OBLIQUE COLLISIONS

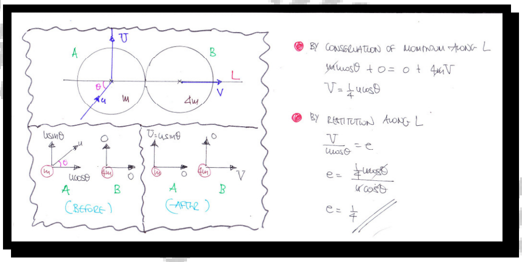
Question 1 (**)

A smooth sphere B of mass $4m$ is at rest on a smooth horizontal surface. Another sphere A is moving with speed u in a straight line, on the same surface. There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact.

The path of A immediately before the collision makes an angle of θ with L, $0° < \theta < 90°$ and immediately after the collision the two spheres move at right angles to one another.

Determine the value of the coefficient of restitution between the two spheres.

$$\boxed{e = \frac{1}{4}}$$



Question 2 (**)

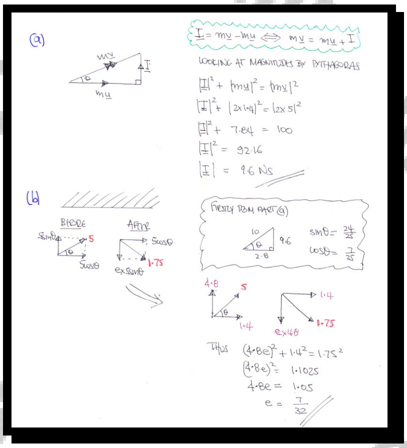
A particle of mass 2 kg is moving on a smooth horizontal plane with speed 1.4 ms^{-1} when it receives an impulse of magnitude $I\text{ Ns}$, in a direction perpendicular to its direction of motion. The speed of the particle, after it receives I, changes to 5 ms^{-1}.

a) Determine the value of I.

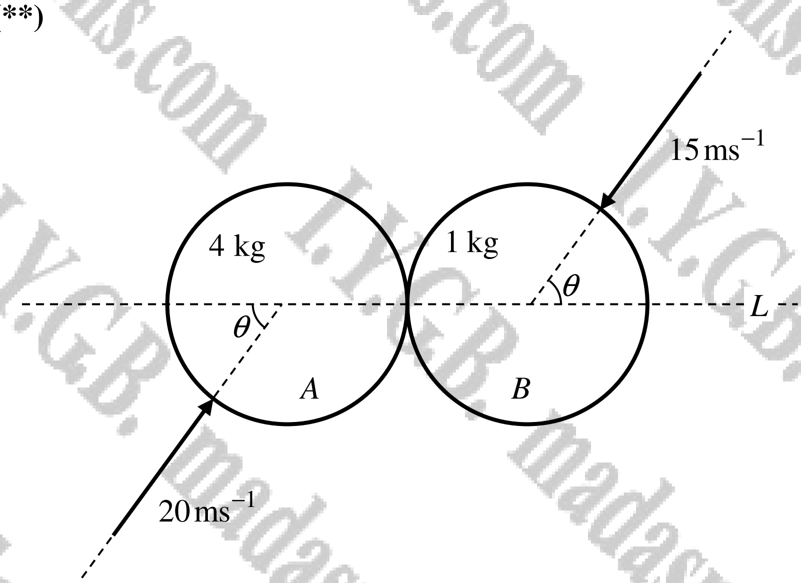
The particle was moving in a direction parallel to a smooth vertical wall in the time period before it received I. After it received I the particle hits the wall and rebounds with speed 1.75 ms^{-1}.

b) Calculate the coefficient of restitution between the particle and the wall.

$\boxed{I = 9.6}$, $\boxed{e = \frac{7}{32}}$



Question 3 (**)

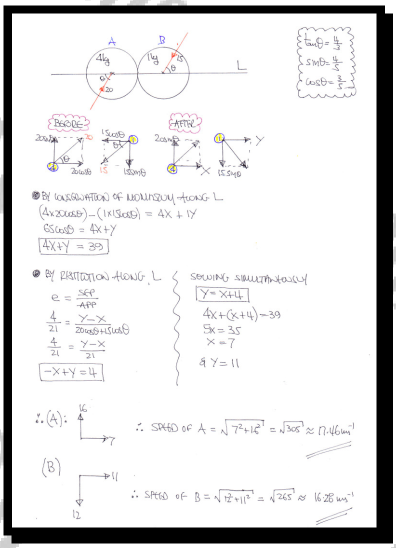


Two uniform smooth spheres A and B, of equal radius, are moving in opposite directions on a smooth horizontal surface when they collide obliquely . The respective masses of A and B are 4 kg and 1 kg.

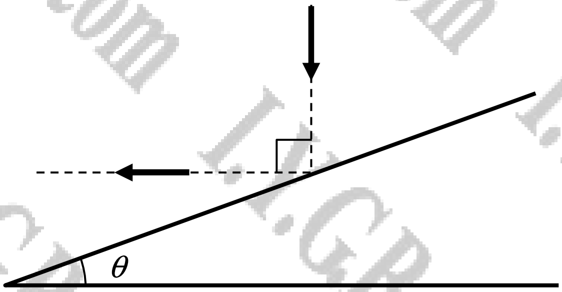
Immediately before the collision the respective speeds of A and B are 20 ms^{-1} and 15 ms^{-1}. On collision the line through the centres of the two spheres is L, and the angle between the speed of each sphere and L is θ, where $\tan\theta = \frac{4}{3}$.

Given that the coefficient of restitution between the two spheres is $\frac{4}{21}$, determine the speed of A and the speed of B, after the collision.

$\boxed{V_A \approx 17.46 \text{ ms}^{-1}}$, $\boxed{V_B \approx 16.28 \text{ ms}^{-1}}$



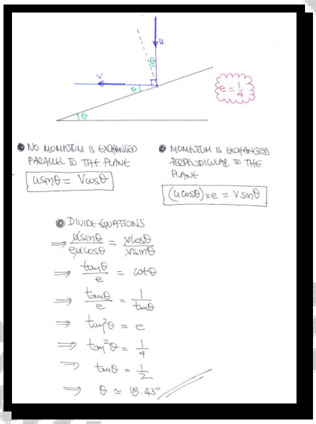
Question 4 (**)



A smooth plane is inclined at an angle θ to the horizontal. A small smooth ball falls vertically and hits the plane and immediately after the impact the direction of the ball is horizontal, as shown in the figure above.

Given the coefficient of restitution between the plane and the ball is 0.25, calculate an approximate value of θ.

$\boxed{\theta \approx 18.43°}$

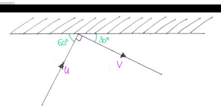


Question 5 (**)

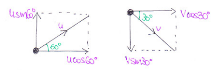
A smooth uniform sphere is moving on a smooth horizontal surface when it collides with a smooth vertical wall. Before the impact the direction of motion of the sphere makes an angle of $60°$ with the wall. After the impact the direction of motion of the sphere makes an angle of $30°$ with the wall.

Determine the value of the coefficient of restitution between the sphere and the wall.

$$\boxed{e = \frac{1}{3}}$$



BY CONSIDERING COMPONENTS OF VELOCITIES BEFORE AND AFTER



NO MOMENTUM IS EXCHANGED PARALLEL TO THE WALL

$$\Longrightarrow \boxed{u\cos 60° = V\cos 30°}$$

BY RESTITUTION, PERPENDICULAR TO THE WALL

$$\Longrightarrow \boxed{e = \frac{V\sin 30°}{u\sin 60°}}$$

SOLVING SIMULTANEOUSLY

$$\left.\begin{aligned} \frac{V}{u} &= \frac{\cos 60°}{\cos 30°} \\ e &= \left(\frac{V}{u}\right)\left(\frac{\sin 30}{\sin 60°}\right) \end{aligned}\right\} \Longrightarrow e = \frac{\cos 60}{\cos 30} \times \frac{\sin 30}{\sin 60}$$

$$\Longrightarrow e = \frac{\sin 30 \cos 60}{\cos 30 \sin 60}$$

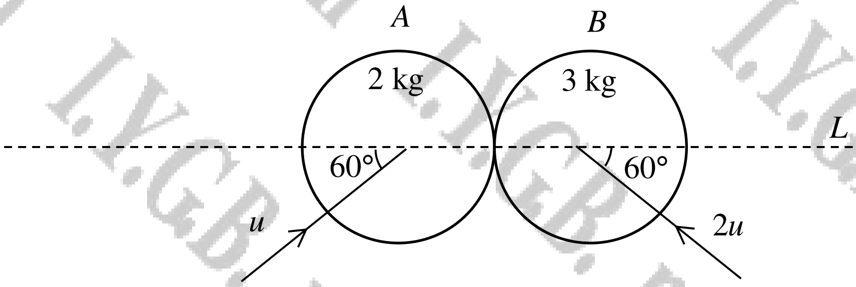
$$\Longrightarrow e = \tan 30 \cot 60$$

$$\Longrightarrow e = \tan 30 \times \frac{1}{\tan 60}$$

$$\Longrightarrow e = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$\Longrightarrow e = \frac{1}{3}$$

Question 6 (**)



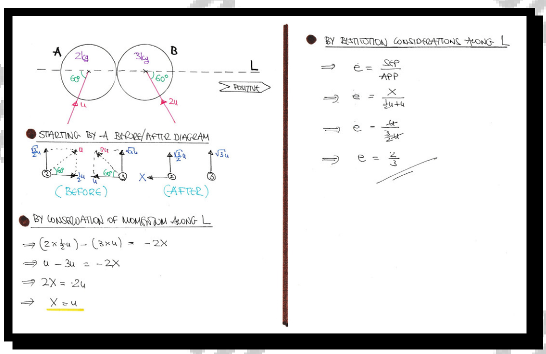
Two smooth spheres, A and B, of equal radius and respective masses 2 kg and 3 kg, are moving on a smooth horizontal plane when they collide obliquely.

When A and B collide the straight line through their centres is denoted by L.

The speeds of A and B before their collision are u and $2u$, respectively and the direction of both speeds is at an angle $60°$ to L, as shown in the figure above.

Given that after the collision B is moving at right angles to L, find the value of the coefficient of restitution between A and B.

, $\boxed{e = \frac{2}{3}}$



Question 7 (**+)

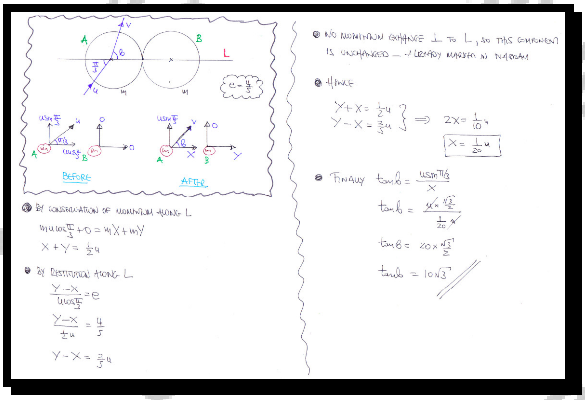
A smooth sphere B is at rest on a smooth horizontal surface. An identical sphere A is moving with speed u in a straight line, on the same surface. There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact.

The path of A immediately before the collision makes an angle of $\frac{\pi}{3}$ with L, and immediately after the collision makes an angle of β with L, where $0 < \beta < \frac{\pi}{2}$.

Given that of the coefficient of restitution between the two spheres is 0.8, show that

$$\tan\beta = 10\sqrt{3}.$$

$\boxed{\text{proof}}$



Question 8 (**+)

A smooth sphere B of mass $3m$ is at rest on a smooth horizontal surface. Another smooth sphere A of mass $2m$ and of equal radius as that of B is moving with speed u in a straight line. There is a collision between the two spheres.

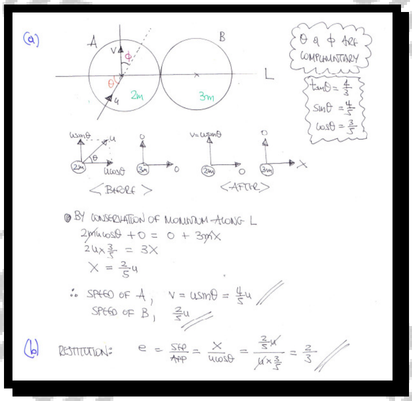
Immediately before the collision the path of A makes an angle θ with the line of the centres of the spheres, where $\tan\theta = \frac{4}{3}$.

As a result of the impact, the direction of motion of A is turned through an angle φ, where $\tan\varphi = \frac{3}{4}$.

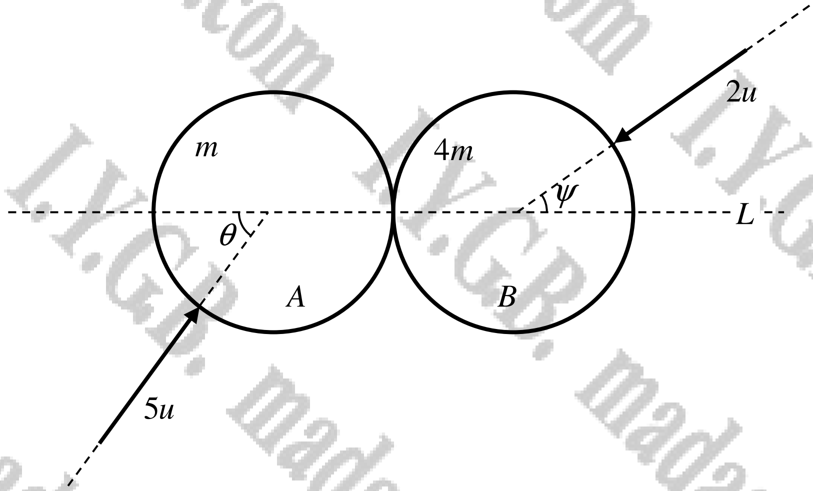
a) Determine the speed of each of the spheres after the collision.

b) Find the value of the coefficient of restitution between the two spheres.

$\boxed{V_A = \frac{4}{5}u}$, $\boxed{V_B = \frac{2}{5}u}$, $\boxed{e = \frac{2}{3}}$



Question 9 (**+)



Two smooth uniform spheres A and B have equal radii, and their respective masses are m and $4m$. The spheres are moving on a smooth horizontal plane when they collide obliquely, with their centres at impact defining the straight line L, as shown in the figure above.

Immediately before the collision, A is moving with speed $5u$ at an acute angle θ to L and B is moving with speed $2u$ at an acute angle ψ to L.

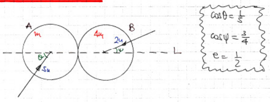
It is further given that $\cos\theta = 0.2$, $\cos\psi = 0.75$ and the coefficient of restitution between the two spheres is 0.5.

a) Determine, in terms of m and u, the magnitude of the impulse on A due to the collision.

b) Express the kinetic energy **gained** by A in the collision, as a percentage of its initial kinetic energy.

