

Created by T. Madas

# POWER

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**Question 1 (\*\*)**

A lorry of mass 6000 kg is travelling on a straight horizontal road. The total air resistance experienced by the lorry is 2800 N, and that remains constant throughout the motion.

Given that the engine of the lorry is working at constant rate of 42 kW, find the acceleration of the lorry when its speed is  $6 \text{ ms}^{-1}$ .

$$a = 0.7 \text{ ms}^{-2}$$

Handwritten solution for Question 1:

- Free-body diagram: A lorry is represented by a dot. Forces acting on it are: Drag force  $D = 2800 \text{ N}$  to the left, Resistance force  $R = 10000 \text{ N}$  to the right, and Weight  $6000g$  downwards. Acceleration  $a$  is shown to the right.
- Equations:
  - Useful:  $P = D \cdot v$
  - $42000 = D \times 6$
  - $D = 7000$
  - $D - 2800 = 6000a$
  - $7000 - 2800 = 6000a$
  - $4200 = 6000a$
  - $a = 0.7 \text{ ms}^{-2}$

**Question 2 (\*\*)**

A small tractor of mass 1000 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle  $\beta$  to the horizontal, where  $\sin \beta = \frac{5}{49}$ .

The total non gravitational resistance experienced by the tractor is 250 N, and that remains constant throughout the motion. The tractor is modelled as a particle.

Find the power generated by the engine of the tractor when is travelling up the plane at a constant speed of  $12 \text{ ms}^{-1}$ .

$$P = 15 \text{ kW}$$

Handwritten solution for Question 2:

- Free-body diagram: A tractor is shown on an inclined plane at angle  $\beta$ . Forces acting on it are: Drag force  $D = 250 \text{ N}$  down the slope, Resistance force  $R = 1000g \sin \beta$  down the slope, Normal force  $N$  perpendicular to the slope, and Weight  $1000g$  vertically downwards. Acceleration  $a$  is shown up the slope.
- Equations:
  - constant speed  $\Rightarrow$  equilibrium of forces in the direction of motion
  - $D = 250 + 1000g \sin \beta$
  - $D = 250 + 1000 \times 9.8 \times \frac{5}{49}$
  - $D = 1230$
  - $P = Dv$
  - $P = 1230 \times 12$
  - $P = 15000 \text{ (watts)}$
  - $P = 15 \text{ kW}$

**Question 3 (\*\*)**

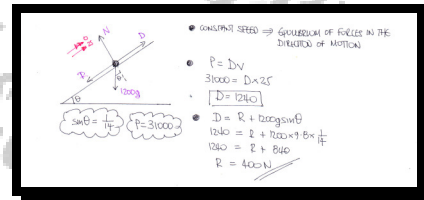
A car of mass 1200 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{1}{14}$ .

The total non gravitational resistance experienced by the car is  $R$  N, and that remains constant throughout the motion. The car is modelled as a particle.

When the engine of the car is working at the constant rate of 31 kW the car is travelling up the plane at a constant speed of  $25 \text{ ms}^{-1}$ .

Find the value of  $R$ .

**$R = 400$**



**Question 4 (\*\*)**

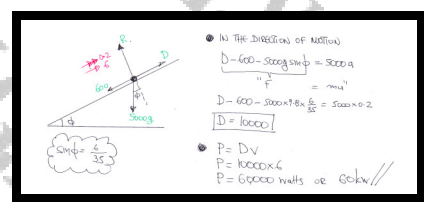
A truck of mass 5000 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle  $\phi$  to the horizontal, where  $\sin \phi = \frac{6}{35}$ .

The total non gravitational resistance experienced by the truck is 600 N, and that remains constant throughout the motion. The truck is modelled as a particle.

At a certain instant, the truck has a speed of  $6 \text{ ms}^{-1}$  and is accelerating up the plane at  $0.2 \text{ ms}^{-2}$ .

Determine the power generated by the engine of the truck at that instant.

**$P = 60 \text{ kW}$**



**Question 5** (\*\*)

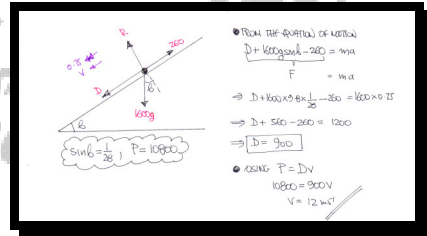
A car of mass 1600 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle  $\beta$  to the horizontal, where  $\sin \beta = \frac{1}{28}$ .

The total non gravitational resistance experienced by the car is 260 N, and that remains constant throughout the motion. The car is modelled as a particle.

