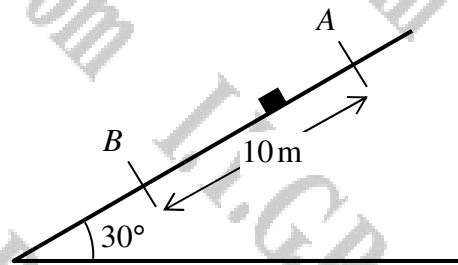


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WORK & ENERGY

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Question 1 (**)



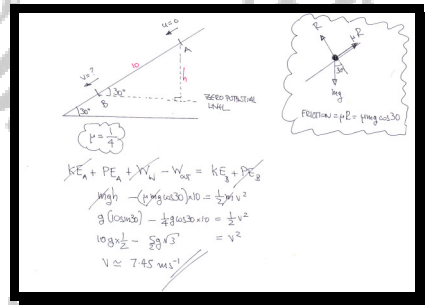
The figure above shows a particle sliding down a rough plane inclined at an angle of 30° to the horizontal. The box is released from rest at the point A and passes through the point B , which lies 10 m further down the plane, with a speed of $v \text{ ms}^{-1}$.

The points A and B lie on a line of greatest slope on the plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$.

Find the value of v , correct to three significant figures.

$$v \approx 7.45 \text{ ms}^{-1}$$

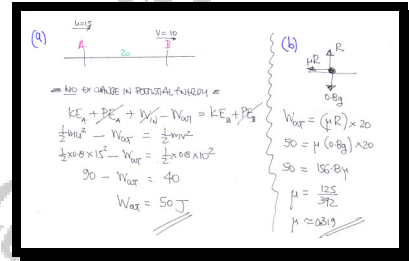


Question 2 ()**

A particle of mass 0.8 kg is sliding on a rough horizontal surface reducing its speed from 15 ms^{-1} at a point A , to 10 ms^{-1} at a point B . It is assumed that the only resistance to the motion of the particle is the ground friction.

- Given that the distance AB is 20 m , determine the work done by the friction in the particle's journey from A to B .
- Find, to three significant figures, the value of the coefficient of friction between the particle and the horizontal surface.

$W_{\text{out}} = 50 \text{ J}$, $\mu \approx 0.319$



Question 3 ()**

A small box of mass 2 kg is sliding on a floor between two points A and B . The box has a speed of 8 ms^{-1} at A and comes to rest at B .

The box is modelled as a particle and the floor as a rough horizontal plane, where μ is the coefficient of friction between the particle and the horizontal plane.

- Calculate the kinetic energy loss as the box moves from A to B .
- Given that $AB = 12 \text{ m}$, find the value of μ .

kinetic energy loss = 64 J, $\mu \approx 0.272$

(a) $KE_{\text{initial}} = \frac{1}{2} \times 2 \times 8^2 = 64 \text{ J}$
 $KE_{\text{final}} = 0$
 \therefore loss of 64 J

(b) $u = 8$ at A , $v = 0$ at B
 (Consideration in potential energy)
 $KE_A + PE_A + W_{\text{fr}} - W_{\text{grav}} = KE_B + PE_B$
 $\frac{1}{2} \times 2 \times 8^2 - 12 \times 2 \times \mu = 0$
 $\frac{1}{2} \times 2 \times 64 - \mu(24) \times 12 = 0$
 $64 - 12\mu = 0$
 $12\mu = 64$
 $\mu = \frac{64}{12} \approx 0.533$

Question 4 (+)**

A particle of mass 2 kg is projected vertically upwards from ground level with a speed of 21 ms^{-1} and comes to instantaneous rest at a height of $d \text{ m}$ above the ground. The particle is subject to a constant non gravitational resistance of 7 N, assumed constant throughout the motion.

Use work and energy considerations to determine the value of d .

$d \approx 16.58 \text{ m}$

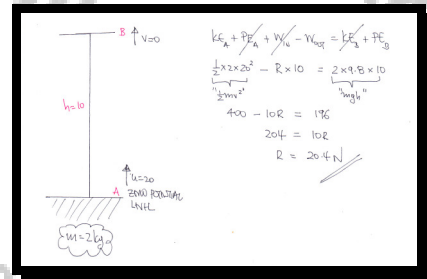
$u = 21$, $v = 0$
 (Consideration in potential energy)
 $KE_A + PE_A + W_{\text{res}} - W_{\text{grav}} = KE_B + PE_B$
 $\frac{1}{2} \times 2 \times 21^2 - 7d = 2 \times 9.8 \times d$
 $441 - 7d = 19.6d$
 $441 = 26.6d$
 $d = 16.58 \text{ m}$

Question 5 (**+)

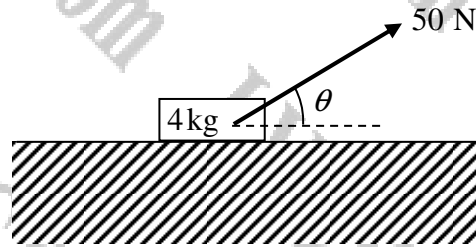
A particle of mass 2 kg is projected vertically upwards from ground level with a speed of 20 ms^{-1} and comes to instantaneous rest at a height of 10 m above the ground. The particle is subject to a constant non gravitational resistance of $R \text{ N}$, throughout the motion.

Use work and energy considerations to determine the value of R .

$$R = 20.4 \text{ N}$$



Question 6 (**+)



The figure above shows a small box of mass 4 kg, pulled by a rope along rough horizontal ground. The box has a speed of 10 ms^{-1} at the point A and 12 ms^{-1} at the point B .

The force supplied by the rope is 50 N and is inclined at an angle θ to the horizontal ground, where $\tan \theta = \frac{4}{3}$.

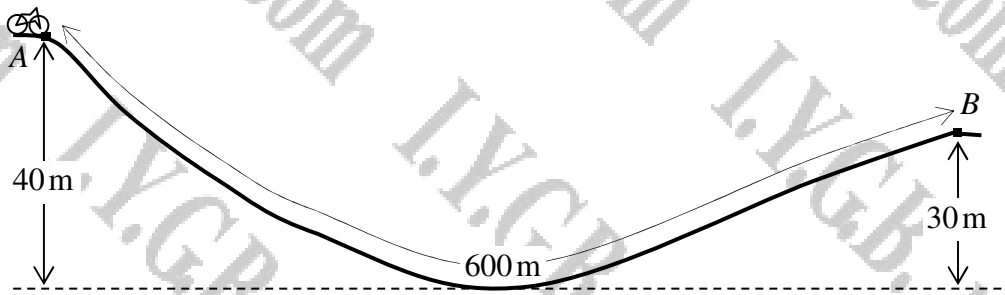
The box is modelled as a particle experiencing a constant ground friction of F N.

