

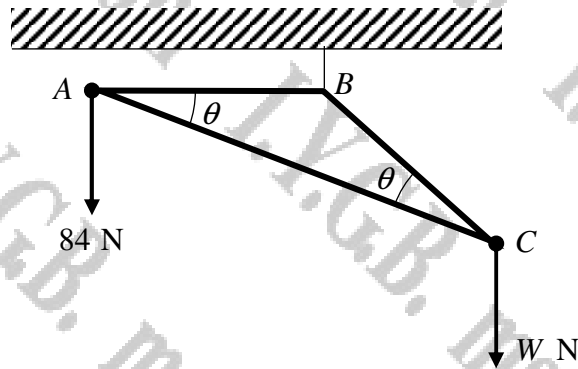
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FRAMEWORKS & MULTISTRUCTURES

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FRAMEWORKS

Question 1 (***)



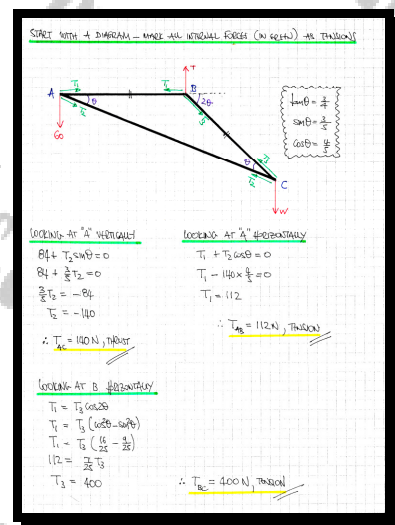
A framework consists of 3 light rigid rods AB , BC and CA smoothly joined at their ends, forming a triangle.

It is also known that $\angle BAC = \angle BCA = \theta$, where $\theta = \arctan\left(\frac{3}{4}\right)$.

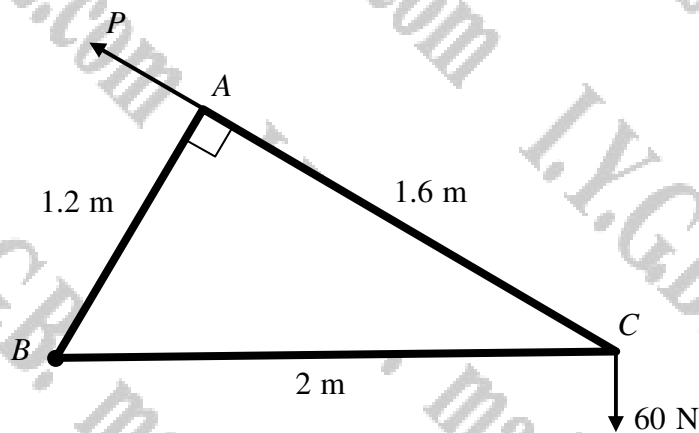
The framework is suspended below a horizontal ceiling by a vertical inextensible wire attached to B . A mass of weight 84 N is placed at A and another mass of weight $W\text{ N}$ is placed at C , so that AB remains horizontal, as shown in the figure above.

Find the magnitude of the internal forces acting on each of the 3 rods, classifying them as tension or thrust.

, $R_{AC} = 140\text{ N}$, thrust, $R_{AB} = 112\text{ N}$, tension, $R_{BC} = 400\text{ N}$, tension



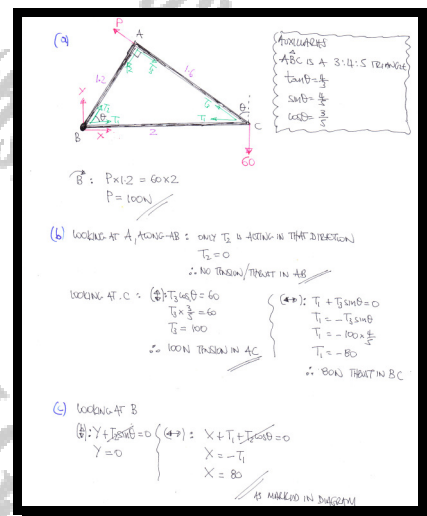
Question 2 (***)



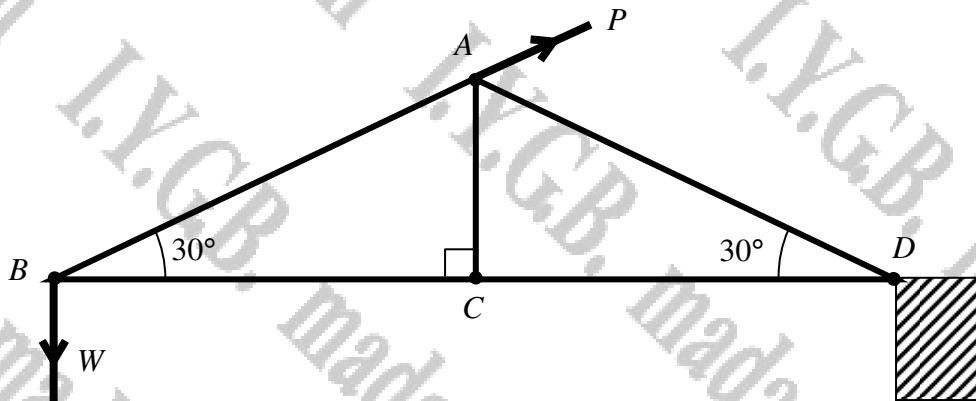
A framework ABC consists of three light pin jointed rods, freely hinged to a rigid support at B . The framework supports a weight of 60 N at C and is kept in equilibrium with BC horizontal by a force P acting in the direction CA . The lengths of the three rods, in m , are marked on the diagram, and $\angle BAC = 90^\circ$, as shown in the figure above.

- Find the value of P .
- Calculate the magnitude of each of the internal forces in the three rods, further classifying each of them as a tension or as a thrust.
- Determine the magnitude and direction of the reaction of the hinge onto the framework at B .

$$P = 100\text{ N}, T_{AB} = 0, T_{AC} = 100\text{ N, tension}, T_{BC} = 80\text{ N, thrust}, R = 80\text{ N, in } BC$$



Question 3 (***)



A light rigid framework consists of 5 light pin jointed rods, AB , AC , AD , BC and BD , where $|AB| = |AD|$, $|BC| = |BD|$, $\angle ABC = \angle ADC = 30^\circ$ and $\angle ACD = 90^\circ$, as shown in the figure above.

The framework is freely hinged at D and a weight W is supported at B .

The framework is supported in equilibrium, with BCD horizontal, by a force P which acts at B in the direction of BA .

- Find the magnitude, in terms of W , and the direction of the reaction force acting on the framework at D .
- Determine, in terms of W , the magnitude of the internal force acting on each of the rods, classifying them where applicable as tension or thrust.

