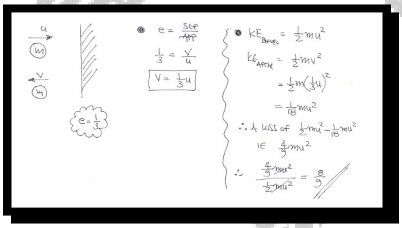
COLLISIONS

Question 1 (**)

A smooth sphere is moving with speed u ms^{-1} on a smooth horizontal plane when it strikes at right angles a fixed smooth vertical wall. The sphere is modelled as a particle. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$.

Find the fraction of the kinetic energy is lost by the sphere, as a result of the impact with the wall.

$$\boxed{\frac{8}{9}}$$



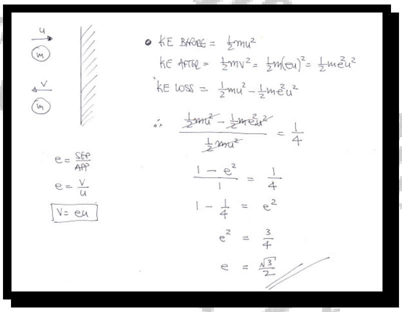
Question 2 (+)**

A smooth sphere is moving with speed u ms^{-1} on a smooth horizontal plane when it strikes at right angles a fixed smooth vertical wall.

One quarter of the kinetic energy is lost by the sphere, as a result of the impact with the wall. The sphere is modelled as a particle.

Find the coefficient of restitution between the sphere and the wall.

$$\boxed{e = \frac{\sqrt{3}}{2}}$$



Question 3 (***)

A smooth sphere A of mass 3 kg is moving with speed 4 ms^{-1} on a smooth horizontal plane when it collides directly with a smooth sphere B of mass 5 kg moving with speed 2 ms^{-1}, in the opposite direction as B. The two spheres are modelled as particles and the coefficient of restitution between them is $\frac{5}{6}$.

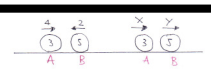
a) Calculate the speed of A and the speed of B after the collision.

After the collision between A and B, sphere B collides directly with a smooth vertical wall. The coefficient of restitution between B and the wall is $\frac{1}{5}$.

b) Find the magnitude of the impulse exerted by the wall onto B.

$\boxed{|V_A| = \frac{23}{8}\ \text{ms}^{-1}}$, $\boxed{|V_B| = \frac{17}{8}\ \text{ms}^{-1}}$, $\boxed{|I| = 12.75\ \text{Ns}}$

(a)



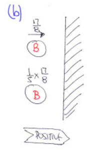
- BY CONSERVATION OF MOMENTUM
  $(4\times3)-(2\times5) = 3X+5Y$
  $\boxed{3X+5Y=2}$
- BY RESTITUTION
  $e = \frac{SEP}{APP}$
  $\frac{5}{6} = \frac{Y-X}{6}$
  $Y-X=5$
  $\boxed{Y=X+5}$

$3X+5(X+5)=2$
$3X+5X+25=2$
$8X=-23$
$X=-\frac{23}{8}$ (MINUS $\Rightarrow$ "A" HAS REBOUNDED)
$X=-2.875$

- $Y=X+5$
  $Y=-\frac{23}{8}+5$
  $Y=\frac{17}{8}=2.125$

$\therefore$ SPEED OF A IS $2.875\ \text{ms}^{-1}$
SPEED OF B IS $2.125\ \text{ms}^{-1}$ (BOTH REBOUNDED)

(b)



- SPEED OF B AFTER REBOUNDING IS "e × APPROACH SPEED"
  IE $\frac{1}{5}\times\frac{17}{8} = \frac{17}{40} = 0.425$
- TAKING "RIGHT" AS POSITIVE
  I = MOM AFTER − MOM BEFORE
  $I = -\frac{17}{40}\times5 - \frac{17}{8}\times5$
  $I = -\frac{51}{4}$
  $|I| = \frac{51}{4} = 12.75\ \text{Ns}$

Question 4 (***)

Two smooth spheres P and Q of respective masses $3m$ and $4m$ are moving towards each other, both with speed u, when they collide directly. As a result of the collision the direction of the motion of Q is reversed and its speed is halved.

The spheres are modelled as particles moving on a smooth horizontal plane.

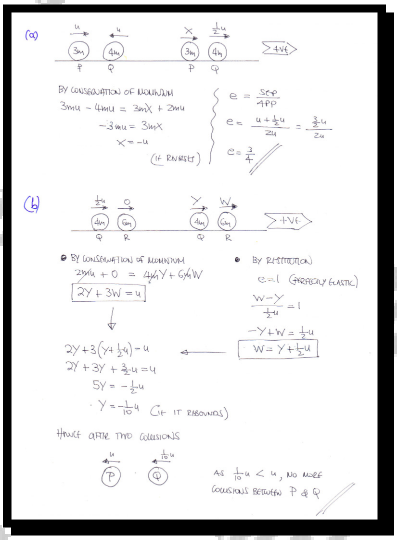
a) Find the coefficient of restitution between the two spheres.