Given that the distance AB is 200 m, use work and energy considerations to find the value of F .

$F = 29.56$

The handwritten solution shows a free-body diagram of the box and a work-energy calculation. The free-body diagram includes forces: tension T at angle θ , weight $4g$, normal reaction N , and friction F . The work-energy calculation starts with $K_A + W_{nc} = K_B$, where $K_A = \frac{1}{2} \times 4 \times 10^2$, $W_{nc} = (50 \cos \theta - F) \times 200$, and $K_B = \frac{1}{2} \times 4 \times 12^2$. It then solves for F to get $F = 29.56 \text{ N}$.

Question 7 (**+)



The figure above shows the path of a cyclist on a section of a road from A to B , where the distance AB is 600 m.

The cyclist leaves point A at the top of a hill with a speed of 10 ms^{-1} and descends a vertical distance of 40 m to the bottom of the hill. The cyclist then ascends a vertical distance of 30 m to the top of another hill at point B . The speed of the cyclist at B is 12 ms^{-1} .

The combined mass of the cyclist and his bike is 80 kg.

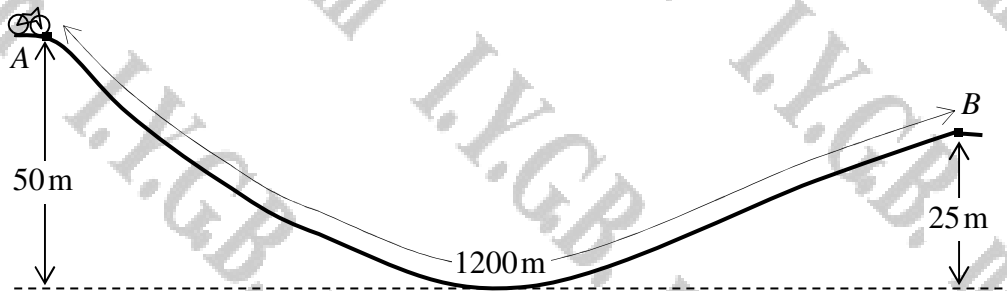
The cyclist and his bike are modelled as a single particle subject to a constant non gravitational resistance of 25 N, throughout the motion.

Find the work done by the cyclist.

$W = 8920 \text{ J}$

$u = 10$
 $v = 12$
 $h = 40$
 $k = 30$
 $d = 600$
 $m = 80$
 CONSTANT RESISTANCE OF 25
 $KE_A + PE_A + W_W - W_{GR} = KE_B + PE_B$
 $\frac{1}{2} \times 80 \times 10^2 + 80 \times 9.8 \times 40 + W_W - 25 \times 600 = \frac{1}{2} \times 80 \times 12^2 + 80 \times 9.8 \times 30$
 $4000 + 31360 + W_W - 15000 = 5760 + 23520$
 $W_W = 8920$

Question 8 (***)



The figure above shows the path of a cyclist on a section of a road from A to B , where the distance AB is 1200 m.

The cyclist leaves point A at the top of a hill with a speed $V \text{ ms}^{-1}$ and descends a vertical distance of 50 m to the bottom of the hill. He then ascends a vertical distance of 25 m to the top of another hill at point B .

The cyclist takes 110 s to travel from A to B and is assumed to be working at the constant rate of 40 W, throughout the motion.

The combined mass of the cyclist and his bike is 80 kg.

The cyclist and his bike are modelled as a single particle subject to a constant non gravitational resistance of 20 N, throughout the motion.

Show that speed of the cyclist at B is $V \text{ ms}^{-1}$.

$V \text{ ms}^{-1}$, proof

LOOKING AT THE DIAGRAM BELOW

$$KE_A + PE_A + W_{\text{cyc}} - W_{\text{res}} = KE_B + PE_B$$

$\frac{1}{2}mV^2 + mgh + \frac{\text{work done}}{\text{time}} - \frac{\text{work done}}{\text{time}} = KE_B + mgh$
 $40 + 20 \times 1200 = KE_B + 19600$
 $40 = KE_B + 19600 - 24000$
 $40 = KE_B - 4400$
 $KE_B = 4440$

RETURNING TO THE ORIGINAL QUESTION

$$\Rightarrow KE_A + 80 \times 9.8 \times 50 + 4400 - 24000 = KE_B + 80 \times 9.8 \times 25$$

$$\Rightarrow KE_A + 39200 + 4400 - 24000 = KE_B + 19600$$

$$\Rightarrow KE_A + 19600 = KE_B + 19600$$

$$\Rightarrow KE_A = KE_B$$

∴ SAME SPEED AS THE CYCLIST AT A IS MAINTAINED

Question 9 (*)**

A car of mass 1500 kg is travelling up a hill on a straight road, with the engine of the car working at the constant rate of 13 kW for 1 minute.

During this minute the car increases its speed from 7 ms^{-1} to 24 ms^{-1} and in addition to the work done against gravity, 80000 J of work is done against resistances to motion parallel to the direction of motion of the car.

Calculate the vertical displacement of the car in this 1 minute interval.

$$h \approx 20.73 \text{ m}$$

• Power = $\frac{W_{in}}{Time}$
 $13000 = \frac{W_{in}}{60}$
 $W_{in} = 780000$

• TRACKING THE LEVEL AT WHICH THE SPEED OF THE CAR IS 7 ms^{-1} , AS THE ZERO POTENTIAL LEVEL.
 $K_E + PE_A + W_{in} - W_{out} = K_E + PE_B$
 $\frac{1}{2} \times 1500 \times 7^2 + 78000 - 80000 = \frac{1}{2} \times 1500 \times 24^2 + 1500gh$
 $36750 + 78000 - 80000 = 45000 + 1500gh$
 $30750 = 1500gh$
 $h = \frac{30750}{1500} \approx 20.5 \text{ m}$

Question 10 (*)**

A particle P of mass 4 kg is moving on the line of greatest slope of a rough plane, inclined at an angle θ to the horizontal, where $\tan \theta = \frac{3}{4}$.

