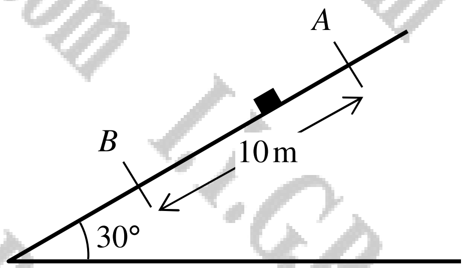
WORK & ENERGY

Question 1 (**)



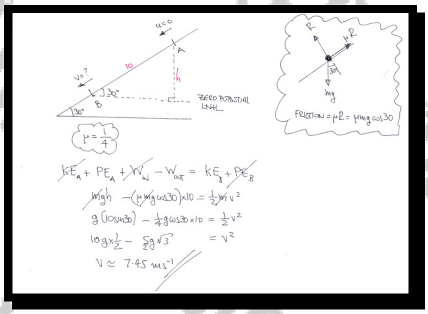
The figure above shows a particle sliding down a rough plane inclined at an angle of $30°$ to the horizontal. The box is released from rest at the point A and passes through the point B, which lies 10 m further down the plane, with a speed of $v \text{ ms}^{-1}$.

The points A and B lie on a line of greatest slope on the plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$.

Find the value of v, correct to three significant figures.

$\boxed{v \approx 7.45 \text{ ms}^{-1}}$



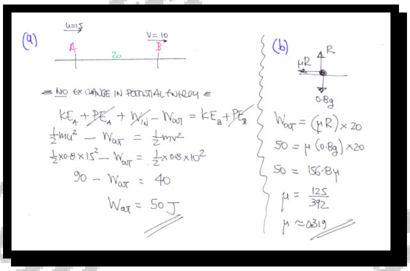
Question 2 (**)

A particle of mass 0.8 kg is sliding on a rough horizontal surface reducing its speed from 15 ms^{-1} at a point A, to 10 ms^{-1} at a point B. It is assumed that the only resistance to the motion of the particle is the ground friction.

a) Given that the distance AB is 20 m, determine the work done by the friction in the particle's journey from A to B.

b) Find, to three significant figures, the value of the coefficient of friction between the particle and the horizontal surface.

$\boxed{W_{\text{out}} = 50 \text{ J}}$, $\boxed{\mu \approx 0.319}$



Question 3 (**)

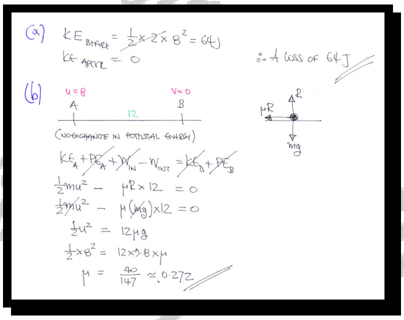
A small box of mass 2 kg is sliding on a floor between two points A and B. The box has a speed of 8 ms^{-1} at A and comes to rest at B.

The box is modelled as a particle and the floor as a rough horizontal plane, where μ is the coefficient of friction between the particle and the horizontal plane.

a) Calculate the kinetic energy loss as the box moves from A to B.

b) Given that $AB = 12\text{ m}$, find the value of μ.

$\boxed{\text{kinetic energy loss} = 64\text{ J}}$, $\boxed{\mu \approx 0.272}$

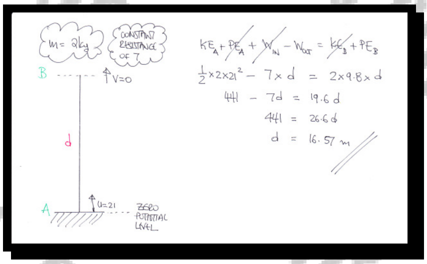


Question 4 (+)**

A particle of mass 2 kg is projected vertically upwards from ground level with a speed of 21 ms^{-1} and comes to instantaneous rest at a height of d m above the ground. The particle is subject to a constant non gravitational resistance of 7 N, assumed constant throughout the motion.

Use work and energy considerations to determine the value of d.

$\boxed{d \approx 16.58\text{ m}}$

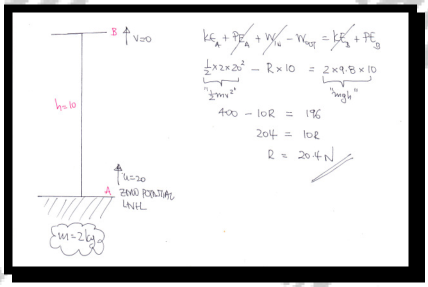


Question 5 (+)**

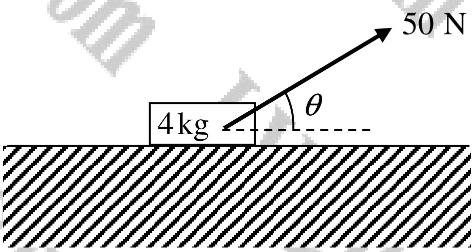
A particle of mass 2 kg is projected vertically upwards from ground level with a speed of 20 ms^{-1} and comes to instantaneous rest at a height of 10 m above the ground. The particle is subject to a constant non gravitational resistance of R N, throughout the motion.

Use work and energy considerations to determine the value of R .

$\boxed{R = 20.4 \text{ N}}$



Question 6 (+)**



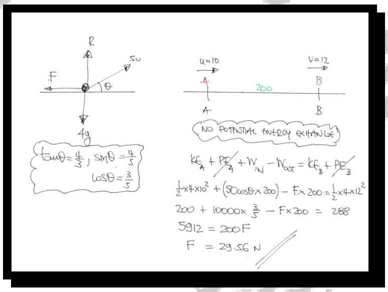
The figure above shows a small box of mass 4 kg, pulled by a rope along rough horizontal ground. The box has a speed of 10 ms^{-1} at the point A and 12 ms^{-1} at the point B.

The force supplied by the rope is $50\,\text{N}$ and is inclined at an angle θ to the horizontal ground, where $\tan\theta = \frac{4}{3}$.