$\boxed{}$, $\boxed{I = 3mu}$, $\boxed{12\%}$

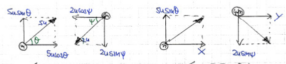
a) STARTING WITH A DIAGRAM



$\cos\theta = \frac{1}{5}$

$\cos\psi = \frac{3}{4}$

$e = \frac{1}{2}$



NO MOMENTUM IS EXCHANGED IN A DIRECTION PERPENDICULAR TO L

- BY CONSERVATION OF LINEAR MOMENTUM ALONG "L"

$\Rightarrow 5\cancel{m}u\cos\theta - 2(4\cancel{m})u\cos\psi = \cancel{m}X + 4\cancel{m}Y$

$\Rightarrow 5u\cos\theta - 8u\cos\psi = X + 4Y$

$\Rightarrow u - 6u = X + 4Y$

$\Rightarrow \boxed{X + 4Y = -5u}$

- BY RESTITUTION ALONG "L"

$e = \frac{SEP}{APP}$

$\frac{1}{2} = \frac{Y - X}{5u\cos\theta + 2u\cos\psi}$

$\frac{1}{2} = \frac{Y - X}{u + \frac{3}{2}u}$

$\boxed{-X + Y = \frac{5}{4}u}$

ADDING YIELDS

$5Y = -\frac{15}{4}u$

$Y = -\frac{3}{4}u$ (REBOUNDED) & $X = -2u$ (ALSO REBOUNDED)

FINALLY THE IMPULSE ON A – JUST ON THE DIRECTION OF L

MOM A AFTER – MOMENTUM OF A BEFORE

$I = mX - m(5u\cos\theta)$

$I = m(-2u) - mu$

$I = -3mu$

$|I| = 3mu$

b) KINETIC ENERGY OF A BEFORE

$\frac{1}{2}m(5u)^2 = \frac{25}{2}mu^2$

KINETIC ENERGY OF A AFTER

$\frac{1}{2}mX^2 + \frac{1}{2}m(5u\sin\theta)^2 = \frac{1}{2}m(-2u)^2 + \frac{1}{2}m(25u^2\sin^2\theta)$

$= 2mu^2 + \frac{25}{2}mu^2(1 - \cos^2\theta)$

$= 2mu^2 + \frac{25}{2}mu^2\left(1 - \frac{1}{25}\right)$

$= 2mu^2 + \frac{\cancel{25}}{2} \times \frac{24}{\cancel{25}}mu^2$

$= 14mu^2$

KINETIC ENERGY GAIN

$14mu^2 - \frac{25}{2}mu^2 = \frac{3}{2}mu^2$

REQUIRED PROPORTION

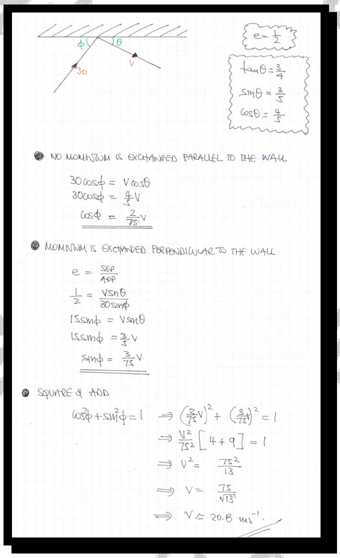
$\frac{\frac{3}{\cancel{2}}mu^2}{\frac{25}{\cancel{2}}mu^2} = \frac{3}{25}$ OR 12%

Question 10 (+)**

A smooth uniform sphere is moving with speed 30 ms^{-1} on a smooth horizontal surface, when it collides with a smooth vertical wall. After the impact the velocity of the sphere makes an angle θ with the wall, where $\tan\theta = 0.75$.

Given that the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, determine the speed of the sphere after the collision.

$$\boxed{v \approx 20.8 \text{ ms}^{-1}}$$



$e = \frac{1}{2}$

$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

- NO MOMENTUM IS EXCHANGED PARALLEL TO THE WALL

$$30\cos\phi = V\cos\theta$$
$$30\cos\phi = \frac{4}{5}V$$
$$\cos\phi = \frac{2}{75}V$$

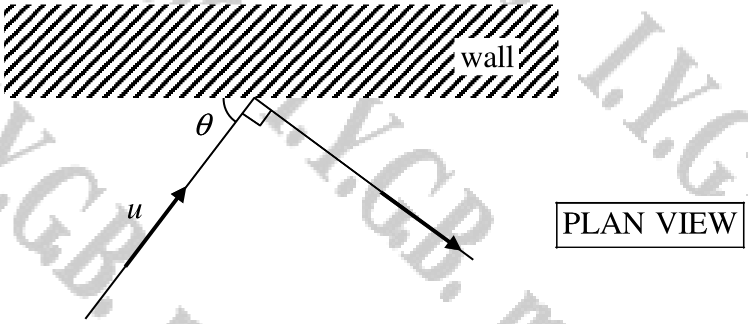
- MOMENTUM IS EXCHANGED PERPENDICULAR TO THE WALL

$$e = \frac{SEP}{APP}$$
$$\frac{1}{2} = \frac{V\sin\theta}{30\sin\phi}$$
$$15\sin\phi = V\sin\theta$$
$$15\sin\phi = \frac{3}{5}V$$
$$\sin\phi = \frac{3}{75}V$$

- SQUARE & ADD

$$\cos^2\phi + \sin^2\phi = 1 \implies \left(\frac{2}{75}V\right)^2 + \left(\frac{3}{75}V\right)^2 = 1$$
$$\implies \frac{V^2}{75^2}\left[4+9\right] = 1$$
$$\implies V^2 = \frac{75^2}{13}$$
$$\implies V = \frac{75}{\sqrt{13}}$$
$$\implies V \simeq 20.8 \text{ ms}^{-1}.$$

Question 11 (+)**



A ball is moving on a smooth horizontal surface, bouncing off a smooth vertical wall, where the coefficient of restitution between the wall and the ball is e.

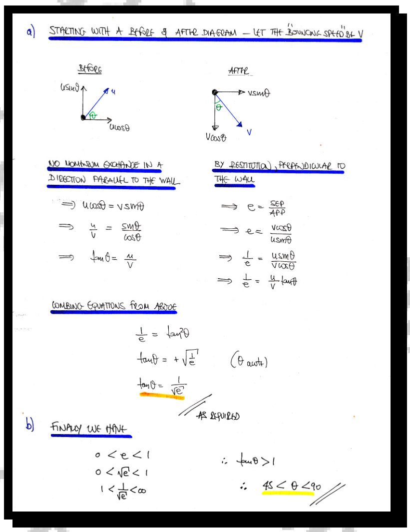
The speed of the ball before hitting the wall is u ms^{-1} and makes an acute angle θ with the wall. After the collision with wall the path of the ball turns by a right angle as shown in the figure above.

a) Show clearly that

$$\tan\theta = \frac{1}{\sqrt{e}}.$$

b) Hence determine the range of possible values of θ.

$\boxed{}$, $\boxed{45° \le \theta < 90°}$



a) STARTING WITH A BEFORE & AFTER DIAGRAM – LET THE "BOUNCING SPEED" BE V

BEFORE

$u\sin\theta$ u θ $u\cos\theta$

AFTER

$V\sin\theta$ θ $V\cos\theta$ V

NO MOMENTUM EXCHANGE IN A DIRECTION PARALLEL TO THE WALL

$\Longrightarrow u\cos\theta = V\sin\theta$

$\Longrightarrow \frac{u}{V} = \frac{\sin\theta}{\cos\theta}$

$\Longrightarrow \tan\theta = \frac{u}{V}$

BY RESTITUTION, PERPENDICULAR TO THE WALL

$\Longrightarrow e = \frac{SEP}{APP}$

$\Longrightarrow e = \frac{V\cos\theta}{u\sin\theta}$

$\Longrightarrow \frac{1}{e} = \frac{u\sin\theta}{V\cos\theta}$

$\Longrightarrow \frac{1}{e} = \frac{u}{V}\tan\theta$

COMBINING EQUATIONS FROM ABOVE

$\frac{1}{e} = \tan^2\theta$

$\tan\theta = +\sqrt{\frac{1}{e}}$ (θ acute)

$\tan\theta = \frac{1}{\sqrt{e}}$ // AS REQUIRED

b) FINALLY WE HAVE

$0 < e < 1$

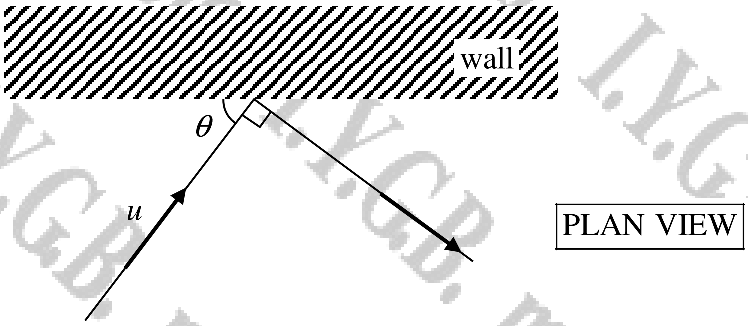
$0 < \sqrt{e} < 1$

$1 < \frac{1}{\sqrt{e}} < \infty$

$\therefore \tan\theta > 1$

$\therefore 45 < \theta < 90$ //

Question 12 (**+)

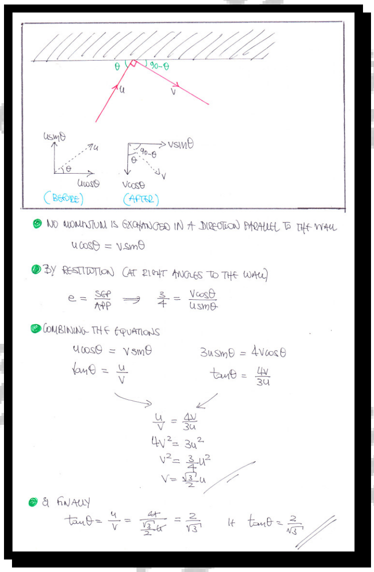


A ball is moving on a smooth horizontal surface, bouncing off a smooth vertical wall, where the coefficient of restitution between the wall and the ball is $\frac{3}{4}$.

The speed of the ball before hitting the wall is u ms^{-1} and makes an acute angle θ with the wall. After the collision with wall the path of the ball turns by a right angle as shown in the figure above.

Find the speed of the ball in terms of u, and the value of $\tan\theta$.

$\boxed{v = \frac{1}{2}\sqrt{3}\,u}$, $\boxed{\tan\theta = \frac{2}{\sqrt{3}}}$



- NO MOMENTUM IS EXCHANGED IN A DIRECTION PARALLEL TO THE WALL

$u\cos\theta = v\sin\theta$

- BY RESTITUTION (AT RIGHT ANGLES TO THE WALL)

$e = \frac{SEP}{APP} \Longrightarrow \frac{3}{4} = \frac{v\cos\theta}{u\sin\theta}$

- COMBINING THE EQUATIONS

$u\cos\theta = v\sin\theta$ $\quad$ $3u\sin\theta = 4v\cos\theta$

$\tan\theta = \frac{u}{v}$ $\quad$ $\tan\theta = \frac{4v}{3u}$

$\frac{u}{v} = \frac{4v}{3u}$

$4v^2 = 3u^2$

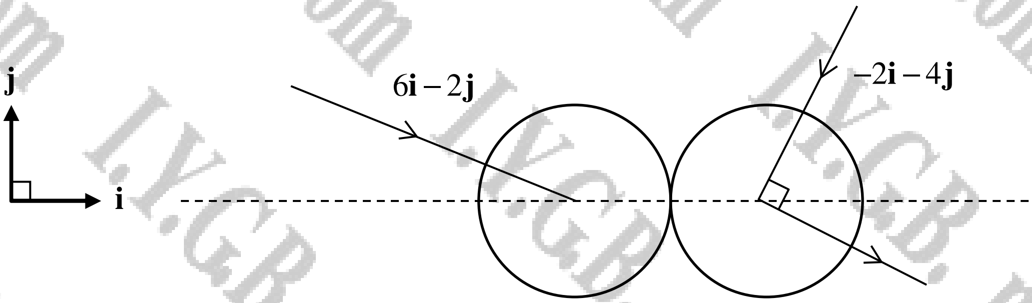
$v^2 = \frac{3}{4}u^2$

$v = \frac{\sqrt{3}}{2}u$

- & FINALLY

$\tan\theta = \frac{u}{v} = \frac{u}{\frac{\sqrt{3}}{2}u} = \frac{2}{\sqrt{3}}$ $\quad$ // $\tan\theta = \frac{2}{\sqrt{3}}$

Question 13 (***)



The vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors perpendicular to each other.*

Two smooth spheres, A and B, of equal radius and respective masses 4 kg and 2 kg, are moving on a smooth horizontal plane when they collide obliquely.

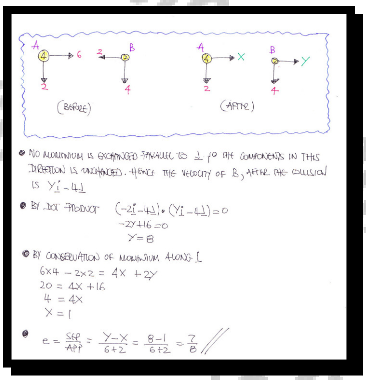
When A and B collide the straight line through their centres is parallel to the unit vector $\mathbf{i}$.

The velocities of A and B before their collision are $(6\mathbf{i}-2\mathbf{j})$ ms^{-1} and $(-2\mathbf{i}-4\mathbf{j})$ ms^{-1}, respectively.

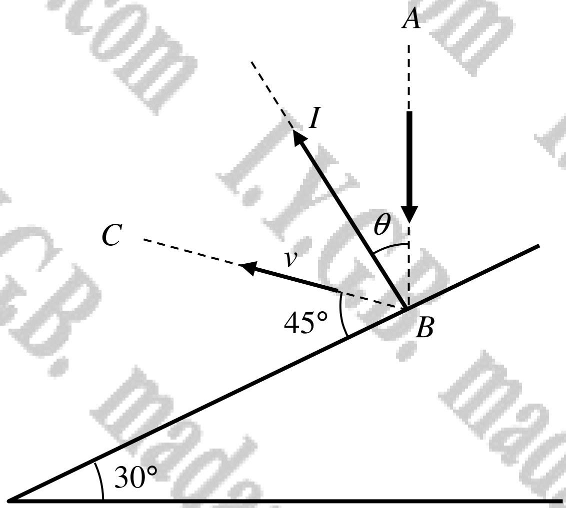
As a result of the collision, the direction of motion of B is changed through $90°$, as shown in the figure above.

Determine the coefficient of restitution between the two spheres.

$$\boxed{e=\frac{7}{8}}$$



Question 14 (***)



The point B lies on a smooth plane inclined at $30°$ to the horizontal. A particle of mass $\frac{1}{7}$ kg is dropped from a point A which lies 10 m vertically above B.

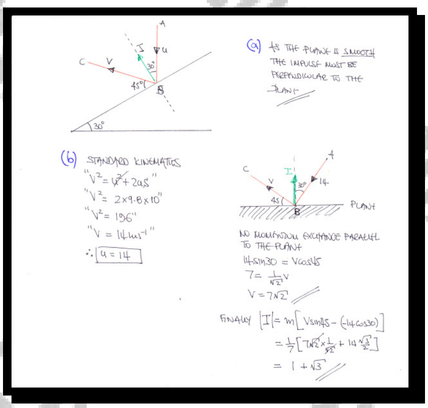
The particle rebounds from the plane in the direction BC with speed v ms^{-1}, at an angle of $45°$ to the plane. The points A, B and C lie in a vertical plane which includes the line of greatest slope of the plane.

The impulse exerted by the plane on the particle has magnitude I Ns, and is inclined at an angle θ to AB.

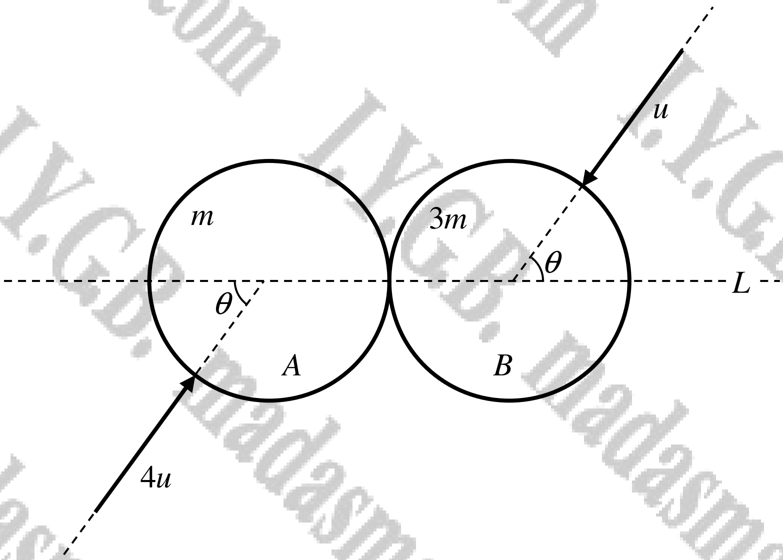
a) Justify why $\theta = 30°$.

b) Determine the exact value of v and the exact value of I.

$\boxed{v = 7\sqrt{2}}$, $\boxed{I = 1+\sqrt{3}}$



Question 15 (***)



Two smooth uniform spheres A and B with equal radii have masses m and $3m$ respectively. The spheres are moving in opposite directions on a smooth horizontal surface, where they collide obliquely.

