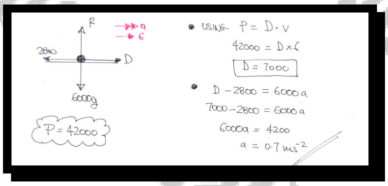
POWER

Question 1 (**)

A lorry of mass 6000 kg is travelling on a straight horizontal road. The total air resistance experienced by the lorry is 2800 N, and that remains constant throughout the motion.

Given that the engine of the lorry is working at constant rate of 42 kW, find the acceleration of the lorry when its speed is 6 ms^{-1}.

$$\boxed{a = 0.7 \text{ ms}^{-2}}$$



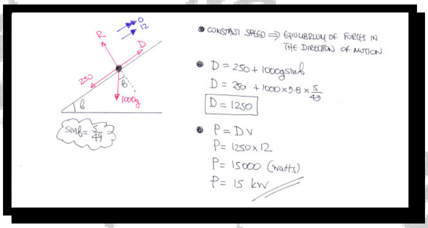
Question 2 (**)

A small tractor of mass 1000 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle β to the horizontal, where $\sin\beta = \frac{5}{49}$.

The total non gravitational resistance experienced by the tractor is 250 N, and that remains constant throughout the motion. The tractor is modelled as a particle.

Find the power generated by the engine of the tractor when is travelling up the plane at a constant speed of 12 ms^{-1}.

$$\boxed{P = 15 \text{ kW}}$$



Question 3 (**)

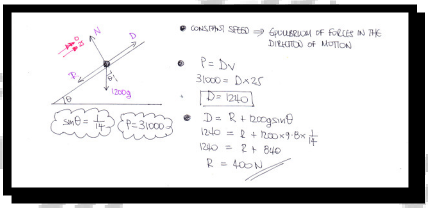
A car of mass 1200 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{14}$.

The total non gravitational resistance experienced by the car is R N, and that remains constant throughout the motion. The car is modelled as a particle.

When the engine of the car is working at the constant rate of 31 kW the car is travelling up the plane at a constant speed of 25 ms^{-1}.

Find the value of R.

$\boxed{R = 400}$



Question 4 (**)

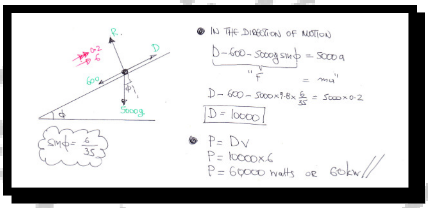
A truck of mass 5000 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle φ to the horizontal, where $\sin\varphi = \frac{6}{35}$.

The total non gravitational resistance experienced by the truck is 600 N, and that remains constant throughout the motion. The truck is modelled as a particle.

At a certain instant, the truck has a speed of 6 ms^{-1} and is accelerating up the plane at 0.2 ms^{-2}.

Determine the power generated by the engine of the truck at that instant.

$\boxed{P = 60 \text{ kW}}$



Question 5 (**)

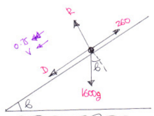
A car of mass 1600 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle β to the horizontal, where $\sin\beta = \frac{1}{28}$.

The total non gravitational resistance experienced by the car is 260 N, and that remains constant throughout the motion. The car is modelled as a particle.

When the engine of the car is working at the rate of 10.8 kW the car is travelling down the plane with a speed v ms^{-1}, accelerating at 0.75 ms^{-2}.

Find the value of v.

$\boxed{v = 12}$



$\sin\beta = \frac{1}{28}$, $P = 10800$

- FROM THE EQUATION OF MOTION

$\underbrace{D + 1600g\sin\beta - 260}_{F} = ma$

$\Rightarrow D + 1600 \times 9.8 \times \frac{1}{28} - 260 = 1600 \times 0.75$

$\Rightarrow D + 560 - 260 = 1200$

$\Rightarrow \boxed{D = 900}$

- USING $P = Dv$

$10800 = 900v$

$v = 12 \text{ ms}^{-1}$

Question 6 (+)**

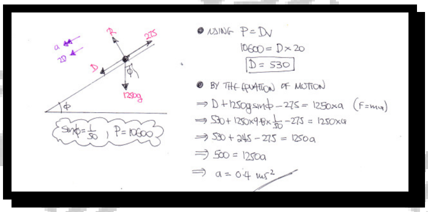
A car of mass 1250 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle φ to the horizontal, where $\sin\varphi = \frac{1}{50}$.