The particle is projected up the plane with a speed $u\text{ ms}^{-1}$ from a point A on the plane, comes to instantaneous rest at a point B , and then slides back down the plane passing through A again.

The coefficient of friction between the particle and the plane is $\frac{2}{7}$ and the distance AB is 2.5 m .

Use work and energy considerations to find ...

- ... the value of u .
- ... the speed of the particle as it passes through A again.

$$u \approx 6.37, \quad \approx 4.27\text{ ms}^{-1}$$

(a) $m=4$
 $\tan \theta = \frac{3}{4}$

Diagram (a) shows a particle of mass m moving up an inclined plane from point A to point B . The distance AB is 2.5 m . The angle of the plane is θ . The particle starts with speed u and comes to rest at B . A force diagram at B shows the weight mg acting vertically downwards, the normal reaction R acting perpendicular to the plane, and the friction force F acting up the plane.

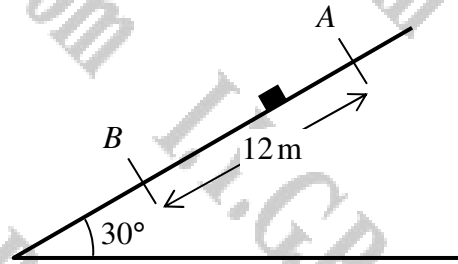
Work done by friction = $4F$
 $= \frac{2}{7} \times 4 \times 9.8 \times \frac{5}{4}$
 $= 89.6$

Energy equation:
 $K_E + PE_A + W_{fr} - W_{gr} = K_E + PE_B$
 $\frac{1}{2}mv^2 - (Friction) \times 2.5 = mgh$
 $\frac{1}{2} \times 4 \times u^2 - 89.6 \times 2.5 = 4 \times 9.8 \times (2.5 \sin \theta)$
 $2u^2 - 22.4 = 4 \times 9.8 \times 2.5 \times 0.6$
 $2u^2 = 83.2$
 $u^2 = 41.6$
 $u \approx 6.37\text{ ms}^{-1}$

(b) Diagram (b) shows the particle sliding back down the plane from point B to point A . The distance BA is 2.5 m . A force diagram at A shows the weight mg acting vertically downwards, the normal reaction R acting perpendicular to the plane, and the friction force F acting down the plane.

Energy equation:
 $K_E + PE_B + W_{fr} - W_{gr} = K_E + PE_A$
 $mgh - (Friction) \times 2.5 = \frac{1}{2}mv^2$
 $4 \times 9.8 \times 2.5 \sin \theta - 22.4 = \frac{1}{2} \times 4 \times v^2$
 $v^2 = 18.2$
 $v \approx 4.27\text{ ms}^{-1}$

Question 11 (***)



The figure above shows a box of mass 0.5 kg sliding down a rough plane inclined at an angle of 30° to the horizontal. The box passes through the point A with a speed of 10 ms^{-1} and through the point B , which lies 12 m further down the plane, with a speed of 9 ms^{-1} .

- a) Find the loss in energy of the box as it moves from A to B .

The coefficient of friction between the box and the plane is μ .

- b) Find the value of μ , correct to three significant figures.

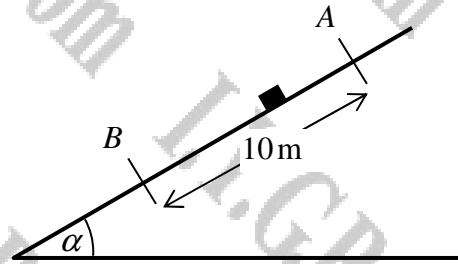
loss = 34.15 J, $\mu \approx 0.671$

Handwritten solution for Question 11:

Part (a):
 $m = 0.5\text{ kg}$
 $v_A = 10$
 $v_B = 9$
 $h = 12 \sin 30^\circ = 6\text{ m}$
 $PE_A = mgh = 0.5 \times 9.8 \times 6 = 29.4\text{ J}$
 $PE_B = 0$
 $KE_A = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 10^2 = 25\text{ J}$
 $KE_B = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 9^2 = 20.25\text{ J}$
 Loss = $29.4 - 20.25 = 9.15\text{ J}$ (Note: This handwritten calculation seems to be for a different part of the question or is a correction. The printed answer is 34.15 J.)

Part (b):
 Work done by friction = 34.15
 $34.15 = (\mu \times mg \cos 30) \times 12$
 $34.15 = \mu \times 0.5 \times 9.8 \times \frac{\sqrt{3}}{2} \times 12$
 $34.15 = \frac{147\sqrt{3}}{2} \mu$
 $\mu \approx 0.671$

Question 12 (***)



The figure above shows a particle sliding down a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{4}{3}$. The box is released from rest at the point A and passes through the point B, which lies 10 m further down the plane, with a speed of 7 ms^{-1} .

Use work and energy considerations, to show that the coefficient of friction between the particle and the plane is $\frac{11}{12}$.

proof

$\tan \alpha = \frac{4}{3}$, $\sin \alpha = \frac{4}{5}$, $\cos \alpha = \frac{3}{5}$
 $K_A + P_{SA} + W_{fr} - W_{gr} = K_B + P_{SB}$
 $mgh - (\mu R) \times 10 = \frac{1}{2}mv^2$
 $9.8 \times 10 \times \frac{4}{5} - \mu \times 9.8 \times \frac{3}{5} \times 10 = \frac{1}{2} \times 7^2$
 $78.4 - 19.6\mu = 24.5$
 $53.9 = 19.6\mu$
 $\mu = \frac{11}{12} \approx 0.917$

Question 13 (***)

A particle of mass 0.5 kg is projected with a speed of 5 ms^{-1} from a point A on a rough plane, inclined at an angle of 20° to the horizontal.

The particle slides up the plane and comes to instantaneous rest at a point B on the plane which is 2 m away from A .

The points A and B lie on the line of greatest slope on the plane. The particle is subject to a constant non gravitational resistance of $R \text{ N}$, throughout the motion.

- a) Determine the value of R , correct to three significant figures.

The angle of the plane is now increased to 40° but the value of R remains unchanged.

The particle is next projected with a speed of 8 ms^{-1} from A and slides up the plane coming to instantaneous rest at a distance $d \text{ m}$ away from A .