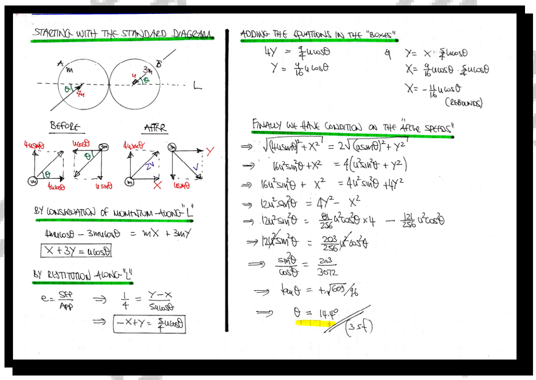
Immediately before the collision, A has speed $4u$ and its direction of motion forms an angle θ to the straight line L, joining the centres of the spheres on impact.

Immediately before the collision B has speed u with its direction of motion also forming an angle θ to L, as shown in the figure above.

Immediately after the collision, the speed of A is twice the speed of B.

Given further that the coefficient of restitution between A and B is 0.25, find the value of θ.

$\boxed{}$, $\boxed{\theta \approx 14.4°}$



Question 16 (***)

The vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors perpendicular to each other.*

Two smooth uniform spheres A and B with equal radii are moving on a smooth horizontal surface.

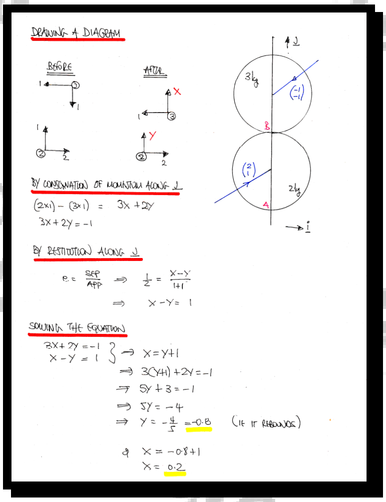
The mass of A is 2 kg and the mass of B is 3 kg.

The velocity of A is $(2\mathbf{i}+\mathbf{j})$ ms^{-1} and the velocity of B is $(-\mathbf{i}-\mathbf{j})$ ms^{-1}.

The spheres collide when the line joining their centres is parallel to $\mathbf{j}$.

Given further that the coefficient of restitution between A and B is 0.5, find kinetic energy lost as a result of the collision.

$\boxed{}$, $\boxed{1.8 \text{ J}}$



DRAWING A DIAGRAM

BY CONSERVATION OF MOMENTUM ALONG $\mathbf{j}$

$(2\times1)-(3\times1) = 3X+2Y$

$3X+2Y=-1$

BY RESTITUTION ALONG $\mathbf{j}$

$e = \frac{SEP}{APP} \Rightarrow \frac{1}{2} = \frac{X-Y}{1+1}$

$\Rightarrow X-Y=1$

SOLVING THE EQUATION

$\left.\begin{array}{l}3X+2Y=-1\\ X-Y=1\end{array}\right\} \Rightarrow X=Y+1$

$\Rightarrow 3(Y+1)+2Y=-1$

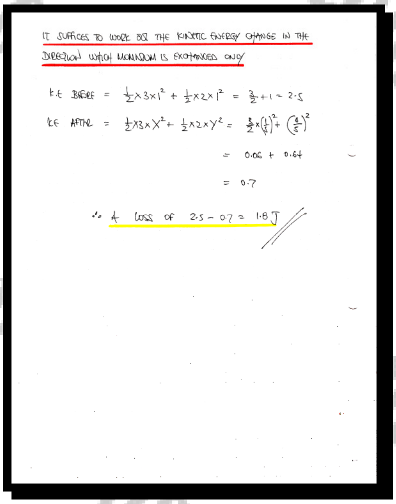
$\Rightarrow 5Y+3=-1$

$\Rightarrow 5Y=-4$

$\Rightarrow Y=-\frac{4}{5}=-0.8$ (IT REBOUNDS)

& $X=-0.8+1$

$X=0.2$



IT SUFFICES TO WORK OUT THE KINETIC ENERGY CHANGE IN THE DIRECTION WHICH MOMENTUM IS EXCHANGED ONLY

K.E BEFORE $= \frac{1}{2}\times3\times1^2+\frac{1}{2}\times2\times1^2 = \frac{3}{2}+1=2.5$

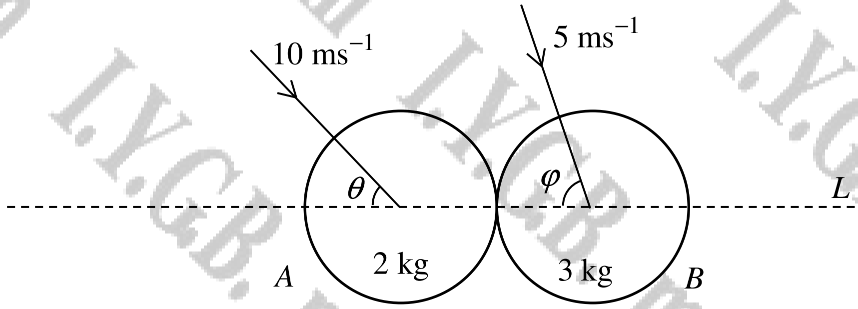
K.E AFTER $= \frac{1}{2}\times3\times X^2+\frac{1}{2}\times2\times Y^2 = \frac{3}{2}\times\left(\frac{1}{5}\right)^2+\left(\frac{4}{5}\right)^2$

$= 0.06+0.64$

$= 0.7$

$\therefore$ A LOSS OF $2.5-0.7=1.8$ J

Question 17 (***)



Two smooth spheres, A and B, of equal radius and respective masses 2 kg and 3 kg, are moving on a smooth horizontal plane when they collide obliquely.

The speeds of A and B before their collision are 10 ms^{-1} and 5 ms^{-1} respectively.

When A and B collide the straight line through their centres is denoted by L.

The direction of the speed of A before the collision is at an acute angle θ to L, where $\tan\theta = \frac{3}{4}$.

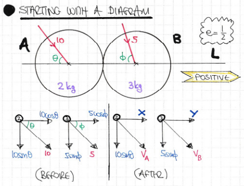
The direction of the speed of B before the collision is at an acute angle φ to L, where $\tan\varphi = \frac{4}{3}$, as shown in the figure above.

The coefficient of restitution between A and B is $\frac{1}{2}$.

Calculate the speed of A and the speed of B, after the collision.

$\boxed{\text{M456}}$, $\boxed{V_A = \sqrt{48.25} \approx 6.95 \text{ ms}^{-1}}$, $\boxed{V_B = \sqrt{52} \approx 7.21 \text{ ms}^{-1}}$

- STARTING WITH A DIAGRAM



- NO MOMENTUM IS EXCHANGED "PERPENDICULARLY" TO L
- BY CONSERVATION OF MOMENTUM OF L

$\Rightarrow 2(10\cos\theta) + 3(5\cos\phi) = 2X + 3Y$

$\Rightarrow 20\cos\theta + 15\cos\phi = 2X + 3Y$

$\Rightarrow 20\left(\frac{4}{5}\right) + 15\left(\frac{3}{5}\right) = 2X + 3Y$

$\Rightarrow 2X + 3Y = 16 + 9$

$\Rightarrow 2X + 3Y = 25$

$\sin\theta = \cos\phi = \frac{3}{5}$

$\cos\theta = \sin\phi = \frac{4}{5}$

- BY CONSIDERING RESTITUTION ALONG L

$\Rightarrow e = \frac{\text{SEP}}{\text{APP}}$

$\Rightarrow \frac{1}{2} = \frac{Y - X}{10\cos\theta - 5\cos\phi}$

$\Rightarrow \frac{1}{2} = \frac{Y - X}{10\left(\frac{4}{5}\right) - 5\left(\frac{3}{5}\right)}$

$\Rightarrow -X + Y = \frac{5}{2}$

$\Rightarrow -2X + 2Y = 5$

- SOLVING SIMULTANEOUSLY

$5Y = 30$ (ADDING)

$Y = 6$

& $2X + 3Y = 25$

$2X + 18 = 25$

$2X = 7$

$X = \frac{7}{2}$

- FINALLY THE "AFTER SPEEDS CAN BE OBTAINED

$V_A = \sqrt{(10\sin\theta)^2 + X^2} = \sqrt{\left[10\left(\frac{3}{5}\right)\right]^2 + 3.5^2} = \sqrt{48.25} \approx 6.95 \text{ ms}^{-1}$

$V_B = \sqrt{(5\sin\phi)^2 + Y^2} = \sqrt{5\left(\frac{4}{5}\right)^2 + 6^2} = \sqrt{52} \approx 7.21 \text{ ms}^{-1}$

Question 18 (***+)

A smooth sphere B of mass $4m$ is at rest on a smooth horizontal surface. Another sphere A, of mass m, is moving with speed u in a straight line, on the same surface.

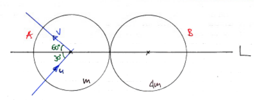
There is a collision between the two spheres where L is the straight line joining the centres of the two spheres at the moment of impact.

The direction of motion of A immediately before the collision makes an angle of $30°$ with L. As a result of the collision, the direction of motion of A is turned through a right angle.

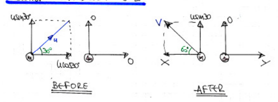
Determine the value of the coefficient of restitution between the two spheres.

$\boxed{}$, $\boxed{e = \frac{2}{3}}$

DRAWING A STANDARD OBLIQUE COLLISION DIAGRAM



DRAW BEFORE & AFTER & NOTE THAT THERE IS NO MOMENTUM EXCHANGE IN A DIRECTION PERPENDICULAR TO "L"



BY CONSERVATION OF MOMENTUM ALONG "L"

$\cancel{m}u\cos 30° + 0 = -\cancel{m}X + 4\cancel{m}Y$

$\boxed{-X + 4Y = \frac{\sqrt{3}}{2}u}$ — (I)

BY RESTITUTION ALONG "L"

$e = \frac{\text{SEP}}{\text{APP}} \implies e = \frac{X+Y}{u\cos 30°}$

$\implies X + Y = eu\cos 30$

$\implies \boxed{X + Y = \frac{\sqrt{3}}{2}eu}$ — (II)

AS THERE IS NO MOMENTUM EXCHANGE PERPENDICULAR TO "L"

$X = V\cos 60$

$\boxed{X = \frac{1}{2}V}$ — (III)

& BY GEOMETRY

$V\sin 60 = u\sin 30$

$\frac{\sqrt{3}}{2}V = \frac{1}{2}u$

$\boxed{u = \sqrt{3}V}$ — (IV)

SOLVING (I) & (II) FOR X & Y BY ADDING

$\implies 5Y = \frac{\sqrt{3}}{2}u + \frac{\sqrt{3}}{2}eu$

$\implies Y = \frac{\sqrt{3}}{10}u + \frac{\sqrt{3}}{10}eu \implies X = \frac{\sqrt{3}}{2}eu - Y$

$\implies X = \frac{\sqrt{3}}{2}eu - \frac{\sqrt{3}}{10}u - \frac{\sqrt{3}}{10}eu$

$\implies X = \frac{2}{5}\sqrt{3}eu - \frac{\sqrt{3}}{10}u$

USING THIS X WITH (III) & (IV)

$\implies X = \frac{1}{2}V$

$\implies 2X = V$

$\implies 2\sqrt{3}X = \sqrt{3}V$

$\implies 2\sqrt{3}X = u$

$\implies 2\sqrt{3}\left[\frac{2}{5}\sqrt{3}eu - \frac{\sqrt{3}}{10}u\right] = u$

$\implies 2\sqrt{3}\left(\frac{2}{5}\sqrt{3}e - \frac{\sqrt{3}}{10}\right) = 1$

$\implies \frac{12}{5}e - \frac{3}{5} = 1$

$\implies 12e - 3 = 5$

$\implies 12e = 8$

$\therefore e = \frac{2}{3}$

Question 19 (***+)

A small smooth sphere B of mass m is at rest on a smooth horizontal surface.

Another smooth sphere A of mass $2m$ and of equal radius as that of B is moving with speed u in a straight line, on the same surface.

There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact.

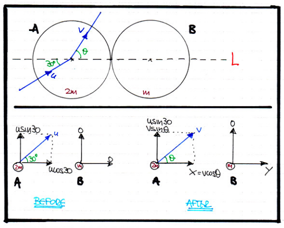
The path of A immediately before the collision makes an angle of $30°$ with L.

a) Determine simplified expressions for the speeds of A and B, in terms of u and e, where e is the coefficient of restitution between the two spheres.

b) Given further that 20% of the kinetic energy of the system is lost due to the impact, show that $e^2 = \frac{1}{5}$.

$\boxed{}$, $\boxed{|V_A| = u\sqrt{\frac{1}{12}\left(e^2 - 4e + 7\right)}}$, $\boxed{|V_B| = \frac{u}{\sqrt{3}}(1+e)}$

a) STARTING WITH A USUAL DIAGRAM FOR THE COLLISION



BY CONSERVATION OF MOMENTUM ALONG L

$\Rightarrow 2\cancel{m}u\cos30 + 0 = 2\cancel{m}X + \cancel{m}Y$

$\Rightarrow 2X + Y = 2u\cos30°$

$\Rightarrow \boxed{2X + Y = \sqrt{3}u}$

BY RESTITUTION ALONG

$\Rightarrow \frac{Y - X}{u\cos30} = e$

$\Rightarrow -X + Y = eu\cos30°$

$\Rightarrow -X + Y = \frac{\sqrt{3}}{2}eu$

$\Rightarrow \boxed{-2X + 2Y = \sqrt{3}eu}$

SOLVING THE EQUATIONS YIELDS BY ADDITION

$\Rightarrow 3Y = \sqrt{3}u + \sqrt{3}eu$

$\Rightarrow 3Y = \sqrt{3}u(1+e)$

$\Rightarrow Y = \frac{\sqrt{3}}{3}u(1+e)$

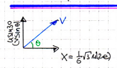
AND $X = Y - \frac{\sqrt{3}}{2}eu$

$\Rightarrow X = \frac{\sqrt{3}}{3}u(1+e) - \frac{\sqrt{3}}{2}eu = \frac{\sqrt{3}}{3}u + \frac{\sqrt{3}}{3}eu - \frac{\sqrt{3}}{2}eu$

$\Rightarrow X = \frac{\sqrt{3}}{3}u - \frac{\sqrt{3}}{6}eu$

$\Rightarrow X = \frac{\sqrt{3}}{6}u(2-e)$

SPEED OF A — NO MOMENTUM EXCHANGE $\perp$ TO L



$|V|^2 = (u\sin30)^2 + X^2$

$|V|^2 = \frac{1}{4}u^2 + \frac{1}{12}u^2(2-e)^2$

$|V|^2 = \frac{1}{12}u^2\left[3 + (2-e)^2\right]$

$|V|^2 = \frac{1}{12}u^2\left[e^2 - 4e + 7\right]$

$|V_A| = u\sqrt{\frac{e^2 - 4e + 7}{12}}$

SPEED OF B

SIMPLY FOUND ABOVE AS THE "PERPENDICULAR TO L" COMPONENT IS ZERO

$|V_B| = \frac{1}{3}\sqrt{3}u(1+e)$

b) K.E. BEFORE THE COLLISION

$\frac{1}{2}(2m)u^2 = mu^2$

K.E AFTER THE COLLISION

$\frac{1}{2}(2m)V_A^2 + \frac{1}{2}mY^2$

$= m\left(\frac{u^2(e^2-4e+7)}{12}\right) + \frac{1}{2}m\left(\frac{u^2}{3}(1+e)^2\right)$

$= \frac{mu^2}{12}(e^2 - 4e + 7) + \frac{mu^2}{6}(e^2 + 2e + 1)$

$= \frac{mu^2}{12}\left[e^2 - \cancel{4e} + 7 + 2e^2 + \cancel{4e} + 2\right]$

$= \frac{mu^2}{12}\left[3e^2 + 9\right]$

$= \frac{1}{4}mu^2(e^2 + 3)$

FINALLY WE HAVE

$\Rightarrow \frac{\frac{1}{4}\cancel{mu^2}(e^2+3)}{\cancel{mu^2}} = \frac{4}{5}$ ← "20% LOST"

$\Rightarrow \frac{1}{4}(e^2 + 3) = \frac{4}{5}$

$\Rightarrow e^2 + 3 = \frac{16}{5}$

$\Rightarrow e^2 = \frac{1}{5}$

AS REQUIRED

Question 20 (*+)**

A smooth uniform sphere P is moving on a smooth horizontal plane when it collides obliquely with an identical sphere Q which is at rest on the plane.

Immediately before the collision P is moving with speed u ms^{-1} in a direction which makes an angle of $60°$ with the line joining the centres of the spheres.