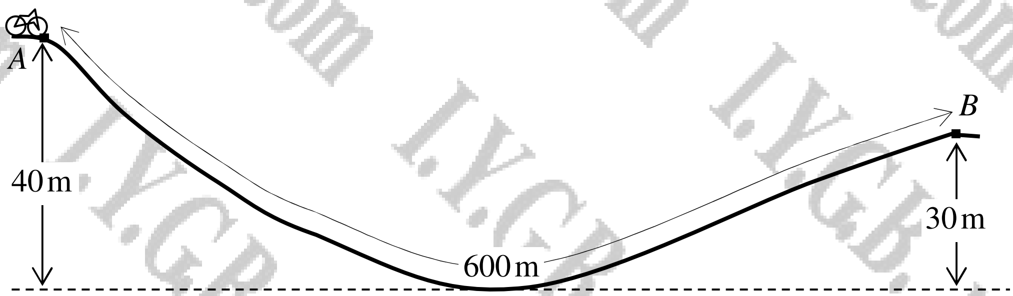
The box is modelled as a particle experiencing a constant ground friction of F N.

Given that the distance AB is 200 m, use work and energy considerations to find the value of F.

$\boxed{F = 29.56}$



Question 7 (+)**



The figure above shows the path of a cyclist on a section of a road from A to B, where the distance AB is 600 m.

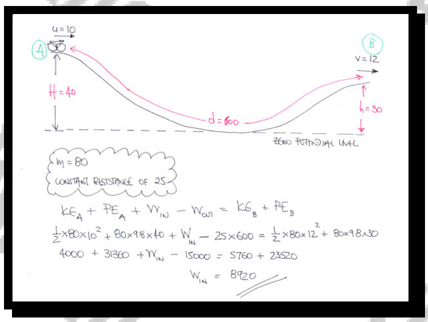
The cyclist leaves point A at the top of a hill with a speed of 10 ms^{-1} and descends a vertical distance of 40 m to the bottom of the hill. The cyclist then ascends a vertical distance of 30 m to the top of another hill at point B. The speed of the cyclist at B is 12 ms^{-1}.

The combined mass of the cyclist and his bike is 80 kg.

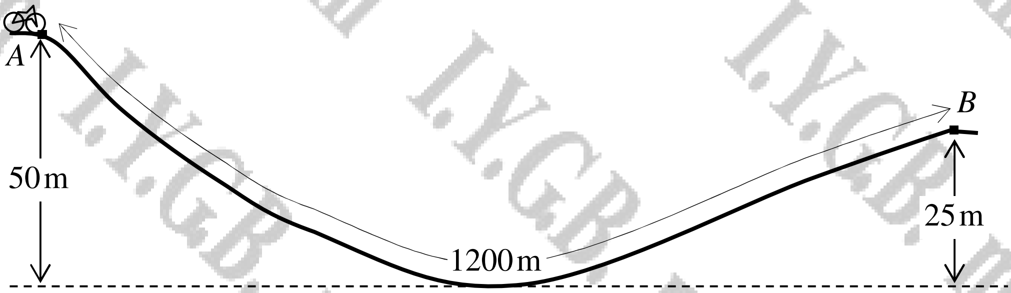
The cyclist and his bike are modelled as a single particle subject to a constant non gravitational resistance of 25 N, throughout the motion.

Find the work done by the cyclist.

$$\boxed{W = 8920 \text{ J}}$$



Question 8 (***)



The figure above shows the path of a cyclist on a section of a road from A to B, where the distance AB is 1200 m.

The cyclist leaves point A at the top of a hill with a speed V ms^{-1} and descends a vertical distance of 50 m to the bottom of the hill. He then ascends a vertical distance of 25 m to the top of another hill at point B.

The cyclist takes 110 s to travel from A to B and is assumed to be working at the constant rate of 40 W, throughout the motion.

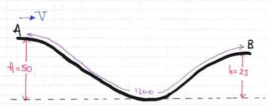
The combined mass of the cyclist and his bike is 80 kg.

The cyclist and his bike are modelled as a single particle subject to a constant non gravitational resistance of 20 N, throughout the motion.

Show that speed of the cyclist at B is V ms^{-1}.

$\boxed{\text{proof}}$

LOOKING AT THE DIAGRAM BELOW



$$KE_A + PE_A + W_{IN} - W_{OUT} = KE_B + PE_B$$

KE_A: $\frac{1}{2}mV^2$; PE_A: mgH; W_{OUT}: 20×1200; KE_B: TO BE FOUND; PE_B: mgh

$$\text{POWER} = \frac{\text{WORK IN}}{\text{TIME}}$$

$$40 = \frac{\text{WORK IN}}{110}$$

$$W_{IN} = 4400$$

RETURNING TO THE ENERGY EQUATION

$\Rightarrow KE_A + 80 \times 9.8 \times 50 + 4400 - 24000 = KE_B + 80 \times 9.8 \times 25$

$\Rightarrow KE_A + 39200 + 4400 - 24000 = KE_B + 19600$

$\Rightarrow KE_A + 19600 = KE_B + 19600$

$\Rightarrow KE_A = KE_B$

$\therefore$ SAME SPEED AS THE KINETIC ENERGY IS UNCHANGED

Question 9 (***)

A car of mass 1500 kg is travelling up a hill on a straight road, with the engine of the car working at the constant rate of 13 kW for 1 minute.

During this minute the car increases its speed from 7 ms^{-1} to 24 ms^{-1} and in addition to the work done against gravity, 80000 J of work is done against resistances to motion parallel to the direction of motion of the car.

Calculate the vertical displacement of the car in this 1 minute interval.

$\boxed{h \approx 20.73 \text{ m}}$

- POWER $= \dfrac{W_{IN}}{\text{TIME}}$

$13000 = \dfrac{W_{IN}}{60}$

$W_{IN} = 780{,}000$

- TAKING THE LEVEL AT WHICH THE SPEED OF THE CAR IS 7 ms^{-1}, AS THE ZERO POTENTIAL LEVEL

$KE_A + \cancel{PE_A} + W_{IN} - W_{OUT} = KE_B + PE_B$

$\frac{1}{2} \times 1500 \times 7^2 + 780000 - 80000 = \frac{1}{2} \times 1500 \times 24^2 + 1500gh$

$36750 + 780000 - 80000 = 432000 + 1500gh$

$304750 = 1500gh$

$h = \frac{6095}{294} \approx 20.73\text{m}$

Question 10 (***)

A particle P of mass 4 kg is moving on the line of greatest slope of a rough plane, inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$.

The particle is projected up the plane with a speed $u \text{ ms}^{-1}$ from a point A on the plane, comes to instantaneous rest at a point B, and then slides back down the plane passing through A again.