- b) Find the value of d , correct to three significant figures.

$R \approx 1.45 \text{ N}$, $d \approx 3.48 \text{ m}$

(a) $m = 0.5$
 $R = \text{constant resistance}$
 Diagram: Incline at 20° , point A to B distance 2m. Initial speed $u = 5$, final speed $v = 0$.

$$KE_A + PE_A + W_{\text{res}} - W_{\text{grav}} = KE_B + PE_B$$

$$\frac{1}{2} \times 0.5 \times 5^2 - R \times 2 = 0.5 \times 9.8 \times (2 \sin 20^\circ)$$

$$6.25 - 2R = 3.25179 \dots$$

$$2.9982 = 2R$$

$$R \approx 1.499 \dots$$

$$R \approx 1.45$$

(b) $u = 8$, $R = 1.45$
 Diagram: Incline at 40° , point A to B distance d . Initial speed $u = 8$, final speed $v = 0$.

$$KE_A + PE_A + W_{\text{res}} - W_{\text{grav}} = KE_B + PE_B$$

$$\frac{1}{2} \times 0.5 \times 8^2 - (1.45 \times d) = 0.5 \times 9.8 \times (d \sin 40^\circ)$$

$$16 - 1.45d = 4.9d \sin 40^\circ$$

$$16 - 1.45d = (3.192 \dots)d$$

$$16 = (4.642 \dots)d$$

$$d \approx 3.48 \text{ m}$$

Question 14 (***)

A woman and her bike are modelled a single particle of combined mass 72 kg.

The woman cycles with constant speed of 5 ms^{-1} , **up** a straight road, which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal, where $\sin \theta = \frac{2}{21}$.

The total non gravitational resistance experienced by the cyclist is assumed to be constant at 25 N.

- a) Find the power generated by the woman when cycling up the hill.

The woman then turns her bike around at some point A on the road. She freewheels down the same road starting with a speed of 5 ms^{-1} . She passes through some point B on that road with a speed $v \text{ ms}^{-1}$.

The total non gravitational resistance experienced by the cyclist is assumed to be the same as in part (a).

- b) Given that the distance AB is 180 m, find the value of v .

$P = 461 \text{ W}$, $v \approx 15.4$

(a) \bullet CONSTANT SPEED \Rightarrow ZERO ACCELERATION \Rightarrow EQUILIBRIUM IN THE DIRECTION OF MOTION

$D = 25 + 72g \sin \theta$
 $D = 25 + 72 \times \frac{2}{21}$
 $D = 461$

$P = Dv$
 $P = 461 \times 5$
 $P = 2305 \text{ W}$

(b)


$KE_A + PE_A + W_{friction} = KE_B + PE_B$
 $\frac{1}{2} \times 72 \times 5^2 + 72 \times 9.8 \times 180 \times \frac{2}{21} - (25 \times 180) = \frac{1}{2} \times 72 \times v^2$
 $900 + 12096 - 4500 = 36v^2$
 $8496 = 36v^2$
 $v^2 = 236$
 $v \approx 15.3622 \dots$
 $v \approx 15.4 \text{ ms}^{-1}$

Question 15 (*)**

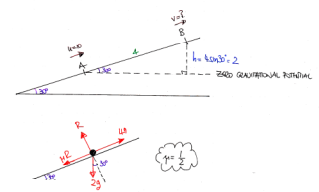
A particle of mass 2 kg is dragged by a constant force of 49 N, up the line of greatest slope of a rough plane, inclined at an angle of 30° to the horizontal. This force is also acting in the line of greatest slope of the plane. The coefficient of friction between the particle and the plane is 0.5.

The particle passes through two points on the plane A and B which are 4 m apart, and point B is at a higher level on the plane than point A .

Given that the particle is passing through A with a speed of 10 ms^{-1} , use work and energy considerations to find the speed of the particle as it passes through B .

 , $v \approx 14.93 \text{ ms}^{-1}$

SIMILAR WITH A ZIMBABWE



FORMING THE ENERGY EQUATION

$$\Rightarrow K.E_A + P.E_A + W_{int} - W_{ext} = K.E_B + P.E_B$$

$$\Rightarrow \frac{1}{2}mv^2 + 0 + (49 \times 4) - 42 \times 4 = \frac{1}{2}mv^2 + mgh$$

$$\Rightarrow \frac{1}{2} \times 2 \times 10^2 + 196 - \frac{1}{2} \times (2 \times 9.8 \times 4) = \frac{1}{2} \times 2 \times v^2 + 2 \times 9.8 \times 2$$

$$\Rightarrow 100 + 196 - 39.2 = v^2 + 39.2$$

$$\Rightarrow v^2 = 222.851042 \dots$$

$$\Rightarrow v = 14.93 \text{ ms}^{-1}$$

Question 16 (***)

A car of mass 1230 kg is moving on a straight road which lies on the line of greatest slope of a rough plane, inclined at an angle θ to the horizontal, where $\tan \theta = \frac{9}{40}$.

There is a constant non gravitational resistance of 400 N acting on the car.

The car passes through two points on the plane A and B , with speeds 6 ms^{-1} and 20 ms^{-1} , respectively, where the point B lies at a higher level on the plane than A .

The distance AB is 615 m and it takes 50 s for the car to travel this distance.

- Find, in MJ, the work done by the engine of the car.
- Determine the average rate at which the engine of the car is working.

2.10 MJ , $P \approx 41.9 \text{ kW}$

(a)

Diagram: A car of mass $m = 1230 \text{ kg}$ moves up an inclined plane from point A to point B . The distance AB is 615 m . The car's speed at A is 6 ms^{-1} and at B is 20 ms^{-1} . A constant non-gravitational resistance of 400 N acts on the car. The angle of the plane is θ , where $\tan \theta = \frac{9}{40}$.

Work done by the engine:

$$W_{\text{eng}} + W_{\text{nc}} + W_{\text{grav}} - W_{\text{fr}} = K_E + P_E$$

$$\Rightarrow \frac{1}{2} \times 1230 \times 6^2 + W_{\text{nc}} - (400 \times 615) = \frac{1}{2} \times 1230 \times 20^2 + 1230 \times 9.8 \times 615 \sin \theta$$

$$\Rightarrow 22140 + W_{\text{nc}} - 246000 = 246000 + 1627290$$

$$\Rightarrow W_{\text{nc}} = 2097150 \text{ J}$$

$$\Rightarrow W_{\text{nc}} = 2.10 \text{ MJ} \quad (3 \text{ s.f.})$$

(b) Rate of work done = Power = $\frac{W_{\text{nc}}}{\text{Time}} = \frac{2097150}{50} = 41943 \text{ W} \approx 41.9 \text{ kW}$

Question (***)

A block of mass 5 kg is pulled along a rough, straight, horizontal road by a constant horizontal force of magnitude 47 N.