The total non gravitational resistance experienced by the car is 275 N, and that remains constant throughout the motion. The car is modelled as a particle.

When the engine of the car is working at the rate of 10.6 kW the car is travelling down the plane at a speed of 20 ms^{-1}, accelerating at $a \text{ ms}^{-2}$.

Find the value of a.

$\boxed{a = 0.4}$



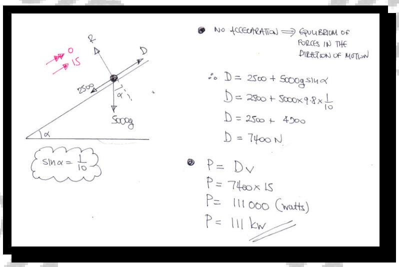
Question 7 (+)**

A lorry of mass 5000 kg is travelling up at constant speed of 15 ms^{-1} on a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{10}$.

The total non gravitational resistance experienced by the lorry is 2500 N, and this is assumed to remain constant throughout the motion.

Find the power developed by the engine of the lorry, in kW.

$\boxed{P = 111 \text{ kW}}$



Question 8 (**+)

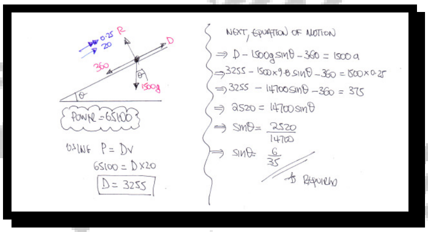
A car of mass 1500 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal.

The total non gravitational resistance experienced by the car is 360 N, and that remains constant throughout the motion. The car is modelled as a particle.

At a certain instant, the power generated by the engine of the car is 65.1 kW, the car has a speed of 20 ms^{-1} and is accelerating up the plane at 0.25 ms^{-2}.

Show clearly that $\sin\theta = \dfrac{6}{35}$.

proof



Question 9 (**+)

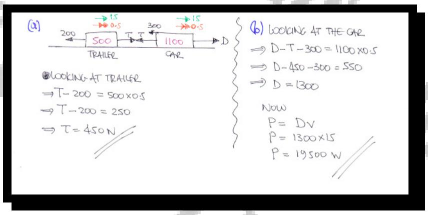
A trailer of mass 500 kg is towed by a car of mass 1100 kg along a straight horizontal road. The total resistance experienced by the car is 300 N, and the total resistance experienced by the trailer is 200 N. The trailer and the car are both modelled as particles, and the towbar joining the trailer to the car is modelled as a light rigid rod parallel to the road.

At a given instant the car and the trailer are moving with speed 15 ms^{-1} and acceleration 0.5 ms^{-2}.

a) Calculate the tension in the towbar.

b) Determine the rate at which the engine of the car is working.

$\boxed{T = 450 \text{ N}}$, $\boxed{P = 19500 \text{ W}}$



Question 10 (***)

A woman and her bike are modelled a single particle of combined mass 72 kg.

The woman cycles with constant speed of 5 ms^{-1}, **up** a straight road, which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{2}{21}$.

The total non gravitational resistance experienced by the cyclist is assumed to be constant at 25 N.