Consequently, sphere Q collides directly with a third sphere R of mass $6m$ which is initially at rest. The collision between Q and R is perfectly elastic.

Sphere R is also modelled as a particle.

b) Show that after the collision between Q and R, they will be no more collisions between P and Q.

$\boxed{e = \frac{3}{4}}$



(a)

BY CONSERVATION OF MOMENTUM

$3mu - 4mu = 3mX + 2mu$

$-3mu = 3mX$

$X = -u$ (IT REVERSES)

$e = \frac{SEP}{APP}$

$e = \frac{u + \frac{1}{2}u}{2u} = \frac{\frac{3}{2}u}{2u}$

$e = \frac{3}{4}$

(b)

• BY CONSERVATION OF MOMENTUM

$2mu + 0 = 4mY + 6mW$

$\boxed{2Y + 3W = u}$

• BY RESTITUTION

$e = 1$ (PERFECTLY ELASTIC)

$\frac{W - Y}{\frac{1}{2}u} = 1$

$-Y + W = \frac{1}{2}u$

$\boxed{W = Y + \frac{1}{2}u}$

$2Y + 3\left(Y + \frac{1}{2}u\right) = u$

$2Y + 3Y + \frac{3}{2}u = u$

$5Y = -\frac{1}{2}u$

$Y = -\frac{1}{10}u$ (IT REBOUNDS)

HENCE AFTER TWO COLLISIONS

AS $\frac{1}{10}u < u$, NO MORE COLLISIONS BETWEEN P & Q

Question 5 (***)

A smooth sphere P of mass m is moving with speed u on a smooth horizontal plane when it collides directly with a smooth sphere Q of mass $4m$ which is initially at rest. The spheres are modelled as particles and the coefficient of restitution between the two spheres is e.

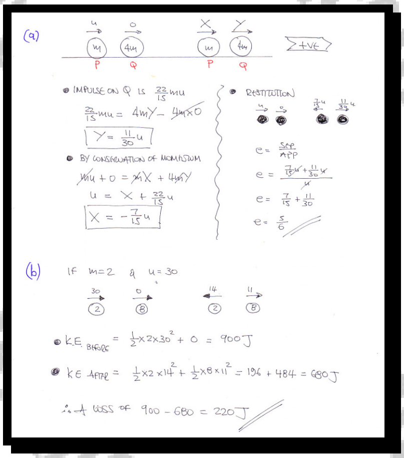
The magnitude of the impulse of P on Q is $\frac{22}{15}mu$.

a) Find the value of e.

It is now given that $m = 2$ kg and $u = 30$ ms^{-1}.

b) Show that the kinetic energy loss due to the collision is 220 J.

$$\boxed{e = \frac{5}{6}}$$



Question 6 (***+)

A smooth sphere A of mass $3m$ is moving with speed $5u$ on a smooth horizontal plane. It collides directly with a smooth sphere B of mass m moving with speed $3u$ in the opposite direction as A. The spheres are modelled as particles and the coefficient of restitution between A and B is e.

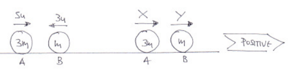
a) Find, in terms of e and u, the speeds of the two spheres after the collision.

b) Show that A cannot possibly reverse direction as a result of the collision.

c) Given further that the total kinetic energy of the two spheres after the collision is $\frac{39}{2}mu^2$, show clearly that $e = \frac{1}{4}$

$\boxed{V_A = u(3-2e)}$, $\boxed{V_B = 3u(1+2e)}$

(a)



- BY CONSERVATION OF MOMENTUM

$15mu - 3mu = 3mX + mY$

$\boxed{12u = 3X + Y}$

- BY RESTITUTION

$e = \frac{\text{SEP}}{\text{APP}}$

$e = \frac{Y - X}{8u}$

$\boxed{-X + Y = 8eu}$

SUBTRACT

$\left.\begin{array}{l} 3X + Y = 12u \\ -X + Y = 8eu \end{array}\right\} \Rightarrow 4X = 12u - 8eu$

$X = 3u - 2eu$

$X = u(3-2e)$

AND $Y = X + 8eu$

$Y = (3u - 2eu) + 8eu$

$Y = 3u + 6eu$

$Y = 3u(1+2e)$

(b)

$0 < e < 1$

$0 < 2e < 2$

$-2 < -2e < 0$

$1 < 3 - 2e < 3$

$u < u(3-2e) < 3u$

$u < X < 3u$

$\therefore$ CANNOT REVERSE DIRECTION AS $X > 0$

OR

- IF $e = 0$

$X = 3u$ (TO THE RIGHT)

- IF $e = 1$

$X = u$ (TO THE RIGHT)