$\boxed{}$, $\boxed{R = W, \text{ at } 60^\circ \text{ to the upward vertical}}$, $\boxed{R_{AB} = 2W, \text{ Tension}}$, $\boxed{R_{AC} = 0}$,
 $\boxed{R_{AD} = W, \text{ Tension}}$, $\boxed{R_{BC} = R_{CD} = \sqrt{3}W, \text{ Thrust}}$

a) START WITH A DIAGRAM

NEXT SOME UNIONS

Let $|AB| = |AD| = 2a$
 $|AC| = |AB| \sin 30^\circ = a$
 $|BC| = |CD| = |AB| \cos 30^\circ = \sqrt{3}a$

WORKING AT D: VERTICALLY

$W|AB| = P \cos 30^\circ |AC| + P \sin 30^\circ |CD|$
 $2\sqrt{3}a W = \frac{\sqrt{3}}{2} P a + \frac{1}{2} P \sqrt{3} a$
 $2W = \frac{P}{2} + \frac{P}{2}$
 $P = W$

WORKING AT B: HORIZONTALLY

$T_2 \sin 30^\circ = 0$
 $X = -P \cos 30^\circ$
 $X = -W \frac{\sqrt{3}}{2}$
 $X = \frac{\sqrt{3}}{2} W$ (to the left)

WORKING AT A: HORIZONTALLY

$Y + P \sin 30^\circ = W$
 $Y + \frac{1}{2} W = W$
 $Y = \frac{1}{2} W$

WORKING AT C: VERTICALLY

$T_3 = 0$

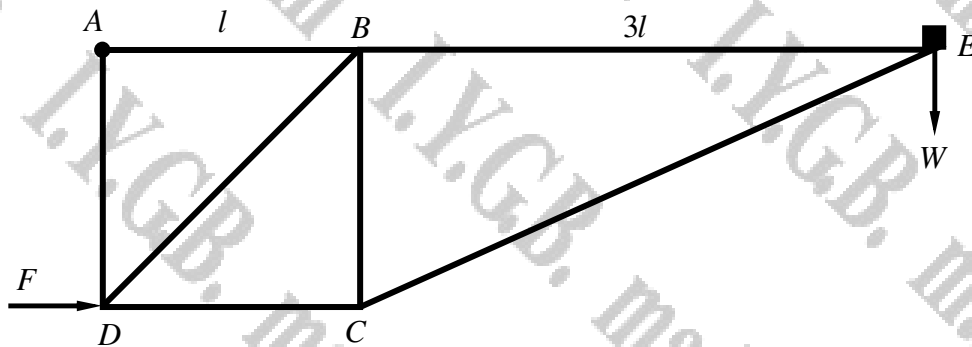
WORKING AT A: VERTICALLY

$T_2 \cos 30^\circ = P \cos 30^\circ + T_3 \cos 30^\circ$
 $2W = W + T_3$
 $T_3 = W$ (Tension)

WORKING AT C: HORIZONTALLY

$T_2 = T_3$
 $T_2 = \sqrt{3} W$ (Thrust)

Question 5 (***)



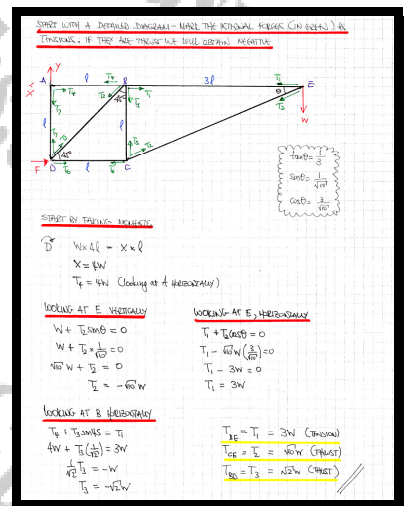
A framework consists of 7 light rigid rods smoothly joined between them as shown in the figure above.

The rods AB , BC , CD and DA form a square of side length l . The rod BE is of length $3l$, so that ABE is a straight line. Two more rods, BD and CE complete a rigid structure smoothly hinged at a fixed point A .

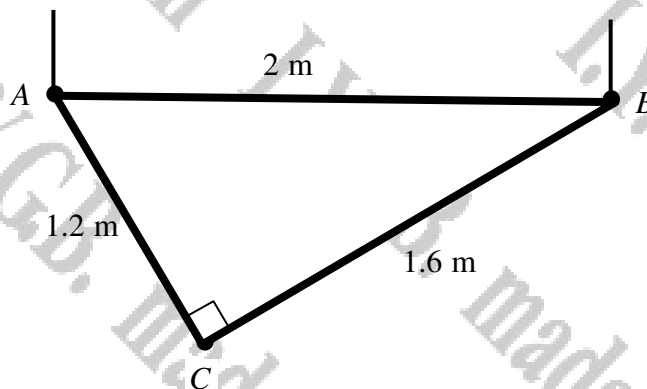
When a mass of weight W is placed at E , it is required that ABE remains in a horizontal position. This is achieved by an external force F acting at D , in the direction DC .

Determine, in terms of W , the magnitude of the forces acting on BE , CE and BD , further classifying them as tension or thrust.

$$\boxed{45.6^\circ}, \quad T_{BE} = 3W, \text{ tension}, \quad T_{CE} = \sqrt{10}W, \text{ thrust}, \quad T_{BD} = \sqrt{2}W, \text{ thrust}$$



Question 6 (***)



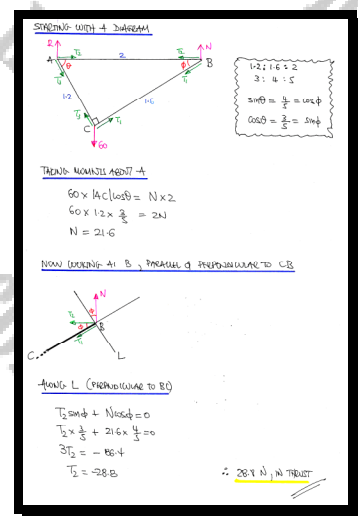
The figure above shows a framework ABC consists of three light pin jointed rods AB , BC and AC , freely hinged to a rigid support at B .

The framework supports a weight of 60 N at C and is kept in equilibrium with AB horizontal by two vertical wires at A and B .