The coefficient of restitution between the spheres is $7 - 4\sqrt{3}$.

a) Show clearly that after the collision P is moving at an angle of $75°$ with the line joining the centres of the spheres.

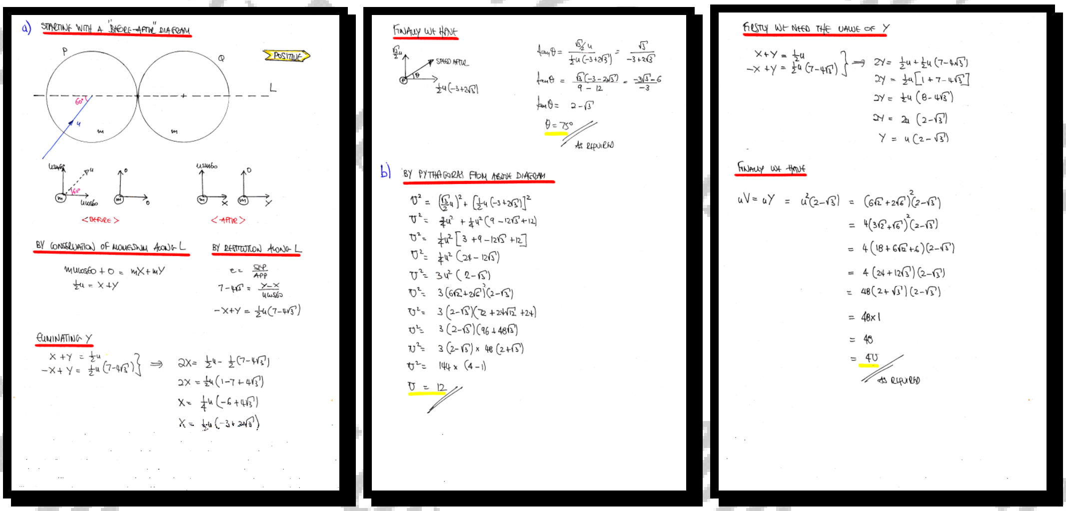
After the collision the respective speeds of P and Q are U ms^{-1} and V ms^{-1}.

It is further given that $u = \left(6\sqrt{2} + 2\sqrt{6}\right)$ ms^{-1}.

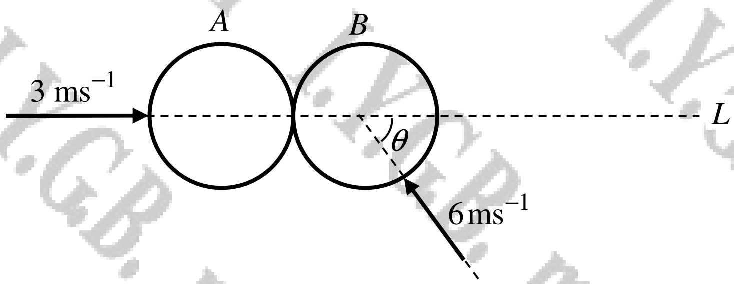
b) Find the value of U.

c) Show further that $uV = 4U$.

$\boxed{}$, $\boxed{U = 12}$



Question 21 (*+)**



Two uniform smooth spheres A and B, of equal radius and equal masses, are moving in opposite directions on a smooth horizontal surface when they collide obliquely.

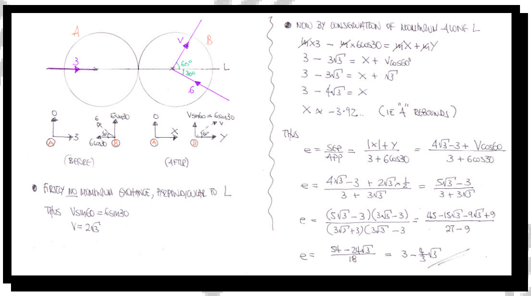
The straight line through the centres of A and B just before the collide is denoted by L and the coefficient of restitution between A and B is e.

Just before the collision the speed of A is 3 ms^{-1} along L and the corresponding speed of B is 6 ms^{-1} at an angle $30°$ to L, as shown in the figure above.

After the collision, the direction of motion of B is at right angles to its original direction of motion.

Show that $e = 3 - k\sqrt{3}$, where k is a rational constant to be found.

$$\boxed{e = 3 - \frac{4}{3}\sqrt{3}}$$



Question 22 (***+)

A small smooth sphere B of mass $2m$ is at rest on a smooth horizontal surface.

Another small smooth sphere A, of mass m and radius equal to that of B, is moving with speed u in a straight line, on the same surface.

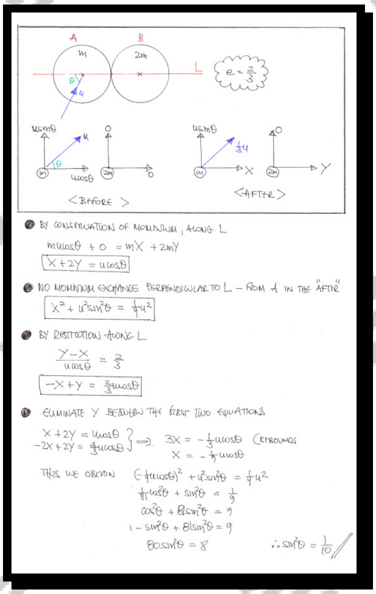
There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact. The path of A immediately before the collision makes an angle of θ with L.

The speed of A after the collision is $\frac{1}{3}u$.

Given further that the coefficient of restitution between A and B is $\frac{2}{3}$ show that

$$\sin^2\theta = \frac{1}{10}.$$

proof



- BY CONSERVATION OF MOMENTUM, ALONG L

$$mu\cos\theta + 0 = mX + 2mY$$
$$\boxed{X + 2Y = u\cos\theta}$$

- NO MOMENTUM EXCHANGE PERPENDICULAR TO L – FROM A IN THE "AFTER"

$$\boxed{X^2 + u^2\sin^2\theta = \frac{1}{9}u^2}$$

- BY RESTITUTION ALONG L

$$\frac{Y - X}{u\cos\theta} = \frac{2}{3}$$
$$\boxed{-X + Y = \frac{2}{3}u\cos\theta}$$

- ELIMINATE Y BETWEEN THE FIRST TWO EQUATIONS

$$\left.\begin{array}{l} X + 2Y = u\cos\theta \\ -2X + 2Y = \frac{4}{3}u\cos\theta \end{array}\right\} \Rightarrow 3X = -\frac{1}{3}u\cos\theta \quad \text{(REBOUNDS)}$$
$$X = -\frac{1}{9}u\cos\theta$$

THUS WE OBTAIN $\left(-\frac{1}{9}u\cos\theta\right)^2 + u^2\sin^2\theta = \frac{1}{9}u^2$

$$\frac{1}{81}\cos^2\theta + \sin^2\theta = \frac{1}{9}$$
$$\cos^2\theta + 81\sin^2\theta = 9$$
$$1 - \sin^2\theta + 81\sin^2\theta = 9$$
$$80\sin^2\theta = 8$$
$$\therefore \sin^2\theta = \frac{1}{10}$$

Question 23 (***+)

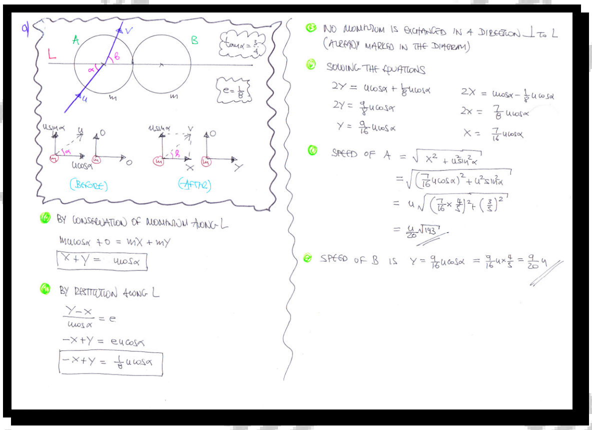
A small smooth sphere B is at rest on a smooth horizontal surface. An identical sphere A is moving with speed u in a straight line, on the same surface. There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact. The path of A immediately before the collision makes an angle of α with L, where $\tan\alpha = 0.75$.

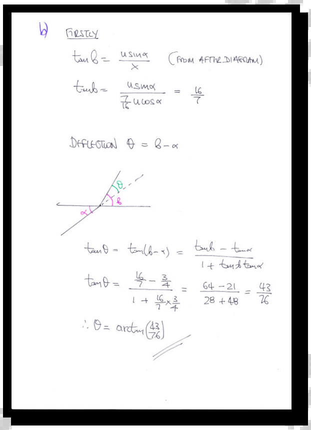
a) Given that the coefficient of restitution between the two spheres is $\frac{1}{8}$ determine simplified expressions for the speeds of A and B, in terms of u.

The path of A is deflected by an angle θ.

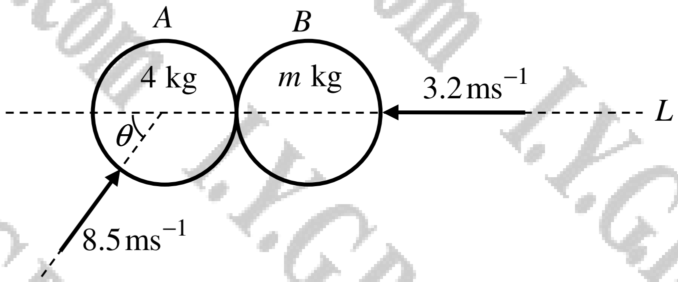
b) Show that $\theta = \arctan\left(\frac{43}{76}\right)$.

$$\boxed{|V_A| = \frac{u}{20}\sqrt{193}}, \quad \boxed{|V_B| = \frac{9u}{20}}$$





Question 24 (***+)



Two uniform smooth spheres A and B, of equal radius, have respective masses of 4 kg and m kg, are moving in opposite directions on a smooth horizontal surface when they collide obliquely.

The straight line through the centres of A and B, just before the collide is denoted by L and the coefficient of restitution between A and B is e.

Immediately before the collision the speed of A is 8.5 ms^{-1} at an angle θ to L, where $\tan\theta = \frac{8}{15}$. The corresponding speed of B is 3.2 ms^{-1} along L, as shown in the figure above.

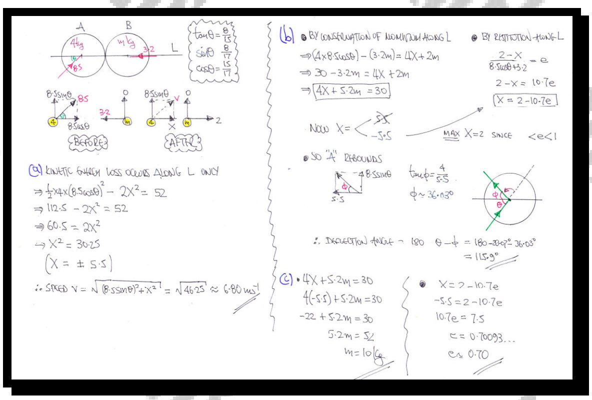
As a result of the impact, B rebounds with speed 2 ms^{-1} and A loses 52 J of kinetic energy.

a) Calculate the speed of A after the collision.

b) Determine the angle through by which the motion of A is deflected.

c) Find the value of m and the value of e.

$\boxed{\text{speed} \approx 6.80 \text{ ms}^{-1}}$, $\boxed{\delta \approx 115.9°}$, $\boxed{m = 10 \text{ kg}}$, $\boxed{e \approx 0.70}$



Question 25 (***+)

A small smooth ball of mass m is falling vertically when it strikes a fixed smooth plane which is inclined at an angle θ to the horizontal, where $\theta > 30°$.

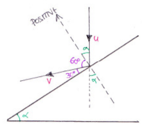
Immediately before striking the plane the ball has speed u.

Immediately after striking the plane the ball moves in a direction which makes an angle of $30°$ with the plane.

The coefficient of restitution between the ball and the plane is e.

Determine, in terms of m, u and e, the magnitude of the impulse exerted by the plane on the ball.

proof



- PARALLEL TO THE PLANE, THERE IS NO MOMENTUM EXCHANGE
  $\boxed{u\sin\alpha = v\sin 60}$
- PERPENDICULAR TO THE PLANE, THERE IS MOMENTUM EXCHANGE
  $\boxed{v\cos\alpha = eu\cos\alpha}$

$$\left.\begin{array}{l} u\sin\alpha = \frac{\sqrt{3}}{2}v \\ eu\cos\alpha = \frac{1}{2}v \end{array}\right\} \Rightarrow \left.\begin{array}{l} \sin\alpha = \frac{\sqrt{3}v}{2u} \\ \cos\alpha = \frac{v}{2eu} \end{array}\right\} \Rightarrow \text{SQUARE AND ADD}$$

$$\Rightarrow 1 = \frac{3v^2}{4u^2} + \frac{v^2}{4e^2u^2}$$
$$\Rightarrow 4e^2u^2 = 3e^2v^2 + v^2$$
$$\Rightarrow 4e^2u^2 = (3e^2+1)v^2$$
$$\Rightarrow v^2 = \frac{4e^2u^2}{3e^2+1}$$

IMPULSE CALCULATION TAKING "AWAY FROM THE PLANE AS POSITIVE"

$$I = (mv\cos 60) - (-mu\cos\alpha)$$
$$I = \frac{1}{2}mv + mu\cos\alpha$$
$$I = \frac{1}{2}m\frac{2eu}{\sqrt{3e^2+1}} + mu \times \frac{1}{\sqrt{3e^2+1}}$$
$$I = \frac{meu}{\sqrt{3e^2+1}} + \frac{mu}{\sqrt{3e^2+1}}$$
$$I = \frac{mu}{\sqrt{3e^2+1}}(e+1)$$
$$I = \frac{mu(e+1)}{\sqrt{3e^2+1}}$$

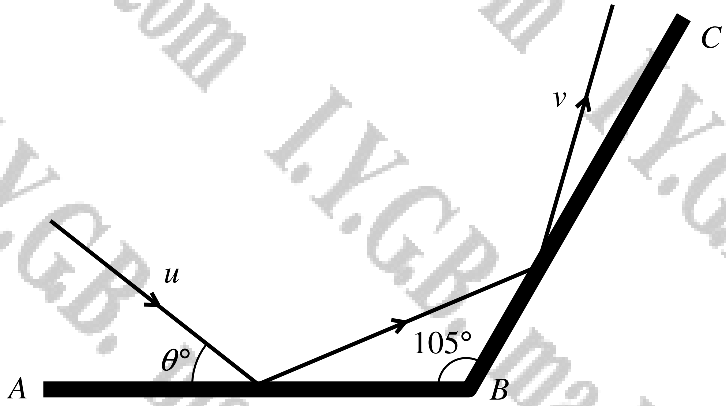
BUT $\left.\begin{array}{l} u\sin\alpha = \frac{\sqrt{3}}{2}v \\ eu\cos\alpha = \frac{1}{2}v \end{array}\right\}$ DIVIDE

$$\frac{\tan\alpha}{e} = \sqrt{3}$$
$$\tan\alpha = e\sqrt{3}$$

$e\sqrt{3}$, $3e^2+1$, α, 1

$$\boxed{\cos\alpha = \frac{1}{3e^2+1}}$$

Question 26 (***+)



The figure above shows the plan view of part of a smooth horizontal floor, where AB and BC are smooth vertical walls. The angle between AB and BC is $105°$.

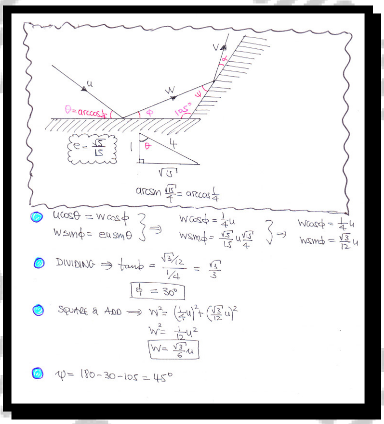
A ball is projected along the floor towards AB with speed u on a path at an angle of $\theta°$ to AB, where $\cos\theta = 0.25$. The ball hits AB, then hits BC and finally rebounds from BC with speed v.

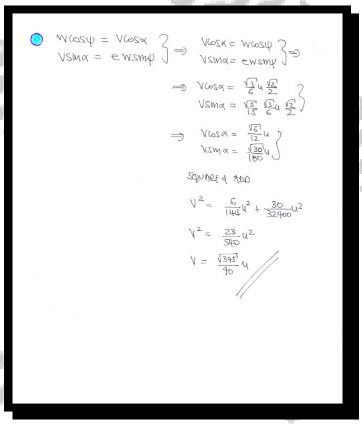
The coefficient of restitution between the ball and each wall is $\frac{\sqrt{5}}{15}$.