WE CANNOT GET A NEGATIVE VALUE FOR X IF $0 < e < 1$

$\therefore$ "ALWAYS TO THE RIGHT"

(c) $KE_{\text{AFTER}} = \frac{39}{2}mu^2$

$\Rightarrow \frac{1}{2}(3m)X^2 + \frac{1}{2}(m)Y^2 = \frac{39}{2}mu^2$

$\Rightarrow 3X^2 + Y^2 = 39u^2$

$\Rightarrow 3\left[u(3-2e)\right]^2 + \left[3u(1+2e)\right]^2 = 39u^2$

$\Rightarrow 3u^2(3-2e)^2 + 9u^2(1+2e)^2 = 39u^2$

$\Rightarrow (3-2e)^2 + 3(1+2e)^2 = 13$

$\Rightarrow 9 - \cancel{12e} + 4e^2 + 12e^2 + \cancel{12e} + 3 = 13$

$\Rightarrow 16e^2 = 1$

$\Rightarrow e^2 = \frac{1}{16}$

$\Rightarrow e = \frac{1}{4}$ $(e > 0)$

AS REQUIRED

Question 7 (***+)

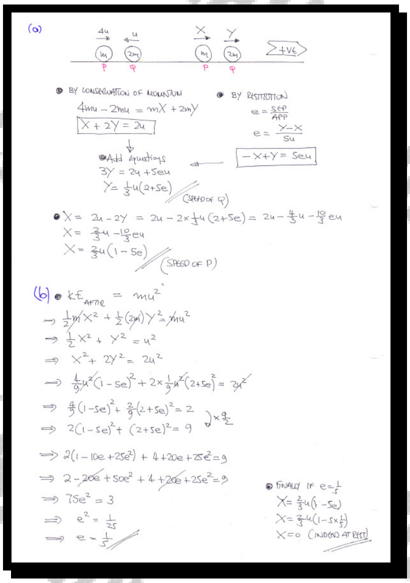
A smooth sphere P of mass m is moving with speed $4u$ on a smooth horizontal plane. It collides directly with a smooth sphere Q of mass $2m$ moving with speed u in the opposite direction as P. The spheres are modelled as particles and the coefficient of restitution between two spheres is e.

a) Find, in terms of e and u, the speeds of the two spheres after their collision.

The total kinetic energy of the two spheres after the collision is mu^2.

b) Find the value of e and hence show that P is at rest after the collision.

$\boxed{V_P = \frac{2}{3}u(1-5e)}$, $\boxed{V_Q = \frac{1}{3}u(5e+2)}$, $\boxed{e = \frac{1}{5}}$


(a)

• BY CONSERVATION OF MOMENTUM

$4mu - 2mu = mX + 2mY$

$\boxed{X + 2Y = 2u}$

• BY RESTITUTION

$e = \frac{SEP}{APP}$

$e = \frac{Y-X}{5u}$

$\boxed{-X+Y = 5eu}$

• Add equations

$3Y = 2u + 5eu$

$Y = \frac{1}{3}u(2+5e)$ (SPEED OF Q)

• $X = 2u - 2Y = 2u - 2\times\frac{1}{3}u(2+5e) = 2u - \frac{4}{3}u - \frac{10}{3}eu$

$X = \frac{2}{3}u - \frac{10}{3}eu$

$X = \frac{2}{3}u(1-5e)$ (SPEED OF P)

(b) • $KE_{AFTER} = mu^2$

$\Rightarrow \frac{1}{2}\cancel{m}X^2 + \frac{1}{2}(2\cancel{m})Y^2 = \cancel{m}u^2$

$\Rightarrow \frac{1}{2}X^2 + Y^2 = u^2$

$\Rightarrow X^2 + 2Y^2 = 2u^2$

$\Rightarrow \frac{4}{9}\cancel{u^2}(1-5e)^2 + 2\times\frac{1}{9}\cancel{u^2}(2+5e)^2 = 2\cancel{u^2}$

$\Rightarrow \frac{4}{9}(1-5e)^2 + \frac{2}{9}(2+5e)^2 = 2 \quad \times\frac{9}{2}$

$\Rightarrow 2(1-5e)^2 + (2+5e)^2 = 9$

$\Rightarrow 2(1-10e+25e^2) + 4 + 20e + 25e^2 = 9$

$\Rightarrow 2 - \cancel{20e} + 50e^2 + 4 + \cancel{20e} + 25e^2 = 9$

$\Rightarrow 75e^2 = 3$

$\Rightarrow e^2 = \frac{1}{25}$

$\Rightarrow e = \frac{1}{5}$

• FINALLY IF $e = \frac{1}{5}$

$X = \frac{2}{3}u(1-5e)$

$X = \frac{2}{3}u(1-5\times\frac{1}{5})$

$X = 0$ (INDEED AT REST)

Question 8 (***+)

A smooth sphere P of mass m is moving with constant speed on a smooth horizontal plane when it collides directly with a smooth sphere Q of mass km, where k is a positive constant, which is initially at rest. After the collision both spheres are moving in the same direction as the original direction of P with speeds u and $4u$.