When the engine of the car is working at the rate of 10.8 kW the car is travelling down the plane with a speed  $v \text{ ms}^{-1}$ , accelerating at  $0.75 \text{ ms}^{-2}$ .

Find the value of  $v$ .

$v = 12$



**Question 6 (\*\*+)**

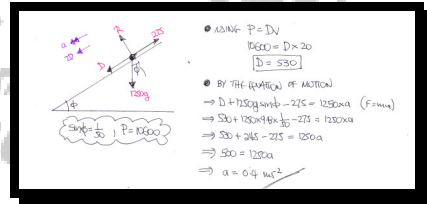
A car of mass 1250 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle  $\phi$  to the horizontal, where  $\sin \phi = \frac{1}{50}$ .

The total non gravitational resistance experienced by the car is 275 N, and that remains constant throughout the motion. The car is modelled as a particle.

When the engine of the car is working at the rate of 10.6 kW the car is travelling down the plane at a speed of  $20 \text{ ms}^{-1}$ , accelerating at  $a \text{ ms}^{-2}$ .

Find the value of  $a$ .

$a = 0.4$



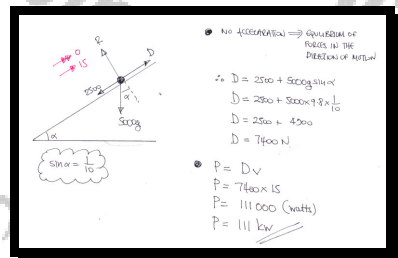
**Question 7 (\*\*+)**

A lorry of mass 5000 kg is travelling up at constant speed of  $15 \text{ ms}^{-1}$  on a straight road inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{1}{10}$ .

The total non gravitational resistance experienced by the lorry is 2500 N, and this is assumed to remain constant throughout the motion.

Find the power developed by the engine of the lorry, in kW .

$P = 111 \text{ kW}$



**Question 8** (\*\*\*)

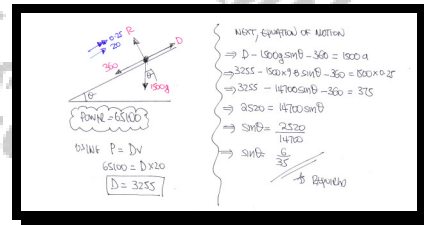
A car of mass 1500 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle  $\theta$  to the horizontal.

The total non gravitational resistance experienced by the car is 360 N, and that remains constant throughout the motion. The car is modelled as a particle.

At a certain instant, the power generated by the engine of the car is 65.1 kW, the car has a speed of  $20 \text{ ms}^{-1}$  and is accelerating up the plane at  $0.25 \text{ ms}^{-2}$ .

Show clearly that  $\sin \theta = \frac{6}{35}$ .

proof



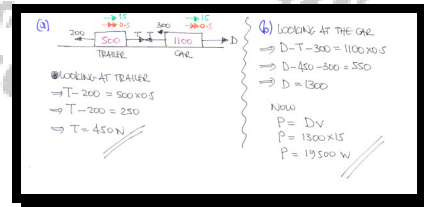
**Question 9** (\*\*+)

A trailer of mass 500 kg is towed by a car of mass 1100 kg along a straight horizontal road. The total resistance experienced by the car is 300 N, and the total resistance experienced by the trailer is 200 N. The trailer and the car are both modelled as particles, and the towbar joining the trailer to the car is modelled as a light rigid rod parallel to the road.

At a given instant the car and the trailer are moving with speed  $15 \text{ ms}^{-1}$  and acceleration  $0.5 \text{ ms}^{-2}$ .

- Calculate the tension in the towbar.
- Determine the rate at which the engine of the car is working.

$$T = 450 \text{ N}, \quad P = 19500 \text{ W}$$



**Question 10** (\*\*\*)

A woman and her bike are modelled a single particle of combined mass 72 kg.

The woman cycles with constant speed of  $5 \text{ ms}^{-1}$ , **up** a straight road, which lies on the line of greatest slope of a plane inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{2}{21}$ .

The total non gravitational resistance experienced by the cyclist is assumed to be constant at 25 N.

- a) Find the power generated by the woman when cycling up the hill.

The woman then turns her bike around at some point  $A$  on the road. She freewheels down the same road starting with a speed of  $5 \text{ ms}^{-1}$ . She passes through some point  $B$  on that road with a speed  $v \text{ ms}^{-1}$ .

The total non gravitational resistance experienced by the cyclist is assumed to be the same as in part (a).

- b) Given that the distance  $AB$  is 180 m, find the value of  $v$ .