By modelling the ball as a particle, show that

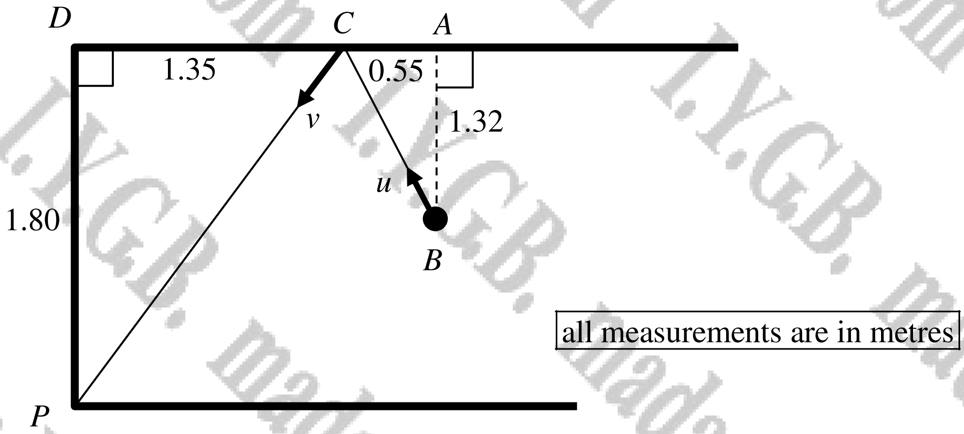
$$v = \frac{\sqrt{345}}{90}u .$$

proof





Question 27 (***+)



The figure above shows the plan of a rectangular smooth surface of a pool table whose cushions are also smooth. A ball of mass 0.13 kg is projected from B with constant speed u ms^{-1} and hits the cushion at C. It rebounds from C with constant speed v ms^{-1} and travels directly to the corner pocket at P.

The respective lengths of AB, AC, CD and DP are 1.32 m, 0.55 m, 1.35 m and 1.80 m, as shown in the figure. The ball is modelled as a particle.

a) Determine the coefficient of restitution between the ball and the cushion.

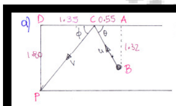
It is given that $u = 9$ and the **total time** the ball takes to travel from B to P is 0.7 s.

b) Calculate the magnitude of the impulse exerted by the cushion on the ball.

c) Find, correct to two decimal places, the magnitude of the force exerted by the cushion on the ball.

$\boxed{e = \frac{5}{9}}$, $\boxed{I = 1.68 \text{ Ns}}$, $\boxed{F = \frac{189}{17} \approx 11.12 \text{ N}}$

a)



$\tan\theta = \frac{1.32}{0.55} = \frac{12}{5}$

$\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

$\tan\phi = \frac{1.80}{1.35} = \frac{4}{3}$

$\sin\phi = \frac{4}{5}$, $\cos\phi = \frac{3}{5}$

- NO MOMENTUM EXCHANGE PARALLEL TO THE CUSHION

$\boxed{u\cos\theta = v\cos\phi}$

- BY RESTITUTION, PERPENDICULAR TO THE CUSHION

$\boxed{v\sin\phi = eu\sin\theta}$

DIVIDE EQUATIONS

$\frac{e\cancel{u}\sin\theta}{\cancel{u}\cos\theta} = \frac{\cancel{v}\sin\phi}{\cancel{v}\cos\phi}$

$e\tan\theta = \tan\phi$

$e \times \frac{12}{5} = \frac{4}{3}$

$e = \frac{5}{9}$

(b) IMPULSE IS PERPENDICULAR TO THE CUSHION; TAKE THE DIRECTION AWAY FROM THE CUSHION AS POSITIVE

$\Rightarrow I = mv\sin\phi - (-mu\sin\theta)$

$\Rightarrow I = m(v\sin\phi + u\sin\theta)$

$\Rightarrow I = m(eu\sin\theta + u\sin\theta)$

$\Rightarrow I = mu\sin\theta(e+1)$

$\Rightarrow I = 0.13 \times 9 \times \frac{12}{13} \times (\frac{5}{9}+1)$

$\Rightarrow I = 1.68$ Ns

(c) DISTANCE $|CP| = 2.25$
DISTANCE $|BC| = 1.43$ PYTHAGORAS

$u = 9$

$v = \frac{u\cos\theta}{\cos\phi} = \frac{75}{13}$

$t_1 = \frac{|BC|}{u} = \frac{1.43}{9} = \frac{143}{900}$

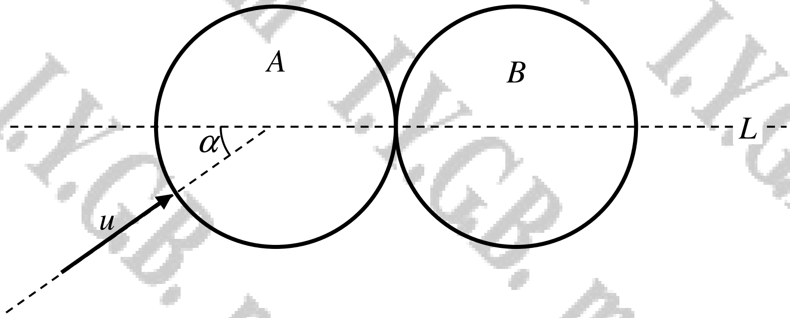
$t_2 = \frac{|CP|}{v} = \frac{2.25}{75/13} = 0.39$

$\therefore 0.7 - \frac{143}{900} - 0.39 = \frac{34}{225} \leftarrow$ TIME IN CONTACT WITH CUSHION

"$Ft = I$"

$F \times \frac{34}{225} = 1.68$ $\therefore F \simeq 11.12$ N

Question 28 (***+)



A uniform smooth sphere B is at rest on a smooth horizontal surface. An identical sphere A moving with speed u on the same surface collides with B obliquely.

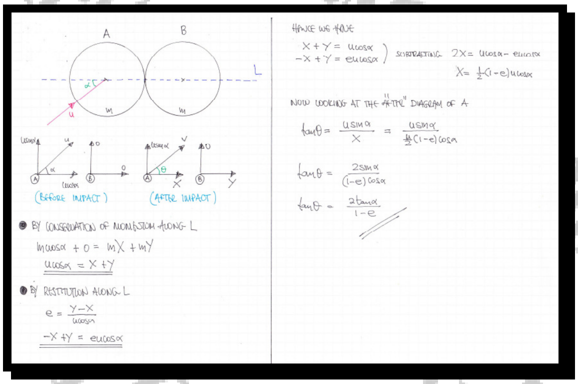
On collision the line through the centres of the two spheres is L, and the angle between u and L on impact is α, as shown in the figure above.

Given that **after** the collision the acute angle between the path of A and L is θ, show that

$$\tan\theta = \frac{2\tan\alpha}{1-e},$$

where e is the coefficient of restitution between the two spheres.

$\boxed{\text{proof}}$



- BY CONSERVATION OF MOMENTUM ALONG L

$mu\cos\alpha + 0 = mX + mY$

$\underline{\underline{u\cos\alpha = X + Y}}$

- BY RESTITUTION ALONG L

$e = \dfrac{Y-X}{u\cos\alpha}$

$\underline{\underline{-X + Y = eu\cos\alpha}}$

HENCE WE HAVE

$\left.\begin{array}{l} X + Y = u\cos\alpha \\ -X + Y = eu\cos\alpha \end{array}\right\}$ SUBTRACTING $2X = u\cos\alpha - eu\cos\alpha$

$X = \frac{1}{2}(1-e)u\cos\alpha$

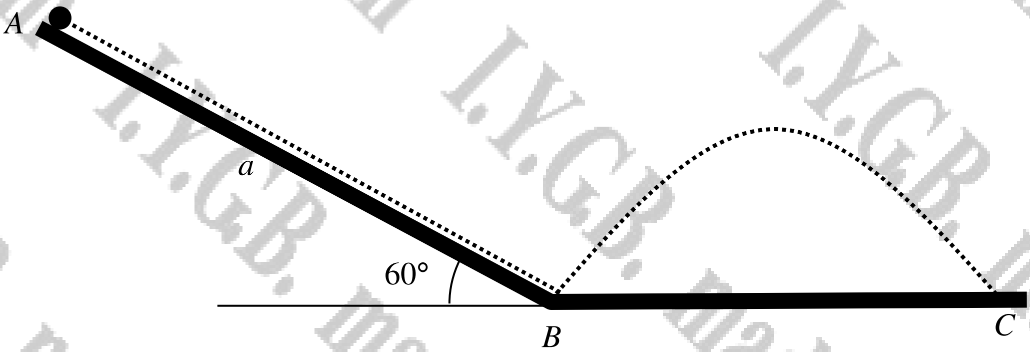
NOW LOOKING AT THE "AFTER" DIAGRAM OF A

$\tan\theta = \dfrac{u\sin\alpha}{X} = \dfrac{u\sin\alpha}{\frac{u}{2}(1-e)\cos\alpha}$

$\tan\theta = \dfrac{2\sin\alpha}{(1-e)\cos\alpha}$

$\tan\theta = \dfrac{2\tan\alpha}{1-e}$

Question 29 (***+)



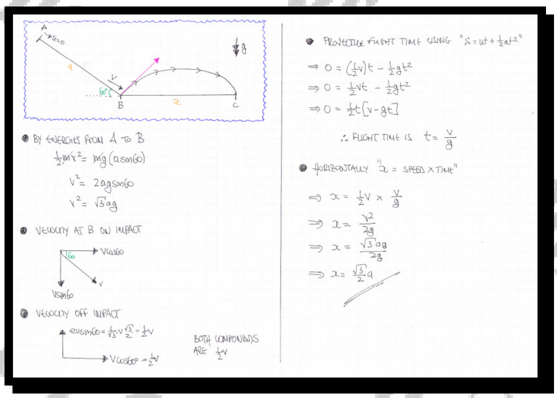
The figure above shows the path of a particle released from rest and moving down a line of greatest slope of a smooth plane inclined at $60°$ to the horizontal.

The particle is released from the point A on the plane and strikes a smooth horizontal plane at the point B, where $|AB| = a$.

The particle rebounds off the horizontal plane at B and first strikes the horizontal plane at the point C.

Given that the coefficient of restitution between the particle and horizontal plane is $\frac{1}{\sqrt{3}}$, show that $|BC| = \frac{\sqrt{3}}{2}a$.

proof



- BY ENERGIES FROM A TO B

$\frac{1}{2}mv^2 = mg(a\sin 60)$

$v^2 = 2ag\sin 60$

$v^2 = \sqrt{3}ag$

- VELOCITY AT B ON IMPACT

$V\cos 60$, $V\sin 60$

- VELOCITY OFF IMPACT

$ev\sin 60 = \frac{1}{\sqrt{3}}v\frac{\sqrt{3}}{2} = \frac{1}{2}v$

$V\cos 60 = \frac{1}{2}V$

BOTH COMPONENTS ARE $\frac{1}{2}v$

- PROJECTILE FLIGHT TIME USING "$s = ut + \frac{1}{2}at^2$"

$\Rightarrow 0 = (\frac{1}{2}v)t - \frac{1}{2}gt^2$

$\Rightarrow 0 = \frac{1}{2}vt - \frac{1}{2}gt^2$

$\Rightarrow 0 = \frac{1}{2}t[v - gt]$

$\therefore$ FLIGHT TIME IS $t = \frac{v}{g}$

- HORIZONTALLY "x = SPEED × TIME"

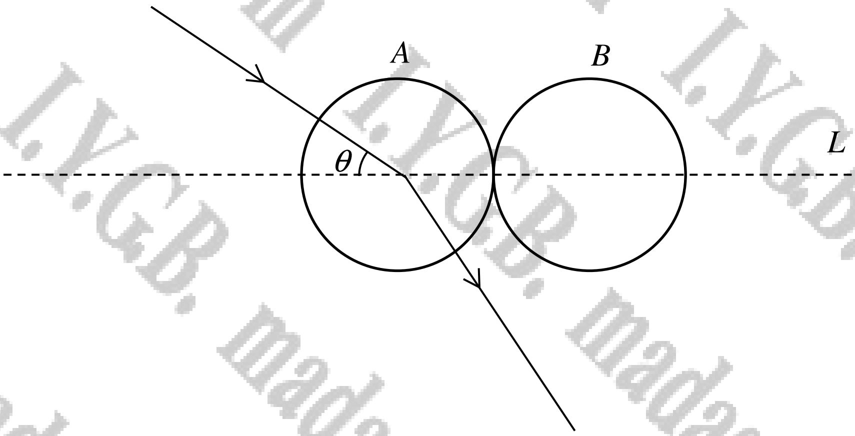
$\Rightarrow x = \frac{1}{2}v \times \frac{v}{g}$

$\Rightarrow x = \frac{v^2}{2g}$

$\Rightarrow x = \frac{\sqrt{3}ag}{2g}$

$\Rightarrow x = \frac{\sqrt{3}}{2}a$

Question 30 (***+)



A smooth sphere A travelling along a smooth horizontal surface collides with an identical sphere B, which lies at rest on the same surface. At the moment of impact the direction of motion of A makes an angle θ with the straight line L, which joins the centres of the sphere at the moment of impact.

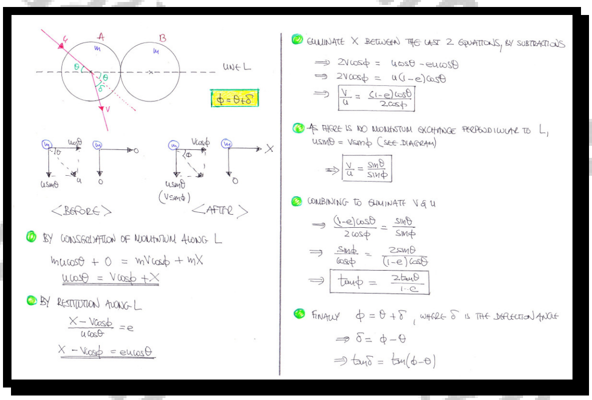
The coefficient of restitution between A and B is e.

The motion of A is deflected as a result of the impact by an angle δ.

