULE

$$\frac{\sin \theta}{12} = \frac{\sin 120}{18} \Rightarrow \sin \theta = \frac{2}{3} \sin 120$$

$$\sin \theta = \frac{\sqrt{3}}{3}$$

$$\theta \approx 35.26^\circ$$

SO BEARING IS $60 - \theta = 60 - 35.26^\circ = 24.74^\circ \approx 025^\circ$

(b)

$V_A = V_B + V_{A/B}$ AND THE VELOCITY TRIANGLE HAS SIDES 12, 18, 10.5

• BY THE COSINE RULE

$$10.5^2 = 18^2 + 12^2 - 2 \times 18 \times 12 \times \cos 60^\circ$$

$$10.5^2 = 252$$

$$10.5 = \sqrt{252} \approx 15.874 \dots$$

• BY THE SINE RULE

$$\frac{\sin \theta}{12} = \frac{\sin 60}{15.874}$$

$$\sin \theta = \frac{12 \times \sin 60}{15.874}$$

$$\theta \approx 46.99^\circ$$

SO FINDING B THIS IS WHERE B IS CLOSEST

SO FINDING B THIS IS WHERE B IS CLOSEST

(i) \therefore SHORTEST DISTANCE = $|BC| = 10.5 \sin \theta = 10.5 \times \frac{1}{\sqrt{3}} = \frac{3\sqrt{7}}{2} \approx 6.87 \text{ km}$

(ii) $|AC| = 10.5 \cos \theta = 10.5 \times \cos(46.99^\circ) = 10.5 \times \frac{2}{3} = 7 \text{ km}$

TIME = $\frac{|AC|}{|V_A|} = \frac{7}{18} = 0.388 \dots \therefore \text{AT } 12:30$

Question 20 (*)**

The radar of a battleship detects a destroyer, 50 km North of the battleship.

The destroyer is moving on a bearing of 120° with constant speed 40 km h^{-1} .

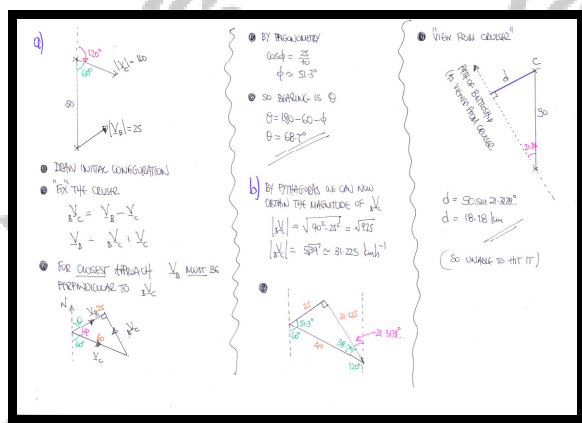
The maximum speed of the battleship is 25 km h^{-1} and on detecting the destroyer, it heads on a bearing θ° with maximum speed, in order to get as close as possible to the destroyer.

- a) Find the value of θ .

The guns of the battleship have a range of 10 km.

- b) Determine whether the destroyer gets within the range of the battleship's guns.

$$\theta \approx 68.7^\circ, \quad d_{\min} \approx 18.2 \text{ km} > 10 \text{ km}$$



Question 21 (***)

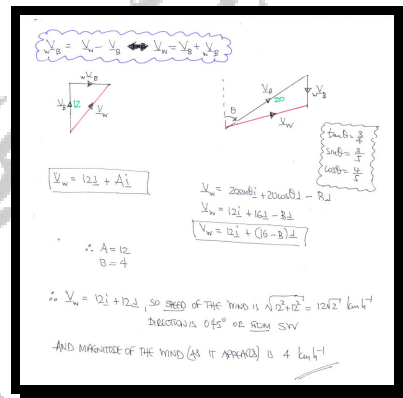
When a boat is sailing due North with constant speed 12 km h^{-1} , the wind appears to the crew on the boat to be blowing from the West.

The boat increases its speed to 20 km h^{-1} and changes its direction to a bearing θ° , where $\tan \theta = \frac{3}{4}$. The wind now appears to the crew on the boat to be blowing from the North.

Assuming the true velocity of the wind is the same throughout the boat's journey determine in any order ...

- ... the **true** speed of the wind.
- ... the **true** direction of the wind.
- ... the **apparent** speed of the wind when the boat is sailing at 20 km h^{-1} on a θ° bearing.

$$\left[|v_w| = 12\sqrt{2} \approx 17.0 \text{ km h}^{-1}, \text{ } 045^\circ \text{ or from SW}, \left[|v_w| = 4 \text{ km h}^{-1}\right.\right.$$



Question 22 (***)

A ship A is sailing due east with constant speed 20 km h^{-1} .

At 15.00 hours, another ship B is 11 km away on a bearing of 160° from A .

Find the latest time by which B can intercept A , assuming that B will set on such course with constant speed 19 km h^{-1} .

, 17:43

SHIP WITH A CONFIGURATION DIAGRAM AND A VELOCITY TRIANGLE

For interception the observation of "A" will be seen as "B" heading directly "towards" "A" at all times, so v_{AB} must be along BA.

BY THE SINE RULE

$$\frac{\sin \phi}{20} = \frac{\sin 70^\circ}{19}$$

$$\sin \phi = 0.9811 \dots$$

$$\phi = 81.5522 \dots \text{ or } 98.4477 \dots$$

For "largest time" we need ϕ to be smaller, so ϕ will be obtuse

$$\phi = 180 - 70 - 98.4477 \dots$$

$$\phi = 11.5522 \dots$$

BY THE SINE RULE AGAIN

$$\frac{|v_A|}{\sin \phi} = \frac{|v_B|}{\sin 70^\circ} \Rightarrow |v_B| = \frac{\sin \phi}{\sin 70^\circ} \times 19$$

$$|v_B| = 4.0494463 \dots \text{ km h}^{-1}$$

NOW B HAS TO COVER THE DISTANCE $|AB|$ WITH CONSTANT SPEED $|v_B| = 4.04 \dots$

$$\Rightarrow \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow 4.04 \dots = \frac{11}{\text{Time}}$$

