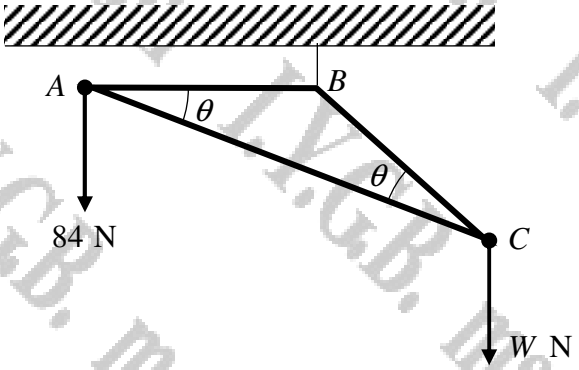
FRAMEWORKS
&
MULTISTRUCTURES

FRAMEWORKS

Question 1 (***)



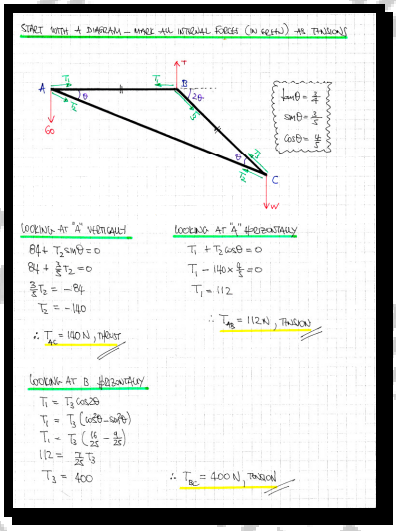
A framework consists of 3 light rigid rods AB, BC and CA smoothly joined at their ends, forming a triangle.

It is also known that $\angle BAC = \angle BCA = \theta$, where $\theta = \arctan\left(\frac{3}{4}\right)$.

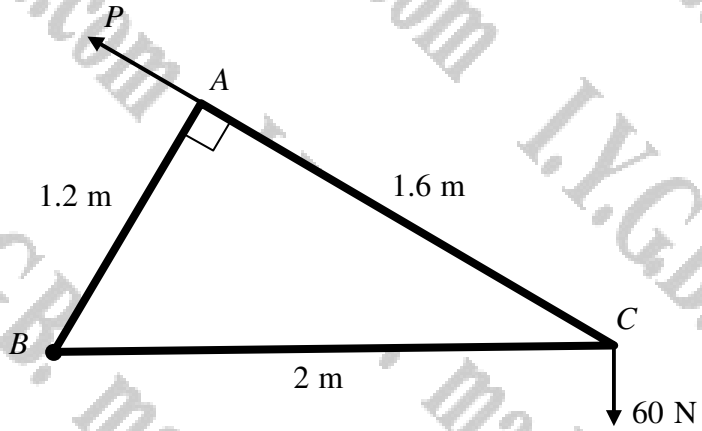
The framework is suspended below a horizontal ceiling by a vertical inextensible wire attached to B. A mass of weight 84 N is placed at A and another mass of weight W N is placed at B, so that AB remains horizontal, as shown in the figure above.

Find the magnitude of the internal forces acting on each of the 3 rods, classifying them as tension or thrust.

$\boxed{}$, $\boxed{R_{AC} = 140 \text{ N, thrust}}$, $\boxed{R_{AB} = 112 \text{ N, tension}}$, $\boxed{R_{BC} = 400 \text{ N, tension}}$



Question 2 (***)



A framework ABC consists of three light pin jointed rods, freely hinged to a rigid support at B. The framework supports a weight of 60 N at C and is kept in equilibrium with BC horizontal by a force P acting in the direction CA. The lengths of the three rods, in m, are marked on the diagram, and $\angle BAC = 90°$, as shown in the figure above.

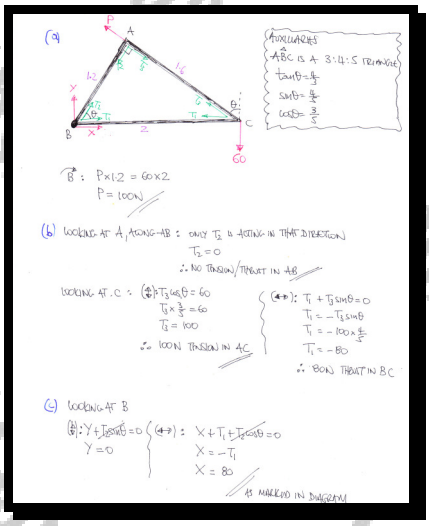
a) Find the value of P.

b) Calculate the magnitude of each of the internal forces in the three rods, further classifying each of them as a tension or as a thrust.

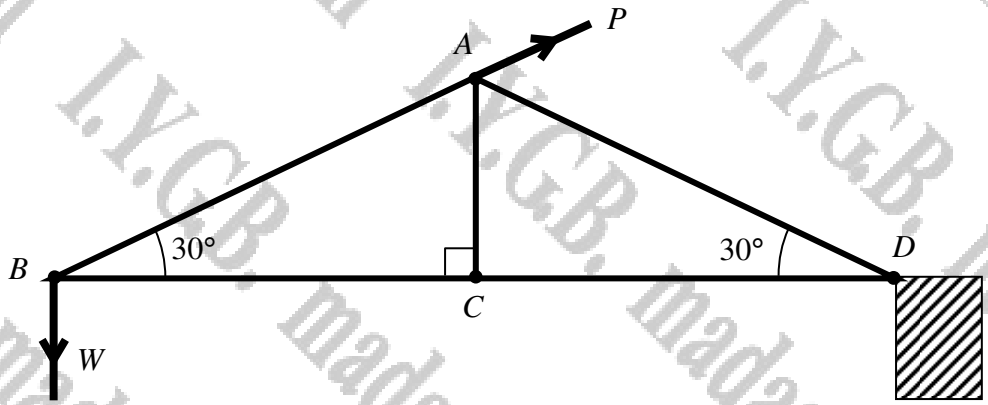
c) Determine the magnitude and direction of the reaction of the hinge onto the framework at B.