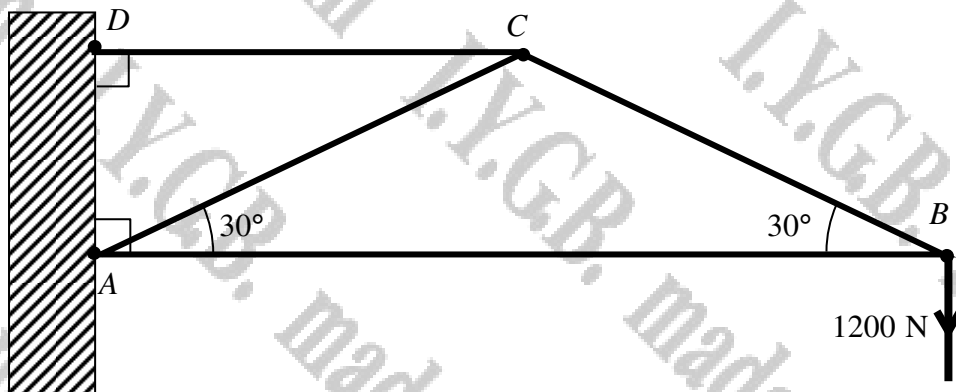
The lengths of the three rods, in metres, are marked on the diagram, and $\angle BAC = 90^\circ$.

Determine the magnitude force acting at AB .

, $T_{AB} = 28.8\text{ N}$, thrust



Question 7 (***)



A light rigid framework consists of 4 light pin jointed rods, AB , AC , BC and CD , where $|AC| = |BC|$, $\angle CAB = \angle ABC = 30^\circ$ and $\angle ADC = \angle BAD = 90^\circ$, as shown in the figure above.

The framework is freely hinged at the points A and D and a weight of 1200 N is supported at B as shown the figure above.

Find the magnitude of the reaction forces acting on the framework at A and D , and the magnitudes of all the internal forces acting on each of the four rods, classifying them as tension or thrust.

$$\boxed{}, \quad R_A = 1200\sqrt{13} \approx 4327 \text{ N},$$

$$\boxed{R_D = 2400\sqrt{3} \approx 4157 \text{ N}} \quad \boxed{R_{AB} = 1200\sqrt{3} \approx 2078 \text{ N, thrust}}, \quad \boxed{R_{BC} = 2400, \text{ tension}},$$

$$\boxed{R_{CD} = 2400\sqrt{3} \approx 4157 \text{ N, tension}}, \quad \boxed{R_{CA} = 2400, \text{ thrust}}$$

• LOOKING AT "A" HORIZONTALLY
 $T_3 + T_4 \cos 30^\circ = 0$
 $T_3 = -T_4 \cos 30^\circ$
 $T_3 = -2400 \cos 30^\circ$
 $T_3 = -2078 \text{ N}$

• LOOKING AT "A" VERTICALLY
 $T_3 \sin 30^\circ + T_4 \sin 30^\circ = 0$
 $T_3 = -T_4$
 $T_3 = -2400 \text{ N}$

• LOOKING AT "C" HORIZONTALLY
 $T_1 + T_4 \cos 30^\circ = T_2 \cos 30^\circ$
 $T_1 + (-2400) \cos 30^\circ = 2400 \cos 30^\circ$
 $T_1 = 2400 \cos 30^\circ$
 $T_1 = 2078 \text{ N}$

• LOOKING AT "C" VERTICALLY
 $T_1 = T_2$
 $T_1 = 2400 \text{ N}$

• LOOKING AT "B" HORIZONTALLY
 $X = T_3 + T_4 \cos 30^\circ$
 $X = -2078 \text{ N} - 2400 \cos 30^\circ$
 $X = -2078 \text{ N} - 2078 \text{ N}$
 $X = -4157 \text{ N}$
 $X = -2400\sqrt{3}$
 (OPPOSITE DIRECTION TO THAT MARKED)

• LOOKING AT "B" VERTICALLY
 $Y + T_4 \sin 30^\circ = 0$
 $Y + (2400) \sin 30^\circ = 0$
 $Y = -1200 \text{ N}$

• LOOKING AT "B" HORIZONTALLY
 $Y = 0$

• LOOKING AT "D" HORIZONTALLY
 $X = T_3 + T_4 \cos 30^\circ$
 $X = -2078 \text{ N} - 2400 \cos 30^\circ$
 $X = -2078 \text{ N} - 2078 \text{ N}$
 $X = -4157 \text{ N}$
 $X = -2400\sqrt{3}$
 (OPPOSITE DIRECTION TO THAT MARKED)

• LOOKING AT "D" VERTICALLY
 $Y + T_4 \sin 30^\circ = 0$
 $Y + (2400) \sin 30^\circ = 0$
 $Y = -1200 \text{ N}$

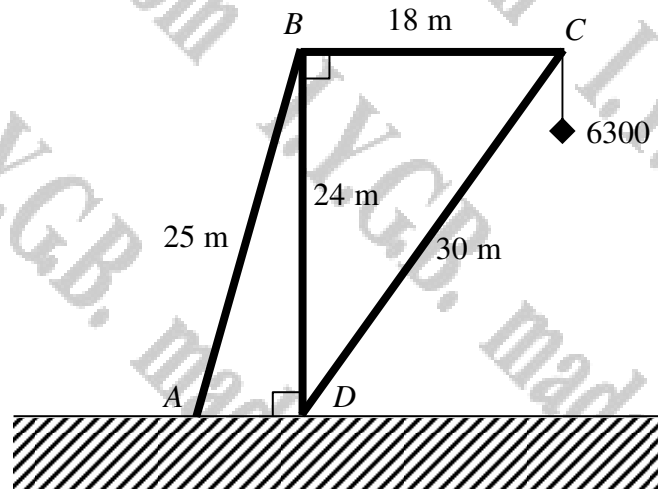
• LOOKING AT "A" HORIZONTALLY
 $X = T_3 + T_4 \cos 30^\circ$
 $X = -2078 \text{ N} - 2400 \cos 30^\circ$
 $X = -2078 \text{ N} - 2078 \text{ N}$
 $X = -4157 \text{ N}$
 $X = -2400\sqrt{3}$
 (OPPOSITE DIRECTION TO THAT MARKED)

• LOOKING AT "A" VERTICALLY
 $Y + T_4 \sin 30^\circ = 0$
 $Y + (2400) \sin 30^\circ = 0$
 $Y = -1200 \text{ N}$

• LOOKING AT "B" HORIZONTALLY
 $X = T_3 + T_4 \cos 30^\circ$
 $X = -2078 \text{ N} - 2400 \cos 30^\circ$
 $X = -2078 \text{ N} - 2078 \text{ N}$
 $X = -4157 \text{ N}$
 $X = -2400\sqrt{3}$
 (OPPOSITE DIRECTION TO THAT MARKED)

• LOOKING AT "B" VERTICALLY
 $Y + T_4 \sin 30^\circ = 0$
 $Y + (2400) \sin 30^\circ = 0$
 $Y = -1200 \text{ N}$

Question 8 (***)



A ground crane is modelled as a light rigid framework consisting of 4 light pin jointed rods, AB , BD , BC and CD , where $|AB| = 25\text{ m}$, $|BD| = 24\text{ m}$, $|BC| = 18\text{ m}$ and $|CD| = 30\text{ m}$, as shown in the figure above.