$$\Rightarrow \text{Time} = \frac{11}{4.04 \dots} = 2.716 \dots \text{ hours}$$

$$\Rightarrow \text{Time} = 2 \text{ hours } 43 \text{ minutes}$$

4 hr 17:43

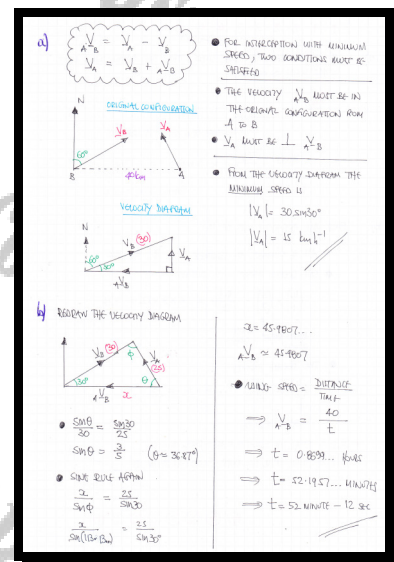
Question 23 (***)

A ship B is moving on a bearing 060° at constant speed 30 km h^{-1} .

Another ship A moving with constant speed $V \text{ km h}^{-1}$ sets on a course to intercept B , when A gets to a position 40 km east of B .

- Find the minimum value of V , required for interception.
- Given further that $V = 25$, determine the time it takes A to intercept B .

52' - 12"



Question 24 (***)

When a jogger is running due North with constant speed 4 ms^{-1} , the wind appears to him to be blowing from the West.

When the jogger is running due North with constant speed 8 ms^{-1} , the wind appears to him to be blowing from the North West.

Assuming the true velocity of the wind is the same throughout the joggers run, determine in any order ...

- ... the **true** speed of the wind.
- ... the **true** direction of the wind.

$$|v_w| = 4\sqrt{2} \approx 5.66 \text{ ms}^{-1}, \quad 045^\circ \text{ or from SW}$$

Handwritten solution for Question 24:

Let v_w be the true wind velocity, v_j the jogger's velocity, and v_{wj} the apparent wind velocity.

Case 1: Jogger runs at 4 ms^{-1} due North. Apparent wind is from the West, so v_{wj} is due East.

Case 2: Jogger runs at 8 ms^{-1} due North. Apparent wind is from the North West, so v_{wj} is from the North West (45° from North).

Vector equations:

$$v_{wj} = v_w - v_j \quad \Leftrightarrow \quad v_w = v_{wj} + v_j$$

For Case 1: $v_w = v_{wj1} + v_{j1}$. $v_{j1} = 4\mathbf{i}$, $v_{wj1} = B\mathbf{j}$ (where B is the speed from the West).

For Case 2: $v_w = v_{wj2} + v_{j2}$. $v_{j2} = 8\mathbf{i}$, v_{wj2} is from the North West, so $v_{wj2} = \frac{8}{\sqrt{2}}\mathbf{i} + \left(8 - \frac{8}{\sqrt{2}}\right)\mathbf{j}$.

Equating the two expressions for v_w :

$$B\mathbf{j} + 4\mathbf{i} = \frac{8}{\sqrt{2}}\mathbf{i} + \left(8 - \frac{8}{\sqrt{2}}\right)\mathbf{j}$$

Equating components:

$$4 = \frac{8}{\sqrt{2}} \quad \Rightarrow \quad B = 8 - \frac{8}{\sqrt{2}}$$

Wait, the handwritten solution shows a different approach. Let's follow the handwritten steps:

From Case 1: $v_w = B\mathbf{j} + 4\mathbf{i}$

From Case 2: $v_w = \frac{8}{\sqrt{2}}\mathbf{i} + \left(8 - \frac{8}{\sqrt{2}}\right)\mathbf{j}$

Equating the two expressions for v_w :

$$B\mathbf{j} + 4\mathbf{i} = \frac{8}{\sqrt{2}}\mathbf{i} + \left(8 - \frac{8}{\sqrt{2}}\right)\mathbf{j}$$

Equating components:

$$4 = \frac{8}{\sqrt{2}} \quad \Rightarrow \quad B = 8 - \frac{8}{\sqrt{2}}$$

Wait, the handwritten solution shows:

$$4 = \frac{8}{\sqrt{2}} \quad \Rightarrow \quad B = 4\sqrt{2}$$

Then the magnitude of v_w is:

$$|v_w| = \sqrt{(4\sqrt{2})^2 + 4^2} = \sqrt{32 + 16} = \sqrt{48} = 4\sqrt{3} \approx 6.93 \text{ ms}^{-1}$$

Wait, the handwritten solution shows:

$$|v_w| = 4\sqrt{2} \approx 5.66 \text{ ms}^{-1}$$

Direction: 045° or from SW.

Question 25 (***)

A coastal base C is on bearing 045° from an army airport base B , 150 km away.

As part of a training exercise, a plane leaves B on a direct path to C . On reaching C , the plane immediately returns directly back to B , along the same path.

The plane is flying with constant speed 700 km h^{-1} relative to the air. During the entire flight there is a wind blowing from a bearing of 105° , with speed 50 km h^{-1} .

Determine the flight time in minutes and seconds.