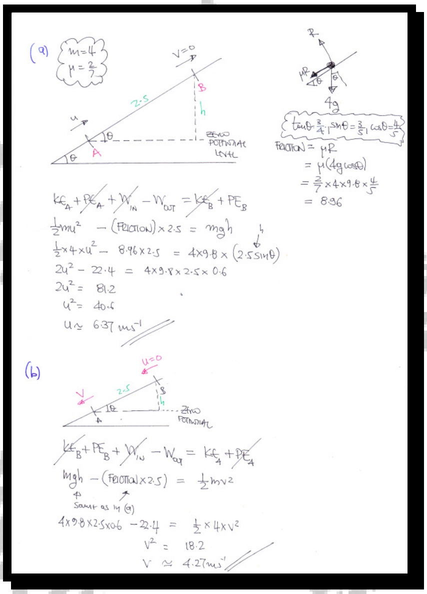
The coefficient of friction between the particle and the plane is $\frac{2}{7}$ and the distance AB is 2.5 m.

Use work and energy considerations to find …

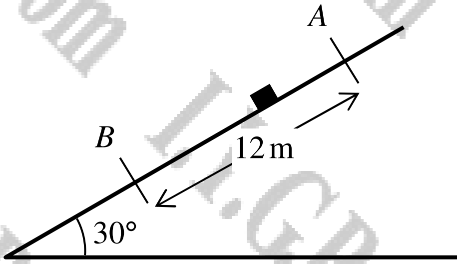
a) … the value of u.

b) … the speed of the particle as it passes through A again.

$\boxed{u \approx 6.37}$, $\boxed{\approx 4.27 \text{ ms}^{-1}}$



Question 11 (***)



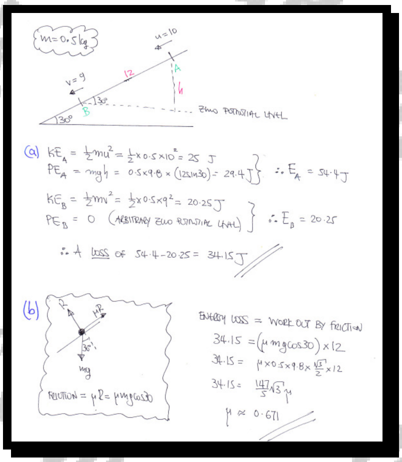
The figure above shows a box of mass $0.5\,\text{kg}$ sliding down a rough plane inclined at an angle of $30°$ to the horizontal. The box passes through the point A with a speed of $10\ \text{ms}^{-1}$ and through the point B, which lies 12 m further down the plane, with a speed of $9\ \text{ms}^{-1}$.

a) Find the loss in energy of the box as it moves from A to B.

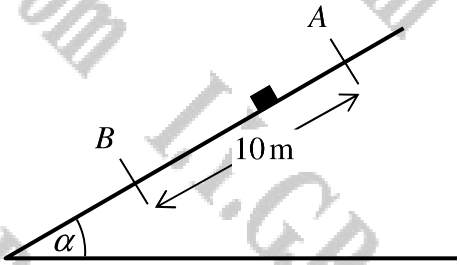
The coefficient of friction between the box and the plane is μ.

b) Find the value of μ, correct to three significant figures.

$\boxed{\text{loss} = 34.15\ \text{J}}$, $\boxed{\mu \approx 0.671}$



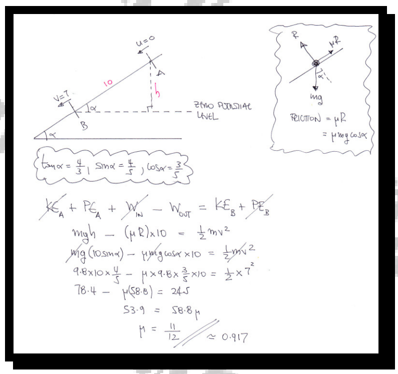
Question 12 (***)



The figure above shows a particle sliding down a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$. The box is released from rest at the point A and passes through the point B, which lies 10 m further down the plane, with a speed of 7 ms^{-1}.

Use work and energy considerations, to show that the coefficient of friction between the particle and the plane is $\frac{11}{12}$.

proof



Question 13 (***)

A particle of mass $0.5\,\text{kg}$ is projected with a speed of $5\ \text{ms}^{-1}$ from a point A on a rough plane, inclined at an angle of $20°$ to the horizontal.

The particle slides up the plane and comes to instantaneous rest at a point B on the plane which is $2\ \text{m}$ away from A.

The points A and B lie on the line of greatest slope on the plane. The particle is subject to a constant non gravitational resistance of R N, throughout the motion.

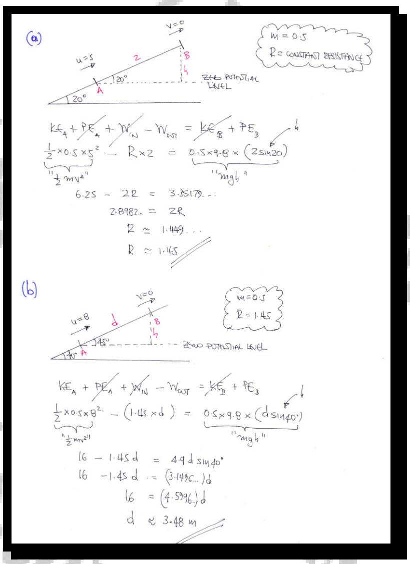
a) Determine the value of R, correct to three significant figures.

The angle of the plane is now increased to $40°$ but the value of R remains unchanged.

The particle is next projected with a speed of $8\ \text{ms}^{-1}$ from A and slides up the plane coming to instantaneous rest at a distance d m away from A.

b) Find the value of d, correct to three significant figures.

$\boxed{R \approx 1.45\,\text{N}}$, $\boxed{d \approx 3.48\,\text{m}}$



Question 14 (***)

A woman and her bike are modelled a single particle of combined mass 72 kg.

The woman cycles with constant speed of 5 ms^{-1} , **up** a straight road, which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{2}{21}$.

The total non gravitational resistance experienced by the cyclist is assumed to be constant at 25 N.

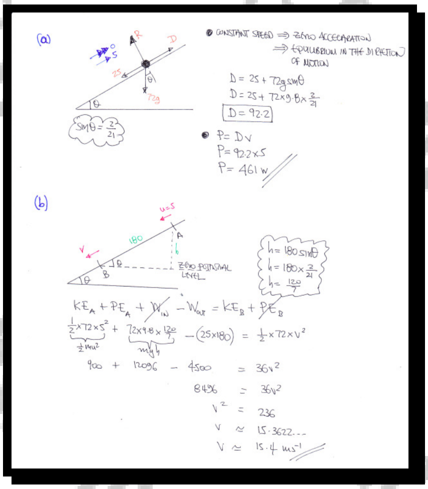
a) Find the power generated by the woman when cycling up the hill.