The crane is attached to the ground at the points A and D and a weight of 6300 N is suspended from C , as shown the figure above.

Find the magnitude of the reaction forces acting on the framework at A and D , and the magnitudes of all the internal forces acting on each of the four rods, classifying them as tension or thrust.

$\boxed{}$, $R_A = 16875\text{ N}$, $R_D \approx 22600\text{ N}$, $R_{AB} = 16875\text{ N}$, tension, $R_{BC} = 4725$, tension, $R_{CD} = 7875\text{ N}$, thrust, $R_{BD} = 16200$, thrust

WORK IN THE DIAGRAM ALL INTERNAL FORCES AS TENSIONS (W GUESSES)

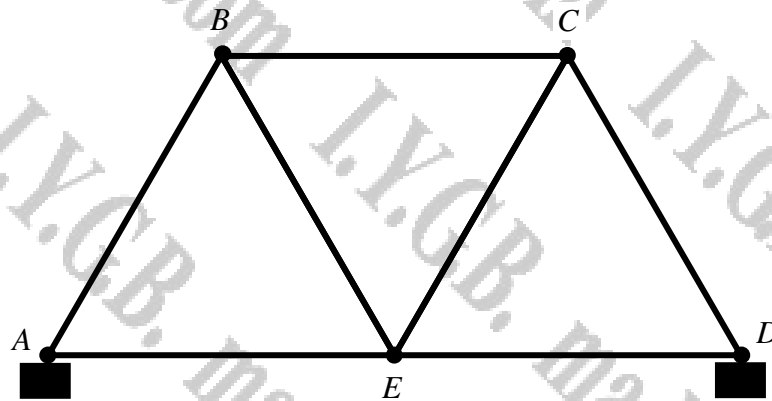
- LOOKING AT C**
 $\sum F_y = 0$
 $T_{BC} - 6300 = 0$
 $T_{BC} = 6300$ (TENSION)
 $\sum F_x = 0$
 $T_{CD} - T_{BC} = 0$
 $T_{CD} = 6300$ (TENSION)
- LOOKING AT B**
 $\sum F_y = 0$
 $T_{BD} - T_{BC} = 0$
 $T_{BD} = 6300$ (TENSION)
 $\sum F_x = 0$
 $T_{AB} - T_{BD} = 0$
 $T_{AB} = 6300$ (TENSION)
- LOOKING AT A**
 $\sum F_y = 0$
 $R_A - T_{AB} = 0$
 $R_A = 6300$ (TENSION)
- LOOKING AT D**
 $\sum F_y = 0$
 $R_D - T_{BD} = 0$
 $R_D = 6300$ (TENSION)

WORKING OUT REACTION FORCES

ROD AB : 16875 N (TENSION)
 ROD BC : 4725 N (TENSION)
 ROD CD : 7875 N (TENSION)
 ROD BD : 16200 N (TENSION)

REACTION AT A : 16875 N
 REACTION AT D : 22600 N

Question 9 (***)



A rigid framework $ABCDE$ consists of seven identical light pin jointed rods as shown in the figure above. The framework rests at two fixed supports at the points A and D .

When the framework supports a weight of W N at the midpoint AE , there is 600 N thrust on AB .

Determine the magnitude of each of the reaction forces at A and at C and the magnitude of each of the internal forces in the rods EA , EB , EC , ED , CB and CD , further classifying each of them as a tension or as a thrust.

$$\boxed{}, R_A = 300\sqrt{3} \text{ N}, R_D = 1050 \text{ N}, T_{EA} = 300 \text{ N, tension},$$

$$T_{EB} = 600 \text{ N, thrust}, T_{EC} = 200 \text{ N, tension}, T_{ED} = 400 \text{ N, tension},$$

$$T_{CB} = 600 \text{ N, thrust}, T_{CD} = 200 \text{ N, thrust}$$

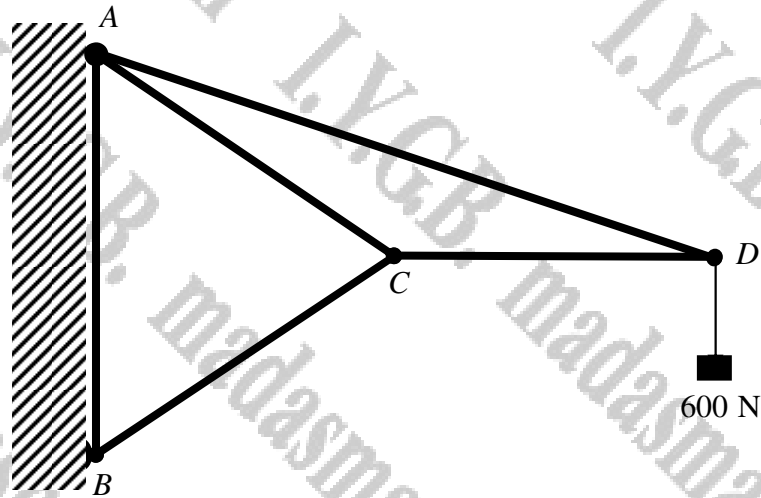
- LOOKING AT "A" VERTICALLY
 $\Rightarrow X + (-600 \sin 60^\circ) = 0$
 $\Rightarrow X = 600 \sin 60^\circ$
 $\Rightarrow X = 300\sqrt{3} \text{ N}$
- TAKING MOMENTS ABOUT "D"
 $\Rightarrow X(a) = W(2a)$
 $\Rightarrow W = \frac{2}{3}X$
 $\Rightarrow W = 400\sqrt{3} \text{ N}$
- LOOKING AT "A" HORIZONTALLY & NOTING THE EXTERNAL HORIZONTAL REACTION IS ZERO
 $\Rightarrow T_1 + (-600 \cos 60^\circ) = 0$
 $\Rightarrow T_1 = 300 \text{ N}$ (TENSION)
- FINDING EXTERNAL FORCE VERTICALLY
 $\Rightarrow X + Y = W$
 $\Rightarrow 300\sqrt{3} + Y = 400\sqrt{3}$
 $\Rightarrow Y = 100\sqrt{3} \text{ N}$
- LOOKING AT "B" VERTICALLY
 $\Rightarrow (-600 \sin 60^\circ) + T_2 \sin 60^\circ = 0$
 $\Rightarrow T_2 = 600 \text{ N}$ (TENSION)
- LOOKING AT "B" HORIZONTALLY
 $\Rightarrow -600 \cos 60^\circ = T_3 + T_2 \cos 60^\circ$
 $\Rightarrow -300 = T_3 + 600 \times \frac{1}{2}$
 $\Rightarrow -600 = T_3$
 $\Rightarrow T_3 = 600 \text{ N}$ (THRUST)
- LOOKING AT "D" VERTICALLY
 $\Rightarrow Y + T_4 \sin 60^\circ = 0$
 $\Rightarrow 100\sqrt{3} + T_4 \frac{\sqrt{3}}{2} = 0$
 $\Rightarrow 200 + T_4 = 0$
 $\Rightarrow T_4 = -200$
 $\Rightarrow T_4 = 200 \text{ N}$ (THRUST)
- LOOKING AT "D" HORIZONTALLY
 $\Rightarrow T_5 + T_6 \cos 60^\circ = 0$
 $\Rightarrow -200 + T_6 \times \frac{1}{2} = 0$
 $\Rightarrow T_6 = 400 \text{ N}$ (TENSION)
- LOOKING AT "C" VERTICALLY
 $\Rightarrow T_7 \sin 60^\circ + T_8 \sin 60^\circ = 0$
 $\Rightarrow T_7 = -T_8$
 $\Rightarrow T_7 = 200 \text{ N}$ (TENSION)