The spheres are modelled as particles and the coefficient of restitution between the two spheres is e.

a) Show clearly that $4k = \frac{3}{e} - 1$.

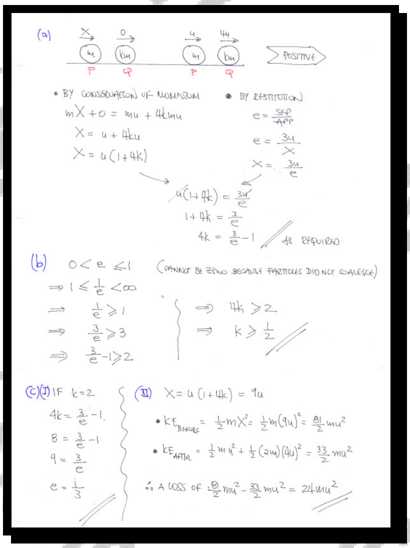
b) Hence, find the range of the possible values of k.

c) Given further that $k = 2$ …

  i. … calculate the value of e

  ii. … find the kinetic energy loss of the system, due to the collision.

$\boxed{k \geq \frac{1}{2}}$, $\boxed{e = \frac{1}{3}}$, $\boxed{\text{loss} = 24mu^2}$



(a) X 0 u 4u, m km m km, P Q P Q, POSITIVE

• BY CONSERVATION OF MOMENTUM

$mX + 0 = mu + 4kmu$

$X = u + 4ku$

$X = u(1+4k)$

• BY RESTITUTION

$e = \frac{\text{SEP}}{\text{APP}}$

$e = \frac{3u}{X}$

$X = \frac{3u}{e}$

$u(1+4k) = \frac{3u}{e}$

$1 + 4k = \frac{3}{e}$

$4k = \frac{3}{e} - 1$ // AS REQUIRED

(b) $0 < e \leq 1$ (CANNOT BE ZERO BECAUSE PARTICLES DID NOT COALESCE)

$\Rightarrow 1 \leq \frac{1}{e} < \infty$

$\Rightarrow \frac{1}{e} \geq 1$

$\Rightarrow \frac{3}{e} \geq 3$

$\Rightarrow \frac{3}{e} - 1 \geq 2$

$\Rightarrow 4k \geq 2$

$\Rightarrow k \geq \frac{1}{2}$ //

(c)(I) IF $k = 2$

$4k = \frac{3}{e} - 1$

$8 = \frac{3}{e} - 1$

$9 = \frac{3}{e}$

$e = \frac{1}{3}$ //

(II) $X = u(1+4k) = 9u$

• $KE_{\text{BEFORE}} = \frac{1}{2}mX^2 = \frac{1}{2}m(9u)^2 = \frac{81}{2}mu^2$

• $KE_{\text{AFTER}} = \frac{1}{2}mu^2 + \frac{1}{2}(2m)(4u)^2 = \frac{33}{2}mu^2$

$\therefore$ A LOSS OF $\frac{81}{2}mu^2 - \frac{33}{2}mu^2 = 24mu^2$ //

Question 9 (***+)

A smooth sphere A of mass m is moving with constant speed on a smooth horizontal plane when it collides directly with a smooth sphere B of mass $3m$, which is initially at rest. After the collision A reverses direction and is speed is v.

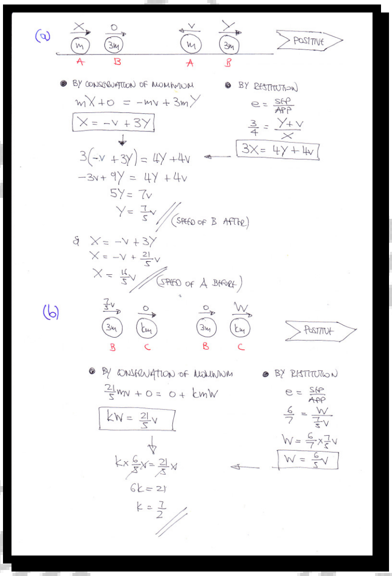
The two spheres are modelled as particles and the coefficient of restitution between the two spheres is $\frac{3}{4}$.

a) Determine, in terms of v, the speed A before the collision and the speed of B after the collision.

Consequently, B collides directly with a third sphere C of mass km, where k is a positive constant. It is further given that C was initially at rest and the coefficient of restitution between B and C is $\frac{6}{7}$.

b) By modelling C as a smooth particle, find the value of k, given that B is brought to rest after the collision.