$P = 461 \text{ W}$ ,  $v \approx 15.4$

(a)  $\bullet$  CONSTANT SPEED  $\Rightarrow$  ZERO ACCELERATION  $\Rightarrow$  EQUILIBRIUM IN THE DIRECTION OF MOTION

$D = 25 + 72g \sin \theta$   
 $D = 25 + 72 \times \frac{2}{21}$   
 $D = 461$

$P = Dv$   
 $P = 461 \times 5$   
 $P = 2305 \text{ W}$

(b)

$KE_A + PE_A + W_{\text{res}} = KE_B + PE_B$   
 $\frac{1}{2} \times 72 \times 5^2 + 72 \times 180 \times \frac{2}{21} - 25 \times 180 = \frac{1}{2} \times 72 \times v^2$   
 $900 + 1200 - 4500 = 36v^2$   
 $8400 = 36v^2$   
 $v^2 = 233$   
 $v \approx 15.26 \text{ ms}^{-1}$



## Question 11 (\*\*\*)

A car of mass 1500 kg is travelling up a hill on a straight road, with the engine of the car working at the constant rate of 13 kW for 1 minute.

During this minute the car increases its speed from  $7 \text{ ms}^{-1}$  to  $24 \text{ ms}^{-1}$  and in addition to the work done against gravity, 80000 J of work is done against resistances to motion parallel to the direction of motion of the car.

Calculate the vertical displacement of the car in this 1 minute interval.

$$h \approx 20.73 \text{ m}$$

• Power =  $\frac{W_{in}}{Time}$   
 $13000 = \frac{W_{in}}{60}$   
 $W_{in} = 780000$

• TRACKING THE LEVEL AT WHICH THE SPEED OF THE CAR IS  $7 \text{ ms}^{-1}$ , AS THE ZERO POTENTIAL LEVEL.  
 $K_E + PE_A + W_{in} - W_{out} = K_E + PE_B$   
 $\frac{1}{2} \times 1500 \times 7^2 + 78000 - 80000 = \frac{1}{2} \times 1500 \times 24^2 + 1500gh$   
 $36750 + 78000 - 80000 = 45000 + 1500gh$   
 $30750 = 1500gh$   
 $h = \frac{30750}{1500} \approx 20.5 \text{ m}$

**Question 12** (\*\*\*)

The total air resistance on a cyclist is given by  $kv^2$ , where  $v$  is his speed in  $\text{ms}^{-1}$  and  $k$  is a positive constant. The cyclist freewheels at a constant speed of  $3.5 \text{ ms}^{-1}$  down a slope inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{1}{20}$ .

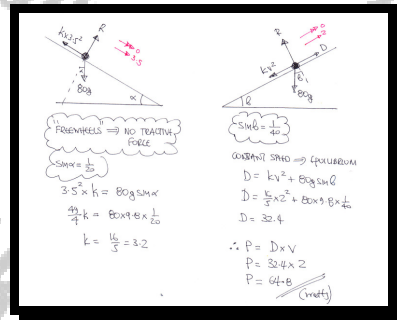
The cyclist and his bike are modelled as a single particle of mass  $80 \text{ kg}$ .

- a) Show that  $k = 3.2$ .

The cyclist next cycles up a different slope inclined at an angle  $\beta$  to the horizontal, where  $\sin \beta = \frac{1}{40}$ .

- b) Given the cyclist's constant speed is now  $2 \text{ ms}^{-1}$ , find the rate at which the cyclist is working.

$P = 64.8 \text{ W}$



**Question 13** (\*\*\*)

A car of mass 1600 kg is travelling up a straight road inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{1}{40}$ .

The car is modelled as a particle travelling at constant speed of  $25 \text{ ms}^{-1}$  and the resistance to its motion due to non-gravitational forces has a constant magnitude of 500 N.

The car travels between two points on the road, A and B in 20 s.

Determine the work done by the engine of the car, as the car moves from A to B.

22300, 446000 J

**STARTING WITH A CONSIDERED DIAGRAM**

Force diagram showing forces on the car: Drag (D), Resistance (R), Weight (W), and Normal force (N). The car is moving up the incline with velocity v.