, 25 minutes – 45 seconds

• STARTING WITH THE STANDARD EQUATION FOR VELOCITY TRIANGLES

$$\vec{V}_A = \vec{V}_P - \vec{V}_W$$

$$\vec{V}_P = \vec{V}_A + \vec{V}_W$$

V_P = PLANE VELOCITY
 V_W = WIND VELOCITY

• FOR THE FIRST PART OF THE JOURNEY

• BY THE COSINE RULE

$$700^2 = 50^2 + 150^2 - 2 \times 50 \times 150 \times \cos 135^\circ$$

$$\Rightarrow 490000 = 2500 + 22500 - 15000\sqrt{2}$$

$$\Rightarrow 0 = 22500 - 15000\sqrt{2}$$

• USING THE QUADRATIC FORMULA

$$22500 = 15000\sqrt{2}$$

$$\sqrt{2} = \frac{22500}{15000} = 1.5$$

$$\sqrt{2} = 1.5$$

• REARRANGING FOR THE OTHER JOURNEY AND USING THE COSINE RULE AGAIN

$$700^2 = 50^2 + 150^2 - 2 \times 50 \times 150 \times \cos 135^\circ$$

$$\Rightarrow 490000 = 2500 + 22500 - 15000\sqrt{2}$$

$$\Rightarrow 0 = 22500 - 15000\sqrt{2}$$

• SOLUTION BY THE QUADRATIC FORMULA AND VECTORS

$$y = -25 \pm 25\sqrt{2}$$

$y = 67.434 \dots$
 $y = -72.434 \dots$

• FIND THE REQUIRED TIME FOR BOTH JOURNEYS IS

$$T = \frac{150}{x} + \frac{150}{y} = \frac{150}{25 + 25\sqrt{2}} + \frac{150}{-25 + 25\sqrt{2}}$$

$$= \frac{6}{1 + \sqrt{2}} + \frac{6}{-1 + \sqrt{2}}$$

$$= 6 \left[\frac{1}{\sqrt{2} + 1} + \frac{1}{\sqrt{2} - 1} \right]$$

$$= \frac{6 \times 2\sqrt{2}}{2 - 1}$$

$$= \frac{12\sqrt{2}}{1}$$

$$\approx 16.97 \dots \text{ hours}$$

$$\approx 25 \text{ minutes} - 45 \text{ seconds}$$

Question 28 (***)

The radar of a battleship detects a destroyer, 50 km west of the battleship.

The destroyer is moving on a bearing of 30° with constant speed 40 km h^{-1} .

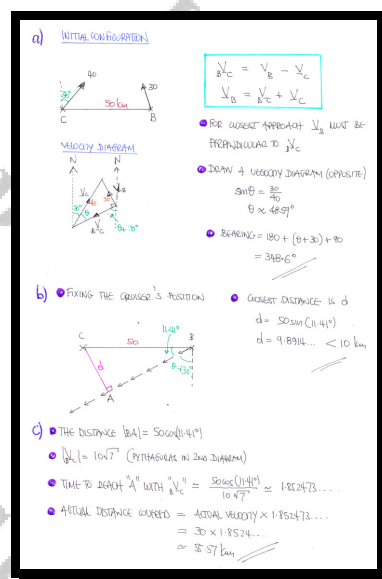
The maximum speed of the battleship is 30 km h^{-1} and on detecting the destroyer, it heads on a bearing θ° with maximum speed, in order to get as close as possible to the destroyer.

- a) Find the value of θ .

The guns of the battleship have a range of 10 km.

- b) Determine whether the destroyer gets within the range of the battleship's guns.
c) Calculate the actual distance the battleship covers from the instant it sets in pursuit of the destroyer until it gets as close to it.

$$\theta \approx 348.6^\circ, \quad d_{\min} \approx 9.89 \text{ km} < 10 \text{ km}, \quad \approx 55.57 \text{ km}$$



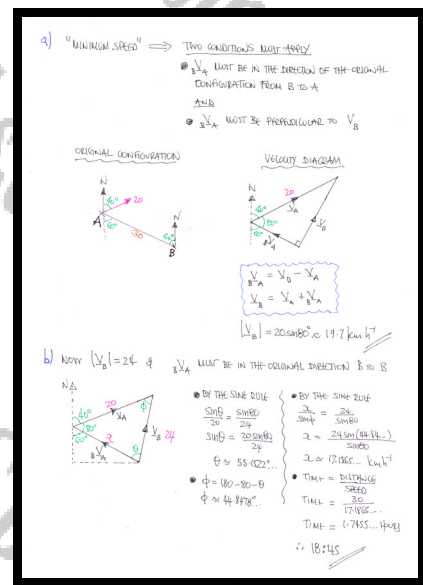
Question 29 (***)

A ship A is travelling at a constant speed of 20 km h^{-1} on a bearing of 040° .

At 17.00 hours, another ship B is 30 km away from A and the bearing of A from B is 300° . Ship B is travelling at a constant speed of $U \text{ km h}^{-1}$ and sets on a course to intercept A .

- Find the least possible value of U .
- Given that $U = 24$, determine the earliest time at which B intercepts A .

$$U \approx 19.7, 18:45$$



Question 30 (***)

At time $t = 0$ two walkers A and B are 250 m apart with B due south of A . The park in which they are taking their walk is wide and its grounds are completely flat.

A is walking due east with constant speed 1.6 ms^{-1} .

B walks with constant speed 1.5 ms^{-1} in a straight line and in such a way so that he passes as close as possible to B .

- a) Find, as a bearing, the direction of the path of B .

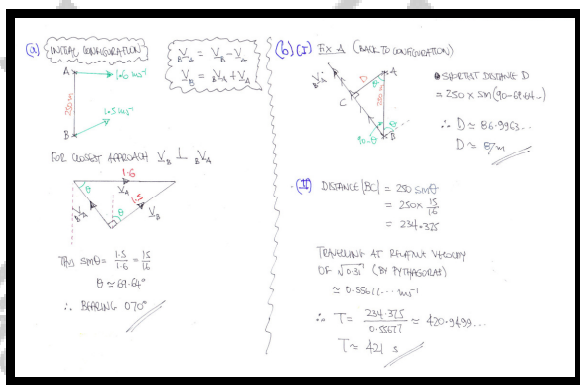
The two walkers are at their closest distance together, D m, at time T s.