The block moves in a straight line between two points A and B , where $|AB| = 7$ m.

The coefficient of friction between the block and the road is 0.5.

Find the speed of the particle at A given that its speed at B is 4 times as large as its speed at A .

, $u = \frac{1}{5}\sqrt{105} \approx 2.05 \text{ ms}^{-1}$

BY ENERGY CONSERVATION

$$\Rightarrow K_E + P.E. + W_{fr} - W_g = K.E. + P.E.$$

$$\Rightarrow \frac{1}{2}mv^2 + F \cdot d - F_f \cdot d = \frac{1}{2}mv^2$$

\uparrow External force \uparrow Friction force = $\mu R = \mu mg$

$$\Rightarrow \frac{1}{2}mv^2 + F \cdot d - \mu mg \cdot d = \frac{1}{2}mv^2$$

SUBSTITUTION: SCALE NUMBERS

$$\Rightarrow \frac{1}{2} \times 5v^2 + 47 \times 7 - \frac{1}{2} \times 5 \times 9.8 \times 7 = \frac{1}{2} \times 5v^2$$

$$\Rightarrow 2.5v^2 + 329 - 343 = 2.5v^2$$

$$\Rightarrow 5v^2 + 315 = 5v^2$$

$$\Rightarrow v^2 + 63 = v^2$$

BUT IT IS GIVEN THAT $v = 4u$

$$\Rightarrow v^2 + 63 = (4u)^2$$

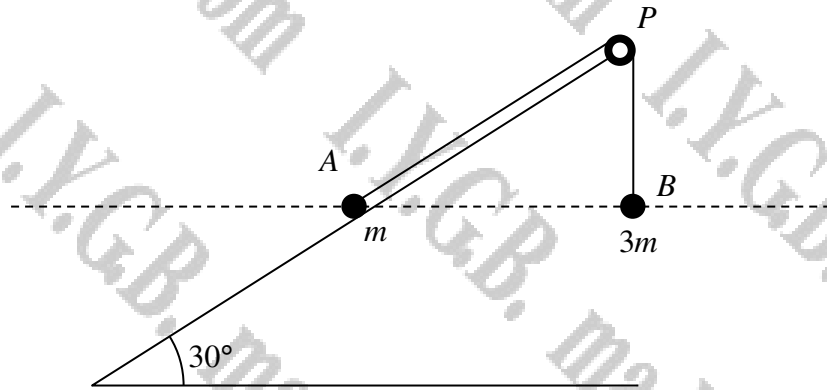
$$\Rightarrow 63 = 15u^2$$

$$\Rightarrow u^2 = \frac{63}{15} = \frac{21}{5}$$

$$\Rightarrow |u| = \frac{1}{5}\sqrt{105}$$

$$\Rightarrow |u| \approx 2.05 \text{ ms}^{-1}$$

Question 17 (***)



Two particles A and B , of mass m and $3m$ respectively, are attached to each of the ends of a light inextensible string. The string passes over a smooth pulley P , at the top of a fixed smooth plane, inclined at 30° to the horizontal.

Particle A is held at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P , as shown in the figure above. The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane.

The particles are released from rest, from the same horizontal level with the string taut. When B has fallen by a distance l , its speed is v , and A has not yet reach P .

By using energy considerations and ignoring air resistance, express v in terms of g and l .

$$v = \frac{1}{2}\sqrt{5gl}$$

STARTING WITH AN ENERGY DIAGRAM

Initial Energy: $KE_A + KE_B + PE_A + PE_B$
 Final Energy: $KE_A + KE_B + PE_A + PE_B$

on release: $0 + 0 + mgl + 3mg(-l)$
 AFTER BOTH HAVENOT MET: $\frac{1}{2}mv^2 + \frac{1}{2}(3m)v^2 + mg(l \sin 30^\circ) + 3mg(-l)$

$\Rightarrow 0 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)v^2 + mg(l \sin 30^\circ) + 3mg(-l)$
 $\Rightarrow 0 = \frac{1}{2}mv^2 + \frac{3}{2}mv^2 + \frac{1}{2}mgl - 3mgl$
 $\Rightarrow 0 = mv^2 + 3mgl - 5mgl$
 $\Rightarrow 0 = 4mv^2 - 5mgl$
 $\Rightarrow 4v^2 = 5gl$
 $\Rightarrow v^2 = \frac{5gl}{4}$
 $\Rightarrow |v| = \frac{1}{2}\sqrt{5gl}$

Question 18 (***)

A car of mass 1300 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{10}$.

The total non gravitational resistance experienced by the car is assumed to be a constant force of magnitude of 400 N. The engine of the car is working at the constant rate of 30 kW.

The car is passing through the point A with a speed 10 ms^{-1} and continues to accelerate up the plane, passing through the point B with speed 30 ms^{-1} , 30 s after passing through A.

By modelling the car as a particle, find ...

- ... the acceleration of the car at A.
- ... the distance AB.

, $a = 1.02 \text{ ms}^{-2}$, $|AB| \approx 227 \text{ m}$

(a) $P = Dv$
 $30000 = D \times 10$
 $D = 3000$
 $N = 400 - (3000 \sin \theta) = 1200a$
 $3000 - 400 - (3000 \times \frac{1}{10}) = 1300a$
 $1300 = 1300a$
 $a = 1.02 \text{ ms}^{-2}$

(b) Power = $\frac{\text{Work Done}}{\text{Time}}$
 $30000 = \frac{W}{30}$
 $W = 900000$

By EKE
 $K.E_A + P.E_A + W_{nc} - W_{fr} = K.E_B + P.E_B$
 $\frac{1}{2} \times 1300 \times 10^2 + 400000 - 400d = \frac{1}{2} \times 1300 \times 30^2 + (1300 \times 6 \times 4)$
 $65000 + 40000 - 400d = 585000 + (1300 \times 6 \times 4)$
 $105000 - 400d = 585000 + 12744$
 $380000 = 1674d$
 $d \approx 227.011947 \dots$
 $d \approx 227 \text{ m}$

Question 19 (***)

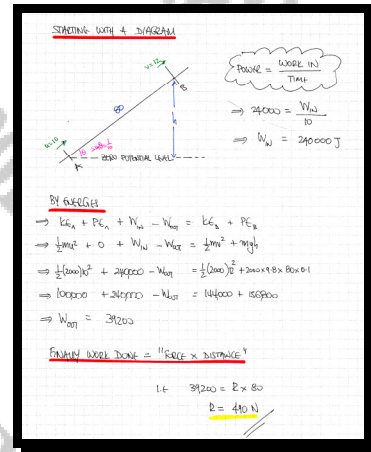
A car of mass 2000 kg is travelling up a line of greatest slope of a hill, inclined at $\arcsin(0.1)$.