$\boxed{V_A = \frac{16}{5}v}$, $\boxed{V_B = \frac{7}{5}v}$, $\boxed{k = \frac{7}{2}}$



Question 10 (***+)

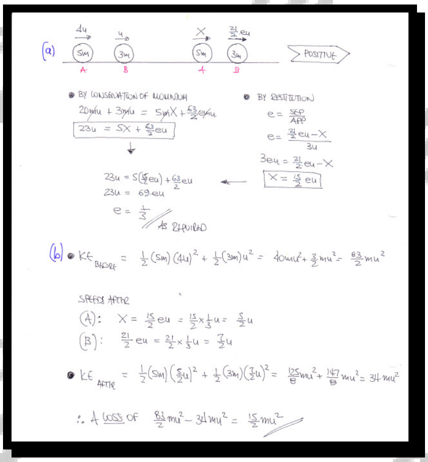
A smooth sphere A of mass $5m$ is moving with speed $4u$ on a smooth horizontal plane when it collides directly with a smooth sphere B of mass $3m$ moving with speed u, in the same direction as A. The two spheres are modelled as particles and the coefficient of restitution between them is e.

After the collision the speed of B is $\frac{21}{2}eu$.

a) Show clearly that $e = \frac{1}{3}$.

b) Find in terms of m and u the kinetic energy lost, as a result of the collision.

$$\boxed{\text{kinetic energy lost} = \frac{15}{2}mu^2}$$



(a)

4u, u, X, $\frac{21}{2}eu$

5m, 3m, 5m, 3m

A, B, A, B

POSITIVE

• BY CONSERVATION OF MOMENTUM

$20mu + 3mu = 5mX + \frac{63}{2}emu$

$\boxed{23u = 5X + \frac{63}{2}eu}$

• BY RESTITUTION

$e = \frac{SEP}{APP}$

$e = \frac{\frac{21}{2}eu - X}{3u}$

$3eu = \frac{21}{2}eu - X$

$\boxed{X = \frac{15}{2}eu}$

$23u = 5\left(\frac{15}{2}eu\right) + \frac{63}{2}eu$

$23u = 69eu$

$e = \frac{1}{3}$ // AS REQUIRED

(b) • $KE_{BEFORE} = \frac{1}{2}(5m)(4u)^2 + \frac{1}{2}(3m)u^2 = 40mu^2 + \frac{3}{2}mu^2 = \frac{83}{2}mu^2$

SPEEDS AFTER

(A): $X = \frac{15}{2}eu = \frac{15}{2} \times \frac{1}{3}u = \frac{5}{2}u$

(B): $\frac{21}{2}eu = \frac{21}{2} \times \frac{1}{3}u = \frac{7}{2}u$

• $KE_{AFTER} = \frac{1}{2}(5m)\left(\frac{5}{2}u\right)^2 + \frac{1}{2}(3m)\left(\frac{7}{2}u\right)^2 = \frac{125}{8}mu^2 + \frac{147}{8}mu^2 = 34mu^2$

$\therefore$ A LOSS OF $\frac{83}{2}mu^2 - 34mu^2 = \frac{15}{2}mu^2$ //

Question 11 (***+)

A smooth sphere A of mass 2 kg is moving with speed 12 ms^{-1} on a smooth horizontal plane when it collides directly with a smooth sphere B of mass m kg, which is initially at rest. After the collision A reverses direction and is speed is v.

The two spheres are modelled as particles and the coefficient of restitution between the two spheres is e.

a) Given that the speed of B after the collision is $2v$, show clearly that

$$m = \frac{v+12}{v}.$$

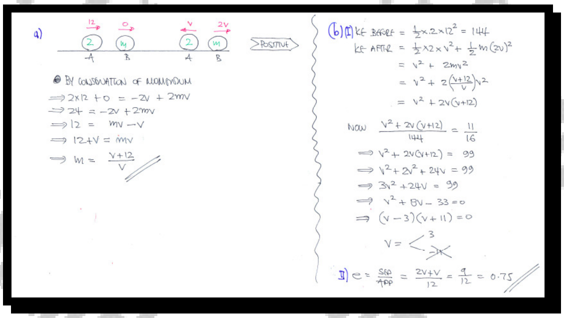
After the collision, $\frac{11}{16}$ of the initial kinetic energy is conserved.

b) Calculate ...

i. … the value of v.

ii. … the value of e.

$\boxed{v=3}$, $\boxed{e=0.75}$



Question 12 (***+)

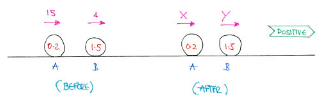
A particle A, of mass 0.2 kg, is travelling in a straight line on a smooth horizontal surface, when it collides with a particle B, of mass 1.5 kg, which is moving on the same surface and in the same direction as A.

The respective speeds of A and B just before the collision are 15 ms^{-1} and 4 ms^{-1}.

The coefficient of restitution between the two particles, e, is such so that the two particles move in the same direction after the collision.

Show that $e < \frac{6}{11}$.