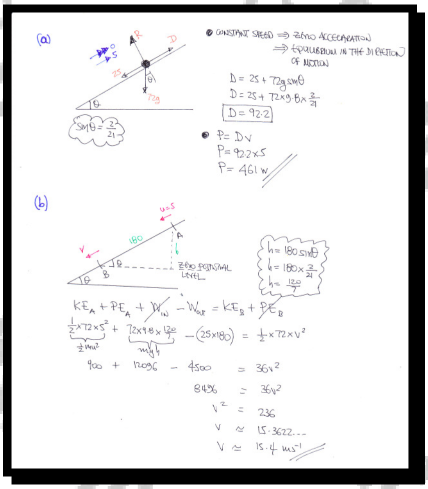
a) Find the power generated by the woman when cycling up the hill.

The woman then turns her bike around at some point A on the road. She freewheels down the same road starting with a speed of 5 ms^{-1}. She passes through some point B on that road with a speed $v \text{ ms}^{-1}$.

The total non gravitational resistance experienced by the cyclist is assumed to be the same as in part (a).

b) Given that the distance AB is 180 m, find the value of v.

$\boxed{P = 461 \text{ W}}$, $\boxed{v \approx 15.4}$



Question 11 (***)

A car of mass 1500 kg is travelling up a hill on a straight road, with the engine of the car working at the constant rate of 13 kW for 1 minute.

During this minute the car increases its speed from 7 ms^{-1} to 24 ms^{-1} and in addition to the work done against gravity, 80000 J of work is done against resistances to motion parallel to the direction of motion of the car.

Calculate the vertical displacement of the car in this 1 minute interval.

$\boxed{h \approx 20.73 \text{ m}}$

- POWER $= \dfrac{W_{IN}}{\text{TIME}}$

$13000 = \dfrac{W_{IN}}{60}$

$W_{IN} = 780{,}000$

- TAKING THE LEVEL AT WHICH THE SPEED OF THE CAR IS 7 ms^{-1}, AS THE ZERO POTENTIAL LEVEL

$KE_A + \cancel{PE_A} + W_{IN} - W_{OUT} = KE_B + PE_B$

$\frac{1}{2}\times1500\times7^2 + 780000 - 80000 = \frac{1}{2}\times1500\times24^2 + 1500gh$

$36750 + 780000 - 80000 = 432000 + 1500gh$

$304750 = 1500gh$

$h = \frac{6095}{294} \approx 20.73\text{m}$

Question 12 (***)

The total air resistance on a cyclist is given by kv^2, where v is his speed in ms^{-1} and k is a positive constant. The cyclist freewheels at a constant speed of $3.5\ \text{ms}^{-1}$ down a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{20}$.

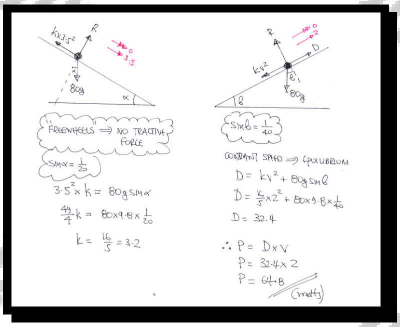
The cyclist and his bike are modelled as a single particle of mass $80\,\text{kg}$.

a) Show that $k = 3.2$.

The cyclist next cycles up a different slope inclined at an angle β to the horizontal, where $\sin\beta = \frac{1}{40}$.

b) Given the cyclist's constant speed is now $2\ \text{ms}^{-1}$, find the rate at which the cyclist is working.

$\boxed{P = 64.8\,\text{W}}$



Question 13 (***)

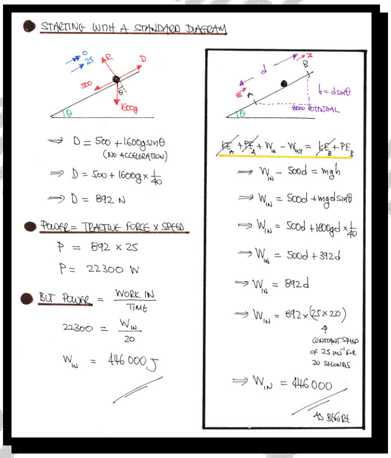
A car of mass 1600 kg is travelling up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{40}$.

The car is modelled as a particle travelling at constant speed of 25 ms^{-1} and the resistance to its motion due to non-gravitational forces has a constant magnitude of 500 N.