$\boxed{P = 100 \text{ N}}$, $\boxed{T_{AB} = 0}$, $\boxed{T_{AC} = 100 \text{ N, tension}}$, $\boxed{T_{BC} = 80 \text{ N, thrust}}$,

$\boxed{R = 80 \text{ N, in } BC}$



Question 3 (***)



A light rigid framework consists of 5 light pin jointed rods, AB, AC, AD, BC and BD, where $|AB|=|AD|$, $|BC|=|BD|$, $\angle ABC=\angle ADC=30°$ and $\angle ACD=90°$, as shown in the figure above.

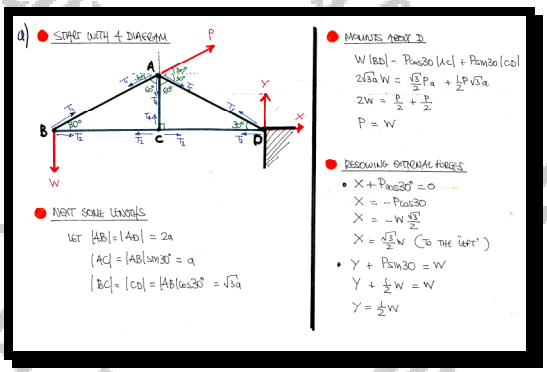
The framework is freely hinged at D and a weight W is supported at B.

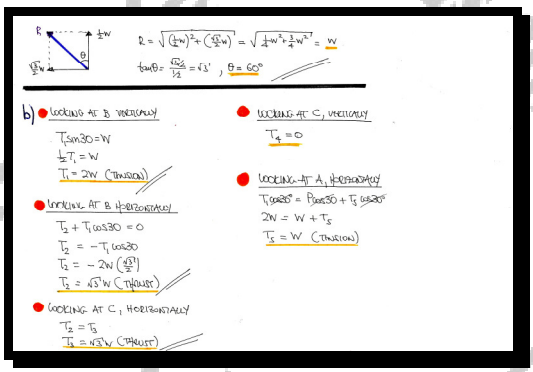
The framework is supported in equilibrium, with BCD horizontal, by a force P which acts at B in the direction of BA

a) Find the magnitude, in terms of W, and the direction of the reaction force acting on the framework at D.

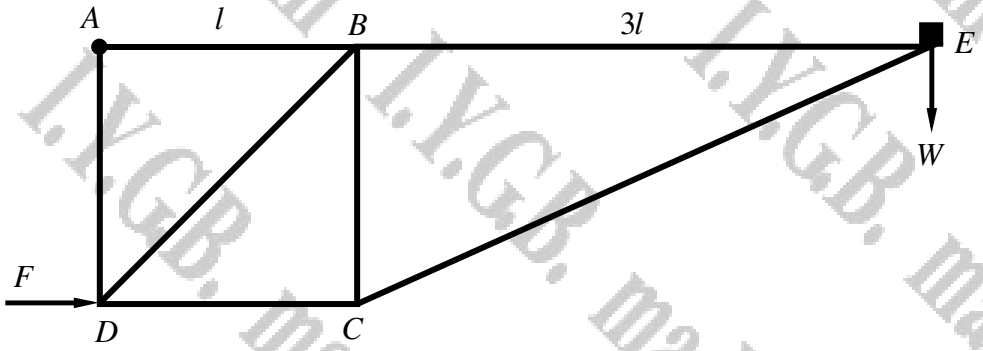
b) Determine, in terms of W, the magnitude of the internal force acting on each of the rods, classifying them where applicable as tension or thrust.

$\boxed{}$, $\boxed{R=W,\text{ at }60°\text{ to the upward vertical}}$, $\boxed{R_{AB}=2W,\text{ Tension}}$, $\boxed{R_{AC}=0}$, $\boxed{R_{AD}=W,\text{ Tension}}$, $\boxed{R_{BC}=R_{CD}=\sqrt{3}W,\text{ Thrust}}$





Question 5 (***)



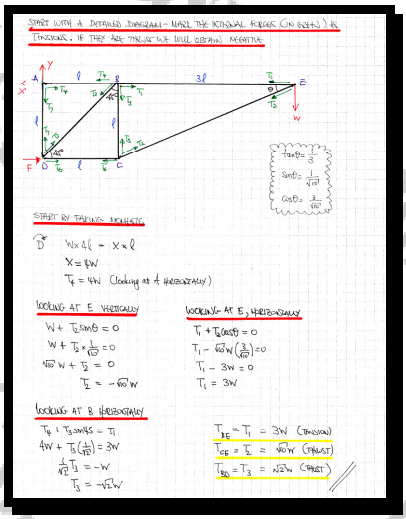
A framework consists of 7 light rigid rods smoothly joined between them as shown in the figure above.

The rods AB, BC, CD and DA form a square of side length l. The rod BE is of length $3l$, so that ABE is a straight line. Two more rods, BD and CE complete a rigid structure smoothly hinged at a fixed point A.

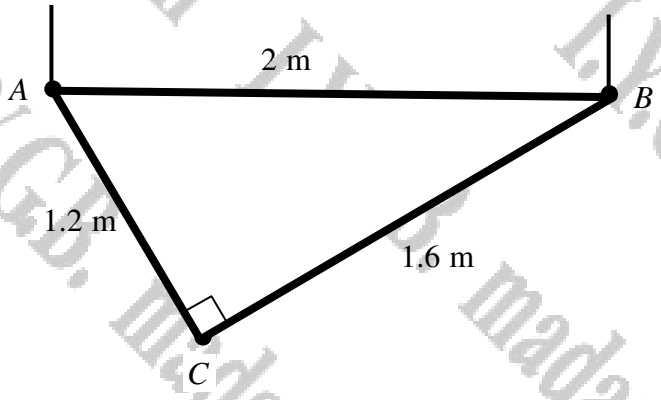
When a mass of weight W is placed at E, it is required that ABE remains in a horizontal position. This is achieved by an external force F acting at D, in the direction DC.

Determine, in terms of W, the magnitude of the forces acting on BE, CE and BD, further classifying them as tension or thrust.