- b) Calculate, in any order, ...

i. ... the value of D .

ii. ... the value of T .

$$\approx 070^\circ, \quad D \approx 87.0 \text{ m}, \quad T \approx 421 \text{ s}$$

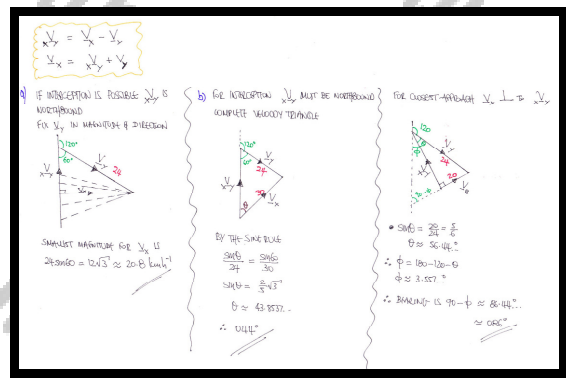


Question 31 (***)

A yacht Y is moving with constant speed 24 km h^{-1} on a straight line course of bearing 120° . At a given instant, a patrol boat X , is due south of Y and sets on a straight line course with constant speed $U \text{ km h}^{-1}$ to intercept Y .

- Calculate the minimum value of U so that X can intercept Y .
- Given that $U = 30$ determine the bearing that X must move on so that interception takes place.
- Given instead that $U = 10$ find the bearing that X must move on so that it passes as close as possible to Y .

$$U_{\min} = 12\sqrt{3} \approx 20.8 \text{ km h}^{-1}, \quad \approx 044^\circ, \quad \approx 086^\circ$$



Question 32 (***)

A patrol boat is due south of a fishing trawler. The trawler is sailing at constant speed of 12 km h^{-1} on a bearing 120° .

The patrol boat decides to intercept the trawler and travels in a straight line with constant speed $U \text{ km h}^{-1}$.

- Find the minimum value of U .
- Given instead that $U = 22$, determine ...
 - ... the bearing of the course that the patrol must take to intercept the trawler.
 - ... the distance the trawler and the distance the patrol boat cover until the interception takes place.

$$U_{\min} = 6\sqrt{3} \approx 10.39 \text{ km h}^{-1}, \quad \approx 028^\circ, \quad D_P \approx 3.47 \text{ km}, \quad D_T \approx 1.89 \text{ km}$$

Q $V_P = V_T - V_R$
 $V_P = V_T + V_R$
 THE INTERSECTION V_P MUST BE IN THE DIRECTION OF THE INITIAL CONVECTION IS DUE NORTH

Diagram showing vectors V_T (Trawler) and V_P (Patrol boat) with V_R (Relative velocity) as the resultant.

BY THE SINE RULE
 $\frac{U}{\sin 60^\circ} = \frac{12}{\sin \theta}$
 $U = \frac{12 \sin 60^\circ}{\sin \theta}$
 $U = \frac{12 \times \frac{\sqrt{3}}{2}}{\sin \theta}$
 $\therefore U_{\min} \text{ occurs when } \sin \theta = 1$
 $\therefore U_{\min} = 6\sqrt{3} \approx 10.39 \text{ km h}^{-1}$
 (IN THIS CASE $\theta = 90^\circ$ SO V_{\min} IS DUE EAST)

6/6 NOW FOR INTERCEPTION WITH $V_P = U = 22$

Diagram showing vectors V_T and V_P with V_R as the resultant.

BY THE SINE RULE
 $\frac{22}{\sin \theta} = \frac{12}{\sin 60^\circ}$
 $\sin \theta = \frac{12 \sin 60^\circ}{22} = \frac{6\sqrt{3}}{11}$
 $\theta \approx 28.2^\circ$
 \therefore BEARING 028°

BY THE SINE RULE AGAIN
 $\frac{a}{\sin \theta} = \frac{22}{\sin 60^\circ}$
 $a = \frac{22 \sin \theta}{\sin 60^\circ}$
 $a = 25.39$
 $\therefore V_P = 25.39 \text{ km h}^{-1}$

SO APPROX. DISTANCES
 • PATROL BOAT
 $0.1514 \times 22 \approx 3.47 \text{ km}$
 • TRAWLER
 $0.1514 \times 12 \approx 1.89 \text{ km}$

SO $t = \frac{a}{V_P} = \frac{25.39}{22} \times T$
 $T = 0.1514 \dots \text{ hours}$

Question 33 (***)

At noon a frigate is 18 km away from a ship and at that time the bearing of the frigate relative to the ship is 120° .

The ship is sailing east at a constant speed of 20 km h^{-1} .

- a) Determine the minimum speed with which the frigate can intercept the ship.

The frigate sets off to intercept the ship by sailing at a constant speed of 15 km h^{-1} .

- b) Calculate, to the nearest degree, the two possible bearings which the frigate can follow, and hence find the shorter of the two possible interception times, correct to the nearest minute.

, $V_{\min} = 10 \text{ km h}^{-1}$, $\theta_1 \approx 78^\circ$, $\theta_2 \approx 342^\circ$, $t \approx 38 \text{ minutes}$

a) START WITH THE INITIAL CONFIGURATION

FOR INTERCEPTION THE VELOCITY OF THE FRIGATE MUST BE PERPENDICULAR TO THE RELATIVE VELOCITY BETWEEN THE TWO VESSELS

LET THE SHIP BE "FIXED" - THEN AN OBSERVER ON THE SHIP WOULD BE SEEING THE FRIGATE TRAVELLING DIRECTLY TOWARDS THEM, IF ALONG THE LINE JOINING THE TWO VESSELS - HENCE WE HAVE

$|V_r| = 20 \sin 30^\circ = 10 \text{ km h}^{-1}$

b) FOR INTERCEPTION AGAIN AN OBSERVER ON THE SHIP WILL BE SEEING THE FRIGATE TRAVELLING DIRECTLY TOWARDS THE SHIP ALONG THE LINE JOINING THEM BUT V_f IS NOT PERPENDICULAR TO V_r

Case A: $\theta_1 \approx 78^\circ$
Case B: $\theta_2 \approx 342^\circ$

BY THE SINE RULE IN EACH OF THE TWO CASES

$\frac{\sin 30^\circ}{15} = \frac{\sin \theta}{20}$
 $\Rightarrow \sin \theta = \frac{2}{3}$
 $\Rightarrow \theta = \begin{cases} 41.81^\circ & \leftarrow \text{CASE A (SHORTEST INTERCEPTION)} \\ 138.19^\circ & \leftarrow \text{CASE B (LONGEST INTERCEPTION)} \end{cases}$