, $\boxed{\text{proof}}$



- BY CONSERVATION OF MOMENTUM

$(15 \times 0.2) + (4 \times 1.5) = 0.2X + 1.5Y$

$(15 \times 2) + (4 \times 15) = 2X + 15Y$

$2X + 15Y = 90$

- BY CONSIDERING RESTITUTION

$\frac{Y - X}{15 - 4} = e$

$Y - X = 11e$

$Y = X + 11e$

- IT IS GIVEN THAT BOTH PARTICLES CONTINUE IN THE ORIGINAL DIRECTION OF MOTION, OF WHICH "B" HAS TO BUT "A" MAY HAVE REBOUNDED

- HENCE WE NEED AN EXPRESSION FOR X, THEN SET IT POSITIVE

$\Rightarrow 2X + 15(X + 11e) = 90$

$\Rightarrow 2X + 15X + 165e = 90$

$\Rightarrow 17X = 90 - 165e$

$\Rightarrow X = \frac{15}{17}(6 - 11e)$

BUT $X > 0$

$\Rightarrow 6 - 11e > 0$

$-11e > -6$

$e < \frac{6}{11}$

Question 13 (****)

Three small smooth spheres A, B and C, are resting on a straight line, and in that order, on a horizontal surface.

The respective masses of A, B and C, are m, $3m$ and $7m$.

A is project towards B with speed u and a direct collision takes place.

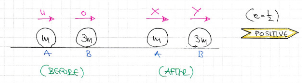
The coefficient of restitution between A and B is 0.5.

The coefficient of restitution between B and C is e.

If there is a second collision between A and B, find the range of possible values of e.

$\boxed{}$, $\boxed{\frac{19}{21} < e \le 1}$

LOOKING AT THE COLLISION BETWEEN A & B



BY CONSERVATION OF MOMENTUM

$\Rightarrow mu + 0 = mX + 3mY$

$\Rightarrow u = X + 3Y$

$\Rightarrow X + 3Y = u$

BY CONSIDERING RESTITUTION

$\Rightarrow e = \frac{\text{SEP}}{\text{APP}}$

$\Rightarrow \frac{1}{2} = \frac{Y - X}{u}$

$\Rightarrow -X + Y = \frac{1}{2}u$

ADDING EQNS

$4Y = \frac{3}{2}u$

$Y = \frac{3}{8}u$

AND USING $X = u - 3Y$

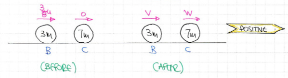
$X = u - 3\left(\frac{3}{8}u\right)$

$X = u - \frac{9}{8}u$

$X = -\frac{1}{8}u$

IE A HAS REBOUNDED (MINUS) WITH SPEED $\frac{1}{8}u$

NEXT THE COLLISION BETWEEN B & C



BY CONSERVATION OF MOMENTUM

$\Rightarrow 3m\left(\frac{3}{8}u\right) + 0 = 3mV + 7mW$

$\Rightarrow \frac{9}{8}u = 3V + 7W$

BY CONSIDERING RESTITUTION

$\Rightarrow e = \frac{\text{SEP}}{\text{APP}}$

$\Rightarrow e = \frac{W - V}{\frac{3}{8}u}$

$\Rightarrow -V + W = \frac{3}{8}ue$

$\Rightarrow -7V + 7W = \frac{21}{8}ue$

$\Rightarrow 7V - 7W = -\frac{21}{8}ue$

ADDING THE EQUATIONS ABOVE (WE ONLY NEED V)

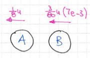
$\Rightarrow 10V = \frac{9}{8}u - \frac{21}{8}eu$

$\Rightarrow 10V = \frac{3}{8}u(3 - 7e)$

$\Rightarrow V = \frac{3}{80}u(3 - 7e)$ ← TO THE "RIGHT"

$\Rightarrow V = \frac{3}{80}u(7e - 3)$ ← TO THE "LEFT"

FOR A COLLISION BETWEEN B & A



$\Rightarrow \frac{3}{80}u(7e - 3) > \frac{1}{8}u$

$\Rightarrow 7e - 3 > \frac{10}{3}$

$\Rightarrow 7e > \frac{19}{3}$

$\Rightarrow e > \frac{19}{21}$ OR $\frac{19}{21} < e \le 1$

Question 14 (****)

A smooth sphere P of mass $2m$ is moving with speed u on a smooth horizontal plane when it collides directly with a smooth sphere Q of mass $3m$ which is initially at rest. The spheres are modelled as particles and the coefficient of restitution between the two spheres is $\frac{1}{4}$.