The car travels between two points on the road, A and B in 20 s.

Determine the work done by the engine of the car, as the car moves from A to B.

$\boxed{446000 \text{ J}}$



Question 14 (***+)

A cyclist and her bike are modelled as a single particle of mass 60 kg, subject to a constant frictional force F, when in motion.

When the cyclist is working at a constant rate of 120 W she is travelling at constant speed of 4 ms^{-1} on a straight horizontal road.

a) Find the value of F.

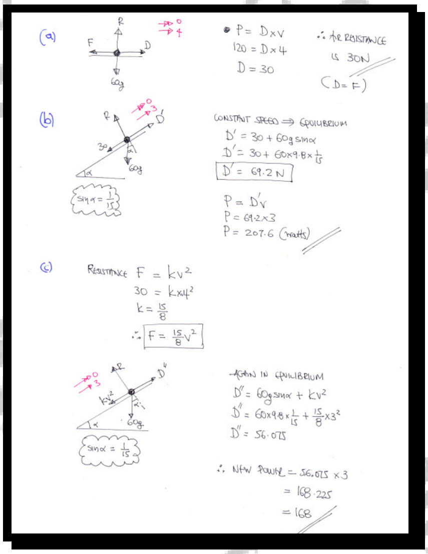
The cyclist next cycles up a straight road inclined at an angle α to the horizontal, where $\sin a = \frac{1}{15}$. The constant frictional force F remains unchanged and the cyclist is now travelling at a constant speed of 3 ms^{-1}.

b) Find the rate at which the cyclist is working.

The model is next refined and it is now assumed that the frictional force F is proportional to the square of the speed of the cyclist.

c) Find an amended value for the power developed by the cyclist in travelling up the same sloped road at the constant speed of 3 ms^{-1}.

$\boxed{F = 30 \text{ N}}$, $\boxed{P = 207.6 \text{ W}}$, $\boxed{P \approx 168 \text{ W}}$



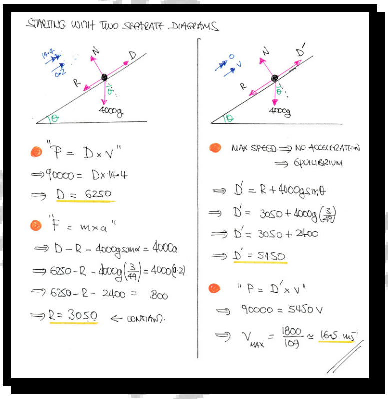
Question 15 (***+)

A lorry, of mass 4000 kg, is travelling up the line of greatest slope of a hill inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{49}$. The engine of the lorry is working at the constant rate of 90 kW.

The motion of the lorry is subject to a **constant** non gravitational resistance.

Determine the greatest speed of the lorry up the hill, given that at some instant during the climb the lorry is accelerating at 0.2 ms^{-2} and its speed is 14.4 ms^{-1}.

EMTLN, $\boxed{|v|_{\text{max}} \approx 16.5 \text{ ms}^{-1}}$



Question 16 (***+)

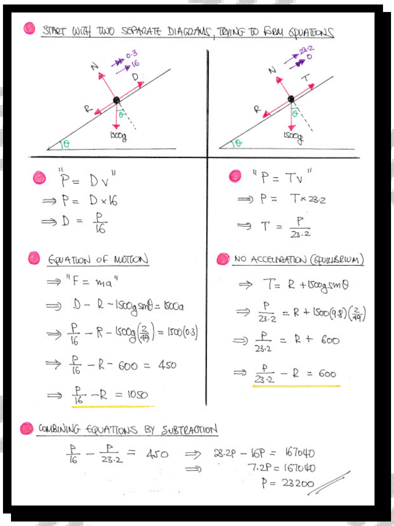
A car, of mass 1500 kg, is travelling up the line of greatest slope of a hill inclined at an angle θ to the horizontal, where $\sin\theta = \frac{2}{49}$. The engine of the car is working at a constant rate of P W.