$\Rightarrow \text{BEARINGS} \begin{cases} 20^\circ + 30^\circ + 41.81^\circ \approx 91.81^\circ \\ 30^\circ + 0^\circ - 90^\circ \approx -60^\circ \end{cases}$

NOW USING THE SINE RULE IN CASE A TO FIND $|V_r|$

$\frac{|V_r|}{\sin(180 - 30 - 41.81)} = \frac{15}{\sin 30^\circ}$ CASE A, $\theta = 41.81^\circ$
 $|V_r| = \frac{15 \sin(108.19^\circ)}{\sin 30^\circ}$
 $|V_r| = 28.5004 \dots \text{ km h}^{-1}$

NOW USING THIS SPEED, THE FRIGATE WILL HAVE TO COVER THE INITIAL DISTANCE OF 18 km

$T = \frac{18}{28.5004 \dots} = 0.63156 \dots \text{ hours} \times 60$
 $\approx 38 \text{ minutes}$

Question 34 (***+)

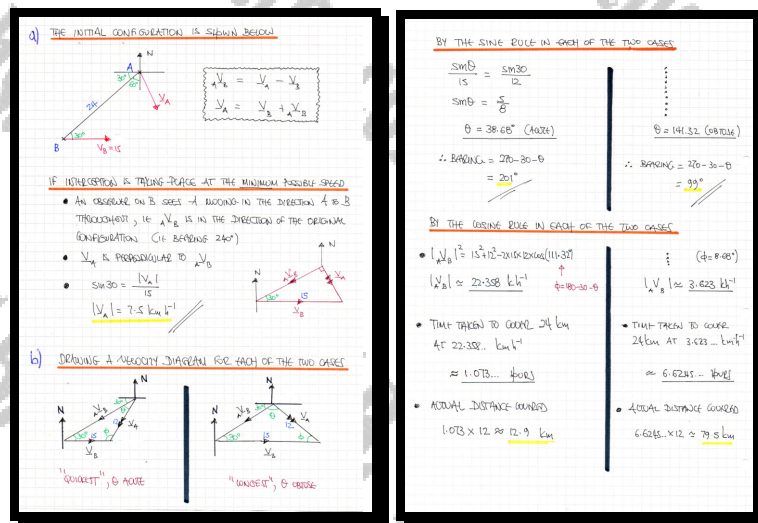
A ship B is travelling due east at a constant speed of 15 km h^{-1} .

At midnight, another ship A is 24 km away from B so that the bearing of B from A is 240° . Ship B is travelling at a constant speed of $U \text{ km h}^{-1}$ and sets on a course to intercept A .

- Find the least possible value of U .
- Given that $U = 12$, determine the two possible bearings at which A can sail so it can intercept B .

Determine the actual distance covered by A in each of these two cases.

$$U = 7.5, \quad \theta \approx 099^\circ, \quad d \approx 79.5 \text{ km}, \quad \theta \approx 201^\circ, \quad d \approx 12.9 \text{ km}$$



Question 35 (**)**

The unit vectors \mathbf{i} and \mathbf{j} are oriented due east and due north, respectively.

Two boats, A and B, are moving in the open sea with velocities $(7\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$ and $(-3\mathbf{i} + 9\mathbf{j}) \text{ km h}^{-1}$, respectively.

At noon, B is on a bearing of 120° from A, 12 km away.

Calculate, correct to the nearest m, the closest distance between the two boats and the time when they are at that closest distance.

, $d \approx 202 \text{ m}$, 13:02

INITIAL CONFIGURATION

VELOCITY TRIANGLE

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$(-3\mathbf{i} + 9\mathbf{j}) = (7\mathbf{i} + 3\mathbf{j}) + \mathbf{v}_{B/A}$

$\mathbf{v}_{B/A} = (-10\mathbf{i} + 6\mathbf{j})$

FIXING A, AN OBSERVER ON A SEES THE FOLLOWING

SMALLEST DISTANCE d IS

$d = 12 \sin \theta$

$d = 12 \sin(44.715^\circ)$

$d = 8.2018 \dots \text{ km}$

$d \approx 202 \text{ m}$

ALTERNATIVE BY VECTORS

THE POSITION OF B AT NOON, TO BE THE ORIGIN - THE POSITION VECTOR OF B AT NOON WOULD BE

$\mathbf{r}_B = (12 \cos 120^\circ)\mathbf{i} + (12 \sin 120^\circ)\mathbf{j}$

$\mathbf{r}_B = (-6\mathbf{i} + 6\sqrt{3}\mathbf{j})$

THE POSITION VECTORS OF THE TWO BOATS, t HOURS AFTER NOON, IS

$\mathbf{r}_A = (0\mathbf{i}) + (7\mathbf{j})t = (7t)\mathbf{j}$

$\mathbf{r}_B = (-6\mathbf{i} + 6\sqrt{3}\mathbf{j}) + (-10\mathbf{i} + 6\mathbf{j})t = (-6 - 10t)\mathbf{i} + (6\sqrt{3} + 6t)\mathbf{j}$

THE POSITION VECTOR OF B, RELATIVE TO A IS GIVEN BY

$\mathbf{r}_B - \mathbf{r}_A = (-6 - 10t)\mathbf{i} + (6\sqrt{3} - 6t)\mathbf{j}$

THE DISTANCE BETWEEN THE BOATS AT TIME t

$d = |\mathbf{r}_B - \mathbf{r}_A|$

$d = \sqrt{(-6 - 10t)^2 + (6\sqrt{3} - 6t)^2}$

$d = \sqrt{36 + 120t + 100t^2 + 108 - 72t + 36t^2}$

$d = \sqrt{144t^2 - 48t + 144}$

$d^2 = 144t^2 - 48t + 144$

LET $f(t) = 144t^2 - 48t + 144$ BY CALCULATING THE MINIMUM OF $f(t)$

$f'(t) = 288t - 48 = 0$

$t = \frac{48}{288} = \frac{1}{6} \text{ hours} = 10 \text{ minutes}$

SO THE MINIMUM DISTANCE

$d_{\min} = \sqrt{144\left(\frac{1}{6}\right)^2 - 48\left(\frac{1}{6}\right) + 144}$