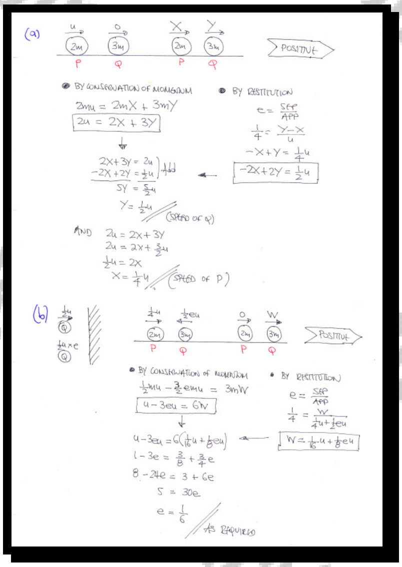
a) Find, in terms of u, the speeds of P and Q after the collision.

Consequently, sphere Q strikes at right angles a fixed smooth vertical wall, and rebounds at right angles. The coefficient of restitution between Q and the wall is e.

After a second collision between the spheres, P is brought to rest.

b) Show clearly that $e = \frac{1}{6}$.

$\boxed{V_P = \frac{1}{4}u}$, $\boxed{V_Q = \frac{1}{2}u}$



(a)

POSITIVE

• BY CONSERVATION OF MOMENTUM

$2mu = 2mX + 3mY$

$\boxed{2u = 2X + 3Y}$

• BY RESTITUTION

$e = \frac{SEP}{APP}$

$\frac{1}{4} = \frac{Y-X}{u}$

$-X + Y = \frac{1}{4}u$

$\boxed{-2X + 2Y = \frac{1}{2}u}$

$2X + 3Y = 2u$
$-2X + 2Y = \frac{1}{2}u$ Add

$5Y = \frac{5}{2}u$

$Y = \frac{1}{2}u$ (SPEED OF Q)

AND $2u = 2X + 3Y$

$2u = 2X + \frac{3}{2}u$

$\frac{1}{2}u = 2X$

$X = \frac{1}{4}u$ (SPEED OF P)

(b)

POSITIVE

• BY CONSERVATION OF MOMENTUM

$\frac{1}{2}mu - \frac{3}{2}emu = 3mW$

$\boxed{u - 3eu = 6W}$

• BY RESTITUTION

$e = \frac{SEP}{APP}$

$\frac{1}{4} = \frac{W}{\frac{1}{4}u + \frac{1}{2}eu}$

$\boxed{W = \frac{1}{16}u + \frac{1}{8}eu}$

$u - 3eu = 6\left(\frac{1}{16}u + \frac{1}{8}eu\right)$

$1 - 3e = \frac{3}{8} + \frac{3}{4}e$

$8 - 24e = 3 + 6e$

$5 = 30e$

$e = \frac{1}{6}$ AS REQUIRED

Question 15 (****)

Two smooth spheres A and B, of respective masses $4m$ and m, are moving in a straight line and in the same direction towards a smooth vertical wall. The speeds of A and B are u and $5u$ respectively. Sphere B hits the wall at right angles and rebounds so that it subsequently collides with A. Immediately after this collision the speed of A is U and the speed of B is V, with the direction of motion of each sphere reversed.

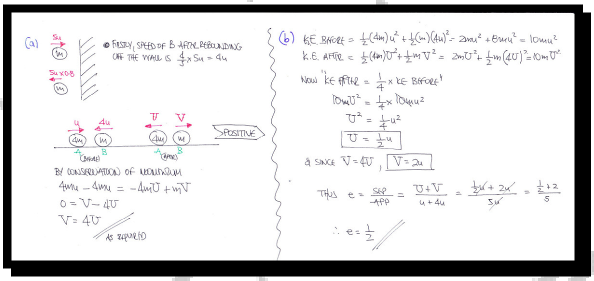
The spheres are modelled as particles and the coefficient of restitution between B and the wall is 0.8.

a) Show clearly that $V = 4U$.

The total kinetic energy of the two spheres immediately after their collision is $\frac{1}{4}$ of the total kinetic energy of the two spheres immediately before their collision.

b) Calculate the coefficient of restitution between the two spheres.

$$\boxed{e = \frac{1}{2}}$$



Question 16 (****)

Two smooth spheres A and B, of respective masses $2m$ and m, are moving with constants speeds on a smooth horizontal plane, when they collide directly.

The respective speeds of A and B after the collision are $2v$ and $11v$. Before the collision the spheres were moving in opposite directions and after the collision both spheres are moving in the original direction of motion of A.

The two spheres are modelled as particles and the coefficient of restitution between them is e.

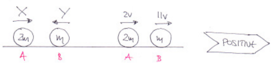
a) Find, in terms of e and v, the speeds of the two spheres before their collision.

The total kinetic energy lost as a result of the collision is $21mv^2$.

b) Find the value of e.