SUMMARY RESULTS

REACTION AT A: $300\sqrt{3} \text{ N}$, UNKNOWNS
 REACTION AT D: 1050 N , UNKNOWNS

AF: 300 N (TENSION)
 ED: 400 N (TENSION)
 BC: 600 N (THRUST)
 EB: 600 N (TENSION)
 EC: 200 N (TENSION)
 CD: 200 N (THRUST)

Question 10 (***)



The figure above shows a light rigid framework $ABCD$, attached to a vertical wall, which consists of 5 light pin jointed coplanar rods, AB , BC , AC , CD and AD , where $|AB| = |BC| = |AC| = |CD| = 1$ m.

The framework is freely hinged at A , where A is on the vertical wall, and touches a smooth support at B , where B is also on the same wall, vertically below A as shown in the figure above. The framework supports a weight of 600 N at D , and remains in equilibrium, with CD horizontal, with the plane $ABCD$ perpendicular to the wall.

Find, in any order, the magnitude of the horizontal and vertical components of the reaction forces acting on the framework at A and B , and the magnitudes the internal forces acting on each of the 5 rods, classifying them as tensions or thrusts.

$\boxed{1120 \text{ N}}$, horizontally at A and B , ≈ 1120 N , vertically at A , 600 N ,
 $R_{AB} \approx 646$ N, Tension , $R_{AC} = R_{BC} \approx 1293$ N, Thrust , $R_{CD} \approx 2239$ N, Thrust ,
 $R_{AD} = 2318$ N, Tension

• START WITH A DIAGRAM WITH ALL KNOWN FORCES MARKED AS TENSIONS

• CALCULATE THE REACTION FORCES

• $\sum \text{moments about B} = 0$

$$600 \times 1 = 600 \times (1 + \cos 30^\circ)$$

$$V = 600(1 + \frac{\sqrt{3}}{2})$$

$$V \approx 1120 \text{ N}$$

• LOOKING AT B HORIZONTALLY

$$\Rightarrow H + T_2 \cos 30^\circ = 0$$

$$\Rightarrow T_2 \cos 30^\circ = -H$$

$$\Rightarrow T_2 = \frac{-300(2 + \sqrt{3})}{\cos 30^\circ}$$

$$\Rightarrow T_2 = -770(2 + \sqrt{3})$$

• LOOKING AT B VERTICALLY

$$\Rightarrow T_1 + T_2 \cos 60^\circ = 0$$

$$\Rightarrow T_1 = -T_2 \cos 60^\circ$$

$$\Rightarrow T_1 = -[-770(2 + \sqrt{3})] \times 2$$

$$\Rightarrow T_1 = 1540(2 + \sqrt{3})$$

• LOOKING AT A HORIZONTALLY

$$\Rightarrow T_1 + T_2 \cos 15^\circ = 0$$

$$\Rightarrow T_2 = -\frac{1540 \cos 15^\circ}{\cos 15^\circ}$$

$$\Rightarrow T_2 = -1540$$

• LOOKING AT A VERTICALLY

$$T_3 \sin 60^\circ = T_2 \sin 30^\circ$$

$$T_3 = -200(2 + \sqrt{3})$$

• LOOKING AT D HORIZONTALLY

$$\Rightarrow T_1 + T_2 \cos 15^\circ = 0$$

$$\Rightarrow T_2 = -\frac{1540 \cos 15^\circ}{\cos 15^\circ}$$

$$\Rightarrow T_2 = -1540$$

• LOOKING AT D VERTICALLY

$$T_3 \sin 60^\circ = T_2 \sin 30^\circ$$

$$T_3 = -200(2 + \sqrt{3})$$

• LOOKING AT C HORIZONTALLY

$$\Rightarrow T_1 + T_2 \cos 60^\circ = 0$$

$$\Rightarrow T_1 = -T_2 \cos 60^\circ$$

$$\Rightarrow T_1 = -[-770(2 + \sqrt{3})] \times 2$$

$$\Rightarrow T_1 = 1540(2 + \sqrt{3})$$

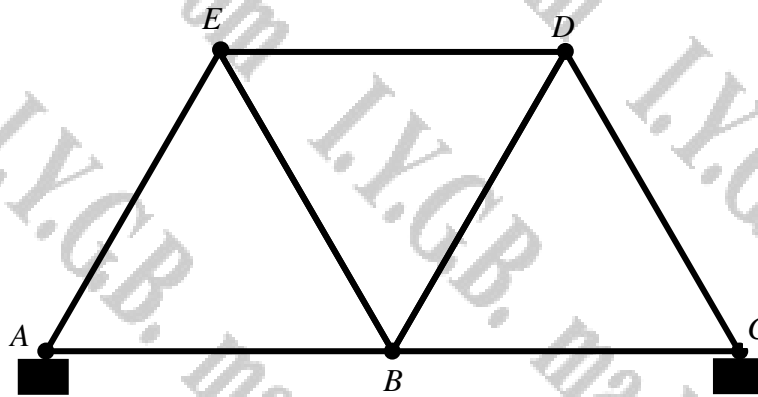
• LOOKING AT C VERTICALLY

$$\Rightarrow T_1 + T_2 \cos 30^\circ = 0$$

$$\Rightarrow T_2 = -\frac{1540 \cos 30^\circ}{\cos 30^\circ}$$

$$\Rightarrow T_2 = -1540$$

Question 11 (***)



A bridge design is modelled by a framework $ABCDE$ consisting of seven identical light pin jointed rods as shown in the figure above.

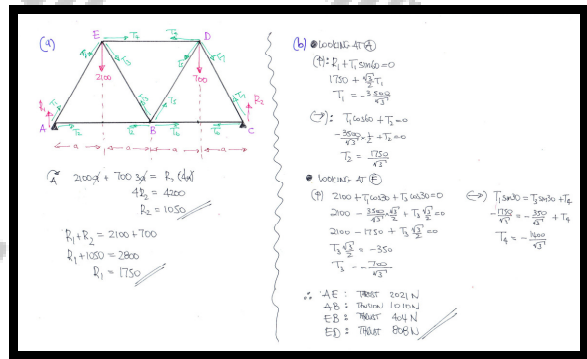
The framework rests at two concrete plinths at A and C .