Show that

$$\tan\delta = \frac{(1+e)\tan\theta}{1-e+2\tan^2\theta}.$$

proof



BY CONSERVATION OF MOMENTUM ALONG L

$mu\cos\theta + 0 = mV\cos\phi + mX$

$u\cos\theta = V\cos\phi + X$

BY RESTITUTION ALONG L

$\frac{X - V\cos\phi}{u\cos\theta} = e$

$X - V\cos\phi = eu\cos\theta$

ELIMINATE X BETWEEN THE LAST 2 EQUATIONS, BY SUBTRACTIONS

$\Rightarrow 2V\cos\phi = u\cos\theta - eu\cos\theta$

$\Rightarrow 2V\cos\phi = u(1-e)\cos\theta$

$\Rightarrow \boxed{\frac{V}{u} = \frac{(1-e)\cos\theta}{2\cos\phi}}$

AS THERE IS NO MOMENTUM EXCHANGE PERPENDICULAR TO L, $u\sin\theta = V\sin\phi$ (SEE DIAGRAM)

$\Rightarrow \boxed{\frac{V}{u} = \frac{\sin\theta}{\sin\phi}}$

COMBINING TO ELIMINATE V & u

$\Rightarrow \frac{(1-e)\cos\theta}{2\cos\phi} = \frac{\sin\theta}{\sin\phi}$

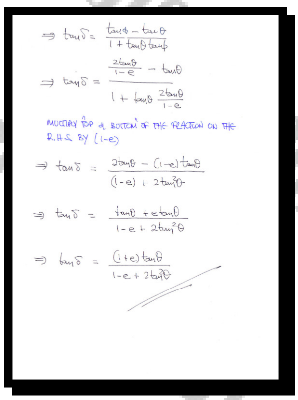
$\Rightarrow \frac{\sin\phi}{\cos\phi} = \frac{2\sin\theta}{(1-e)\cos\theta}$

$\Rightarrow \boxed{\tan\phi = \frac{2\tan\theta}{1-e}}$

FINALLY $\phi = \theta + \delta$, WHERE δ IS THE DEFLECTION ANGLE

$\Rightarrow \delta = \phi - \theta$

$\Rightarrow \tan\delta = \tan(\phi - \theta)$



$\Rightarrow \tan\delta = \frac{\tan\phi - \tan\theta}{1 + \tan\theta\tan\phi}$

$\Rightarrow \tan\delta = \frac{\frac{2\tan\theta}{1-e} - \tan\theta}{1 + \tan\theta\frac{2\tan\theta}{1-e}}$

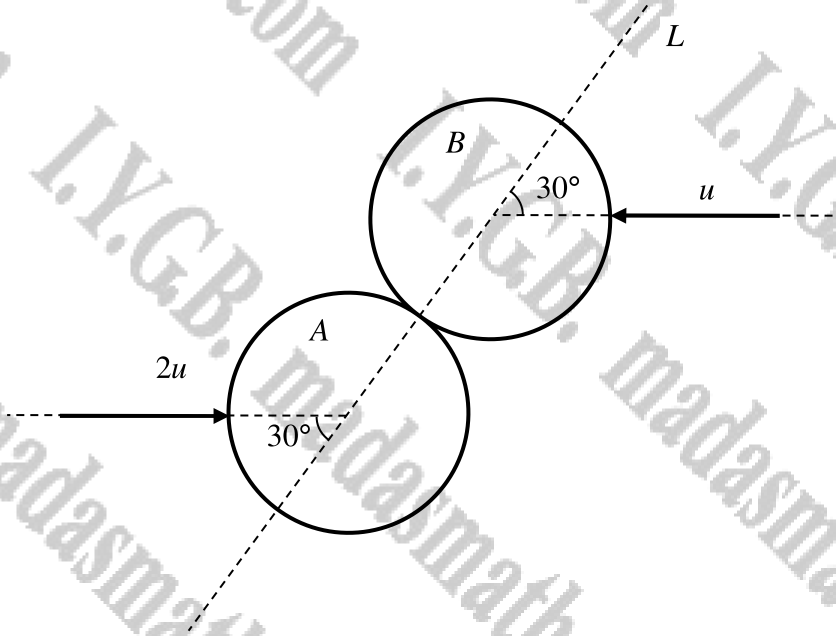
MULTIPLY "TOP" & "BOTTOM" OF THE FRACTION ON THE R.H.S BY $(1-e)$

$\Rightarrow \tan\delta = \frac{2\tan\theta - (1-e)\tan\theta}{(1-e) + 2\tan^2\theta}$

$\Rightarrow \tan\delta = \frac{\tan\theta + e\tan\theta}{1-e+2\tan^2\theta}$

$\Rightarrow \tan\delta = \frac{(1+e)\tan\theta}{1-e+2\tan^2\theta}$

Question 31 (*+)**

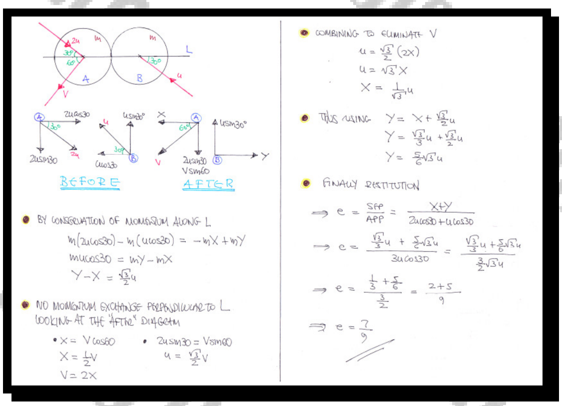


Two identical smooth uniform spheres A and B are moving in opposite directions on a smooth horizontal surface, where they collide obliquely.

Immediately before the collision, A has speed $2u$ with its direction of motion at an angle of $30°$ to the straight line L joining the centres of the spheres on impact, and B has speed u with its direction of motion also at an angle of $30°$ to L, as shown in the figure above.

Given further that the direction of motion of A turns through a right angle as a result of the collision determine the coefficient of restitution between A and B.

$$\boxed{e = \frac{7}{9}}$$



- BY CONSERVATION OF MOMENTUM ALONG L

$$m(2u\cos 30) - m(u\cos 30) = -mX + mY$$
$$mu\cos 30 = mY - mX$$
$$Y - X = \frac{\sqrt{3}}{2}u$$

- NO MOMENTUM EXCHANGE PERPENDICULAR TO L LOOKING AT THE "AFTER" DIAGRAM

• $X = V\cos 60$, $X = \frac{1}{2}V$, $V = 2X$

• $2u\sin 30 = V\sin 60$, $u = \frac{\sqrt{3}}{2}V$

- COMBINING TO ELIMINATE V

$$u = \frac{\sqrt{3}}{2}(2X)$$
$$u = \sqrt{3}X$$
$$X = \frac{1}{\sqrt{3}}u$$

- THUS USING $Y = X + \frac{\sqrt{3}}{2}u$

$$Y = \frac{\sqrt{3}}{3}u + \frac{\sqrt{3}}{2}u$$
$$Y = \frac{5}{6}\sqrt{3}u$$

- FINALLY RESTITUTION

$$\Rightarrow e = \frac{\text{SEP}}{\text{APP}} = \frac{X+Y}{2u\cos 30 + u\cos 30}$$
$$\Rightarrow e = \frac{\frac{\sqrt{3}}{3}u + \frac{5}{6}\sqrt{3}u}{3u\cos 30} = \frac{\frac{\sqrt{3}}{3}u + \frac{5}{6}\sqrt{3}u}{\frac{3}{2}\sqrt{3}u}$$
$$\Rightarrow e = \frac{\frac{1}{3} + \frac{5}{6}}{\frac{3}{2}} = \frac{2+5}{9}$$
$$\Rightarrow e = \frac{7}{9}$$

Question 32 (****)

Two identical small smooth uniform spheres, A and B are travelling towards each other in parallel but opposite directions, on a smooth horizontal surface.

The speed of A is $2u$ and the speed of B is u.

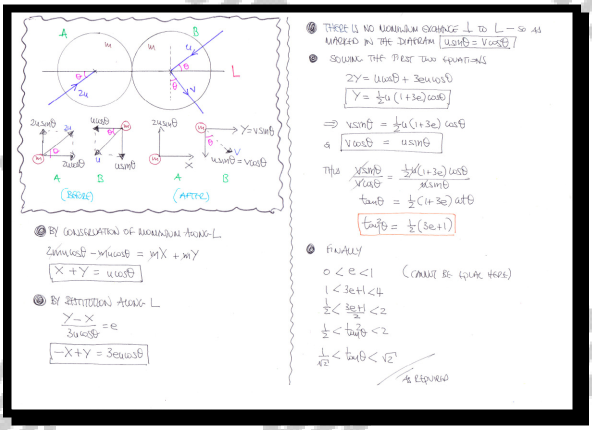
There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact. The acute angle θ is the angle that L with the original direction of the spheres.

After the collision, B moves at right angles to its original direction of motion.

Show that

$$\frac{1}{\sqrt{2}} < \tan\theta < \sqrt{2}.$$

proof



Question 33 (****)

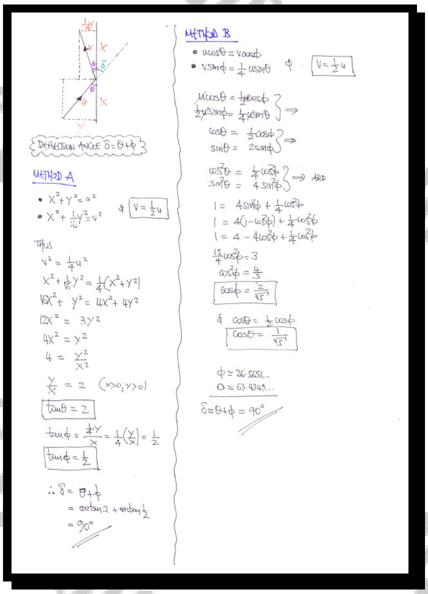
A ball is moving on a smooth horizontal plane when it collides obliquely with a smooth plane vertical wall.

The coefficient of restitution between the ball and the wall is $\frac{1}{4}$.

The speed of the ball immediately after the collision is half the speed of the ball immediately before the collision.

By modelling the ball as a particle, calculate the angle through which the path of the ball is deflected by the collision.

$\boxed{\delta = 90°}$



DEFLECTION ANGLE $\delta = \theta + \phi$

METHOD A

- $x^2 + y^2 = u^2$
- $x^2 + \frac{1}{16}y^2 = v^2$ & $\boxed{v = \frac{1}{2}u}$

Thus

$v^2 = \frac{1}{4}u^2$

$x^2 + \frac{1}{16}y^2 = \frac{1}{4}(x^2 + y^2)$

$16x^2 + y^2 = 4x^2 + 4y^2$

$12x^2 = 3y^2$

$4x^2 = y^2$

$4 = \frac{y^2}{x^2}$

$\frac{y}{x} = 2 \quad (x>0, y>0)$

$\boxed{\tan\theta = 2}$

$\tan\phi = \frac{\frac{1}{4}y}{x} = \frac{1}{4}\left(\frac{y}{x}\right) = \frac{1}{2}$

$\boxed{\tan\phi = \frac{1}{2}}$

$\therefore \delta = \theta + \phi$
$= \arctan 2 + \arctan\frac{1}{2}$
$= 90°$

METHOD B

- $u\cos\theta = v\cos\phi$
- $v\sin\phi = \frac{1}{4}u\sin\theta$ & $\boxed{v = \frac{1}{2}u}$

$\left.\begin{array}{l} u\cos\theta = \frac{1}{2}u\cos\phi \\ \frac{1}{2}u\sin\phi = \frac{1}{4}u\sin\theta \end{array}\right\} \Rightarrow$

$\left.\begin{array}{l} \cos\theta = \frac{1}{2}\cos\phi \\ \sin\theta = 2\sin\phi \end{array}\right\} \Rightarrow$

$\left.\begin{array}{l} \cos^2\theta = \frac{1}{4}\cos^2\phi \\ \sin^2\theta = 4\sin^2\phi \end{array}\right\} \Rightarrow$ ADD

$1 = 4\sin^2\phi + \frac{1}{4}\cos^2\phi$

$1 = 4(1-\cos^2\phi) + \frac{1}{4}\cos^2\phi$

$1 = 4 - 4\cos^2\phi + \frac{1}{4}\cos^2\phi$

$\frac{15}{4}\cos^2\phi = 3$

$\cos^2\phi = \frac{4}{5}$

$\boxed{\cos\phi = \frac{2}{\sqrt{5}}}$

& $\cos\theta = \frac{1}{2}\cos\phi$

$\boxed{\cos\theta = \frac{1}{\sqrt{5}}}$

$\phi \approx 26.5651...$
$\theta \approx 63.4349...$

$\delta \approx \theta + \phi = 90°$

Question 34 (****)

A particle of mass 0.5 kg is moving on a smooth horizontal plane with constant speed in a direction parallel to a long straight wall.

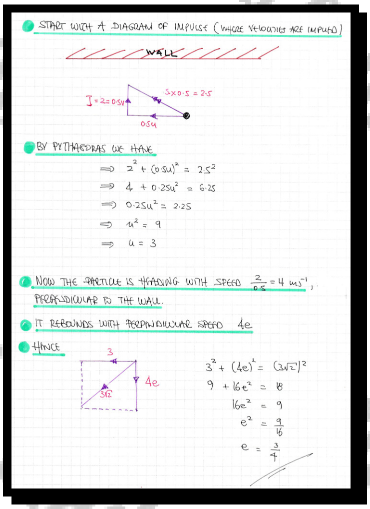
The particle receives an impulse of 2 Ns in a direction perpendicular to its motion.

Consequently, the particle's speed changes to 5 ms^{-1} and a collision with wall follows.

The speed of the particle after its collision with the wall is $3\sqrt{2} \text{ ms}^{-1}$.

Assuming that the contact between the particle and the wall is smooth, determine the coefficient of restitution between the particle and the wall.

$\boxed{\text{M456-F}}$, $\boxed{e = \frac{3}{4}}$



START WITH A DIAGRAM OF IMPULSE (WHERE VELOCITIES ARE IMPULSED)

BY PYTHAGORAS WE HAVE

$\Rightarrow 2^2 + (0.5u)^2 = 2.5^2$

$\Rightarrow 4 + 0.25u^2 = 6.25$

$\Rightarrow 0.25u^2 = 2.25$

$\Rightarrow u^2 = 9$

$\Rightarrow u = 3$

NOW THE PARTICLE IS HEADING WITH SPEED $\frac{2}{0.5} = 4 \text{ ms}^{-1}$, PERPENDICULAR TO THE WALL.

IT REBOUNDS WITH PERPENDICULAR SPEED $4e$

HENCE

$3^2 + (4e)^2 = (3\sqrt{2})^2$

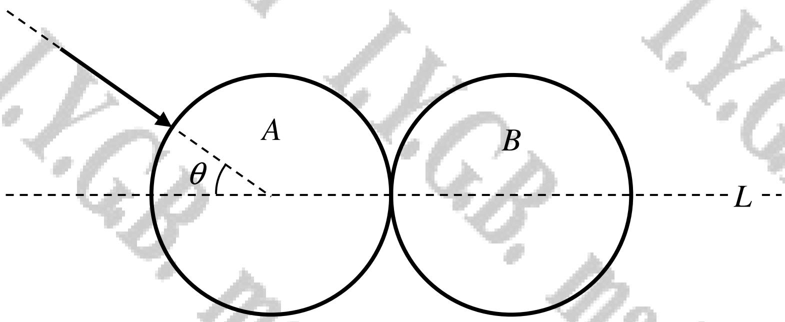
$9 + 16e^2 = 18$

$16e^2 = 9$

$e^2 = \frac{9}{16}$

$e = \frac{3}{4}$

Question 35 (****)

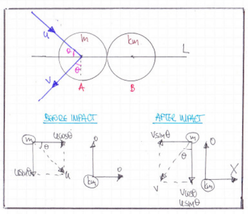


A uniform smooth sphere B of mass km, $k > 0$, is at rest on a smooth horizontal surface. Another sphere A, of equal radius as that of B but of mass m, is moving on the same surface. The two spheres collide obliquely. On collision the line through the centres of the two spheres is L. The angle between the direction of motion of A and L on impact is θ, as shown in the figure above.

As a result of the collision the direction of motion of A is deflected through a right angle

Determine a simplified expression for $\tan^2\theta$, in terms of k and e, where e is the coefficient of restitution between A and B, and hence deduce $k > 1$

$$\boxed{\tan^2\theta = \frac{ek-1}{k+1}}$$



- BY CONSERVATION OF MOMENTUM ALONG L

$mu\cos\theta + 0 = -mV\sin\theta + kmX$

$u\cos\theta = -V\sin\theta + kX$

- BY RESTITUTION ALONG L

$e = \dfrac{X + V\sin\theta}{u\cos\theta}$

- NO MOMENTUM IS EXCHANGED PERPENDICULAR TO L

$V\cos\theta = u\sin\theta$ (AFTER IMPACT DIAGRAM OF "A")

- ELIMINATE X FROM THE FIRST TWO EQUATIONS

$\Rightarrow X + V\sin\theta = eu\cos\theta$

$\Rightarrow X = eu\cos\theta - V\sin\theta$

THUS WE HAVE

$\Rightarrow u\cos\theta = -V\sin\theta + k(eu\cos\theta - V\sin\theta)$

$\Rightarrow u\cos\theta = -V\sin\theta + euk\cos\theta - kV\sin\theta$

$\Rightarrow u = -V\tan\theta + euk - kV\tan\theta$

BUT $V\cos\theta = u\sin\theta$

$V = u\tan\theta$

$\Rightarrow u = -(u\tan\theta)\tan\theta + euk - k(u\tan\theta)\tan\theta$

$\Rightarrow 1 = -\tan^2\theta + ek - k\tan^2\theta$

$\Rightarrow k\tan^2\theta + \tan^2\theta = ek - 1$

$\Rightarrow (k+1)\tan^2\theta = ek - 1$

$\Rightarrow \tan^2\theta = \dfrac{ek-1}{k+1}$

NOW $ek - 1 > 0$

$ek > 1$

$e > \frac{1}{k}$

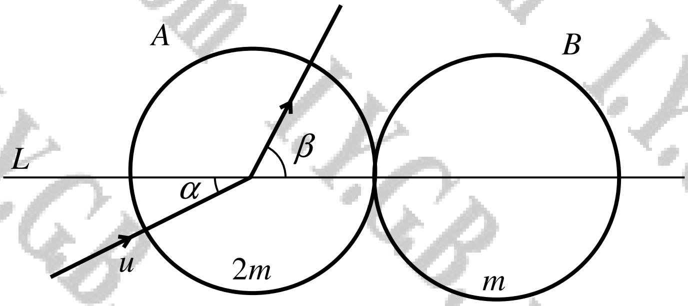
BUT

$0 < e < 1$

$0 < \frac{1}{k} < 1$

$k > 1$

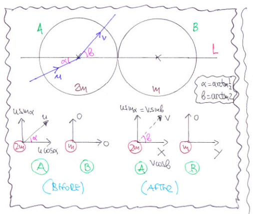
Question 36 (****)



A small smooth sphere B of mass m is at rest on a smooth horizontal surface. Another smooth sphere A of mass $2m$ and of equal radius as that of B is moving with speed u in a straight line, on the same surface. There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact. The direction of motion of A immediately before and immediately after the collision makes acute angles α and β with L, as shown in the figure above.