The motion of the car is subject to a **constant** non gravitational resistance.

At some instant during the climb, the car is accelerating at 0.3 ms^{-2} when its speed is 16 ms^{-1}, reaching a maximum speed of 23.2 ms^{-1}.

Determine the value of P.

$\boxed{P = 23200 \text{ W}}$



Question 17 (***+)

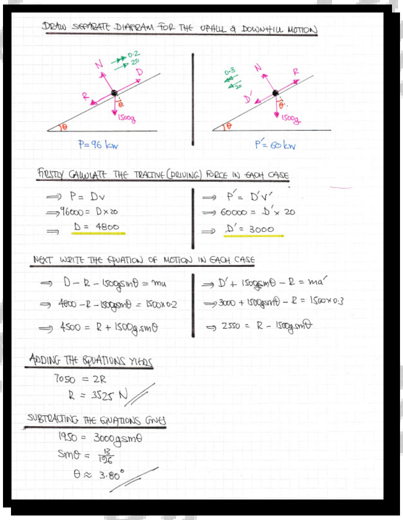
A car, which is modelled as a particle of mass 1500 kg, is travelling on a straight road inclined at an angle θ to the horizontal.

When the engine of the car is working at the constant rate of 96 kW, at an instant when the car is travelling **up** this road with speed 20 ms^{-1} the car is experiencing an acceleration of 0.2 ms^{-2}.

When the engine of the car is working at the constant rate of 60 kW, at an instant when the car is travelling **down** the same road with speed 20 ms^{-1} the car is experiencing an acceleration of 0.3 ms^{-2}.

If the resistance to the motion of the car R, due to non-gravitational forces, has constant magnitude, determine the value of R and the value of θ.

$\boxed{}$, $\boxed{R = 3525 \text{ N}}$, $\boxed{\theta \approx 3.80°}$



Question 18 (***+)

A motorbike, which is modelled as a particle of mass 300 kg, is travelling on a straight road with its engine working at its maximum rate of 54 kW.

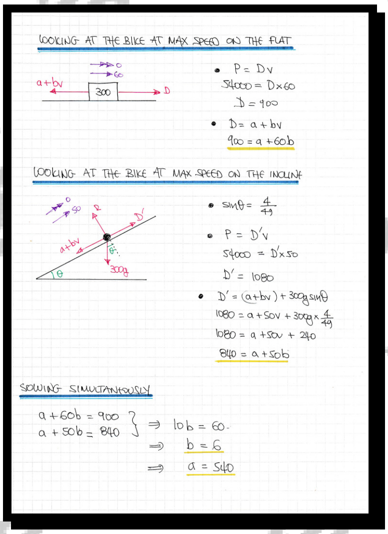
When in motion the motorbike experiences non gravitational resistances of magnitude $a + bv$ N, where v ms^{-1} is its speed.

When travelling on a horizontal stretch of the road, at maximum power, the maximum speed of the motorbike is 60 ms^{-1}.

When travelling, at maximum power, on a stretch of the road inclined at $\arcsin\left(\frac{4}{49}\right)$ to the horizontal, the maximum speed of the motorbike is 50 ms^{-1}.

Determine the acceleration of the motorbike when travelling on a horizontal stretch of the road, at maximum power, at the instant when its speed is 30 ms^{-1}.