The woman then turns her bike around at some point A on the road. She freewheels down the same road starting with a speed of 5 ms^{-1} . She passes through some point B on that road with a speed $v \text{ ms}^{-1}$.

The total non gravitational resistance experienced by the cyclist is assumed to be the same as in part (a).

b) Given that the distance AB is 180 m, find the value of v.

$\boxed{P = 461 \text{ W}}$, $\boxed{v \approx 15.4}$



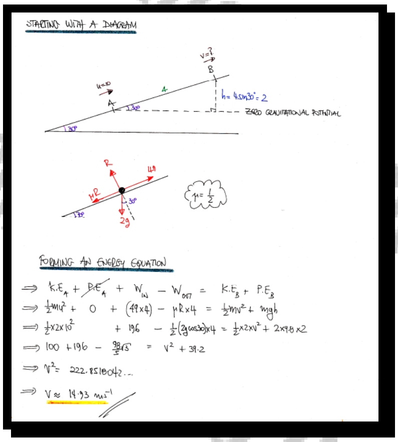
Question 15 (***+)

A particle of mass 2 kg is dragged by a constant force of 49 N, up the line of greatest slope of a rough plane, inclined at an angle of $30°$ to the horizontal. This force is also acting in the line of greatest slope of the plane. The coefficient of friction between the particle and the plane is 0.5.

The particle passes through two points on the plane A and B which are 4 m apart, and point B is at a higher level on the plane than point A.

Given that the particle is passing through A with a speed of 10 ms^{-1}, use work and energy considerations to find the speed of the particle as it passes through B.

, $\boxed{v \approx 14.93 \text{ ms}^{-1}}$



Question 16 (*+)**

A car of mass 1230 kg is moving on a straight road which lies on the line of greatest slope of a rough plane, inclined at an angle θ to the horizontal, where $\tan\theta = \frac{9}{40}$.

There is a constant non gravitational resistance of 400 N acting on the car.

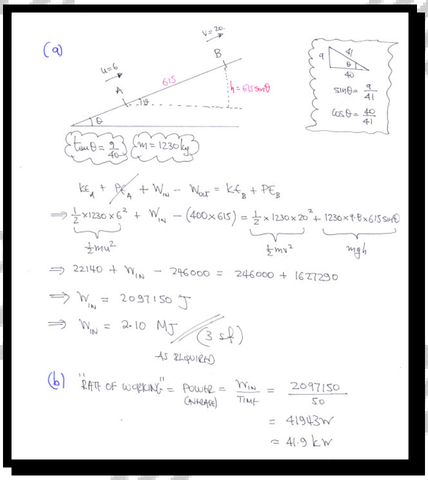
The car passes through two points on the plane A and B, with speeds 6 ms^{-1} and 20 ms^{-1}, respectively, where the point B lies at a higher level on the plane than A.

The distance AB is 615 m and it takes 50 s for the car to travel this distance.

a) Find, in MJ, the work done by the engine of the car.

b) Determine the average rate at which the engine of the car is working.

$\boxed{2.10 \text{ Mj}}$, $\boxed{P \approx 41.9 \text{ kW}}$



(a)

$KE_A + \cancel{PE_A} + W_{IN} - W_{OUT} = KE_B + PE_B$

$\Rightarrow \underbrace{\frac{1}{2}\times 1230 \times 6^2}_{\frac{1}{2}mu^2} + W_{IN} - (400 \times 615) = \underbrace{\frac{1}{2}\times 1230 \times 20^2}_{\frac{1}{2}mv^2} + \underbrace{1230 \times 9.8 \times 615 \sin\theta}_{mgh}$

$\Rightarrow 22140 + W_{IN} - 246000 = 246000 + 1627290$

$\Rightarrow W_{IN} = 2097150 \text{ J}$

$\Rightarrow W_{IN} = 2.10 \text{ MJ}$ (3 sf)

AS REQUIRED

(b) "RATE OF WORKING" = POWER (AVERAGE) = $\frac{W_{IN}}{\text{TIME}} = \frac{2097150}{50}$

$= 41943 \text{ W}$

$\approx 41.9 \text{ kW}$

Question (***+)

A block of mass 5 kg is pulled along a rough, straight, horizontal road by a constant horizontal force of magnitude 47 N.

The block moves in a straight line between two points A and B, where $|AB| = 7 \text{ m}$.

The coefficient of friction between the block and the road is 0.5.

Find the speed of the particle at A given that its speed at B is 4 times as large as its speed at A.

$$\boxed{}, \ \boxed{u = \frac{1}{5}\sqrt{105} \approx 2.05 \text{ ms}^{-1}}$$

BY ENERGY CONSIDERATIONS

$\Rightarrow KE_A + \cancel{PE_A} + W_{IN} - W_{OUT} = KE_B + \cancel{PE_B}$

$\Rightarrow \frac{1}{2}mu^2 + F_1 \times d - F_2 \times d = \frac{1}{2}mv^2$

EXTERNAL FORCE

FRICTION $= \mu R = \mu mg$

$\Rightarrow \frac{1}{2}mu^2 + F_1 d - \mu mgd = \frac{1}{2}mv^2$

SUBSTITUTING SOME NUMBERS

$\Rightarrow \frac{1}{2} \times 5u^2 + 47 \times 7 - \frac{1}{2} \times 5 \times 9.8 \times 7 = \frac{1}{2} \times 5 \times v^2$