Given that $\alpha = \arctan\frac{1}{2}$ and $\beta = \arctan 2$ find the value of the coefficient of restitution between A and B, and hence determine the speed of B in terms of u.

$$\boxed{e = \frac{2}{3}}$$



- BY CONSERVATION OF MOMENTUM ALONG L

$2\cancel{m}u\cos\alpha + 0 = 2\cancel{m}X + \cancel{m}Y$

$\boxed{2X + Y = 2u\cos\alpha}$ (I)

- BY RESTITUTION ALONG L

$\frac{Y - X}{u\cos\alpha} = e$

$\boxed{-X + Y = eu\cos\alpha}$ (II)

- NO MOMENTUM EXCHANGE $\perp$ TO L

$\boxed{u\sin\alpha = v\sin\beta}$ (III) $\boxed{X = v\cos\beta}$ (IV)

- FIND X & Y

$\left.\begin{array}{l} 2X + Y = 2u\cos\alpha \\ -X + Y = eu\cos\alpha \end{array}\right\} \Rightarrow 3X = 2u\cos\alpha - eu\cos\alpha$

$\Rightarrow \boxed{X = \frac{1}{3}u(2-e)\cos\alpha}$

$\left.\begin{array}{l} 2X + Y = 2u\cos\alpha \\ -2X + 2Y = 2eu\cos\alpha \end{array}\right\} \Rightarrow 3Y = 2u\cos\alpha + 2eu\cos\alpha$

$\boxed{Y = \frac{2}{3}u(1+e)\cos\alpha}$

- NOW $\left.\begin{array}{l} X = v\cos\beta \\ X = \frac{1}{3}u(2-e)\cos\alpha \end{array}\right\} \Rightarrow \boxed{\frac{1}{3}u(2-e)\cos\alpha = v\cos\beta}$

- NEXT $\left.\begin{array}{l} v\sin\beta = u\sin\alpha \\ v\cos\beta = \frac{1}{3}u(2-e)\cos\alpha \end{array}\right\}$ DIVIDING

$\tan\beta = 3(2-e)\tan\alpha$

$2 = 3(2-e) \times \frac{1}{2}$

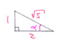
$\frac{4}{3} = 2 - e$

$e = \frac{2}{3}$

- FINALLY THE SPEED OF B

$V = \frac{2}{3}u(1+e)\cos\alpha$

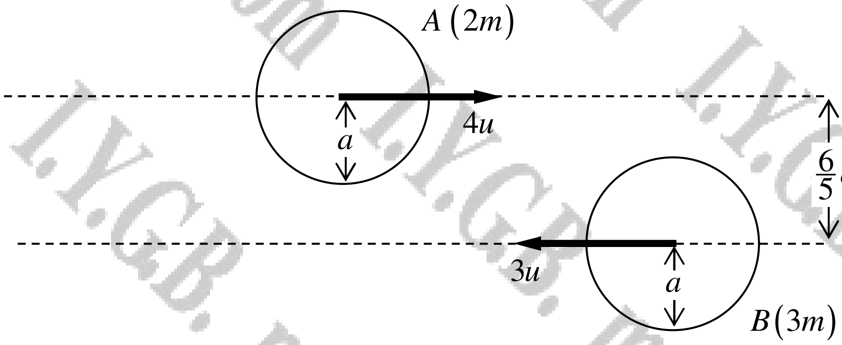
$V = \frac{2}{3}u\left(1 + \frac{2}{3}\right)\frac{2}{\sqrt{5}}$



$V = \frac{2}{3} \times \frac{5}{3} \times \frac{2}{\sqrt{5}}u$

$V = \frac{4}{9}\sqrt{5}u$

Question 37 (****)



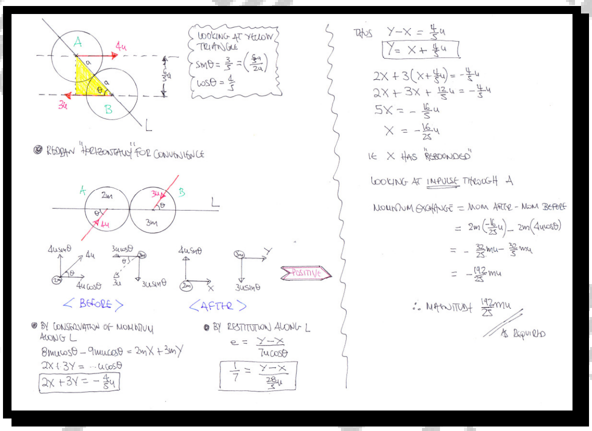
Two smooth spheres A and B, of equal radii, have respective masses of $2m$ kg and $3m$ kg, and are moving on a smooth horizontal surface when they collide.

Leading up to the collision, the centres of the two spheres are tracing parallel paths, $\frac{6}{5}a$ apart, where the respective speeds of A and B are $4u$ and $3u$, as shown in the figure above.

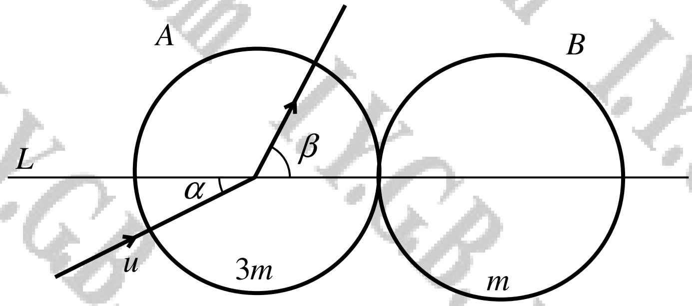
The coefficient of restitution between the two spheres is $\frac{1}{7}$

Show that the momentum exchanged during the collision has magnitude $\frac{192}{25}mu$.

$\boxed{\text{proof}}$



Question 38 (****)



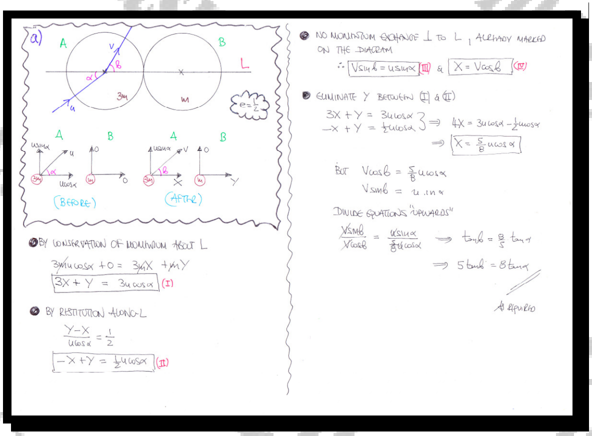
A small smooth sphere B of mass m is at rest on a smooth horizontal surface. Another smooth sphere A of mass $3m$ and of equal radius as that of B is moving with speed u in a straight line, on the same surface. There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact. The direction of motion of A immediately before and immediately after the collision makes acute angles α and β with L, as shown in the figure above.

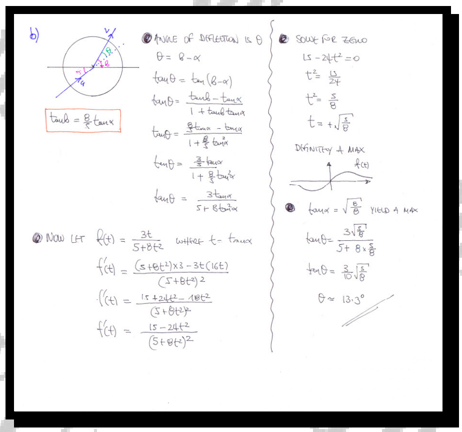
a) Given that the coefficient of restitution between A and B is $\frac{1}{2}$, show that

$$5\tan\beta = 8\tan\alpha .$$

b) Calculate, as α varies, the maximum angle of deflection of A caused by the impact.

$$\boxed{(\beta-\alpha)_{\max} \approx 13.3°}$$





Question 39 (****+)

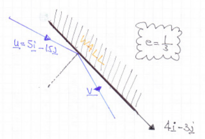
In this question $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular unit vectors in a horizontal plane.*

A smooth sphere is moving on a smooth horizontal surface when it collides with a smooth vertical wall, which is parallel to the direction $(4\mathbf{i}-3\mathbf{j})$.

The velocity of the sphere before the impact with the wall is $(5\mathbf{i}-15\mathbf{j})\ \text{ms}^{-1}$.

Given that the coefficient of restitution between the sphere and the wall is $\frac{1}{3}$, find the velocity of the sphere after the impact.

$$\boxed{v = \left(\frac{61}{5}\mathbf{i} - \frac{27}{5}\mathbf{j}\right)}$$



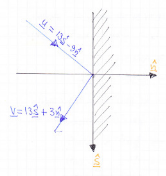
- THE NORMAL DIRECTION "TOWARDS" THE WALL IS $3\underline{i} + 4\underline{j}$
- PARALLEL UNIT VECTOR $\hat{\underline{s}} = \frac{1}{5}(4\underline{i} - 3\underline{j})$
  PERPENDICULAR UNIT VECTOR $\hat{\underline{n}} = \frac{1}{5}(3\underline{i} + 4\underline{j})$
- RESOLVE THE "BEFORE VELOCITY"

$$5\underline{i} - 15\underline{j} = A\hat{\underline{s}} + B\hat{\underline{n}}$$
$$5\underline{i} - 15\underline{j} = \frac{A}{5}(4\underline{i} - 3\underline{j}) + \frac{B}{5}(3\underline{i} + 4\underline{j})$$
$$25\underline{i} - 75\underline{j} = (4A + 3B)\underline{i} + (-3A + 4B)\underline{j}$$

$$\left.\begin{array}{l} 4A + 3B = 25 \\ -3A + 4B = -75 \end{array}\right\} \Rightarrow \begin{array}{r} 12A + 9B = 75 \\ -12A + 16B = -300 \\ \hline 25B = -225 \\ B = -9 \end{array}$$

$$\Rightarrow 4A + 3(-9) = 25$$
$$\Rightarrow 4A - 27 = 25$$
$$\Rightarrow 4A = 52$$
$$\Rightarrow A = 13$$

THUS $\underline{u} = 5\underline{i} - 15\underline{j} = 13\hat{\underline{s}} - 9\hat{\underline{n}}$

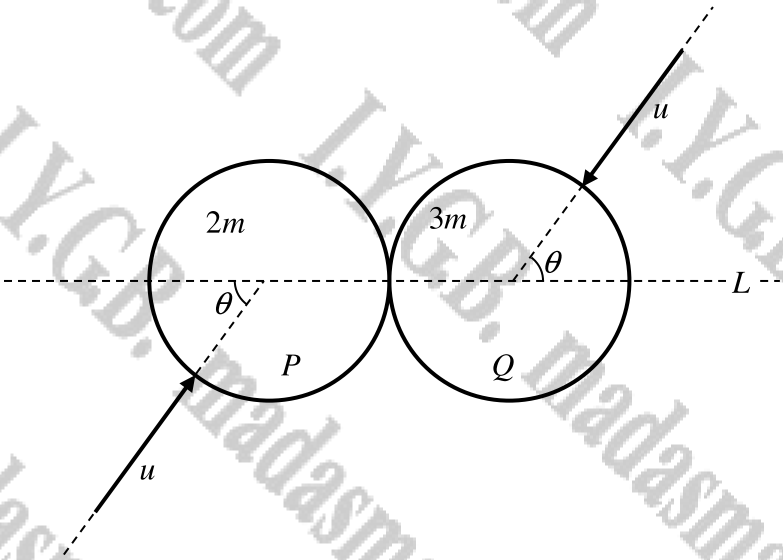


NO MOMENTUM IS EXCHANGED IN THE TANGENTIAL DIRECTION, IE PARALLEL TO THE WALL ($\hat{\underline{s}}$)

MOMENTUM IS EXCHANGED, IN A DIRECTION PERPENDICULAR TO THE WALL $\hat{\underline{n}}$ – REBOUNDS WITH SPEED $\frac{1}{3}$

$$\therefore\ \underline{v} = 13\hat{\underline{s}} + 3\hat{\underline{n}}$$
$$\underline{v} = 13\left[\frac{1}{5}(4\underline{i} - 3\underline{j})\right] + 3\left[\frac{1}{5}(3\underline{i} + 4\underline{j})\right]$$
$$\underline{v} = \frac{52}{5}\underline{i} - \frac{39}{5}\underline{j} + \frac{9}{5}\underline{i} + \frac{12}{5}\underline{j}$$
$$\underline{v} = \frac{61}{5}\underline{i} - \frac{27}{5}\underline{j}$$

Question 40 (****+)



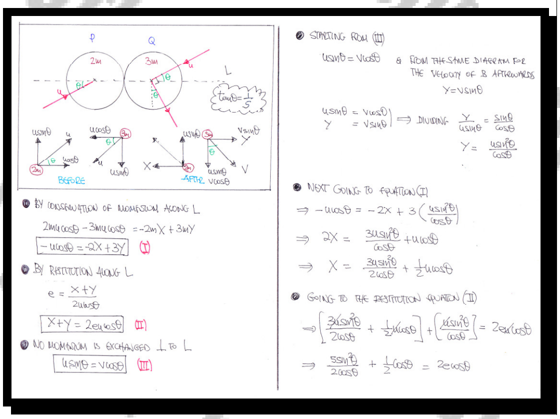
Two smooth uniform spheres P and Q with equal radii have masses $2m$ and $3m$ respectively. The spheres are moving in opposite directions on a smooth horizontal surface, where they collide obliquely.

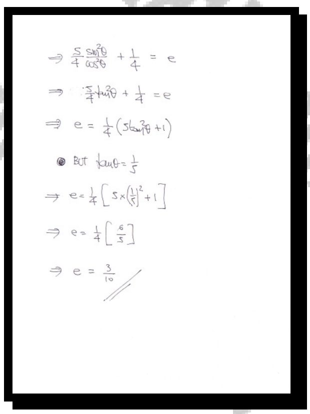
Immediately before the collision, P has speed u, and its direction of motion forms an angle θ with the straight line L, joining the centres of the spheres on impact.

Immediately before the collision Q has speed u with its direction of motion also forming an angle θ to L, as shown in the figure above.

Given that $\tan\theta = \frac{1}{5}$ and the direction of motion of Q is deflected by $90°$, as a result of the collision, find the value of the coefficient of restitution between P and Q.

$\boxed{e = 0.3}$





Question 41 (****+)

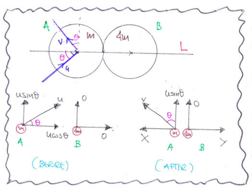
A smooth sphere B of mass $4m$ is at rest on a smooth horizontal surface. Another sphere A is moving with speed u in a straight line, on the same surface. There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact.

The direction of motion of A immediately before the collision makes an angle of θ with L, where $0° < \theta < 90°$.

Immediately after the collision, the direction of motion of A is at right angles to direction of motion of A immediately before the collision.

Determine the greatest possible value of θ.