$\boxed{}$, $\boxed{a = 3.6 \text{ ms}^{-2}}$



LOOKING AT THE BIKE AT MAX SPEED ON THE FLAT

- $P = Dv$
  $54000 = D \times 60$
  $D = 900$
- $D = a + bv$
  $900 = a + 60b$

LOOKING AT THE BIKE AT MAX SPEED ON THE INCLINE

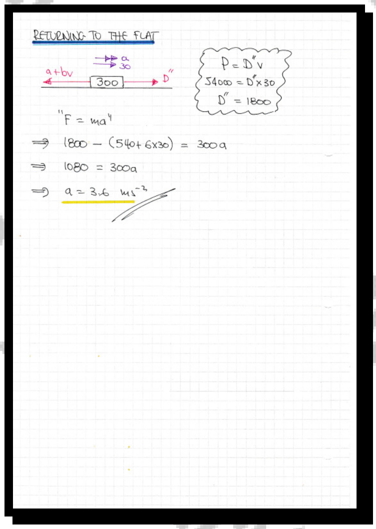
- $\sin\theta = \frac{4}{49}$
- $P = D'v$
  $54000 = D' \times 50$
  $D' = 1080$
- $D' = (a+bv) + 300g\sin\theta$
  $1080 = a + 50v + 300g \times \frac{4}{49}$
  $1080 = a + 50v + 240$
  $840 = a + 50b$

SOLVING SIMULTANEOUSLY

$\left.\begin{array}{l} a + 60b = 900 \\ a + 50b = 840 \end{array}\right\} \Rightarrow 10b = 60$

$\Rightarrow b = 6$

$\Rightarrow a = 540$



RETURNING TO THE FLAT

$P = D''v$
$54000 = D'' \times 30$
$D'' = 1800$

"$F = ma$"

$\Rightarrow 1800 - (540 + 6 \times 30) = 300a$

$\Rightarrow 1080 = 300a$

$\Rightarrow a = 3.6 \text{ ms}^{-2}$

Question 19 (***+)

A car of mass 1300 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{10}$.

The total non gravitational resistance experienced by the car is assumed to be a constant force of magnitude of 400 N. The engine of the car is working at the constant rate of 30 kW.

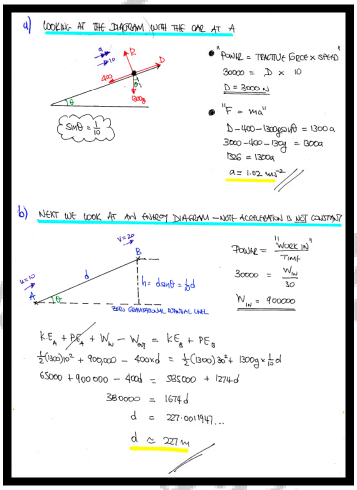
The car is passing through the point A with a speed 10 ms^{-1} and continues to accelerate up the plane, passing through the point B with speed 30 ms^{-1}, 30 s after passing through A.

By modelling the car as a particle, find …

a) … the acceleration of the car at A.

b) ... the distance AB.

$\boxed{}$, $\boxed{a = 1.02 \text{ ms}^{-2}}$, $\boxed{|AB| \approx 227 \text{ m}}$


a) LOOKING AT THE DIAGRAM WITH THE CAR AT A

$\sin\theta = \frac{1}{10}$

- "POWER = TRACTIVE FORCE × SPEED"
  $30000 = D \times 10$
  $D = 3000 \text{ N}$
- "$F = ma$"
  $D - 400 - 1300g\sin\theta = 1300a$
  $3000 - 400 - 130g = 1300a$
  $1326 = 1300a$
  $a = 1.02 \text{ ms}^{-2}$

b) NEXT WE LOOK AT AN ENERGY DIAGRAM — NOTE ACCELERATION IS NOT CONSTANT

POWER = "WORK IN" / TIME

$30000 = \frac{W_{IN}}{30}$

$W_{IN} = 900000$

$K.E_A + \cancel{PE_A} + W_{IN} - W_{OUT} = KE_B + PE_B$

$\frac{1}{2}(1300)10^2 + 900{,}000 - 400 \times d = \frac{1}{2}(1300)30^2 + 1300g \times \frac{1}{10}d$

$65000 + 900000 - 400d = 585000 + 1274d$

$380000 = 1674d$

$d = 227.0011947\ldots$

$d \simeq 227 \text{ m}$

Question 20 (****)

A car of mass 1000 kg is travelling on straight road sections.

The car experiences a constant air resistance when in motion and the car's engine is working at a constant rate, at all times.