Two weights of 2100 N and 700 N are placed at E and D , respectively.

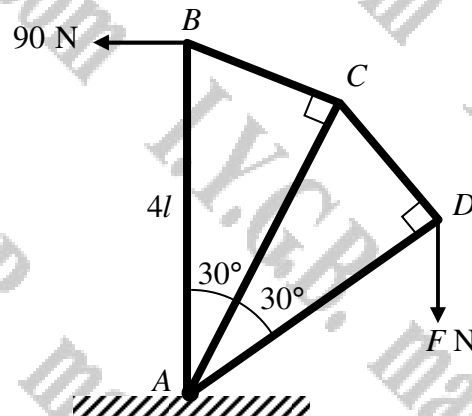
- Determine the magnitude of the reaction force at A and at C .
- Calculate the magnitude of each of the internal forces in the rods AE , AB , EB and ED , further classifying each of them as a tension or as a thrust.

$$R_A = 1750\text{ N}, R_B = 1050\text{ N}, T_{AE} \approx 2021\text{ N, thrust}, T_{AB} \approx 1010\text{ N, tension},$$

$$T_{EB} \approx 404\text{ N, thrust}, T_{ED} \approx 808\text{ N, thrust}$$



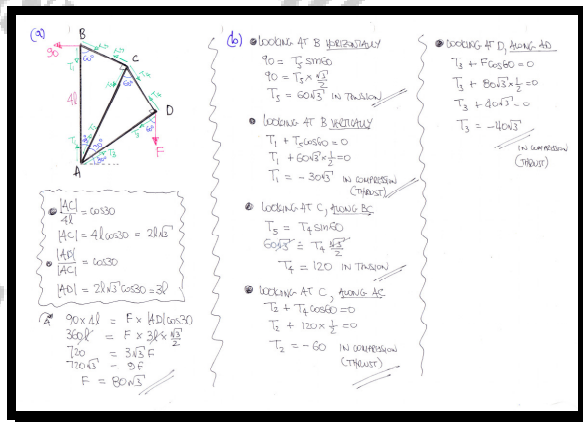
Question 12 (***)



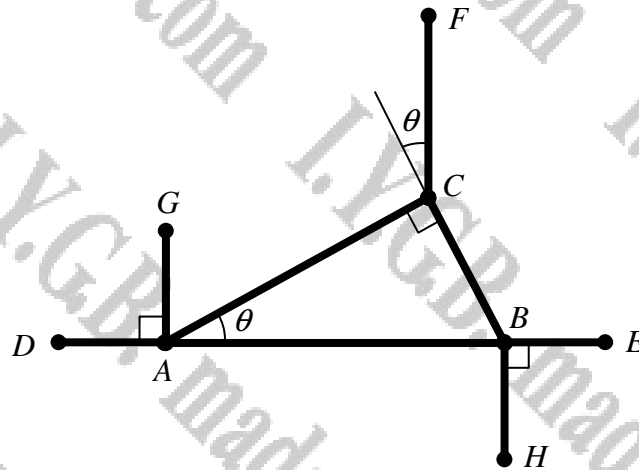
A framework $ABCD$, consists of five light pin jointed rods, freely hinged to a rigid support at A . The framework supports a weight of F N at D and is kept in equilibrium with AB vertical by a horizontal force of 90 N as shown in the figure above. It is further given that the length of the rod AB is $4l$, $\angle ACB = \angle ADC = 90^\circ$ and $\angle BAC = \angle CAD = 30^\circ$.

- Find the exact value of F .
- Calculate, in exact form where appropriate, the internal forces in the five rods, further classifying each of them as a tension or as a thrust.

$$F = 80\sqrt{3} \text{ N}, \quad T_{AB} = 30\sqrt{3} \text{ N, thrust}, \quad T_{AC} = 60 \text{ N, thrust}, \quad T_{AD} = 40\sqrt{3} \text{ N, thrust}, \\ T_{BC} = 60\sqrt{3} \text{ N, tension}, \quad T_{CD} = 120 \text{ N, tension}$$



Question 13 (***)



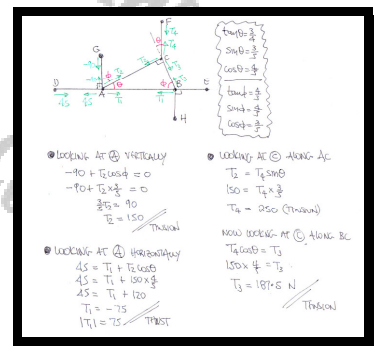
The figure above shows part of a framework of several light rigid rods, freely pin jointed at their ends. All the rods jointed at A , B and C are shown in the figure but only some of the rods jointed at D , E , F , G and H are shown.

The joints D , A , B and E lie in a straight line, $\angle DAG = \angle EBH = \angle ACB = 90^\circ$, and $\angle BAC = \theta$, where $\tan \theta = \frac{3}{4}$. The rod FC is also inclined to BC at θ , as shown in the figure.

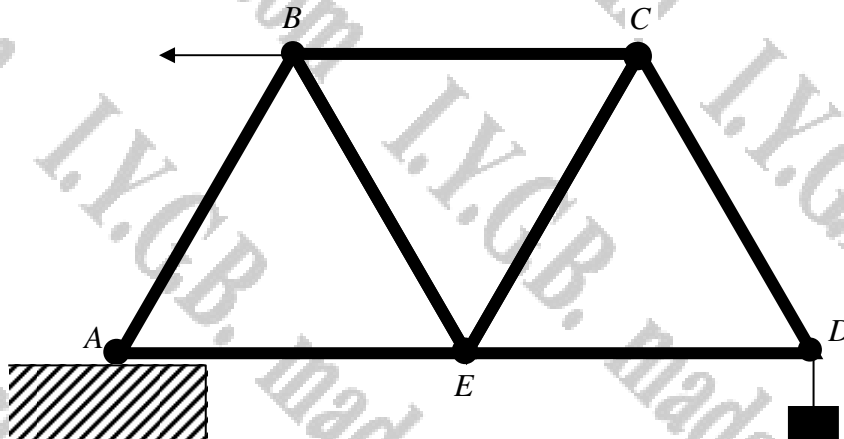
There is a tension of 45 N in AD and a thrust of 90 N in AG .

Calculate, the internal forces in AB , BC and AC , further classifying each of them as a tension or as a thrust.

$$T_{AB} = 75 \text{ N, thrust}, T_{AC} = 150 \text{ N, tension}, T_{BC} = 187.5 \text{ N, tension},$$



Question 14 (***)



A rigid framework $ABCDE$ consists of seven identical light pin jointed rods as shown in the figure above. The framework rests on a fixed support at the point A .