$\Rightarrow \frac{5}{2}u^2 + 329 - \frac{343}{2} = \frac{5}{2}v^2$

$\Rightarrow 5u^2 + 658 - 343 = 5v^2$

$\Rightarrow 5u^2 + 315 = 5v^2$

$\Rightarrow u^2 + 63 = v^2$

BUT IT IS GIVEN THAT $v = 4u$

$\Rightarrow u^2 + 63 = (4u)^2$

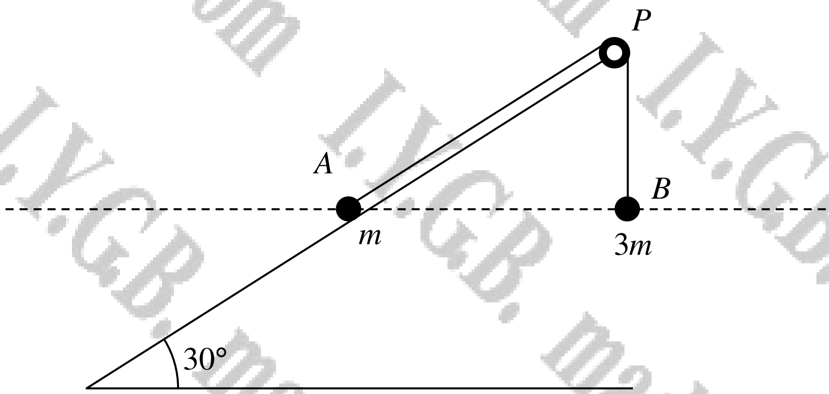
$\Rightarrow 63 = 15u^2$

$\Rightarrow u^2 = 63/15 = \frac{21}{5}$

$\Rightarrow |u| = \frac{1}{5}\sqrt{105}$

$\Rightarrow |u| \approx 2.05 \text{ ms}^{-1}$

Question 17 (***+)



Two particles A and B, of mass m and $3m$ respectively, are attached to each of the ends of a light inextensible string. The string passes over a smooth pulley P, at the top of a fixed smooth plane, inclined at $30°$ to the horizontal.

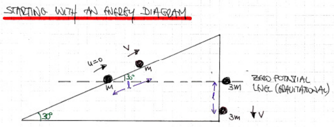
Particle A is held at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P, as shown in the figure above. The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane.

The particles are released from rest, from the same horizontal level with the string taut. When B has fallen by a distance l, its speed is v, and A has not yet reach P.

By using energy considerations and ignoring air resistance, express v in terms of g and l.

$\boxed{v = \frac{1}{2}\sqrt{5gl}}$

STARTING WITH AN ENERGY DIAGRAM



$$\underbrace{\cancel{KE_A} + \cancel{KE_B} + \cancel{PE_A} + \cancel{PE_B}}_{\text{ON RELEASE}} = \underbrace{KE_A + KE_B + PE_A + P.E_B}_{\text{AFTER BOTH MOVING BY } l}$$

$\Rightarrow 0 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)v^2 + mg(l\sin 30) + 3mg(-l)$

$\Rightarrow 0 = \frac{1}{2}mv^2 + \frac{3}{2}mv^2 + \frac{1}{2}mgl - 3mgl$

$\Rightarrow 0 = mv^2 + 3mv^2 + mgl - 6mgl$

$\Rightarrow 0 = 4mv^2 - 5mgl$

$\Rightarrow 0 = 4v^2 - 5gl$

$\Rightarrow 5gl = 4v^2$

$\Rightarrow v^2 = \frac{5}{4}gl$

$\Rightarrow |v| = \frac{1}{2}\sqrt{5gl}$

Question 18 (***+)

A car of mass 1300 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{10}$.

The total non gravitational resistance experienced by the car is assumed to be a constant force of magnitude of 400 N. The engine of the car is working at the constant rate of 30 kW.

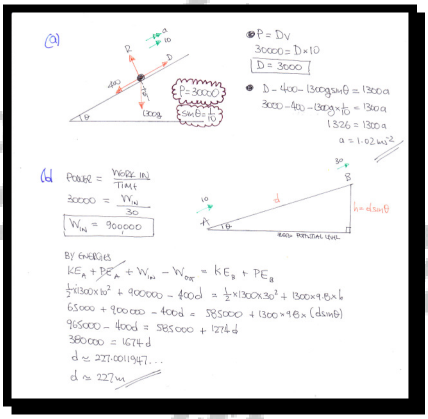
The car is passing through the point A with a speed 10 ms^{-1} and continues to accelerate up the plane, passing through the point B with speed 30 ms^{-1}, 30 s after passing through A.

By modelling the car as a particle, find …

a) … the acceleration of the car at A.

b) ... the distance AB.

$\boxed{}$, $\boxed{a = 1.02 \text{ ms}^{-2}}$, $\boxed{|AB| \approx 227 \text{ m}}$



Question 19 (***+)

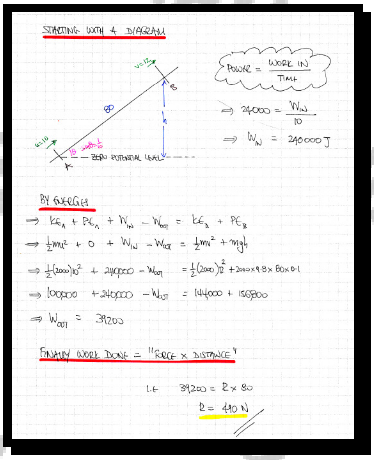
A car of mass 2000 kg is travelling up a line of greatest slope of a hill, inclined at $\arcsin(0.1)$.

At a given time the speed of the car is 10 ms^{-1} and 10 s later the speed of the car is 12 ms^{-1}, having covered a distance of 80 m.

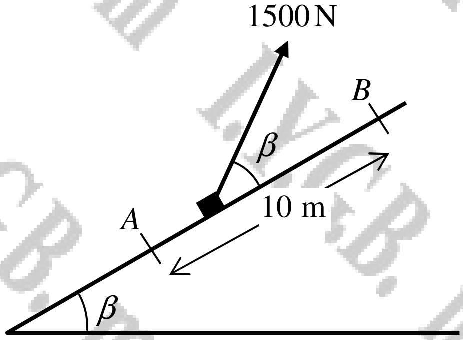
The engine of the car is assumed to be working at a constant rate of 24 kW, and it is assumed that the truck experiences a constant air resistance, R N.

Calculate the value of R.

$\boxed{\text{IYGB}}$, $\boxed{R = 490}$



Question 20 (****)



The figure above shows a box of mass 120 kg being pulled up the line of greatest slope of the plane inclined at an angle β to the horizontal, by an electrically operated cable. The cable is supplying a constant tension of 1500 N and is inclined at an angle β to the plane.

The box passes through the point A with speed 5 ms^{-1} and through the point B which is higher up the plane with speed v ms^{-1}. The distance AB is 10 m.

There is a constant non gravitational resistance of 180 N acting on the box.

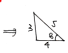
The box is modelled as a particle and the cable as a light inextensible string.

Given that $\tan\beta = \frac{3}{4}$ find the value of v.