At a given time the speed of the car is 10 ms^{-1} and 10 s later the speed of the car is 12 ms^{-1} , having covered a distance of 80 m.

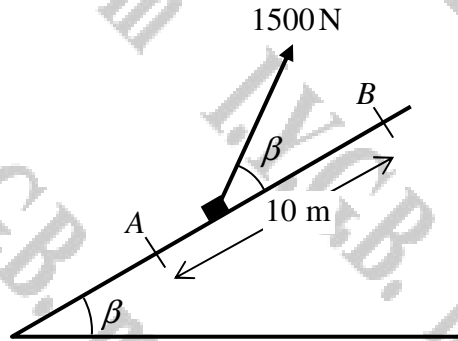
The engine of the car is assumed to be working at a constant rate of 24 kW, and it is assumed that the truck experiences a constant air resistance, $R \text{ N}$.

Calculate the value of R .

, $R = 490$



Question 20 (****)



The figure above shows a box of mass 120 kg being pulled up the line of greatest slope of the plane inclined at an angle β to the horizontal, by an electrically operated cable. The cable is supplying a constant tension of 1500 N and is inclined at an angle β to the plane.

The box passes through the point A with speed 5 ms^{-1} and through the point B which is higher up the plane with speed $v \text{ ms}^{-1}$. The distance AB is 10 m.

There is a constant non gravitational resistance of 180 N acting on the box.

The box is modelled as a particle and the cable as a light inextensible string.

Given that $\tan \beta = \frac{3}{4}$ find the value of v .

, $v \approx 8.80$

ANALYSIS FIRST

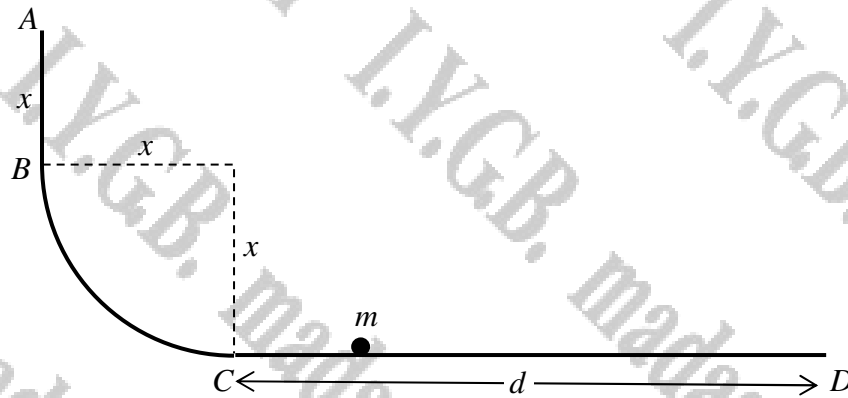
$\tan \beta = \frac{3}{4} \Rightarrow \frac{3}{4} = \frac{3}{4} \Rightarrow \begin{cases} \sin \beta = \frac{3}{5} \\ \cos \beta = \frac{4}{5} \end{cases}$

ONLY THE COMPONENT OF THE TENSION PARALLEL TO THE PLANE DOES WORK FOR THE SYSTEM, I.E.
 $1500 \cos \beta = 1500 \times \frac{4}{5} = 1200$

NOW INCLUDE IN AN ENERGY SYSTEM, ALSO TAKE THE LEVEL OF A AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL.

$\Rightarrow KE_A + PE_A + W_{nc} - W_{mg} = KE_B + PE_B$
 $\Rightarrow \frac{1}{2}(120)v^2 + 0 + (120)(5) - (120)(10) = \frac{1}{2}(120)v^2 + 120g \times 10$
 $\Rightarrow 600 + 6000 - 1200 = 60v^2 + 7056$
 $\Rightarrow 4644 = 60v^2$
 $\Rightarrow v^2 = 77.4$
 $\Rightarrow v \approx 8.80 \text{ ms}^{-1}$

Question 21 (****)



A particle of mass m travels along a path $ABCD$, whose cross section is shown in the figure above.

Section AB is vertical and of length x . Section BC is an arc of a quarter circle of radius x . Section CD is horizontal and of length d .

The particle is released from rest from A and comes to rest at D .

The particle experiences a constant frictional force only when travelling along the straight section CD .

Find the speed of the particle, in terms of g and x , when is at the midpoint of CD .

, $v = \sqrt{2gx}$

START BY OBTAINING THE COEFFICIENT OF FRICTION - OR THE CONSTANT FRICTIONAL FORCE

TRYING THE LEVEL OF 'CD' AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL

$$\Rightarrow KE_A + PE_A + W_N - W_{fr} = KE_B + PE_B$$

$$\Rightarrow mg(2x) - Fd = 0$$

$$\Rightarrow Fd = 2mgx$$

NOW BY ENERGIES FROM A TO THE MIDDLE OF CD

$$\Rightarrow KE_A + PE_A + W_N - W_{fr} = KE_M + PE_M$$

$$\Rightarrow mg(x) - F \cdot \frac{1}{2}d = \frac{1}{2}mv^2$$

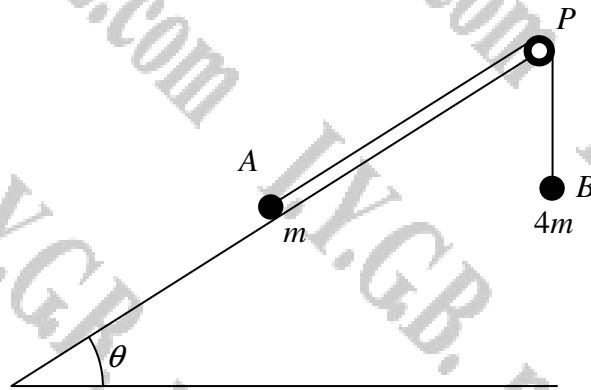
$$\Rightarrow 4mgx - Fd = mv^2$$

$$\Rightarrow 2mgx = mv^2$$

$$\Rightarrow v = \sqrt{2gx}$$

$$\Rightarrow M = \sqrt{2gx}$$

Question 22 (****+)



Two particles A and B , of mass m and $4m$ respectively, are attached to each of the ends of a light inextensible string.