A weight of 600 N is suspended from D , and there is an external force acting at B acting in the direction CB .

Determine the magnitude of internal force acting in the rod BC .

$$\boxed{}, T_{BC} = 400\sqrt{3} \text{ N, tension}$$

SIMPLE: USE A TRIANGLE - MAKE ALL THE INTERNAL FORCES (IN GREEN)
AS THINGS - MAKE EXTERNAL FORCES IN BLUE

$\angle BAE = 60^\circ$
 $\angle BAE = 60^\circ$

TAKE THE MOMENTS ABOUT A

$$F \times 1.5 = 600 \times 2.5$$

$$\Rightarrow F = 1000 \text{ N}$$

$$\Rightarrow 3F = 3000 \text{ N}$$

$$\Rightarrow F = 1000 \text{ N}$$

LOOKING AT D, VERTICALLY

$$T_7 \sin 60^\circ = 600$$

$$\Rightarrow T_7 \times \frac{\sqrt{3}}{2} = 600$$

$$\Rightarrow 3T_7 = 1200\sqrt{3}$$

$$\Rightarrow T_7 = 400\sqrt{3} \text{ (Tension)}$$

LOOKING AT D, HORIZONTALLY

$$\Rightarrow T_6 + T_7 \cos 60^\circ = 0$$

$$\Rightarrow T_6 = -T_7 \times \frac{1}{2}$$

$$\Rightarrow T_6 = -200\sqrt{3} \text{ (Compression)}$$

LOOKING AT C, VERTICALLY

$$\Rightarrow T_2 \cos 60^\circ + T_3 \sin 60^\circ = 0$$

$$\Rightarrow T_2 = -T_3$$

$$\Rightarrow T_2 = -1000 \text{ (Tension)}$$

LOOKING AT C, HORIZONTALLY

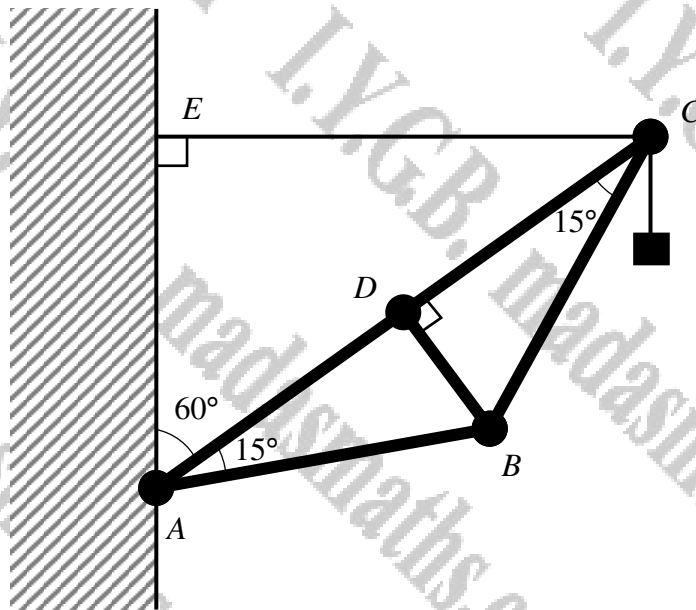
$$\Rightarrow T_1 + T_2 \sin 60^\circ = T_4 \cos 60^\circ$$

$$\Rightarrow T_1 + \frac{1}{2}(-1000\sqrt{3}) = \frac{1}{2}T_4$$

$$\Rightarrow T_1 = 200\sqrt{3} + \frac{1}{2}T_4$$

$$\Rightarrow T_1 = 1000 \text{ (Tension)}$$

Question 15 (****)



The figure above shows a crane, modelled as a framework of light rigid rods, freely pin jointed at their ends.

The rods are jointed at A , B , C and D .

The framework is freely hinged to a rigid support on a vertical wall at A .

A horizontal cable EC has its end E attached to the same above mentioned vertical wall, with E directly above A .

The points A , D and C lie in a straight line such that $|AC| = 30 \text{ m}$, $\angle EAC = 60^\circ$, $\angle DAB = \angle DCB = 15^\circ$ and $\angle AEC = \angle CDB = 90^\circ$.

There is a load of 36000 N hanging freely from C .

Calculate, the internal forces in BC and AC .

$$\boxed{}, \quad \boxed{R_{BC} = 0}, \quad \boxed{R_{AC} = 72000 \text{ N, Thrust}}$$

[solution overleaf]

STRUCTURE WITH A DETAILED DIAGRAM - MAKE NODAL VALUES (AS GIVEN) AS TRUSSES

TAKING MOMENTS (CHOOSE STRUCTURE) ABOUT A

$$\Rightarrow T \times 10 = 30000 \times 10$$

$$\Rightarrow T = \frac{30000 \times 10}{10}$$

$$\Rightarrow T = \frac{30000 \times 100000}{1000000}$$

$$\Rightarrow T = 30000 \text{ N}$$

NOT LOOKING AT C - VERTICALLY AND HORIZONTALLY

(P): $T_{\text{vertical}} + T_{\text{horizontal}} + 30000 = 0$ -I
 (M): $T + T_{\text{vertical}} + T_{\text{horizontal}} = 0$ -II

AS THESE ARE NOT THAT LIGHT TO SOLVE REMOVED POINT C, AND
 RESOLVE PARALLEL & PERPENDICULAR TO AC

Along 'L'

$$30000 \cos 60^\circ = T_{\text{vertical}} + 30000 \sin 30^\circ$$

$$15000 = T_{\text{vertical}} + 15000$$

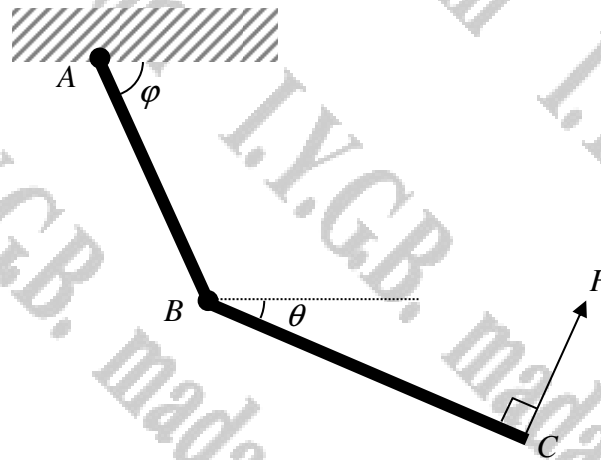
$$T_{\text{vertical}} = 0$$

Created by T. Madas

MULTISTRUCTURES

Created by T. Madas

Question 1 (***)



Two identical uniform rigid rods AB and BC , each of weight 100 N , are freely joined at B and lie in the same vertical plane.