$\boxed{}$, $\boxed{v \approx 8.80}$

AUXILIARIES FIRST

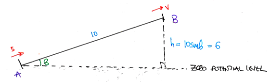
$\tan\beta = \frac{3}{4} \Longrightarrow$



$\Longrightarrow \begin{cases} \sin\beta = \frac{3}{5} \\ \cos\beta = \frac{4}{5} \end{cases}$

ONLY THE COMPONENT OF THE TENSION PARALLEL TO THE PLANE PUTS ENERGY INTO THE SYSTEM, I.E. $1500\cos\beta = 1500 \times \frac{4}{5} = 1200$

NOW LOOKING AT AN ENERGY DIAGRAM, AND TAKING THE LEVEL OF "A", AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL



$\Longrightarrow K.E_A + \cancel{P.E_A} + W_{IN} - W_{OUT} = K.E_B + P.E_B$

$\Longrightarrow \frac{1}{2}(120)\times 5^2 + 0 + (1200\times 10) - (180\times 10) = \frac{1}{2}(120)v^2 + 120g\times 6 \leftarrow \text{"mgh"}$

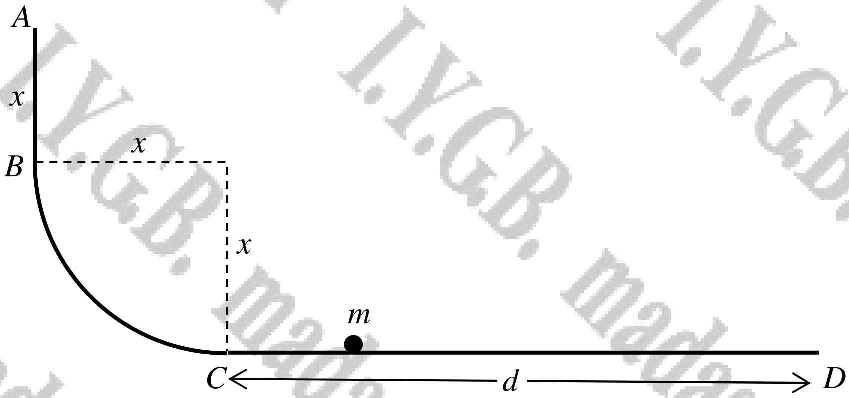
$\Longrightarrow 1500 + 12000 - 1800 = 60v^2 + 7056$

$\Longrightarrow 4644 = 60v^2$

$\Longrightarrow v^2 = 77.4$

$\Longrightarrow |v| \simeq 8.80\ \text{ms}^{-1}$

Question 21 (****)



A particle of mass m travels along a path $ABCD$, whose cross section is shown in the figure above.

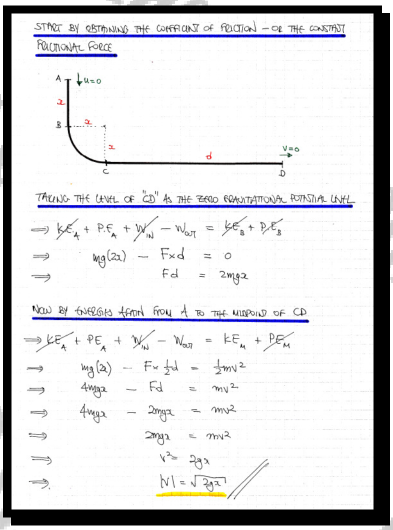
Section AB is vertical and of length x. Section BC is an arc of a quarter circle of radius x. Section CD is horizontal and of length d.

The particle is released from rest from A and comes to rest at D.

The particle experiences a constant frictional force only when travelling along the straight section CD.

Find the speed of the particle, in terms of g and x, when is at the midpoint of CD.

$\boxed{}$, $\boxed{v = \sqrt{2gx}}$



START BY OBTAINING THE COEFFICIENT OF FRICTION – OR THE CONSTANT FRICTIONAL FORCE

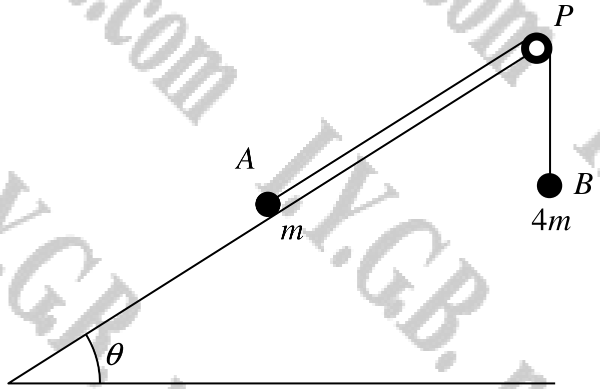
TAKING THE LEVEL OF "CD" AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL

$$\Rightarrow \cancel{KE_A} + PE_A + \cancel{W_{IN}} - W_{OUT} = \cancel{KE_B} + \cancel{PE_B}$$
$$\Rightarrow mg(2x) - F \times d = 0$$
$$\Rightarrow Fd = 2mgx$$

NOW BY ENERGIES AGAIN FROM A TO THE MIDPOINT OF CD

$$\Rightarrow \cancel{KE_A} + PE_A + \cancel{W_{IN}} - W_{OUT} = KE_M + \cancel{PE_M}$$
$$\Rightarrow mg(2x) - F \times \tfrac{1}{2}d = \tfrac{1}{2}mv^2$$
$$\Rightarrow 4mgx - Fd = mv^2$$
$$\Rightarrow 4mgx - 2mgx = mv^2$$
$$\Rightarrow 2mgx = mv^2$$
$$\Rightarrow v^2 = 2gx$$
$$\Rightarrow |v| = \sqrt{2gx}$$

Question 22 (****+)



Two particles A and B, of mass m and $4m$ respectively, are attached to each of the ends of a light inextensible string.

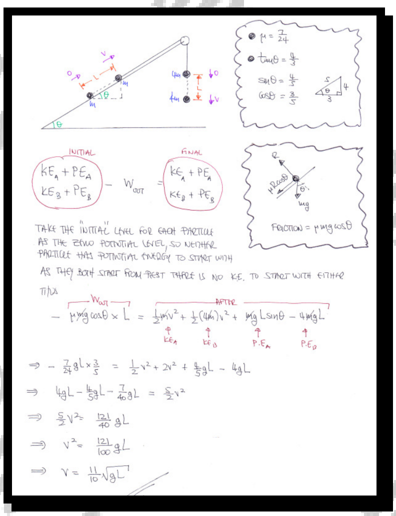
The string passes over a smooth pulley P, at the top of a rough plane, inclined at θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

The coefficient of friction between A and the plane is $\frac{7}{24}$.