$d_{\min} = \sqrt{36 - 8 + 144}$

$d_{\min} = \sqrt{172}$

$d_{\min} \approx 13.11 \text{ km}$

$d_{\min} \approx 13110 \text{ m}$

Question 36 (****)

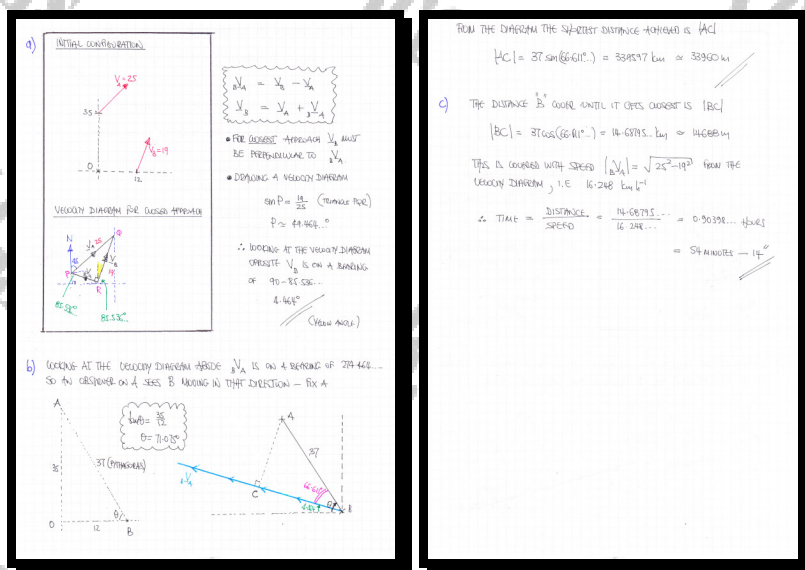
At a certain instant, a ship A is sighted 35 km north of a fixed observation point O , sailing with constant speed 25 km h^{-1} on a bearing 045° .

At the same instant another ship B is sighted 12 km east of O .

B sails with a maximum constant speed of 19 km h^{-1} , in a direction so that it passes as close as possible to A .

- Determine, correct to three decimal places, the bearing at which B is sailing.
- Find, correct to the nearest metre, the shortest distance between A and B .
- Calculate, in minutes and seconds, the time B takes to pass closest to A .

$$\approx 4.464^\circ, \approx 33960 \text{ m}, \approx 54' - 14''$$



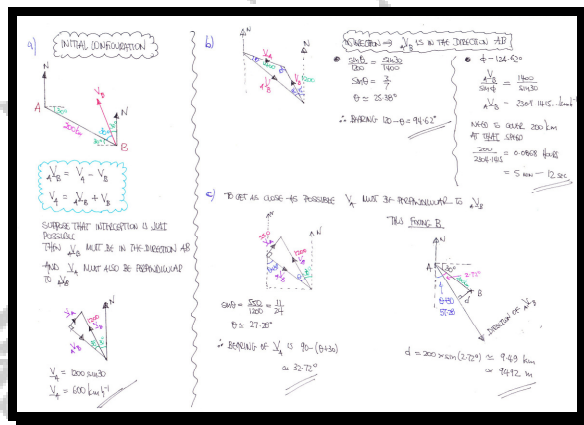
Question 37 (**)**

On the radar screen of a plane A , an enemy aircraft B is observed on a bearing 120° , 200 km away. The speed of the enemy craft is 1200 km h^{-1} on a bearing 330° . The two aircraft are at the same altitude.

A immediately sets with constant speed V_A to intercept B .

- Determine the minimum value of V_A which makes the interception possible.
- Given that $V_A = 1400 \text{ km h}^{-1}$, determine the bearing A must follow in order to intercept B , in the shortest possible time T , and find the value of T , correct to the nearest second.
- Given instead that $V_A = 550 \text{ km h}^{-1}$, determine the bearing A must follow in order to pass as close as possible to B , and find this closest distance correct to the nearest metre.

600 km h^{-1} , 94.62° , $5' - 12''$, 32.72° , 9492 m



Question 38 (****)

The vectors \mathbf{i} and \mathbf{j} are unit vectors mutually perpendicular to one another.

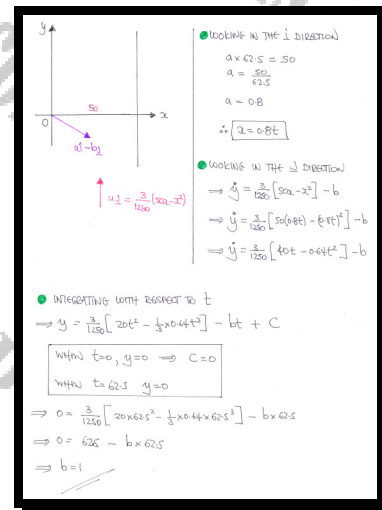
A man is about to swim across a river, starting at a fixed point O on one river bank to a point A in the opposite river bank. The position vector of A relative to O is $50\mathbf{i}$ m. The river flows parallel to \mathbf{j} and the speed of the flow is given by

$$\frac{3}{125}(50x - x^2)\mathbf{j} \text{ ms}^{-1}.$$

The man swims with constant velocity $(a\mathbf{i} - b\mathbf{j}) \text{ ms}^{-1}$, taking 62.5 s to reach A .

Determine the value of b .

$$\boxed{b=1}$$



Question 39 (****)

When a boat is sailing due North with constant speed 15 km h^{-1} , the wind appears to the crew on the boat to be blowing **from** the direction 030° .

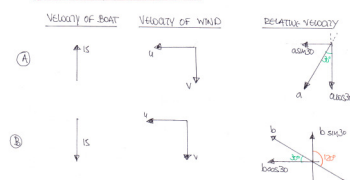
When another boat is sailing due South with constant speed 15 km h^{-1} , the wind appears to the crew on that boat to be blowing **from** the direction 120° .