[MA56], $\boxed{T_{BE}=3W,\text{ tension}}$, $\boxed{T_{CE}=\sqrt{10}\,W,\text{ thrust}}$, $\boxed{T_{BD}=\sqrt{2}\,W,\text{ thrust}}$



Question 6 (***)



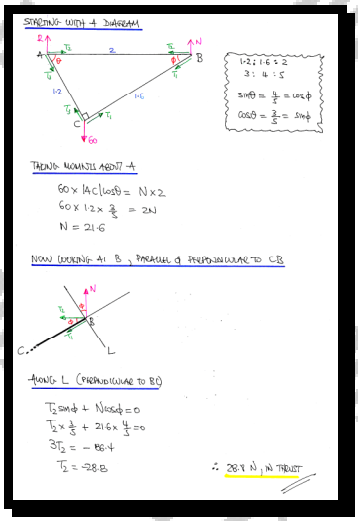
The figure above shows a framework ABC consists of three light pin jointed rods AB, BC and AC, freely hinged to a rigid support at B.

The framework supports a weight of 60 N at C and is kept in equilibrium with AB horizontal by two vertical wires at A and B.

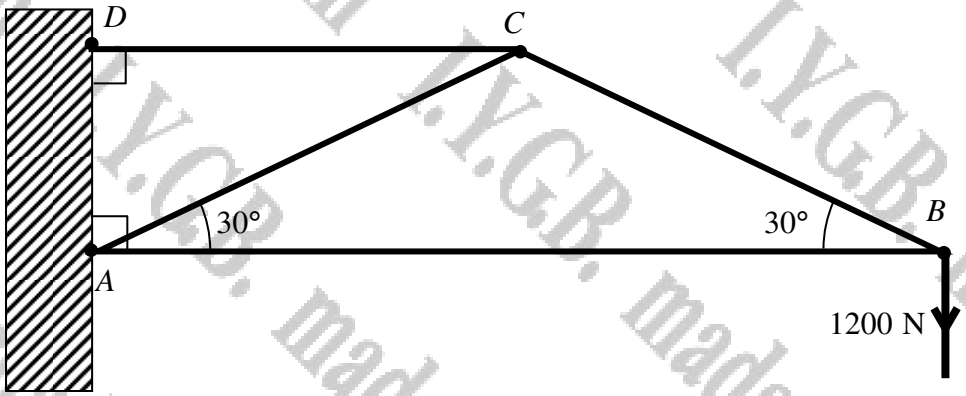
The lengths of the three rods, in metres, are marked on the diagram, and $\angle BAC = 90°$.

Determine the magnitude force acting at AB.

[redacted], $T_{AB} = 28.8$ N, thrust



Question 7 (***+)



A light rigid framework consists of 4 light pin jointed rods, AB, AC, BC and CD, where $|AC|=|BC|$, $\measuredangle CAB=\measuredangle ABC=30°$ and $\measuredangle ADC=\measuredangle BAD=90°$, as shown in the figure above.

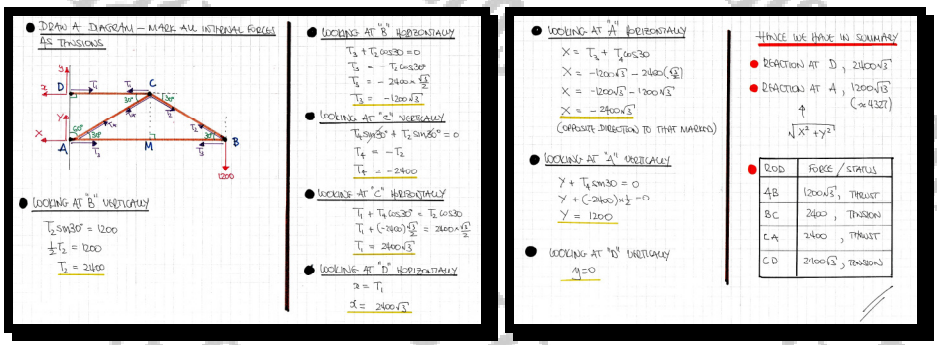
The framework is freely hinged at the points A and D and a weight of 1200 N is supported at B as shown the figure above.

Find the magnitude of the reaction forces acting on the framework at A and D, and the magnitudes of all the internal forces acting on each of the four rods, classifying them as tension or thrust.

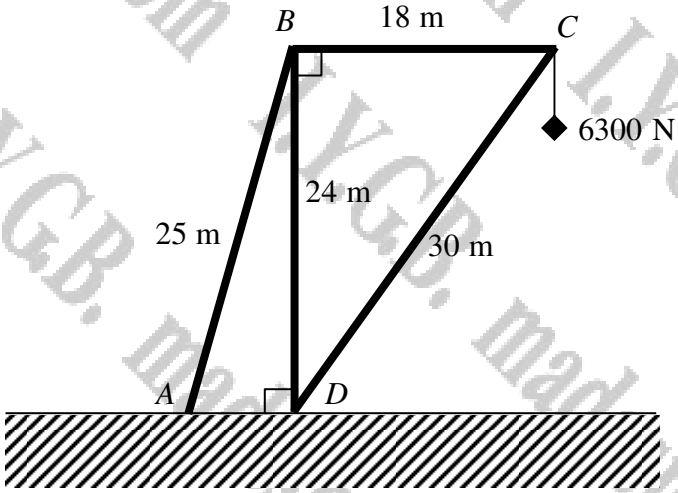
$\boxed{}$, $\boxed{R_A=1200\sqrt{13}\approx 4327\text{ N}}$,

$\boxed{R_D=2400\sqrt{3}\approx 4157\text{ N}}$ $\boxed{R_{AB}=1200\sqrt{3}\approx 2078\text{ N, thrust}}$, $\boxed{R_{BC}=2400\text{, tension}}$,

$\boxed{R_{CD}=2400\sqrt{3}\approx 4157\text{ N, tension}}$, $\boxed{R_{CA}=2400\text{, thrust}}$



Question 8 (***+)