$D = 500 + 1600g \sin \theta$   
 (on acceleration)  
 $\Rightarrow D = 500 + 1600g \times \frac{1}{40}$   
 $\Rightarrow D = 892 \text{ N}$

**Power = TRACTIVE FORCE  $\times$  SPEED**

$P = 892 \times 25$   
 $P = 22300 \text{ W}$

**WKT: Power =  $\frac{\text{Work Done}}{\text{Time}}$**

$22300 = \frac{W_{\text{in}}}{20}$   
 $W_{\text{in}} = 446000 \text{ J}$

Alternative method using energy:

$\frac{1}{2}mv^2 + W_{\text{in}} - W_{\text{out}} = \frac{1}{2}mv^2 + PE$   
 $\Rightarrow W_{\text{in}} - 500d = mgh$   
 $\Rightarrow W_{\text{in}} = 500d + mgh$   
 $\Rightarrow W_{\text{in}} = 500d + 1600gd \times \frac{1}{40}$   
 $\Rightarrow W_{\text{in}} = 500d + 392d$   
 $\Rightarrow W_{\text{in}} = 892d$   
 $\Rightarrow W_{\text{in}} = 892 \times (25 \times 20)$   
 (distance of 25ms for 20 seconds)  
 $\Rightarrow W_{\text{in}} = 446000$

**Question 14** (\*\*\*)

A cyclist and her bike are modelled as a single particle of mass 60 kg, subject to a constant frictional force  $F$ , when in motion.

When the cyclist is working at a constant rate of 120 W she is travelling at constant speed of  $4 \text{ ms}^{-1}$  on a straight horizontal road.

- a) Find the value of  $F$ .

The cyclist next cycles up a straight road inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{1}{15}$ . The constant frictional force  $F$  remains unchanged and the cyclist is now travelling at a constant speed of  $3 \text{ ms}^{-1}$ .

- b) Find the rate at which the cyclist is working.

The model is next refined and it is now assumed that the frictional force  $F$  is proportional to the square of the speed of the cyclist.

- c) Find an amended value for the power developed by the cyclist in travelling up the same sloped road at the constant speed of  $3 \text{ ms}^{-1}$ .

$F = 30 \text{ N}$ ,  $P = 207.6 \text{ W}$ ,  $P \approx 168 \text{ W}$

(a)  $P = D \times v$   
 $120 = D \times 4$   
 $D = 30$   
 $\therefore$  THE RESISTANCE IS 30N  
 $(D = F)$

(b) CONSTANT SPEED  $\Rightarrow$  EQUILIBRIUM  
 $D' = 30 + 60g \sin \alpha$   
 $D' = 30 + 60 \times 9.8 \times \frac{1}{15}$   
 $D' = 69.2 \text{ N}$   
 $P = D'v$   
 $P = 69.2 \times 3$   
 $P = 207.6 \text{ (check)}$

(c) RESISTANCE  $F = kv^2$   
 $30 = k \times 4^2$   
 $k = \frac{15}{8}$   
 $\therefore F = \frac{15}{8}v^2$

$\therefore$  AGAIN IN EQUILIBRIUM  
 $D'' = 60g \sin \alpha + kv^2$   
 $D'' = 60 \times 9.8 \times \frac{1}{15} + \frac{15}{8} \times 3^2$   
 $D'' = 56.015$   
 $\therefore$  NEW POWER =  $56.015 \times 3$   
 $= 168.045$   
 $= 168$

**Question 15** (\*\*\*)

A lorry, of mass 4000 kg, is travelling up the line of greatest slope of a hill inclined at an angle  $\theta$  to the horizontal, where  $\sin\theta = \frac{3}{49}$ . The engine of the lorry is working at the constant rate of 90 kW.

The motion of the lorry is subject to a **constant** non gravitational resistance.

Determine the greatest speed of the lorry up the hill, given that at some instant during the climb the lorry is accelerating at  $0.2 \text{ ms}^{-2}$  and its speed is  $14.4 \text{ ms}^{-1}$ .

$v_{\text{max}} \approx 16.5 \text{ ms}^{-1}$

SPENDING WITH TWO SEPARATE DIAGRAM

$P = D \times v$   
 $\Rightarrow 90000 = D \times 14.4$   
 $\Rightarrow D = 6250$

$F = m \times a$   
 $\Rightarrow D - R - 4000g \sin\theta = 4000a$   
 $\Rightarrow 6250 - R - 4000g \left(\frac{3}{49}\right) = 4000(0.2)$   
 $\Rightarrow 6250 - R - 2400 = 800$   
 $\Rightarrow R = 3050 \leftarrow \text{CONSTANT}$

**MAX SPEED  $\Rightarrow$  NO ACCELERATION  $\Rightarrow$  EQUILIBRIUM**  
 $D = R + 4000g \sin\theta$   
 $D = 3050 + 4000g \left(\frac{3}{49}\right)$   
 $D = 3050 + 2400$   
 $D = 5450$

$P = D \times v$   
 $\Rightarrow 90000 = 5450 v$   
 $\Rightarrow v_{\text{max}} = \frac{90000}{5450} \approx 16.5 \text{ ms}^{-1}$

**Question 16** (\*\*\*)

A car, of mass 1500 kg, is travelling up the line of greatest slope of a hill inclined at an angle  $\theta$  to the horizontal, where  $\sin\theta = \frac{2}{49}$ . The engine of the car is working at a constant rate of  $P$  W.