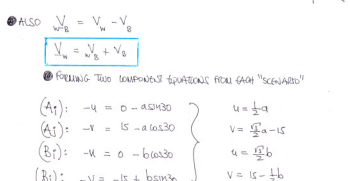
Assuming the true velocity of the wind is the same relative to the earth for both boat crews determine the velocity of the wind.

$$\mathbf{v}_w = -\frac{15}{2}(\sqrt{3}\mathbf{i} + \mathbf{j}) \quad \text{or} \quad |\mathbf{v}_w| = 15, \text{ from bearing } 060^\circ$$

USING CARTESIAN VECTOR COMPONENTS

VELOCITY OF BOAT **VELOCITY OF WIND** **RELATIVE VELOCITY**

(A) 

(B) 

• ALSO $\mathbf{v}_{w/b} = \mathbf{v}_w - \mathbf{v}_b$

$\mathbf{v}_w = \mathbf{v}_{w/b} + \mathbf{v}_b$

• FINDING TWO UNKNOWN QUANTITIES FROM EACH "SCENARIO"

(A): $-u = 0 - a \sin 30$ $u = \frac{1}{2}a$
 (A): $-v = 15 - a \cos 30$ $v = \frac{\sqrt{3}}{2}a - 15$
 (B): $-u = 0 - b \sin 30$ $u = \frac{1}{2}b$
 (B): $-v = -15 + b \sin 30$ $v = 15 - \frac{1}{2}b$

• WE HAVE 4 EQUATIONS & 4 UNKNOWN — SOLVING

$\left\{ \begin{matrix} u = \frac{1}{2}a \\ u = \frac{1}{2}b \end{matrix} \right\} \Rightarrow \frac{1}{2}a = \frac{1}{2}b \Rightarrow a = b$

• 4th CASE $\left\{ \begin{matrix} v = \frac{\sqrt{3}}{2}a - 15 \\ v = 15 - \frac{1}{2}b \end{matrix} \right\} \Rightarrow \frac{\sqrt{3}}{2}a - 15 = 15 - \frac{1}{2}a \Rightarrow \frac{\sqrt{3}+1}{2}a = 30 \Rightarrow a = \frac{60}{\sqrt{3}+1} = 15\sqrt{3}$

$\frac{3}{2}b - 15 = 15 - \frac{1}{2}b$
 $2b = 30$
 $b = 15$ $a = 15\sqrt{3}$

• 4th CASE $\left\{ \begin{matrix} u = \frac{1}{2}a = \frac{15\sqrt{3}}{2} \\ v = 15 - \frac{1}{2}b = 15 - \frac{15}{2} = \frac{15}{2} \end{matrix} \right\}$

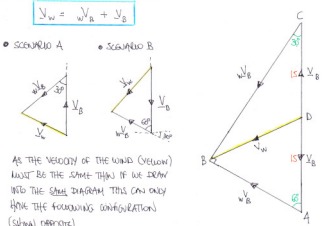
• VELOCITY OF THE WIND IS $-\frac{15\sqrt{3}}{2}\mathbf{i} - \frac{15}{2}\mathbf{j}$
 $= -\frac{15}{2}(\sqrt{3}\mathbf{i} + \mathbf{j})$

• MAGNITUDE IS ON A BEARING 240°
 (OR BEARING 060°)

ALTERNATIVE BY GEOMETRIC METHODS

$\mathbf{v}_w = \mathbf{v}_b + \mathbf{v}_{w/b}$

• SCENARIO A • SCENARIO B



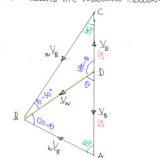
AS THE VELOCITY OF THE WIND (YELLOW) MUST BE THE SAME THING IF WE DRAW THE SAME TRIANGLE, THIS CAN ONLY HOLD THE FOLLOWING CONFIGURATION (SINCE OPPOSITE)

• AS THE TRIANGLE IS RIGHT ANGLED

$|AB| = |AC| \cos 60$
 $|AB| = 30 \times \frac{1}{2}$
 $|AB| = 15$

• AND BY SIMPLE GEOMETRY ON $\triangle ABD$ WE DEDUCE THAT $\triangle ABD$ IS ISOSCELES
 $\therefore \mathbf{v}_w = 15$ & $\angle BDA = 60^\circ$
 \therefore BEARING 240°

• FAILURE TO "SEE" THE RIGHT ANGLE CAN PRODUCE THE FOLLOWING "SOLUTION"



• IN THE SINE RULE ON $\triangle BCD$

$\frac{|v_w|}{\sin 60} = \frac{15}{\sin 60}$

• IN THE SINE RULE ON $\triangle ABD$

$\frac{|v_w|}{\sin 60} = \frac{15}{\sin 120}$

• SOLVING SIMULTANEOUSLY

$\left\{ \begin{matrix} |v_w| = \frac{15 \sin 30}{\sin (60-30)} \\ |v_w| = \frac{15 \sin 60}{\sin (120-60)} \end{matrix} \right\} \Rightarrow \frac{15 \sin 30}{\sin (60-30)} = \frac{15 \sin 60}{\sin (120-60)}$

$\Rightarrow \frac{1}{\sin (60-30)} = \frac{\sqrt{3}}{\sin (120-60)}$

$\Rightarrow \sin (120-60) = \sqrt{3} \sin (60-30)$

• BY THE COSINE RULE

$\Rightarrow \sin^2 (120-60) = \sin^2 (60-30) + \sin^2 (60-30) - 2 \sin (60-30) \sin (60-30) \cos 60$

$\Rightarrow \frac{3}{4} \cos^2 30 + \frac{3}{4} \sin^2 30 = \frac{3}{4} \cos^2 30 - \frac{3}{4} \sin^2 30$

$\Rightarrow \frac{3}{4} \cos^2 30 + \frac{3}{4} \sin^2 30 = \frac{3}{4} \cos^2 30 - \frac{3}{4} \sin^2 30$

$\Rightarrow \sqrt{3} \cos 30 = \sin 60$

$\tan 60 = \sqrt{3}$

$\theta = 60^\circ$

• $|v_w| = \frac{15 \sin 30}{\sin (60-30)} = 15$

\therefore SPEED IS ON A BEARING

Question 40 (****)

Three particles A , B and C are moving on a horizontal plane.