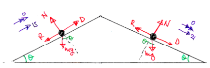
When the car is travelling up a hill inclined at a constant angle $\arcsin\left(\frac{1}{14}\right)$ to the horizontal, its maximum speed is 15 ms^{-1}.

When the car is travelling down the same hill its maximum speed is 21 ms^{-1}.

Calculate the acceleration of the car when it is moving on level ground with a speed of 14 ms^{-1}.

$\boxed{}$, $\boxed{a = 1.05 \text{ ms}^{-2}}$

LOOKING AT TWO DIAGRAMS ON THE HILL



$P = Dv$

$D = \frac{P}{v}$

$m = 1000$

$\theta = \arcsin\frac{1}{14}$

UP THE HILL

$D = R + mg\sin\theta$

$\frac{P}{15} = R + mg\sin\theta$

DOWN THE HILL

$D + mg\sin\theta = R$

$\frac{P}{21} + mg\sin\theta = R$

$\Rightarrow \frac{P}{15} = \left(\frac{P}{21} + mg\sin\theta\right) + mg\sin\theta$

$\Rightarrow \frac{P}{15} - \frac{P}{21} = 2mg\sin\theta$

$\Rightarrow \frac{21P - 15P}{315} = 2 \times 1000 \times 9.8 \times \frac{1}{14}$

$\Rightarrow \frac{6P}{315} = 1400$

$\Rightarrow P = 73500$

FIND THE RESISTANCE NEXT

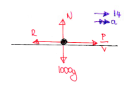
$\Rightarrow R = \frac{P}{21} + mg\sin\theta$

$\Rightarrow R = \frac{73500}{21} + 1000 \times 9.8 \times \frac{1}{14}$

$\Rightarrow R = 3500 + 700$

$\Rightarrow R = 4200$

FINALLY A DIAGRAM ON LEVEL GROUND



$P = 73500$
$R = 4200$

$\Rightarrow$ "$F = ma$"

$\Rightarrow \frac{P}{V} - R = ma$

$\Rightarrow \frac{73500}{14} - 4200 = 1000a$

$\Rightarrow 1050 = 1000a$

$\Rightarrow a = 1.05 \text{ ms}^{-2}$

Question 21 (****)

A man and his bike are modelled a single particle of combined mass 90 kg.

When the man is working at the constant rate of 560 joules per second he achieves a constant cycling speed of 3.2 ms^{-1} on horizontal ground.

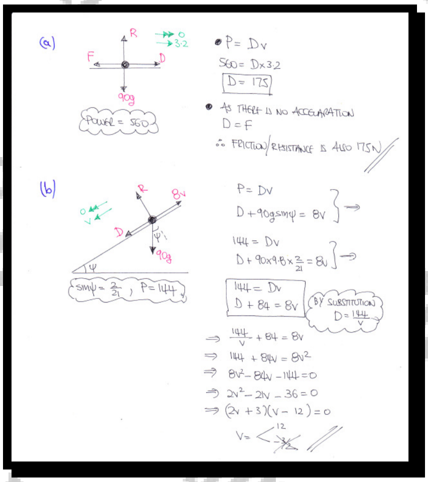
a) Find the magnitude of the resistance experienced by the cyclist, assumed constant throughout his motion.

The man next cycles down a straight road, which lies on the line of greatest slope of a plane inclined at an angle ψ to the horizontal, where $\sin\psi = \dfrac{2}{21}$.

He achieves a constant speed of $v \text{ ms}^{-1}$ when his power is 144 W. The total non gravitational resistance experienced by the cyclist is now assumed to be $8v$ N.

b) Find the value of v.

$\boxed{\text{Resistance} = 175 \text{ N}}$, $\boxed{v = 12}$



Question 22 (****)

A track of mass 6000 kg, when travelling with speed v ms^{-1} it experiences air resistance directly proportional to v^2.