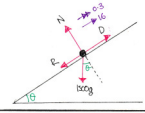
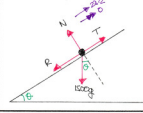
The motion of the car is subject to a **constant** non gravitational resistance.

At some instant during the climb, the car is accelerating at  $0.3 \text{ ms}^{-2}$  when its speed is  $16 \text{ ms}^{-1}$ , reaching a maximum speed of  $23.2 \text{ ms}^{-1}$ .

Determine the value of  $P$ .

45/1P,  $P = 23200 \text{ W}$

• START WITH TWO SEPARATE DIAGRAM, TRYS TO FORM EQUATIONS

• " $P = Dv$ "  
 $\Rightarrow P = D \times 16$   
 $\Rightarrow D = \frac{P}{16}$

• EQUATION OF MOTION  
 $\Rightarrow \sum F = ma$   
 $\Rightarrow D - R - 1500g \sin\theta = 1500(0.3)$   
 $\Rightarrow \frac{P}{16} - R - 1500(\frac{2}{49}) = 1500(0.3)$   
 $\Rightarrow \frac{P}{16} - R - 600 = 450$   
 $\Rightarrow \frac{P}{16} - R = 1050$

• " $P = Tv$ "  
 $\Rightarrow P = T \times 23.2$   
 $\Rightarrow T = \frac{P}{23.2}$

• NO ACCELERATION (EQUILIBRIUM)  
 $\Rightarrow T = R + 1500g \sin\theta$   
 $\Rightarrow \frac{P}{23.2} = R + 1500(\frac{2}{49})$   
 $\Rightarrow \frac{P}{23.2} = R + 600$   
 $\Rightarrow \frac{P}{23.2} - R = 600$

• SUBTRACT EQUATIONS BY SUBTRACTION

$$\frac{P}{16} - \frac{P}{23.2} = 450 \Rightarrow 28.2P - 16P = 167400$$

$$\Rightarrow 7.2P = 167400$$

$$\Rightarrow P = 23200$$

**Question 17** (\*\*\*)

A car, which is modelled as a particle of mass 1500 kg, is travelling on a straight road inclined at an angle  $\theta$  to the horizontal.

When the engine of the car is working at the constant rate of 96 kW, at an instant when the car is travelling **up** this road with speed  $20 \text{ ms}^{-1}$  the car is experiencing an acceleration of  $0.2 \text{ ms}^{-2}$ .

When the engine of the car is working at the constant rate of 60 kW, at an instant when the car is travelling **down** the same road with speed  $20 \text{ ms}^{-1}$  the car is experiencing an acceleration of  $0.3 \text{ ms}^{-2}$ .

If the resistance to the motion of the car  $R$ , due to non-gravitational forces, has constant magnitude, determine the value of  $R$  and the value of  $\theta$ .

,  $R = 3525 \text{ N}$  ,  $\theta \approx 3.80^\circ$

SHOW SEPARATE WORK FOR THE UPHILL & DOWNHILL MOTION

$P = 96 \text{ kW}$        $P = 60 \text{ kW}$

**FIRSTLY CALCULATE THE TRACTIVE (DRIVING) FORCE IN EACH CASE**

$\Rightarrow P = Dv$	$\Rightarrow P' = D'v'$
$\Rightarrow 96000 = D \times 20$	$\Rightarrow 60000 = D' \times 20$
$\Rightarrow D = 4800$	$\Rightarrow D' = 3000$

**NEXT WRITE THE EQUATION OF MOTION IN EACH CASE**

$\Rightarrow D - R - 1500g \sin \theta = ma$	$\Rightarrow D' + 1500g \sin \theta - R = ma'$
$\Rightarrow 4800 - R - 1500g \sin \theta = 1500 \times 0.2$	$\Rightarrow 3000 + 1500g \sin \theta - R = 1500 \times 0.3$
$\Rightarrow 4500 = R + 1500g \sin \theta$	$\Rightarrow 2550 = R - 1500g \sin \theta$

**ADDING THE EQUATIONS YIELDS**

$7050 = 2R$   
 $R = 3525 \text{ N}$

**SUBSTITUTING THE EQUATIONS GIVES**

$1950 = 3000g \sin \theta$   
 $\sin \theta = \frac{1950}{3000g}$   
 $\theta \approx 3.80^\circ$

**Question 18** (\*\*\*)

A motorbike, which is modelled as a particle of mass 300 kg, is travelling on a straight road with its engine working at its maximum rate of 54 kW.