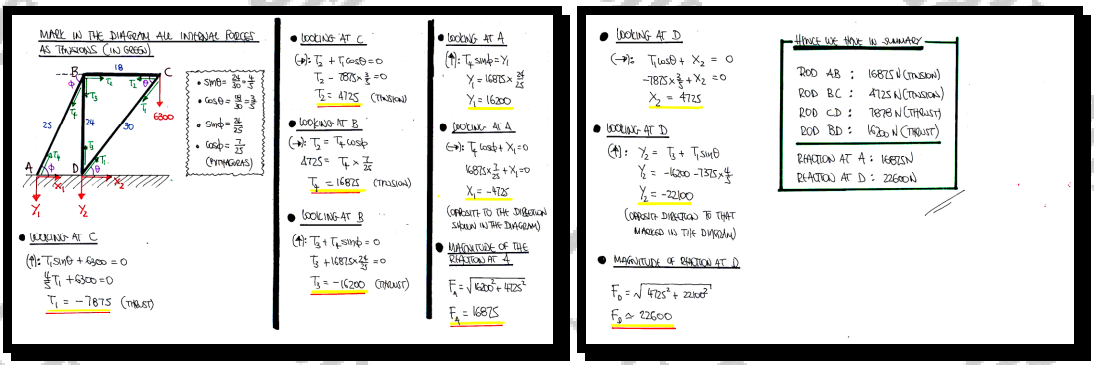
A ground crane is modelled as a light rigid framework consisting of 4 light pin jointed rods, AB, BD, BC and CD, where $|AB| = 25\,\text{m}$, $|BD| = 24\,\text{m}$, $|BC| = 18\,\text{m}$ and $|CD| = 30\,\text{m}$, as shown in the figure above.

The crane is attached to the ground at the points A and D and a weight of 6300 N is suspended from C, as shown the figure above.

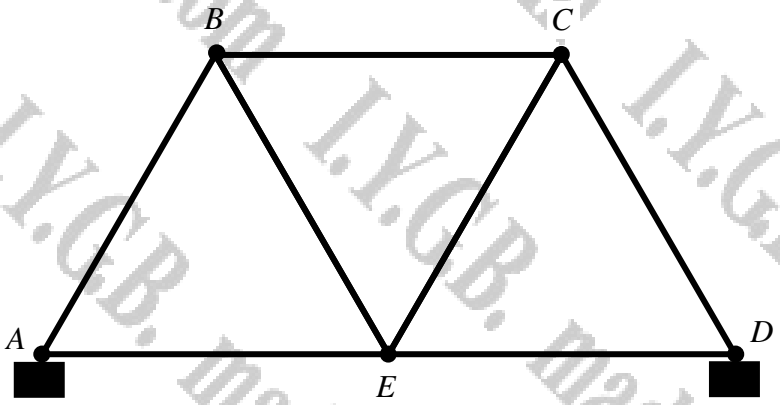
Find the magnitude of the reaction forces acting on the framework at A and D, and the magnitudes of all the internal forces acting on each of the four rods, classifying them as tension or thrust.

$\boxed{}$, $\boxed{R_A = 16875\text{ N}}$, $\boxed{R_D \approx 22600\text{ N}}$ $\boxed{R_{AB} = 16875\text{ N, tension}}$,

$\boxed{R_{BC} = 4725\text{, tension}}$, $\boxed{R_{CD} = 7875\text{ N, thrust}}$, $\boxed{R_{BD} = 16200\text{, thrust}}$



Question 9 **(***+)**



A rigid framework $ABCDE$ consists of seven identical light pin jointed rods as shown in the figure above. The framework rests at two fixed supports at the points A and D.

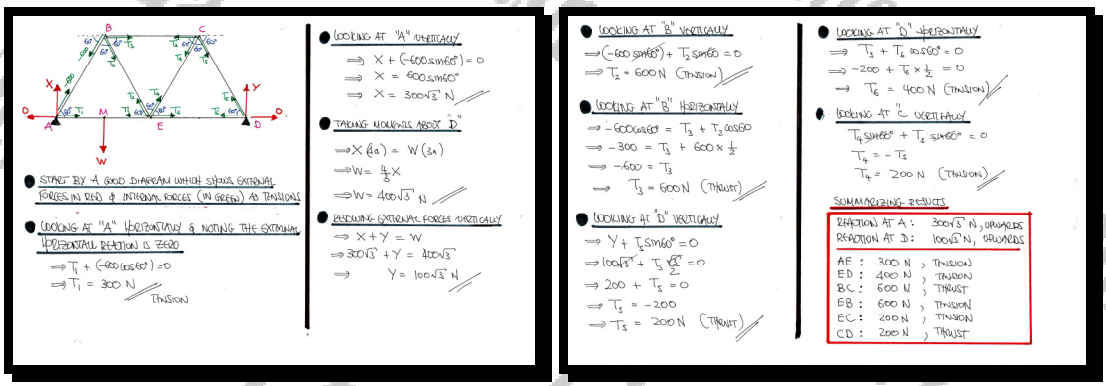
When the framework supports a weight of W N at the midpoint AE, there is 600 N thrust on AB.

Determine the magnitude of each of the reaction forces at A and at C and the magnitude of each of the internal forces in the rods EA, EB, EC, ED, CB and CD, further classifying each of them as a tension or as a thrust.

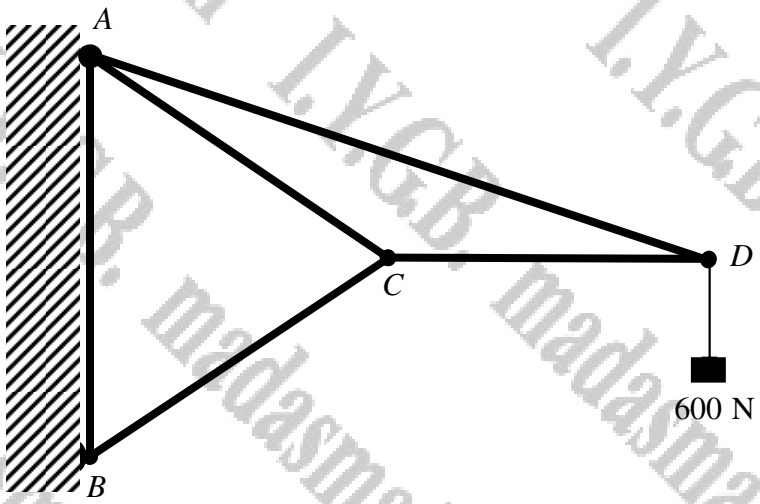
$\boxed{\quad}$, $\boxed{R_A = 300\sqrt{3}\text{ N}}$, $\boxed{R_D = 1050\text{ N}}$, $\boxed{T_{EA} = 300\text{ N, tension}}$,