The rod AB is freely joined at A , a fixed point on a horizontal ceiling. The system is held in equilibrium by a force $F\text{ N}$ acting at C , in a perpendicular direction to BC , as shown in the figure above.

AB and BC form angles φ and θ to the horizontal, respectively, where $\tan \theta = \frac{3}{4}$.

- Find the value of F .
- Calculate the magnitudes of the horizontal and vertical reaction forces, acting on BC at B .
- Determine, in degrees, the size of the angle φ .

$$\boxed{F = 40}, \quad \boxed{R_{\rightarrow} = 24\text{ N}}, \quad \boxed{R_{\uparrow} = 68\text{ N}}, \quad \boxed{\varphi \approx 78.5^\circ}$$

STARTING WITH A DETAILED DIAGRAM

Let length of each rod be 2 m

$\tan \theta = \frac{3}{4}, \sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}$

1) TAKING MOMENTS OF BC ABOUT B

$$\begin{aligned} \Rightarrow F \times 2 &= 100 \cos \theta \times 1 \\ \Rightarrow 2F &= 100 \times \frac{4}{5} \\ \Rightarrow F &= 40\text{ N} \end{aligned}$$

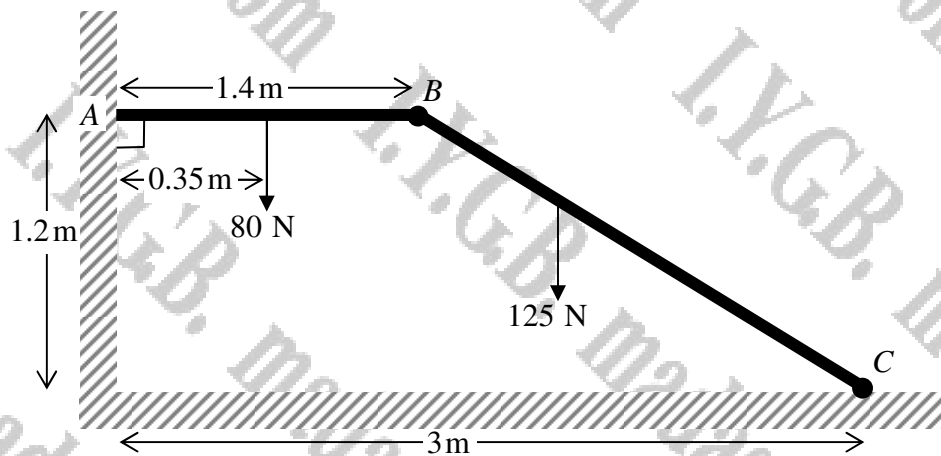
2) RESOLVING FORCES ON THE ROD BC (GREEN RODS)

(1): $Y + F \cos \theta = 100$ (2): $F \sin \theta = X$
 $Y + 40 \times \frac{4}{5} = 100$ $X = 40 \times \frac{3}{5}$
 $Y = 68\text{ N}$ $X = 24\text{ N}$

3) TAKING MOMENTS OF AB ABOUT A (BLUE RODS)

$$\begin{aligned} \Rightarrow 100 \cos \theta \times 1 + Y \cos \theta \times 2 &= X \sin \theta \times 2 \\ 100 \cos \theta + 136 \cos \theta &= 48 \sin \theta \\ \Rightarrow 236 \cos \theta &= 48 \sin \theta \\ \Rightarrow \frac{\sin \theta}{\cos \theta} &= \frac{236}{48} \\ \Rightarrow \tan \theta &= \frac{59}{12} \\ \Rightarrow \theta &\approx 78.5^\circ \end{aligned}$$

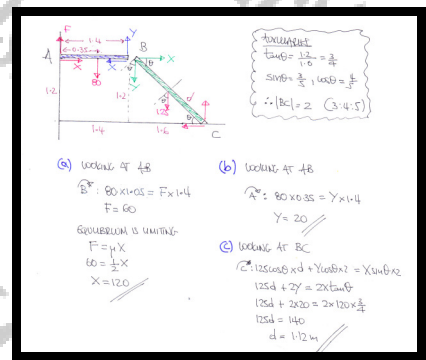
Question 2 (***)



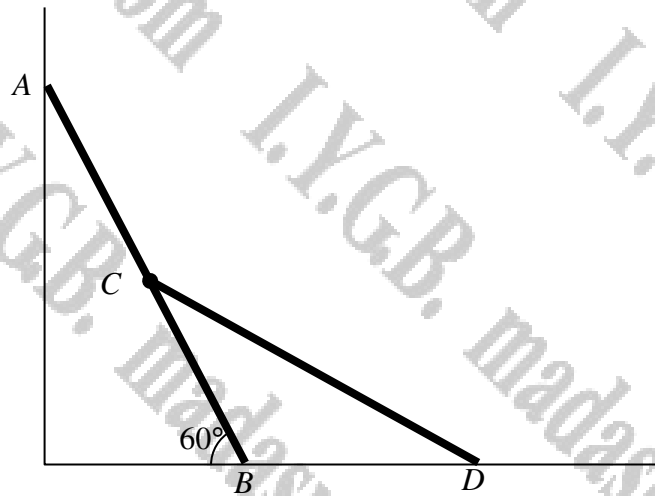
The figure above shows two **non uniform** rigid rods AB and BC , of respective weights 80 N and 125 N , freely joined at B . The rod BC is also freely joined at C , a fixed point on level horizontal ground, which lies at a distance of 3 m from a rough vertical wall. The rod AB has length 1.4 m and rest in a horizontal position in **limiting equilibrium** with its end A against the rough wall. The distance of A from the ground is 1.2 m and the coefficient of friction between the wall and AB is 0.5 . The position of the centre of mass of AB is 0.35 m from A . The two rods lies in the same vertical plane.

- Calculate the magnitude of the horizontal reaction, acting on AB at B .
- Calculate the magnitude of the vertical reaction, acting on AB at B .
- Determine the distance of the position of the centre of mass of BC from C .

$$R_{\rightarrow} = 120\text{ N}, \quad R_{\uparrow} = 20\text{ N}, \quad d = 1.12\text{ m}$$



Question 3 (***)



A uniform rod AB , of weight W and length $4a$, rests in equilibrium at 60° to the horizontal, with A against a wall and B on horizontal ground. Another uniform rod CD , of weight $2W$, is freely joined with its end C at the midpoint of AB and D on horizontal ground. The two rods lie in the same vertical plane so that $BD = 2a$.

The contacts at A and D are smooth but the contact at B is rough.

Given that the system of the two rods is in limiting equilibrium, determine in exact form the coefficient of friction between AB and the ground at B .

$$\mu = \frac{1}{2\sqrt{3}} = \frac{1}{6}\sqrt{3}$$