The string passes over a smooth pulley P , at the top of a rough plane, inclined at θ to the horizontal, where $\tan \theta = \frac{4}{3}$.

The coefficient of friction between A and the plane is $\frac{7}{24}$.

Particle A is held at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P , as shown in the figure above. The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane. The particles are released from rest with the string taut. When B has fallen by a distance L , its speed is v , and A has not yet reach P .

Ignoring air resistance, express v in terms of g and L .

$$v = \frac{11}{10} \sqrt{gL}$$

$\theta = \frac{4}{3}$
 $\tan \theta = \frac{4}{3}$
 $\sin \theta = \frac{4}{5}$
 $\cos \theta = \frac{3}{5}$

INITIAL: $KE_A + PE_A$
 FINAL: $KE_A + PE_A$
 WORK DONE BY GRAVITY: $W_{grav} = mgs \sin \theta$
 WORK DONE BY FRICTION: $W_{fric} = \mu R s = \mu mg \cos \theta s$

TAKE THE INITIAL LEVEL FOR EACH PARTICLE AS THE ZERO POTENTIAL LEVEL, AND NOTE THAT THE TWO PARTICLES MOVE THE SAME DISTANCE ALONG THE STRING.
 AS THEY ARE CONNECTED BY A STRING THERE IS NO KE TO START WITH EITHER.

INITIAL: $KE_B + PE_B$
 FINAL: $KE_B + PE_B$
 WORK DONE BY GRAVITY: $W_{grav} = 4mgL$

$W_{grav} = W_{fric} + \Delta KE$
 $4mgL = \mu mg \cos \theta s + \frac{1}{2}mv^2 + \frac{1}{2}(4m)v^2 + mgL \sin \theta - 4mgL$
 $4gL = \frac{7}{24}mg \cdot \frac{3}{5}L + \frac{1}{2}v^2 + 2v^2 + \frac{1}{5}gL - 4gL$
 $4gL - \frac{1}{5}gL + 4gL = \frac{7}{40}gL + \frac{5}{2}v^2$
 $8\frac{1}{5}gL = \frac{7}{40}gL + \frac{5}{2}v^2$
 $\frac{39}{20}gL = \frac{5}{2}v^2$
 $v^2 = \frac{15}{100}gL$
 $v = \frac{11}{10}\sqrt{gL}$

Question 24 (****+)

A particle is projected from a point A , with speed U , up the line of greatest slope of a rough plane inclined at angle θ to the horizontal.

The particle moves up the plane comes to instantaneous rest at before it starts sliding down the plane, passing through A with speed V .

Show that

$$\frac{V^2}{U^2} = \frac{\tan \theta - \mu}{\tan \theta + \mu}, \quad \mu < \tan \theta,$$

where μ is the coefficient of friction between the particle and the plane.

proof

CONSIDERING THE JOURNEY FROM A TO B , TAKING THE LEVEL OF A AS THE ZERO POTENTIAL LEVEL

$$\Rightarrow kE_A + PE_A + W_{fr} - W_{top} = kE_B + PE_B$$

$$\Rightarrow \frac{1}{2}mU^2 - (\mu R) \times d = mgh$$

$$\Rightarrow \frac{1}{2}mU^2 - \mu(mg \cos \theta)d = \mu g(d \sin \theta)$$

$$\Rightarrow U^2 - 2\mu g d \cos \theta = 2\mu g d \sin \theta$$

$$\Rightarrow U^2 = 2\mu g d (\sin \theta + \cos \theta)$$

$$\Rightarrow 2\mu g d (\sin \theta + \cos \theta) = U^2$$

$$\Rightarrow d = \frac{U^2}{2\mu g (\sin \theta + \cos \theta)}$$

NEXT WE CONSIDER THE RETURN JOURNEY FROM B TO A WITH THE SAME ZERO POTENTIAL LEVEL. NOTE THAT THE REACTION HAS THE OPPOSITE DIRECTION

$$\Rightarrow kE_B + PE_B + W_{fr} - W_{top} = kE_A + PE_A$$

$$\Rightarrow mgh - (\mu R) \times d = \frac{1}{2}mV^2$$

$$\Rightarrow \mu g h - \mu(mg \cos \theta)d = \frac{1}{2}mV^2$$

$$\Rightarrow 2\mu g h - 2\mu g d \cos \theta = V^2$$

$$\Rightarrow V^2 = 2\mu g (d \sin \theta) - 2\mu g d \cos \theta$$

$$\Rightarrow V^2 = 2\mu g (d \sin \theta - d \cos \theta)$$

$$\Rightarrow V^2 = 2\mu g d (\sin \theta - \cos \theta)$$

$$\Rightarrow V^2 = \frac{U^2 (\sin \theta - \mu \cos \theta)}{\sin \theta + \mu \cos \theta}$$

$$\Rightarrow V^2 = \frac{(\tan \theta - \mu) U^2}{\tan \theta + \mu}$$

Question 25 (****+)

A particle is projected from a point P , with speed U , up the line of greatest slope of a rough plane inclined at angle θ to the horizontal.

The particle moves up the plane comes to instantaneous rest at the point Q before it starts sliding down the plane, passing through the point R with speed U .

Show that

$$|PR| = \frac{\mu U^2 \cos \theta}{g(\sin^2 \theta - \mu^2 \cos^2 \theta)}, \quad \mu < \tan \theta,$$

where μ is the coefficient of friction between the particle and the plane.

proof

The handwritten solution is divided into two columns. The left column contains diagrams and energy calculations for the upward and downward journeys. The right column shows algebraic manipulation to solve for the distance x .