When in motion the motorbike experiences non gravitational resistances of magnitude  $a + bv$  N, where  $v$  ms<sup>-1</sup> is its speed.

When travelling on a horizontal stretch of the road, at maximum power, the maximum speed of the motorbike is 60 ms<sup>-1</sup>.

When travelling, at maximum power, on a stretch of the road inclined at  $\arcsin\left(\frac{4}{49}\right)$  to the horizontal, the maximum speed of the motorbike is 50 ms<sup>-1</sup>.

Determine the acceleration of the motorbike when travelling on a horizontal stretch of the road, at maximum power, at the instant when its speed is 30 ms<sup>-1</sup>.

,  $a = 3.6 \text{ ms}^{-2}$

The image shows two pages of handwritten work on grid paper. The left page is titled 'LOOKING AT THE BIKE AT MAX SPEED ON THE FLAT' and 'LOOKING AT THE BIKE AT MAX SPEED ON THE INCLINE'. It includes force diagrams for both cases and a system of equations to solve for 'a' and 'b'. The right page is titled 'RETURNING TO THE FLAT' and shows the calculation for the acceleration 'a' at a speed of 30 ms<sup>-1</sup> on a horizontal surface.

**LOOKING AT THE BIKE AT MAX SPEED ON THE FLAT**

- Force diagram: A horizontal force  $D$  to the right and a resistance force  $a + bv$  to the left. Mass is 300 kg.
- Equations:
  - $P = Dv$
  - $54000 = D \times 60$
  - $D = 900$
  - $D = a + bv$
  - $900 = a + 60b$

**LOOKING AT THE BIKE AT MAX SPEED ON THE INCLINE**

- Force diagram: A force  $D'$  up the incline, a resistance force  $a + bv$  down the incline, a weight force  $300g$  acting vertically down, and a normal force  $N$  acting perpendicular to the incline. The incline angle is  $\arcsin(4/49)$ .
- Equations:
  - $\sin \theta = \frac{4}{49}$
  - $P = D'v$
  - $54000 = D' \times 50$
  - $D' = 1080$
  - $D' = (a + bv) + 300g \sin \theta$
  - $1080 = a + 50v + 300g \times \frac{4}{49}$
  - $1080 = a + 50v + 240$
  - $840 = a + 50b$

**SOLVING SIMULTANEOUSLY**

$$\begin{cases} a + 60b = 900 \\ a + 50b = 840 \end{cases} \Rightarrow \begin{cases} 10b = 60 \\ b = 6 \\ a = 540 \end{cases}$$

**RETURNING TO THE FLAT**

- Force diagram: A force  $D''$  to the right and a resistance force  $a + bv$  to the left. Mass is 300 kg.
- Equations:
  - $P = D''v$
  - $54000 = D'' \times 30$
  - $D'' = 1800$
  - $F = ma$
  - $1800 - (540 + 6 \times 30) = 300a$
  - $1080 = 300a$
  - $a = 3.6 \text{ ms}^{-2}$



**Question 19** (\*\*\*)

A car of mass 1300 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{1}{10}$ .

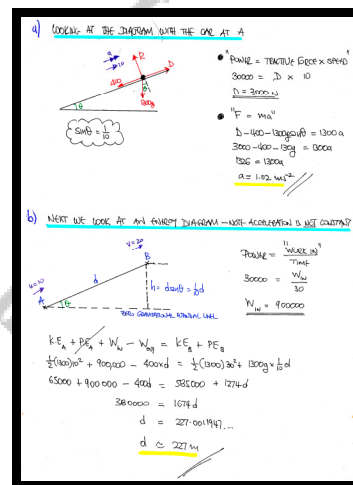
The total non gravitational resistance experienced by the car is assumed to be a constant force of magnitude of 400 N. The engine of the car is working at the constant rate of 30 kW.

The car is passing through the point A with a speed  $10 \text{ ms}^{-1}$  and continues to accelerate up the plane, passing through the point B with speed  $30 \text{ ms}^{-1}$ , 30 s after passing through A.

By modelling the car as a particle, find ...

- a) ... the acceleration of the car at A.
- b) ... the distance AB.

,  $a = 1.02 \text{ ms}^{-2}$  ,  $|AB| \approx 227 \text{ m}$



**Question 20** (\*\*\*\*)

A car of mass 1000 kg is travelling on straight road sections.