Particle A is held at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P, as shown in the figure above. The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane. The particles are released from rest with the string taut. When B has fallen by a distance L, its speed is v, and A has not yet reach P.

Ignoring air resistance, express v in terms of g and L.

$$\boxed{v = \frac{11}{10}\sqrt{gl}}$$



Question 23 (****+)

A smooth bead car of mass 0.1 kg is threaded onto a smooth, straight, vertical wire.

The bead is threaded to one end of a light, inextensible string of length 2 m.

The string is passing over a smooth fixed peg and a particle of mass 0.25 kg is attached to the other end of the string.

The string is taut and the particle lies undisturbed vertically below the peg while the bead is held as the same vertical level as the peg.

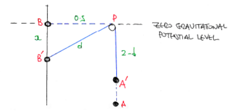
The horizontal distance between the peg and the wire is 0.5 m.

When the bead is released from rest, it descends a vertical distance x m, before it comes momentarily to rest.

Determine the value of x.

$\boxed{}$, $\boxed{x = \frac{8}{15}}$

STARTING WITH A DETAILED DIAGRAM



NO FRICTION INVOLVED, NO KINETIC ENERGIES AS IT IS REST TO REST

$\Rightarrow$ P.E BEFORE = P.E AFTER

$\Rightarrow 0 - 0.25g|PA| = -0.1gx - 0.25g|PA'|$

$\Rightarrow -0.25\cancel{g}(1.5) = -0.1\cancel{g}x - 0.25\cancel{g}(2-d)$

$\Rightarrow \frac{3}{8} = \frac{1}{10}x + \frac{1}{4}(2-d)$

$\Rightarrow 15 = 4x + 10(2-d)$

$\Rightarrow 15 = 4x + 20 - 10d$

$\Rightarrow 0 = 4x + 5 - 10\sqrt{x^2+0.5^2}$ ← PYTHAGORAS ON PBB'

$\Rightarrow 10\sqrt{x^2+0.25} = 4x+5$

$\Rightarrow 100(x^2+0.25) = (4x+5)^2$

$\Rightarrow 100x^2 + \cancel{25} = 25x^2 + 40x + \cancel{25}$

$\Rightarrow 75x^2 = 40x$

$\Rightarrow 75x = 40 \quad (x \neq 0)$

$\therefore x = \frac{8}{15}$

Question 24 (****+)

A particle is projected from a point A, with speed U, up the line of greatest slope of a rough plane inclined at angle θ to the horizontal.

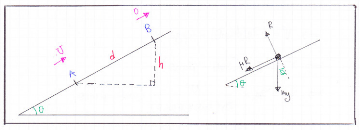
The particle moves up the plane comes to instantaneous rest at before it starts sliding down the plane, passing through A with speed V.

Show that

$$\frac{V^2}{U^2} = \frac{\tan\theta - \mu}{\tan\theta + \mu}, \qquad \mu < \tan\theta,$$

where μ is the coefficient of friction between the particle and the plane.

proof



CONSIDERING THE JOURNEY FROM A TO B, TAKING THE LEVEL OF A AS THE ZERO POTENTIAL LEVEL

$$\Rightarrow KE_A + \cancel{PE_A} + \cancel{W_{IN}} - W_{OUT} = \cancel{KE_B} + P.E_B$$

$$\Rightarrow \frac{1}{2}mU^2 - (\mu R)\times d = mgh$$

$$\Rightarrow \frac{1}{2}\cancel{m}U^2 - \mu(\cancel{m}g\cos\theta)d = \cancel{m}g(d\sin\theta)$$

$$\Rightarrow U^2 - 2\mu g d\cos\theta = 2gd\sin\theta$$

$$\Rightarrow U^2 = 2gd\sin\theta + 2\mu g d\cos\theta$$

$$\Rightarrow 2gd(\sin\theta + \mu\cos\theta) = U^2$$

$$\Rightarrow d = \frac{U^2}{2g(\sin\theta + \mu\cos\theta)}$$

NEXT WE CONSIDER THE RETURN JOURNEY FROM B TO A, WITH THE SAME ZERO POTENTIAL LEVEL. NOTE THAT THE FRICTION HAS THE SAME MAGNITUDE

$$\Rightarrow \cancel{KE_B} + PE_B + \cancel{W_{IN}} - W_{OUT} = KE_A + \cancel{P.E_A}$$

$$\Rightarrow mgh - (\mu R)\times d = \frac{1}{2}mV^2$$

$$\Rightarrow \cancel{m}gh - \mu(\cancel{m}g\cos\theta)d = \frac{1}{2}\cancel{m}V^2$$

$$\Rightarrow 2gh - 2\mu g d\cos\theta = V^2$$

$$\Rightarrow V^2 = 2g(d\sin\theta) - 2\mu g d\cos\theta$$

$$\Rightarrow V^2 = 2gd(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow V^2 = \cancel{2g}\times\frac{U^2}{\cancel{2g}(\sin\theta + \mu\cos\theta)}(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow V^2 = \frac{U^2(\sin\theta - \mu\cos\theta)}{\sin\theta + \mu\cos\theta}$$

$$\Rightarrow V^2 = \frac{(\tan\theta - \mu)U^2}{\tan\theta + \mu}$$

Question 25 (****+)

A particle is projected from a point P, with speed U, up the line of greatest slope of a rough plane inclined at angle θ to the horizontal.

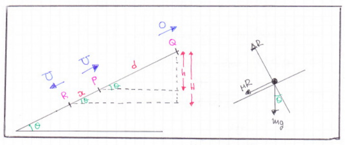
The particle moves up the plane comes to instantaneous rest at the point Q before it starts sliding down the plane, passing through the point R with speed U.

Show that

$$|PR| = \frac{\mu U^2 \cos\theta}{g\left(\sin^2\theta - \mu^2\cos^2\theta\right)}, \qquad \mu < \tan\theta,$$

where μ is the coefficient of friction between the particle and the plane.