$\boxed{T_{EB} = 600\text{ N, thrust}}$, $\boxed{T_{EC} = 200\text{ N, tension}}$, $\boxed{T_{ED} = 400\text{ N, tension}}$,

$\boxed{T_{CB} = 600\text{ N, thrust}}$, $\boxed{T_{CD} = 200\text{ N, thrust}}$



Question 10 **(***+)**

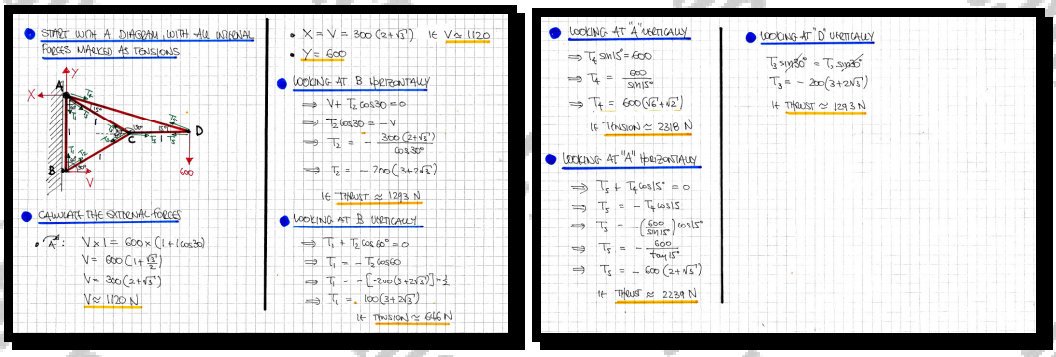


The figure above shows a light rigid framework $ABCD$, attached to a vertical wall, which consists of 5 light pin jointed coplanar rods, AB, BC, AC, CD and AD, where $|AB|=|BC|=|AC|=|CD|=1$ m.

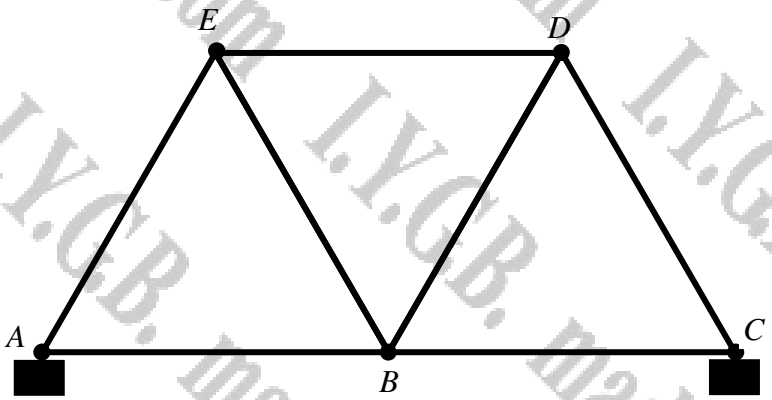
The framework is freely hinged at A, where A is on the vertical wall, and touches a smooth support at B, where B is also on the same wall, vertically below A as shown in the figure above. The framework supports a weight of 600 N at D, and remains in equilibrium, with CD horizontal, with the plane $ABCD$ perpendicular to the wall.

Find, in any order, the magnitude of the horizontal and vertical components of the reaction forces acting on the framework at A and B, and the magnitudes the internal forces acting on each of the 5 rods, classifying them as tensions or thrusts.

$\boxed{\text{MADAS}}$, $\boxed{\text{horizontally at } A \text{ and } B,\ \approx 1120 \text{ N}}$, $\boxed{\text{vertically at } A,\ 600 \text{ N}}$,
$\boxed{R_{AB} \approx 646 \text{ N},\ \text{Tension}}$, $\boxed{R_{AC}=R_{BC} \approx 1293 \text{ N},\ \text{Thrust}}$, $\boxed{R_{CD} \approx 2239 \text{ N},\ \text{Thrust}}$,
$\boxed{R_{AD}=2318 \text{ N},\ \text{Tension}}$



Question 11 **(***+)**



A bridge design is modelled by a framework $ABCDE$ consisting of seven identical light pin jointed rods as shown in the figure above.

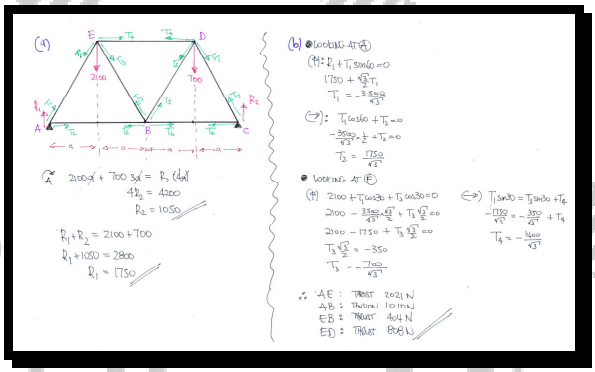
The framework rests at two concrete plinths at A and C.

Two weights of 2100 N and 700 N are placed at E and D, respectively.

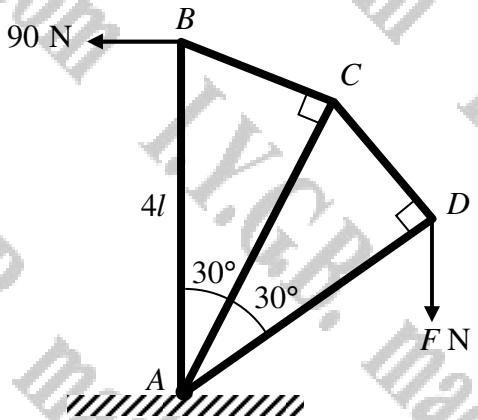
a) Determine the magnitude of the reaction force at A and at C.

b) Calculate the magnitude of each of the internal forces in the rods AE, AB, EB and ED, further classifying each of them as a tension or as a thrust.