Left Column:

- Diagram 1:** Shows a particle at point P on an inclined plane at angle θ . It is projected up to point Q at distance d along the plane. The vertical height is $d \sin \theta$. Forces shown are weight mg , normal reaction R , and friction F .
- Diagram 2:** Shows the particle at point R on the plane. It is moving down with speed U . Forces shown are weight mg , normal reaction R , and friction F .
- Energy Calculations:**
 - Upward Journey (P to Q):**
 - Initial KE: $\frac{1}{2}mU^2$
 - Final KE: 0
 - Initial PE: 0
 - Final PE: $mgd \sin \theta$
 - Work done against friction: $Fd = \mu R d = \mu mg \cos \theta d$
 - Energy equation: $\frac{1}{2}mU^2 - mgd \sin \theta - \mu mg \cos \theta d = 0$
 - Simplify: $U^2 - 2gd \sin \theta - 2\mu g d \cos \theta = 0$
 - Solve for d : $d = \frac{U^2}{2g(\sin \theta + \mu \cos \theta)}$
 - Downward Journey (Q to R):**
 - Initial KE: 0
 - Final KE: $\frac{1}{2}mU^2$
 - Initial PE: $mgd \sin \theta$
 - Final PE: 0
 - Work done against friction: $Fd = \mu R d = \mu mg \cos \theta d$
 - Energy equation: $mgd \sin \theta - \mu mg \cos \theta d = \frac{1}{2}mU^2$
 - Simplify: $2gd \sin \theta - 2\mu g d \cos \theta = U^2$
 - Solve for d : $d = \frac{U^2}{2g(\sin \theta - \mu \cos \theta)}$

Right Column:

- Equation 1: $2g(x+d)\sin \theta - 2\mu g(x+d)\cos \theta = U^2$
- Equation 2: $2g(x+d)[\sin \theta - \mu \cos \theta] = U^2$
- Equation 3: $x+d = \frac{U^2}{2g(\sin \theta - \mu \cos \theta)}$
- Equation 4: $x = \frac{U^2}{2g(\sin \theta - \mu \cos \theta)} - d$
- Equation 5: $x = \frac{U^2}{2g(\sin \theta - \mu \cos \theta)} - \frac{U^2}{2g(\sin \theta + \mu \cos \theta)}$
- Equation 6: $x = \frac{U^2}{2g} \left[\frac{1}{\sin \theta - \mu \cos \theta} - \frac{1}{\sin \theta + \mu \cos \theta} \right]$
- Equation 7: $x = \frac{U^2}{2g} \left[\frac{\sin \theta + \mu \cos \theta - \sin \theta + \mu \cos \theta}{(\sin \theta - \mu \cos \theta)(\sin \theta + \mu \cos \theta)} \right]$
- Equation 8: $x = \frac{U^2}{2g} \times \frac{2\mu \cos \theta}{\sin^2 \theta - \mu^2 \cos^2 \theta}$
- Equation 9: $x = \frac{\mu U^2 \cos \theta}{g(\sin^2 \theta - \mu^2 \cos^2 \theta)}$
- Note: "As required"

Question 26 (*****)

A pump draws water from a tank and pours it at the end of a pipe, of constant cross section, which is located 10 m vertically above the point where the water is drawn.

When the pump is working at the constant rate of 900 W the water is pouring at the end of the pipe with constant speed 20 ms^{-1} .

Determine the cross sectional area of the pipe, in cm^2 .

You may assume that the density of the water is 1000 kg m^{-3}

$A \approx 1.51 \text{ cm}^2$

DENSITY = $\frac{\text{MASS}}{\text{VOLUME}}$ (kg m^{-3})

$\Rightarrow \text{MASS} = \text{DENSITY} \times \text{VOLUME}$

$\Rightarrow \frac{\text{MASS}}{\text{TIME}} = \text{DENSITY} \times \frac{\text{VOLUME}}{\text{TIME}}$

$\Rightarrow \frac{\text{MASS}}{\text{TIME}} = \text{DENSITY} \times \frac{\text{LENGTH} \times \text{CROSS SECTIONAL AREA}}{\text{TIME}}$

$\Rightarrow \frac{\text{MASS}}{\text{TIME}} = \text{DENSITY} \times \frac{\text{LENGTH} \times \text{CROSS SECTIONAL AREA}}{\text{TIME}}$

$\Rightarrow \frac{\text{MASS}}{\text{TIME}} = \text{DENSITY} \times \text{SPEED} \times \text{CROSS SECTIONAL AREA}$

THE ABOVE NEED NOT BE DERIVED AS IT IS COMMONLY KNOWN THAT THE FLOW RATE IN $\text{m}^3 \text{ s}^{-1}$ IS

$\frac{\text{VOLUME}}{\text{TIME}} = \text{SPEED} \times \text{CROSS SECTIONAL AREA}$

NOW GAIN IN KINETIC ENERGY IS POTENTIAL ENERGY MUST EQUAL TO THE WORK IN SUPPLIED BY THE PUMP

$\Rightarrow \text{WORK IN} = \text{GAIN IN KE} + \text{GAIN IN PE}$

$\Rightarrow \text{WORK IN} = \frac{1}{2} M v^2 + M g h$

$\Rightarrow \frac{\text{WORK IN}}{\text{TIME}} = \frac{1}{2} \frac{\text{MASS}}{\text{TIME}} v^2 + \frac{\text{MASS}}{\text{TIME}} g h$

$\Rightarrow \text{POWER} = \frac{1}{2} (\text{DENSITY} \times \text{SPEED} \times \text{CROSS SECTIONAL AREA}) \times \text{SPEED}^2 + (\text{DENSITY} \times \text{SPEED} \times \text{CROSS SECTIONAL AREA}) \times g \times \text{HEIGHT}$

IN SIMILAR FASHION

$\Rightarrow P = \frac{1}{2} \rho A v^3 + \rho v g h$

$\Rightarrow 900 = \frac{1}{2} \times 1000 A \times 20^3 + 1000 \times 20 \times A \times 9.8 \times 10$

$\Rightarrow 900 = 400000 A + 196000 A$

$\Rightarrow 9 = 40000 A + 19600 A$

$\Rightarrow 59600 A = 9$

$\Rightarrow A \approx 0.00015100671 \text{ m}^2 \times 100^2$

HENCE THE CROSS SECTIONAL AREA IS 1.51 cm^2

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