The car experiences a constant air resistance when in motion and the car's engine is working at a constant rate, at all times.

When the car is travelling up a hill inclined at a constant angle  $\arcsin\left(\frac{1}{14}\right)$  to the horizontal, its maximum speed is  $15 \text{ ms}^{-1}$ .

When the car is travelling down the same hill its maximum speed is  $21 \text{ ms}^{-1}$ .

Calculate the acceleration of the car when it is moving on level ground with a speed of  $14 \text{ ms}^{-1}$ .

,  $a = 1.05 \text{ ms}^{-2}$

The image shows two pages of handwritten work. The left page is titled 'LOOKING AT TWO SITUATIONS ON THE HILL'. It contains two force diagrams for a car on an inclined plane. The first diagram is for the car going up the hill, showing forces: weight (mg) acting vertically down, normal force (N) perpendicular to the slope, and air resistance (R) acting up the slope. The second diagram is for the car going down the hill, showing forces: weight (mg) acting vertically down, normal force (N) perpendicular to the slope, and air resistance (R) acting down the slope. Below the diagrams are calculations. For the car going up, it sets  $D = R + mg \sin \theta$  and  $\frac{P}{v} = R + mg \sin \theta$ . It then uses the given maximum speed of  $15 \text{ ms}^{-1}$  to find  $P = 73500$ . For the car going down, it sets  $D = mg \sin \theta = R$  and  $\frac{P}{v} = R + mg \sin \theta$ . It uses the given maximum speed of  $21 \text{ ms}^{-1}$  to find  $R = 4200$ . The right page is titled 'SKETCH A DIAGRAM ON LEVEL GROUND'. It shows a force diagram for a car on level ground with forces: weight (mg) acting vertically down, normal force (N) acting vertically up, air resistance (R) acting to the left, and engine force (F) acting to the right. Below the diagram are calculations:  $F = ma$ ,  $\frac{P}{v} - R = ma$ ,  $\frac{73500}{14} - 4200 = 1000a$ ,  $1050 = 1000a$ , and  $a = 1.05 \text{ ms}^{-2}$ .

**Question 21** (\*\*\*\*)

A man and his bike are modelled a single particle of combined mass 90 kg.

When the man is working at the constant rate of 560 joules per second he achieves a constant cycling speed of  $3.2 \text{ ms}^{-1}$  on horizontal ground.

- a) Find the magnitude of the resistance experienced by the cyclist, assumed constant throughout his motion.

The man next cycles down a straight road, which lies on the line of greatest slope of a plane inclined at an angle  $\psi$  to the horizontal, where  $\sin \psi = \frac{2}{21}$ .

He achieves a constant speed of  $v \text{ ms}^{-1}$  when his power is 144 W. The total non gravitational resistance experienced by the cyclist is now assumed to be  $8v \text{ N}$ .

- b) Find the value of  $v$ .

Resistance = 175 N ,  $v = 12$

**(a)**

Free-body diagram for horizontal motion:  $F$  (forward),  $R$  (backward),  $D$  (backward),  $90g$  (down).  
 Power = 560  
 $P = Dv$   
 $560 = D \times 3.2$   
 $D = 175$

As there is no acceleration  $D = F$   
 Friction/Resistance is also 175N

**(b)**

Free-body diagram for motion down an inclined plane:  $D$  (up the slope),  $8v$  (up the slope),  $90g \sin \psi$  (down the slope),  $90g \cos \psi$  (perpendicular to the slope).  
 Power = 144  
 $\sin \psi = \frac{2}{21}$

$P = Dv$   
 $D + 90g \sin \psi = 8v$   
 $144 = Dv$   
 $D + 90 \times 9.8 \times \frac{2}{21} = 8v$   
 $144 = Dv$   
 $D + 84 = 8v$  (By substitution  $D = \frac{144}{v}$ )  
 $\frac{144}{v} + 84 = 8v$   
 $144 + 84v = 8v^2$   
 $8v^2 - 84v - 144 = 0$   
 $2v^2 - 21v - 36 = 0$   
 $(2v + 3)(v - 12) = 0$   
 $v = 12$

**Question 22** (\*\*\*\*)

A track of mass 6000 kg, when travelling with speed  $v \text{ ms}^{-1}$  it experiences air resistance directly proportional to  $v^2$ .

When the track is travelling at constant speed of  $20 \text{ ms}^{-1}$  along a straight horizontal road, the engine of the track is working at a rate of  $P \text{ W}$ .