$\boxed{R_A = 1750 \text{ N}}$, $\boxed{R_B = 1050 \text{ N}}$, $\boxed{T_{AE} \approx 2021 \text{ N, thrust}}$, $\boxed{T_{AB} \approx 1010 \text{ N, tension}}$,
$\boxed{T_{EB} \approx 404 \text{ N, thrust}}$, $\boxed{T_{ED} \approx 808 \text{ N, thrust}}$



Question 12 **(***+)**



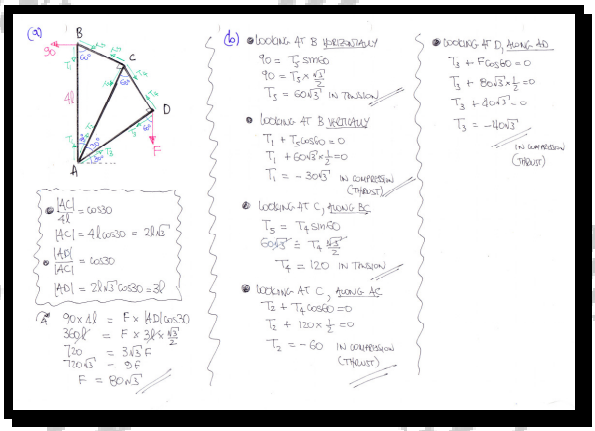
A framework $ABCD$, consists of five light pin jointed rods, freely hinged to a rigid support at A. The framework supports a weight of F N at D and is kept in equilibrium with AB vertical by a horizontal force of 90 N as shown in the figure above. It is further given that the length of the rod AB is $4l$, $\measuredangle ACB = \measuredangle ADC = 90°$ and $\measuredangle BAC = \measuredangle CAD = 30°$.

a) Find the exact value of F.

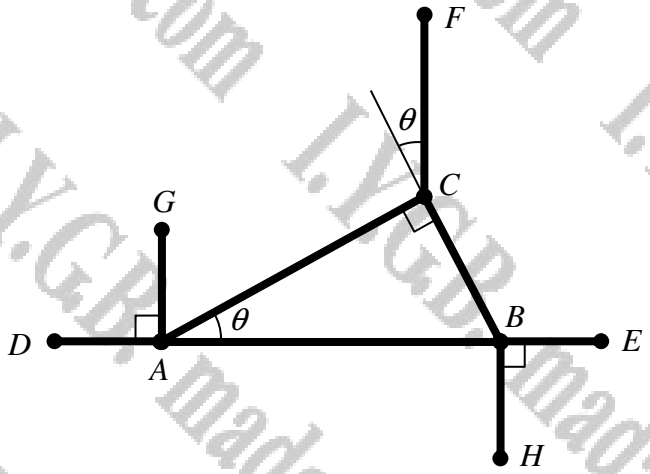
b) Calculate, in exact form where appropriate, the internal forces in the five rods, further classifying each of them as a tension or as a thrust.

$\boxed{F = 80\sqrt{3}\text{ N}}$, $\boxed{T_{AB} = 30\sqrt{3}\text{ N, thrust}}$, $\boxed{T_{AC} = 60\text{ N, thrust}}$, $\boxed{T_{AD} = 40\sqrt{3}\text{ N, thrust}}$,

$\boxed{T_{BC} = 60\sqrt{3}\text{ N, tension}}$, $\boxed{T_{CD} = 120\text{ N, tension}}$



Question 13 **(***+)**



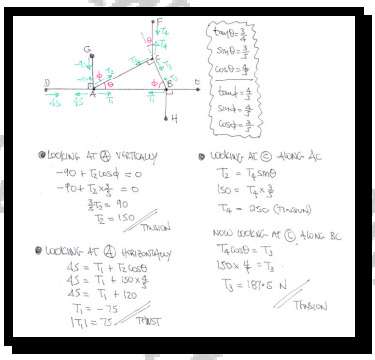
The figure above shows part of a framework of several light rigid rods, freely pin jointed at their ends. All the rods jointed at A, B and C are shown in the figure but only some of the rods jointed at D, E, F, G and H are shown.

The joints D, A, B and E lie in a straight line, $\angle DAG = \angle EBH = \angle ACB = 90°$, and $\angle BAC = \theta$, where $\tan\theta = \frac{3}{4}$. The rod FC is also inclined to BC at θ, as shown in the figure.

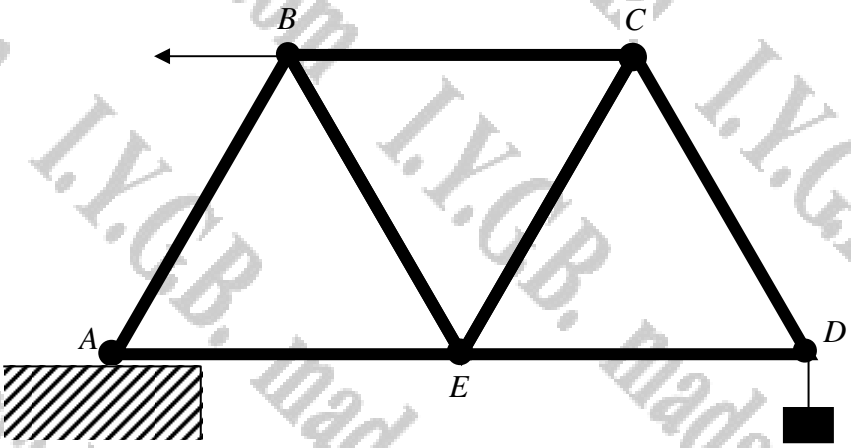
There is a tension of 45 N in AD and a thrust of 90 N in AG.

Calculate, the internal forces in AB, BC and AC, further classifying each of them as a tension or as a thrust.

$\boxed{T_{AB} = 75 \text{ N, thrust}}$, $\boxed{T_{AC} = 150 \text{ N, tension}}$, $\boxed{T_{BC} = 187.5 \text{ N, tension}}$,



Question 14 **(***+)**

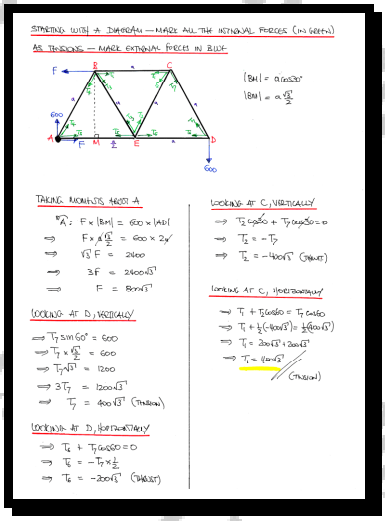


A rigid framework $ABCDE$ consists of seven identical light pin jointed rods as shown in the figure above. The framework rests on a fixed support at the point A.