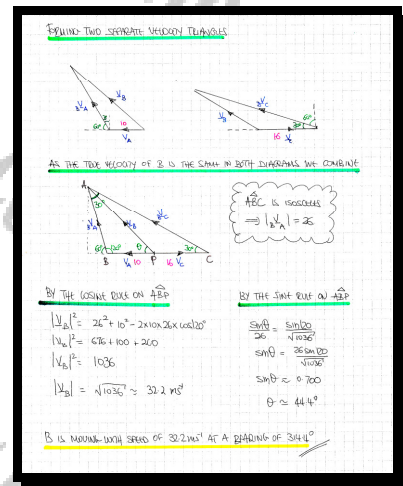
The speed of A is 10 ms^{-1} due west and the speed of C is 16 ms^{-1} due east.

Relative to A , B is moving on a bearing of 330° .

Relative to C , B is moving on a bearing of 300° .

Determine the speed and direction of motion of B .

, $\approx 32.2 \text{ ms}^{-1}$ on a bearing $\approx 314.4^\circ$



Question 41 (****+)

Two particles A and B are moving on a horizontal plane with constant velocities.

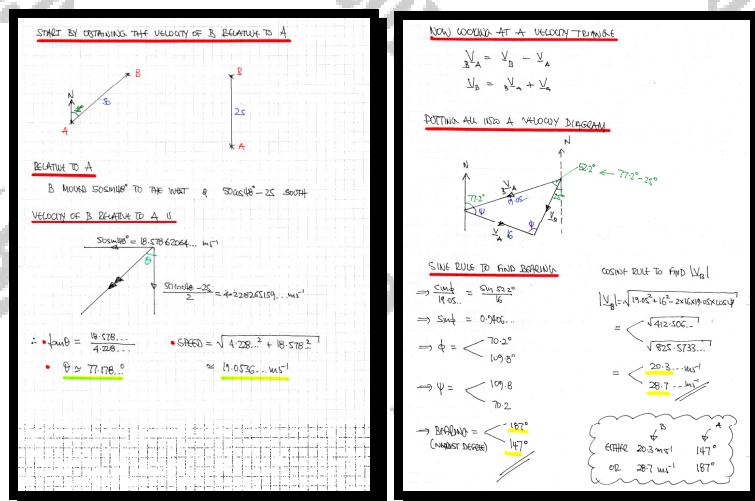
At a given instant B is on a bearing of 048° relative to A and the distance between A and B is 50 m.

The distance between A and B reduces to 25 m after 2 s with B due north from A .

It is further given that the actual speed of A is 16 ms^{-1} and B is moving on a bearing of 205° .

Determine the two possible bearings in which A could be moving and the two possible speeds of B .

$$|\mathbf{v}_B| \approx 20.3 \text{ ms}^{-1} \cap \mathbf{v}_A \text{ on a bearing } \approx 147^\circ \cup |\mathbf{v}_B| \approx 28.7 \text{ ms}^{-1} \cap \mathbf{v}_A \text{ on a bearing } \approx 187^\circ$$



Question 42 (****+)

A ship A is sailing West at 8 mph and a ship B is sailing East at 28 mph.

To a man on A the wind appears to be blowing on a bearing of 170° and to a man on B the wind appears to be blowing on a bearing of 120° .

Find the direction, as a bearing, and the speed of the wind.

$$\approx 204^\circ, \approx 14.1 \text{ mph}$$

1. Firstly $V_w = V_w - V_A$ or similarly $V_w = V_A + V_D$

2. Now velocity triangles separately to start with

3. Set the velocity of the wind is common to both observers (diagrams)

By sine rule on $\triangle ABD$

$$\frac{36}{\sin 50^\circ} = \frac{8}{\sin(180^\circ - 50^\circ)}$$

$$\boxed{2 = \frac{8 \sin 50^\circ}{\sin(180^\circ - 50^\circ)}}$$

By sine rule on $\triangle BCD$

$$\frac{36}{\sin 30^\circ} = \frac{28}{\sin 50^\circ}$$

$$\boxed{2 = \frac{28 \sin 30^\circ}{\sin 50^\circ}}$$

$\Rightarrow \frac{8 \sin 50^\circ}{\sin(180^\circ - 50^\circ)} = \frac{14 \sin 30^\circ}{\sin 50^\circ}$

$\Rightarrow 8 \sin 50^\circ \sin 50^\circ = 14 \sin(180^\circ - 50^\circ) \sin 30^\circ$

$\Rightarrow 8 \sin^2 50^\circ = 14 \sin 50^\circ \sin 30^\circ$

$\Rightarrow 8 \sin 50^\circ = 14 \sin 30^\circ$

$\Rightarrow 8 \sin 50^\circ = 7$

$\Rightarrow \sin 50^\circ = \frac{7}{8}$

$\Rightarrow 50^\circ = \sin^{-1}(\frac{7}{8})$

$\Rightarrow 50^\circ = 70.4^\circ$

$\Rightarrow 50^\circ + 30^\circ + \theta = 204^\circ$

$\Rightarrow \theta = 204^\circ$

$\Rightarrow \alpha = \frac{8 \sin 50^\circ}{\sin(180^\circ - 50^\circ)} = \frac{8 \sin 50^\circ}{\sin(109.6^\circ)} \approx 14.07$

$\Rightarrow \alpha \approx 14.1$

Question 43 (****)

To a motorist P driving South on a level road with constant speed u , the wind appears to be blowing from a bearing $(90 + \theta)^\circ$.

To a motorist Q driving North on a level road with constant speed u , the wind appears to be blowing from a bearing $(90 + \varphi)^\circ$.

To a motorist R driving North on a level road with constant speed $2u$, the wind appears to be blowing from a bearing $(90 + \psi)^\circ$.

Assuming that the true speed and direction of the wind is the same for all three motorists, show that

$$2 \tan \psi = 3 \tan \varphi - \tan \theta.$$

proof