$\boxed{\text{proof}}$



CONSIDERING THE JOURNEY FROM P TO Q, TAKING THE LEVEL OF Q AS THE ZERO POTENTIAL LEVEL

$$\Rightarrow KE_P + PE_P + \cancel{W_{IN}} - W_{OUT} = \cancel{KE_Q} + \cancel{PE_Q}$$

$$\Rightarrow \tfrac{1}{2}mU^2 - mgh - (\mu R)\times d = 0 + 0$$

$$\Rightarrow \tfrac{1}{2}\cancel{m}U^2 - \cancel{m}gd\sin\theta - \mu\cancel{m}g\cos\theta \times d = 0$$

$$\Rightarrow U^2 - 2gd\sin\theta - 2\mu gd\cos\theta = 0$$

$$\Rightarrow U^2 = 2gd(\sin\theta + \mu\cos\theta)$$

$$\Rightarrow d = \frac{U^2}{2g(\sin\theta + \mu\cos\theta)}$$

NEXT CONSIDER THE JOURNEY FROM Q TO R TAKING THE LEVEL OF Q AS THE ZERO POTENTIAL — NOTE THE FRICTION HAS THE SAME MAGNITUDE "UPWARDS"

$$\Rightarrow \cancel{KE_Q} + \cancel{PE_Q} + \cancel{W_{IN}} - W_{OUT} = KE_R + P.E_R$$

$$\Rightarrow -\mu R(x+d) = \tfrac{1}{2}mU^2 - mgH$$

$$\Rightarrow -\mu(\cancel{m}g\cos\theta)(x+d) = \tfrac{1}{2}\cancel{m}U^2 - \cancel{m}g(x+d)\sin\theta$$

$$\Rightarrow -2\mu g(x+d)\cos\theta = U^2 - 2g(x+d)\sin\theta$$

$$\Rightarrow 2g(x+d)\sin\theta - 2\mu g(x+d)\cos\theta = U^2$$

$$\Rightarrow 2g(x+d)\left[\sin\theta - \mu\cos\theta\right] = U^2$$

$$\Rightarrow x + d = \frac{U^2}{2g(\sin\theta - \mu\cos\theta)}$$

$$\Rightarrow x = \frac{U^2}{2g(\sin\theta - \mu\cos\theta)} - d$$

$$\Rightarrow x = \frac{U^2}{2g(\sin\theta - \mu\cos\theta)} - \frac{U^2}{2g(\sin\theta + \mu\cos\theta)}$$

$$\Rightarrow x = \frac{U^2}{2g}\left[\frac{1}{\sin\theta - \mu\cos\theta} - \frac{1}{\sin\theta + \mu\cos\theta}\right]$$

$$\Rightarrow x = \frac{U^2}{2g}\left[\frac{\cancel{\sin\theta} + \mu\cos\theta - \cancel{\sin\theta} + \mu\cos\theta}{(\sin\theta - \mu\cos\theta)(\sin\theta + \mu\cos\theta)}\right]$$

$$\Rightarrow x = \frac{U^2}{\cancel{2}g} \times \frac{\cancel{2}\mu\cos\theta}{\sin^2\theta - \mu^2\cos^2\theta}$$

$$\Rightarrow x = \frac{\mu U^2\cos\theta}{g(\sin^2\theta - \mu^2\cos^2\theta)}$$

AS REQUIRED

Question 26 (*****)

A pump draws water from a tank and pours it at the end of a pipe, of constant cross section, which is located 10 m vertically above the point where the water is drawn.

When the pump is working at the constant rate of 900 W the water is pouring at the end of the pipe with constant speed 20 ms^{-1}.

Determine the cross sectional area of the pipe, in cm^2.

You may assume that the density of the water is 1000 kg m^{-3}

$\boxed{A \approx 1.51 \text{ cm}^2}$

$\boxed{\text{DENSITY} = \frac{\text{MASS}}{\text{VOLUME}} \text{ (IN kg m}^{-3}\text{)}}$

$\Rightarrow \text{MASS} = \text{DENSITY} \times \text{VOLUME}$

$\Rightarrow \frac{\text{MASS}}{\text{TIME}} = \text{DENSITY} \times \frac{\text{VOLUME}}{\text{TIME}}$

$\Rightarrow \frac{\text{MASS}}{\text{TIME}} = \text{DENSITY} \times \frac{\text{LENGTH} \times \text{CROSS SECTIONAL AREA}}{\text{TIME}}$

$\Rightarrow \frac{\text{MASS}}{\text{TIME}} = \text{DENSITY} \times \frac{\text{LENGTH}}{\text{TIME}} \times \text{CROSS-SECTIONAL AREA}$

$\Rightarrow \frac{\text{MASS}}{\text{TIME}} = \text{DENSITY} \times \text{SPEED} \times \text{CROSSSECTIONAL AREA}$

THE ABOVE NEED NOT BE DERIVED AS IT IS COMMONLY KNOWN THAT THE FLOW RATE IN $m^3 s^{-1}$ IS

$\frac{\text{VOLUME}}{\text{TIME}} = \text{SPEED} \times \text{CROSS SECTIONAL AREA}$

NOW GAIN IN KINETIC ENERGY & POTENTIAL ENERGY MUST EQUAL TO THE WORK IN, SUPPLIED BY THE PUMP

$\Rightarrow \text{WORK IN} = \text{GAIN IN K.E} + \text{GAIN IN P.E}$

$\Rightarrow \text{WORK IN} = \frac{1}{2}Mv^2 + Mgh$

$\Rightarrow \frac{\text{WORK IN}}{\text{TIME}} = \frac{1}{2}\frac{\text{MASS}}{\text{TIME}}v^2 + \frac{\text{MASS}}{\text{TIME}}gh$

$\Rightarrow \text{POWER} = \frac{1}{2}(\text{DENSITY} \times \text{SPEED} \times \text{CROSS SECTIONAL AREA}) \times \text{SPEED}^2$
$+ (\text{DENSITY} \times \text{SPEED} \times \text{CROSS SECTIONAL AREA}) \times g \times \text{HEIGHT}$

IN SYMBOLIC FORM

$\Rightarrow P = \frac{1}{2}\rho A v^3 + \rho v A g h$

$\Rightarrow 900 = \frac{1}{2} \times 1000 A \times 20^3 + 1000 \times 20 \times A \times 9.8 \times 10$

$\Rightarrow 900 = 4000000A + 1960000A$

$\Rightarrow 9 = 40000A + 19600A$

$\Rightarrow 59600A = 9$

$\Rightarrow A \approx 0.00015100671 \text{ m}^2$

$\times 100^2$

HENCE THE CROSS SECTIONAL AREA IS 1.51 cm^2

Created by T. Madas