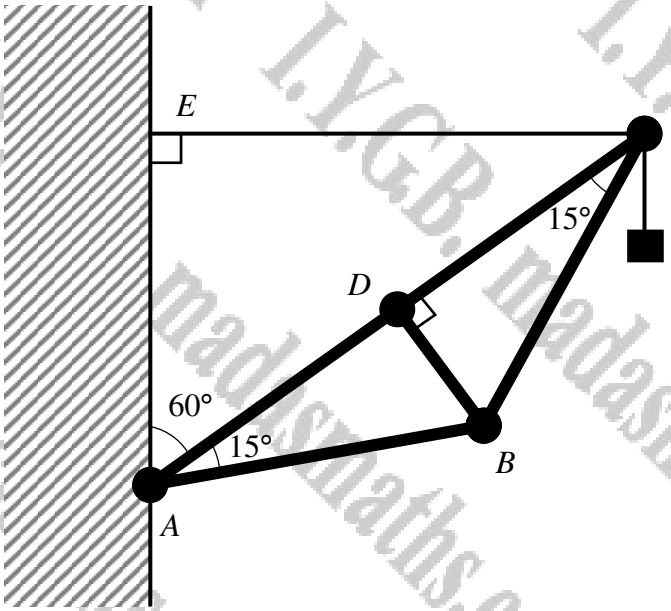
A weight of 600 N is suspended from D, and there is an external force acting at B acting in the direction CB.

Determine the magnitude of internal force acting in the rod BC.

, $\boxed{T_{BC}=400\sqrt{3}\text{ N, tension}}$



Question 15 (****)



The figure above shows a crane, modelled as a framework of light rigid rods, freely pin jointed at their ends.

The rods are jointed at A, B, C and D.

The framework is freely hinged to a rigid support on a vertical wall at A.

A horizontal cable EC has its end E attached to the same above mentioned vertical wall, with E directly above A.

The points A, D and C lie in a straight line such that $|AC| = 30$ m, $\measuredangle EAC = 60°$, $\measuredangle DAB = \measuredangle DCB = 15°$ and $\measuredangle AEC = \measuredangle CDB = 90°$.

There is a load of 36000 N hanging freely from C.

Calculate, the internal forces in BC and AC.

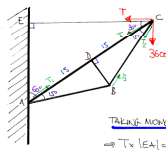
$\boxed{}$, $\boxed{R_{BC} = 0}$, $\boxed{R_{AC}\ 72000\,\text{N},\ \text{Thrust}}$

[solution overleaf]

Created by T. Madas

STARTING WITH A DETAILED DIAGRAM – MAKE INTERNAL FORCES (IN GREEN)

AS TENSIONS



TAKING MOMENTS (ENTIRE STRUCTURE) ABOUT "A"

$\Rightarrow T_2 \times |EA| = 36000 \times |EC|$

$\Rightarrow T = \dfrac{36000 \times |EC|}{|EA|}$

$\Rightarrow T = \dfrac{36000 \times 20\sin 30}{20\cos 30}$

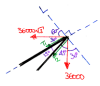
$\Rightarrow T = 36000\tan 30°$

NEXT LOOKING AT C, VERTICALLY AND HORIZONTALLY

$(\uparrow)$: $T_1 \sin 30 + T_2 \sin 45 + 36000 = 0 \quad -I$

$(\rightarrow)$: $T_1 \cos 30 + T_2 \cos 45 = 0 \quad -II$

AS THERE ARE NOT THAT URGENT TO SOLVE IMMEDIATELY POINT C, AND RESOLVE PARALLEL & PERPENDICULAR TO AC



ALONG "L"

$36000\sqrt{3}\cos 60° = T_2 \sin 45 + 36000\cos 30$

$18000\sqrt{3} = T_2 \sin 45 + 18000\sqrt{3}$

$T_2 = 0$

NOW RETURNING TO ANY OF THE TWO EQUATIONS

$T_1 \sin 30 + T_2 \sin 45 + 36000 = 0$

$\frac{1}{2} T_1 = -36000$

$T_1 = -72000 \quad$ (THRUST)

$\therefore$ INTERNAL FORCE IN BC IS ZERO

INTERNAL FORCE IN AC IS 72000 (THRUST)

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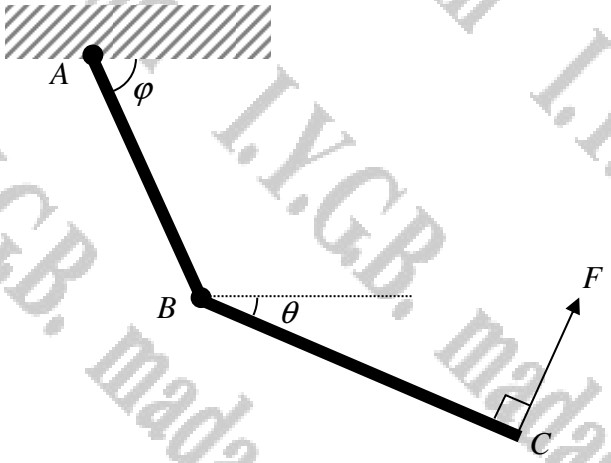
MULTISTRUCTURES

Question 1 (***)



Two identical uniform rigid rods AB and BC, each of weight 100 N, are freely joined at B and lie in the same vertical plane.

The rod AB is freely joined at A, a fixed point on a horizontal ceiling. The system is held in equilibrium by a force F N acting at C, in a perpendicular direction to BC, as shown in the figure above.