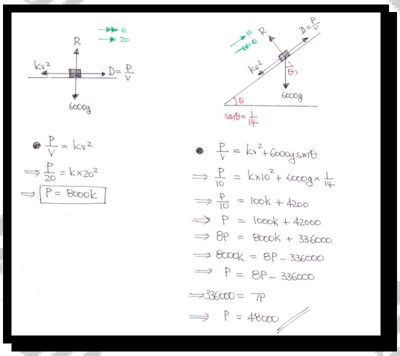
When the track is travelling at constant speed of 20 ms^{-1} along a straight horizontal road, the engine of the track is working at a rate of P W.

When the track is travelling at constant speed of 10 ms^{-1} along a straight horizontal road, up a line of greatest slope of a hill which is inclined at θ to the horizontal, the engine of the track is working at a rate of P W.

It is further given that $\sin\theta = \frac{1}{14}$ and the air resistance to the motion of the track when it travels up the hill, is still directly proportional to v^2.

Determine the value of P.

$\boxed{P = 48000 \text{ W}}$



Question 23 (****)

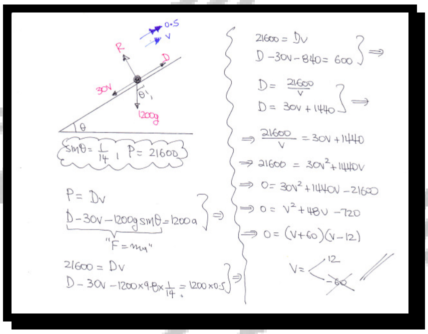
A car of mass 1200 kg is travelling on a straight road which lies on the line of greatest slope of a plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{14}$.

At a given instant the engine of the car is working at the rate of 21.6 kW, the car having a speed $v\text{ ms}^{-1}$, accelerating up the plane at 0.5 ms^{-2}.

The total non gravitational resistance experienced by the car is assumed to be $30v$ N.

By modelling the car as a particle, find the value of v.

$\boxed{v = 12}$



Question 24 (****+)

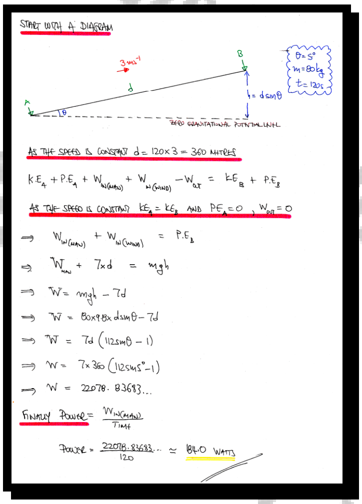
A man of mass 80 kg, is jogging up a hill with constant speed of 3 ms^{-1}, in a straight line inclined at $5°$ to the horizontal.

The man experiences wind assistance during his jog which is modelled as a constant force of magnitude 7 N.

The man is working at a constant rate of P W and takes 2 minutes to jog up the hill.

Assuming no other resistance on the motion of the man when it jogs up the hill, except gravitational resistances, calculate the value of P.

$\boxed{P = 184 \text{ W}}$


START WITH A DIAGRAM

AS THE SPEED IS CONSTANT $d = 120 \times 3 = 360$ METRES

$K.E_A + P.E_A + W_{IN(MAN)} + W_{IN(WIND)} - W_{OUT} = KE_B + P.E_B$

AS THE SPEED IS CONSTANT $KE_A = KE_B$ AND $PE_A = 0$, $W_{OUT} = 0$

$\Rightarrow W_{IN(MAN)} + W_{IN(WIND)} = P.E_B$

$\Rightarrow W_{MAN} + 7 \times d = mgh$

$\Rightarrow W = mgh - 7d$

$\Rightarrow W = 80 \times 9.8 \times d\sin\theta - 7d$

$\Rightarrow W = 7d(112\sin\theta - 1)$

$\Rightarrow W = 7 \times 360(112\sin 5° - 1)$

$\Rightarrow W = 22078.83683\ldots$

FINALLY POWER $= \frac{W_{IN(MAN)}}{TIME}$

POWER $= \frac{22078.83683\ldots}{120} \simeq 184.0$ WATTS