When the track is travelling at constant speed of  $10 \text{ ms}^{-1}$  along a straight horizontal road, up a line of greatest slope of a hill which is inclined at  $\theta$  to the horizontal, the engine of the track is working at a rate of  $P \text{ W}$ .

It is further given that  $\sin \theta = \frac{1}{14}$  and the air resistance to the motion of the track when it travels up the hill, is still directly proportional to  $v^2$ .

Determine the value of  $P$ .

$P = 48000 \text{ W}$

The handwritten solution includes two force diagrams and a series of calculations. The first diagram shows a track on a horizontal surface with forces: weight  $600g$  acting downwards, normal reaction  $R$  acting upwards, and drag force  $k v^2$  acting to the left. The second diagram shows the track on an inclined plane at angle  $\theta$  to the horizontal. Forces shown are: weight  $600g$  acting vertically downwards, normal reaction  $R$  acting perpendicular to the slope, drag force  $k v^2$  acting up the slope, and engine force  $F$  acting up the slope. The calculations are as follows:

$\bullet \frac{P}{v} = k v^2$   
 $\Rightarrow \frac{P}{20} = k \times 20^2$   
 $\Rightarrow P = 800k$

$\bullet \frac{P}{10} = k v^2 + 600g \sin \theta$   
 $\Rightarrow \frac{P}{10} = k \times 10^2 + 600g \times \frac{1}{14}$   
 $\Rightarrow \frac{P}{10} = 100k + 4200$   
 $\Rightarrow P = 100k + 42000$   
 $\Rightarrow 800k = 100k + 42000$   
 $\Rightarrow 700k = 42000$   
 $\Rightarrow k = 60$   
 $\Rightarrow P = 800 \times 60$   
 $\Rightarrow P = 48000$

**Question 23** (\*\*\*\*)

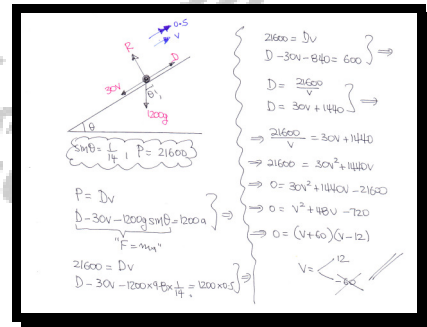
A car of mass 1200 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{1}{14}$ .

At a given instant the engine of the car is working at the rate of 21.6 kW, the car having a speed  $v \text{ ms}^{-1}$ , accelerating up the plane at  $0.5 \text{ ms}^{-2}$ .

The total non gravitational resistance experienced by the car is assumed to be  $30v \text{ N}$ .

By modelling the car as a particle, find the value of  $v$ .

$v = 12$



**Question 24** (\*\*\*\*+)

A man of mass 80 kg, is jogging up a hill with constant speed of  $3 \text{ ms}^{-1}$ , in a straight line inclined at  $5^\circ$  to the horizontal.

The man experiences wind assistance during his jog which is modelled as a constant force of magnitude 7 N.

The man is working at a constant rate of  $P \text{ W}$  and takes 2 minutes to jog up the hill.

Assuming no other resistance on the motion of the man when it jogs up the hill, except gravitational resistances, calculate the value of  $P$ .

,  $P = 184 \text{ W}$

**SPEED UP IN DIAGRAM**

Diagram: A right-angled triangle representing a hill. The hypotenuse is the hill, inclined at  $5^\circ$  to the horizontal. The vertical side is labeled  $h = 40 \text{ m}$ . The horizontal side is labeled  $d$ . A man is shown at the top of the hill. A note says "WIND ASSISTANCE FORCE = 7 N".

As the speed is constant  $d = 120 \times 3 = 360 \text{ METRES}$

$$K.E_s + P.E_s + W_{\text{wind}} + W_{\text{grav}} - W_{\text{net}} = K.E_e + P.E_e$$

As the speed is constant  $K.E_s = K.E_e$  AND  $P.E_s = 0$ ,  $W_{\text{net}} = 0$

$$\Rightarrow W_{\text{wind}} + W_{\text{grav}} = P.E_e$$

$$\Rightarrow W_{\text{wind}} + 7d = mgh$$

$$\Rightarrow W = mgh - 7d$$

$$\Rightarrow W = 80 \times 9.81 \times 40 - 7d$$

$$\Rightarrow W = 7d (112.045 - 1)$$

$$\Rightarrow W = 7 \times 360 (112.045 - 1)$$

$$\Rightarrow W = 22078.8363 \dots$$

Final Power =  $\frac{W_{\text{wind}}}{Time}$

$$Power = \frac{22078.8363}{120} \approx 184 \text{ WATT}$$