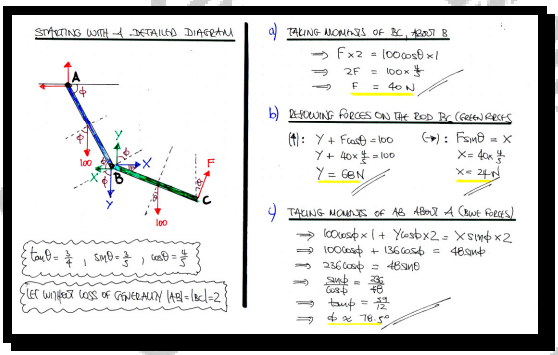
AB and BC form angles φ and θ to the horizontal, respectively, where $\tan\theta = \frac{3}{4}$.

a) Find the value of F.

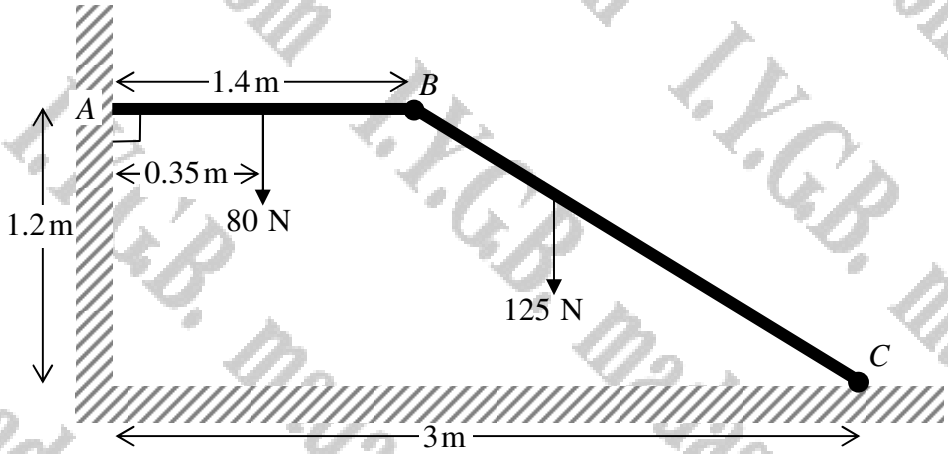
b) Calculate the magnitudes of the horizontal and vertical reaction forces, acting on BC at B.

c) Determine, in degrees, the size of the angle φ.

$\boxed{\quad}$, $\boxed{F = 40}$, $\boxed{R_{\rightarrow} = 24 \text{ N}}$, $\boxed{R_{\uparrow} = 68 \text{ N}}$, $\boxed{\varphi \approx 78.5^\circ}$



Question 2 (*+)**



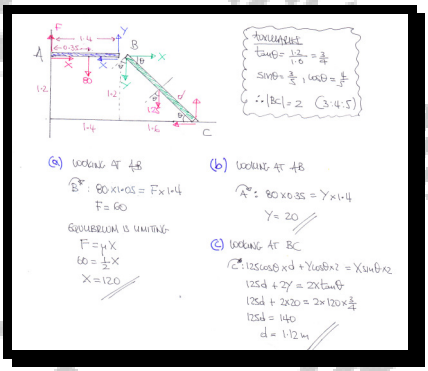
The figure above shows two **non uniform** rigid rods AB and BC, of respective weights 80 N and 125 N, freely joined at B. The rod BC is also freely joined at C, a fixed point on level horizontal ground, which lies at a distance of 3 m from a rough vertical wall. The rod AB has length 1.4 m and rest in a horizontal position in **limiting equilibrium** with its end A against the rough wall. The distance of A from the ground is 1.2 m and the coefficient of friction between the wall and AB is 0.5. The position of the centre of mass of AB is 0.35 m from A. The two rods lies in the same vertical plane.

a) Calculate the magnitude of the horizontal reaction, acting on AB at B.

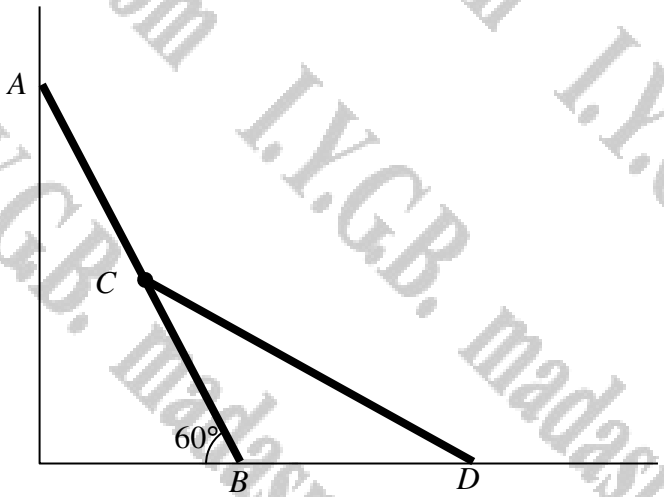
b) Calculate the magnitude of the vertical reaction, acting on AB at B.

c) Determine the distance of the position of the centre of mass of BC from C.

$\boxed{R_{\rightarrow}=120\text{ N}}$, $\boxed{R_{\uparrow}=20\text{ N}}$, $\boxed{d=1.12\text{ m}}$

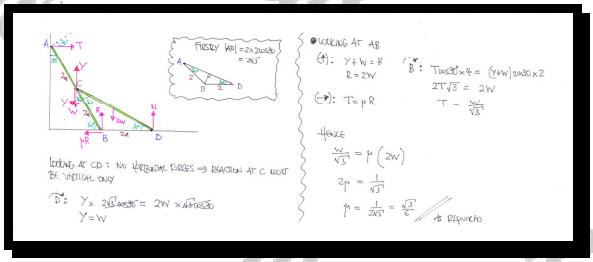


Question 3 (****)



A uniform rod AB, of weight W and length $4a$, rests in equilibrium at $60°$ to the horizontal, with A against a wall and B on horizontal ground. Another uniform rod CD, of weight $2W$, is freely joined with its end C at the midpoint of AB and D on horizontal ground. The two rods lie in the same vertical plane so that $BD = 2a$.

The contacts at A and D are smooth but the contact at B is rough.

Given that the system of the two rods is in limiting equilibrium, determine in exact form the coefficient of friction between AB and the ground at B.

$$\boxed{\mu = \frac{1}{2\sqrt{3}} = \frac{1}{6}\sqrt{3}}$$