$\boxed{\theta \approx 37.76°}$



- BY CONSERVATION OF MOMENTUM ALONG L

$mu\cos\theta + 0 = -mX + 4mY$

$\boxed{u\cos\theta = -X + 4Y}$

- BY RESTITUTION ALONG L

$\frac{X+Y}{u\cos\theta} = e$

$\boxed{X + Y = eu\cos\theta}$

- NO MOMENTUM EXCHANGE $\perp$ TO L

$\boxed{V\cos\theta = u\sin\theta}$ & $\boxed{X = V\sin\theta}$

- SOLVING THE FIRST TWO EQUATIONS TO FIND X & Y

$\left.\begin{array}{l} X+Y = eu\cos\theta \\ -X+4Y = u\cos\theta \end{array}\right)$ ELIMINATE $Y \Rightarrow$

$\left.\begin{array}{l} 4X+4Y = 4eu\cos\theta \\ -X+4Y = u\cos\theta \end{array}\right)$ SUBTRACT $\Rightarrow$

$\Rightarrow 5X = 4eu\cos\theta - u\cos\theta$

$\Rightarrow 5X = u(4e-1)\cos\theta$

$\Rightarrow \boxed{X = \frac{1}{5}u(4e-1)\cos\theta}$

- NOW $V\sin\theta = \frac{1}{5}u(4e-1)\cos\theta$

$\Rightarrow V\sin\theta\cos\theta = \frac{1}{5}u(4e-1)\cos^2\theta$

$\Rightarrow \sin\theta(V\cos\theta) = \frac{1}{5}u(4e-1)\cos^2\theta$

$\Rightarrow \sin\theta(u\sin\theta) = \frac{1}{5}u(4e-1)\cos^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{5}(4e-1)\cos^2\theta$

$\Rightarrow 5\tan^2\theta = 4e-1$

$\Rightarrow 4e = 1 + 5\tan^2\theta$

$\Rightarrow e = \frac{1}{4}(1+5\tan^2\theta)$

- BUT $e \leqslant 1$

$\Rightarrow \frac{1}{4}(1+5\tan^2\theta) \leqslant 1$

$\Rightarrow 1 + 5\tan^2\theta \leqslant 4$

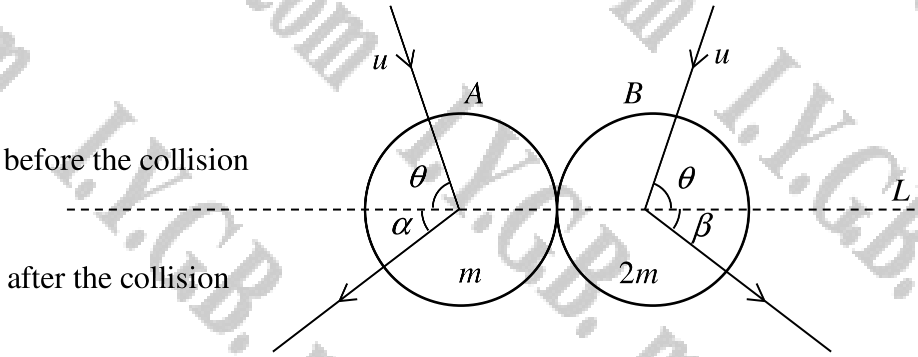
$\Rightarrow 5\tan^2\theta \leqslant 3$

$\tan^2\theta \leqslant \frac{3}{5}$

$\tan\theta \leqslant +\sqrt{\frac{3}{5}}$

$\theta \leqslant 37.76°$

Question 42 (****+)



Two smooth spheres, A and B, of equal radius and respective masses m and $2m$, are moving on a smooth horizontal plane when they collide obliquely.

When A and B collide the straight line through their centres is denoted by L.

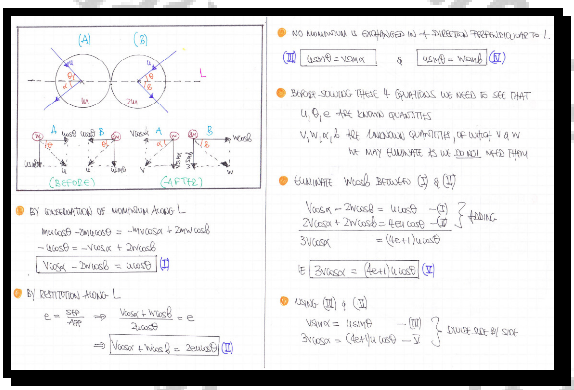
The speeds of A and B before their collision are both u and the direction of both speeds is at an acute angle θ to L.

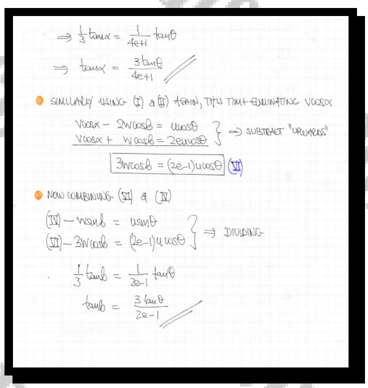
After the collision, the direction of motion of A is at an acute angle α to L, and the direction of motion of B is at an acute angle β to L, as shown in the figure above.

The coefficient of restitution between A and B is e.

Determine simplified expressions for $\tan\alpha$ and $\tan\beta$ in terms of e and θ.

$$\boxed{\tan\alpha = \frac{3\tan\theta}{4e+1}}, \quad \boxed{\tan\beta = \frac{3\tan\theta}{2e-1}}$$





Question 43 (****+)

In this question $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular unit vectors in a horizontal plane.*

Two smooth spheres A and B, of equal radii, have respective masses of $3m$ kg and m kg, and are moving on a smooth horizontal surface when they collide obliquely.

In standard Cartesian vector notation, the respective velocities of A and B just before their collision are $(2\mathbf{i}+4\mathbf{j})\ \text{ms}^{-1}$ and $(3\mathbf{i}-2\mathbf{j})\ \text{ms}^{-1}$.

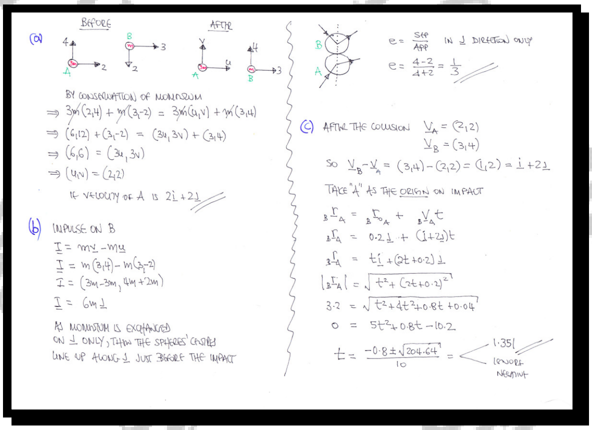
a) Given that the velocity of B after the collision is $(3\mathbf{i}+4\mathbf{j})\ \text{ms}^{-1}$, determine the velocity of A after the collision.

b) Calculate the coefficient of restitution between the two spheres.

The radius of each of the two spheres is 0.1 m. At time T s after the collision the centres of the spheres are 3.2 m apart.

c) Show that $T = 1.351$ s, correct to three decimal places.

$\boxed{e=\frac{5}{9}}$, $\boxed{I = 1.68\ \text{Ns}}$, $\boxed{F=\frac{189}{17}\approx 11.12\ \text{N}}$



(a) BEFORE AFTER

BY CONSERVATION OF MOMENTUM

$\Rightarrow 3m(2,4) + m(3,-2) = 3m(u,v) + m(3,4)$

$\Rightarrow (6,12) + (3,-2) = (3u,3v) + (3,4)$

$\Rightarrow (6,6) = (3u,3v)$

$\Rightarrow (u,v) = (2,2)$

IE VELOCITY OF A IS $2\mathbf{i} + 2\mathbf{j}$

(b) IMPULSE ON B

$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$\mathbf{I} = m(3,4) - m(3,-2)$

$\mathbf{I} = (3m-3m, 4m+2m)$

$\mathbf{I} = 6m\mathbf{j}$

AS MOMENTUM IS EXCHANGED ON $\mathbf{j}$ ONLY, THEN THE SPHERES' CENTRES LINE UP ALONG $\mathbf{j}$ JUST BEFORE THE IMPACT

$e = \frac{\text{SEP}}{\text{APP}}$ IN $\mathbf{j}$ DIRECTION ONLY

$e = \frac{4-2}{4+2} = \frac{1}{3}$

(c) AFTER THE COLLISION $\mathbf{V}_A = (2,2)$, $\mathbf{V}_B = (3,4)$

SO $\mathbf{V}_B - \mathbf{V}_A = (3,4) - (2,2) = (1,2) = \mathbf{i} + 2\mathbf{j}$

TAKE "A" AS THE ORIGIN ON IMPACT

${}_B\mathbf{r}_A = {}_B\mathbf{r}_{0A} + {}_B\mathbf{V}_A t$

${}_B\mathbf{r}_A = 0.2\mathbf{j} + (\mathbf{i}+2\mathbf{j})t$

${}_B\mathbf{r}_A = t\mathbf{i} + (2t+0.2)\mathbf{j}$

$|{}_B\mathbf{r}_A| = \sqrt{t^2 + (2t+0.2)^2}$

$3.2 = \sqrt{t^2+4t^2+0.8t+0.04}$

$0 = 5t^2 + 0.8t - 10.2$

$t = \frac{-0.8 \pm \sqrt{204.64}}{10} = \begin{cases} 1.351 \\ \text{IGNORE NEGATIVE} \end{cases}$

Question 44 (****+)

In this question $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular unit vectors in a horizontal plane.*

A smooth sphere is moving on a smooth horizontal surface when it strikes a fixed smooth vertical wall. The wall is parallel to the vector $3\mathbf{i}+\mathbf{j}$.

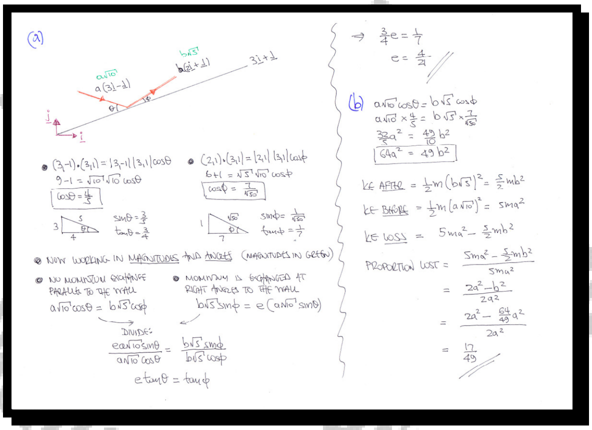
The velocity of the sphere immediately before the impact is $a(3\mathbf{i}-\mathbf{j})\ \text{ms}^{-1}$, where a is a positive scalar constant.

The velocity of the sphere immediately after the impact is $b(2\mathbf{i}+\mathbf{j})\ \text{ms}^{-1}$, where b is a positive scalar constant.

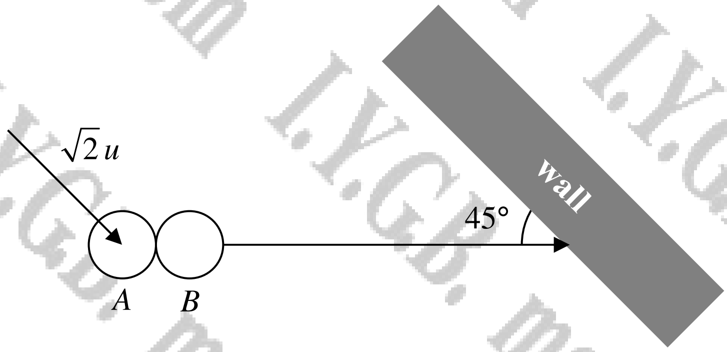
a) Calculate the coefficient of restitution between the two spheres.

b) Determine the fraction of the kinetic energy of the sphere that is lost due to the collision with the wall.

$\boxed{e=\frac{4}{21}}$, $\boxed{\frac{17}{49}}$



Question 45 (****+)



A smooth sphere B is at rest on a smooth horizontal surface at a fixed distance from a smooth, long vertical wall. An identical sphere A is moving with speed $\sqrt{2}u$ in a straight line, on the same surface, in a direction parallel to the wall.

There is a collision between the two spheres and L is the straight line joining the centres of the two spheres at the moment of impact, which is at 45° to the wall as shown in the figure above.

The coefficient of restitution between the two spheres is e.

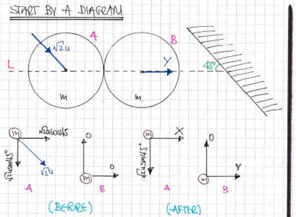
After B collides with the wall, its direction of motion is parallel to the direction of motion of A after the collision.

Show that the coefficient of restitution between B and the wall is

$$\frac{1+e}{3-e}.$$

$\boxed{\text{proof}}$

START BY A DIAGRAM



BY CONSERVATION OF MOMENTUM ALONG L

$m\sqrt{2}u\cos45 + 0 = mX + mY$

$X + Y = u$

BY RESTITUTION ALONG L

$\frac{Y-X}{\sqrt{2}u\cos45} = e$

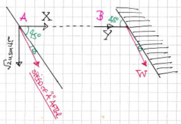
$-X + Y = eu$

SOLVING THE EQUATIONS

$2Y = u + eu$ $\quad$ $2X = u - eu$

$Y = \frac{1}{2}u(1+e)$ $\quad$ $X = \frac{1}{2}u(1-e)$

LOOKING AT ANOTHER DIAGRAM



NEXT WE LOOK AT THE COLLISION WITH THE WALL

- $Y\cos45 = W\cos\theta$ (NO MOMENTUM EXCHANGE)
- $\frac{W\sin\theta}{Y\sin45} = E$ ← RESTITUTION WITH THE WALL

$\left.\begin{aligned} W\sin\theta &= EY\sin45 \\ W\cos\theta &= Y\cos45 \end{aligned}\right)$ DIVIDE, NOTING THAT $\sin45 = \cos45$, GIVES $\tan\theta = E$

FINALLY LOOKING AT THE SPEED OF OF A, AFTER THE COLLISION

$\Rightarrow \tan(45+\theta) = \frac{\sqrt{2}u\sin45}{X} = \frac{\sqrt{2}u \times \frac{1}{\sqrt{2}}}{\frac{1}{2}u(1-e)} = \frac{2}{1-e}$

$\Rightarrow \frac{\tan45 + \tan\theta}{1 - \tan45\tan\theta} = \frac{2}{1-e}$

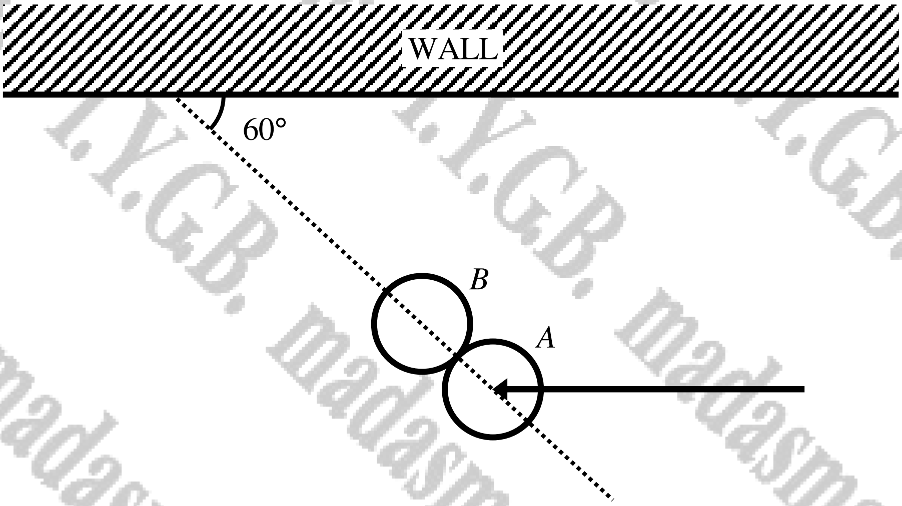
$\Rightarrow \frac{1+E}{1-E} = \frac{2}{1-e}$

$\Rightarrow (1-e) + E(1-e) = 2 - 2E$

$\Rightarrow (3-e)E = 2 - (1-e)$

$\Rightarrow E = \frac{1+e}{3-e}$ AS REQUIRED

Question 46 (*****)



A small smooth sphere A is moving with constant speed, in a straight line on a smooth horizontal floor. It collides obliquely with an identical sphere B which is at rest on the same horizontal floor. The direction of motion of A just before the collision is parallel to a smooth vertical wall. At the instant of impact between the two spheres, a straight line passing through the centres of the spheres makes an acute angle of $60°$ with the wall, as shown in the figure above.

The coefficient of restitution between the two spheres is e.

After the spheres collide, B collides with the vertical wall and rebounds.

If the spheres now move in parallel directions, show that the coefficient of restitution between B and the wall is $\dfrac{1+e}{7-e}$.

, proof