$\boxed{U_A = v\left(5+\frac{3}{e}\right)}$, $\boxed{U_B = v\left(\frac{6}{e}-5\right)}$, $\boxed{e = \frac{3}{4}}$

(a)



- BY CONSERVATION OF MOMENTUM

$2mX - mY = 4mv + 11mv$

$\boxed{2X - Y = 15v}$

- BY RESTITUTION

$e = \frac{SEP}{APP}$

$e = \frac{9v}{X+Y}$

$\boxed{X + Y = \frac{9v}{e}}$

$$\left.\begin{aligned} 2X - Y &= 15v \\ X + Y &= \frac{9v}{e} \end{aligned}\right\} \text{Add}$$

$3X = 15v + \frac{9v}{e}$

$X = 5v + \frac{3v}{e}$

$X = v\left(5 + \frac{3}{e}\right)$ // SPEED OF A

- ALSO BY ELIMINATION

$$\left.\begin{aligned} 2X - Y &= 15v \\ 2X + 2Y &= \frac{18v}{e} \end{aligned}\right) \text{SUBTRACT}$$

$3Y = \frac{18v}{e} - 15v$

$Y = \frac{6v}{e} - 5v$

$Y = v\left(\frac{6}{e} - 5\right)$ // SPEED OF B

(b) $KE_{BEFORE} = \frac{1}{2}(2m)X^2 + \frac{1}{2}(m)Y^2 = m\left[v\left(5+\frac{3}{e}\right)\right]^2 + \frac{1}{2}m\left[v\left(\frac{6}{e}-5\right)\right]^2$

$= mv^2\left(5+\frac{3}{e}\right)^2 + \frac{1}{2}mv^2\left(\frac{6}{e}-5\right)^2$

$= mv^2\left(25 + \frac{30}{e} + \frac{9}{e^2}\right) + \frac{1}{2}mv^2\left(\frac{36}{e^2} - \frac{60}{e} + 25\right)$

$= mv^2\left(25 + \frac{30}{e} + \frac{9}{e^2}\right) + mv^2\left(\frac{18}{e^2} - \frac{30}{e} + \frac{25}{2}\right)$

$= mv^2\left[\frac{75}{2} + \frac{27}{e^2}\right]$

$KE_{AFTER} = \frac{1}{2}(2m)(2v)^2 + \frac{1}{2}m(11v)^2 = 4mv^2 + \frac{121}{2}mv^2 = \frac{129}{2}mv^2$

NOW LOSS $= 21mv^2$

$\Rightarrow mv^2\left[\frac{75}{2} + \frac{27}{e^2}\right] - \frac{129}{2}mv^2 = 21mv^2$

$\Rightarrow \frac{75}{2} + \frac{27}{e^2} - \frac{129}{2} = 21$

$\Rightarrow \frac{27}{e^2} = 48$

$\Rightarrow e^2 = \frac{27}{48}$

$\Rightarrow e^2 = \frac{9}{16}$

$\Rightarrow e = \frac{3}{4}$ //

Question 17 **(****)**

A smooth sphere A of mass m is moving with speed $4u$ on a smooth horizontal plane. It collides directly with a smooth sphere B of mass $6m$ moving with speed u in the same direction as A. As a result of the impact the direction of motion of A is reversed. The spheres are modelled as particles.

The coefficient of restitution between the two spheres is e.

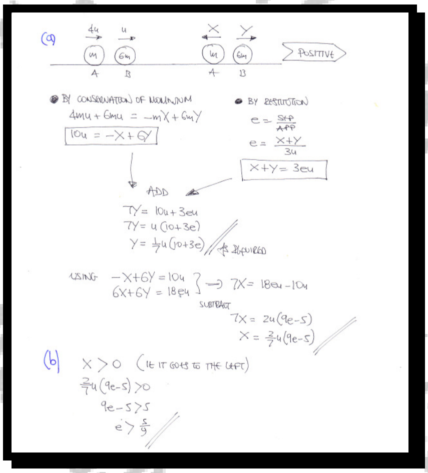
a) Show that after the collision the speed of B is $\frac{1}{7}u(10+3e)$, and find a similar expression for the speed of A.

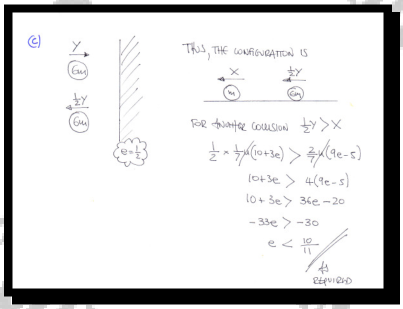
b) Deduce that $e > \frac{5}{9}$.

After the collision, B strikes at right angles a fixed smooth vertical wall, and rebounds also at right angles to the wall. The coefficient of restitution between B and the wall is $\frac{1}{2}$.

c) Given that A and B collide again show further that $e < \frac{10}{11}$.

$$\boxed{\frac{2}{7}u(9e-5)}$$





Question 18 (****)

A smooth sphere A of mass m is moving with speed u on a smooth horizontal plane when it collides directly with a smooth sphere B of mass $3m$ which is at rest.

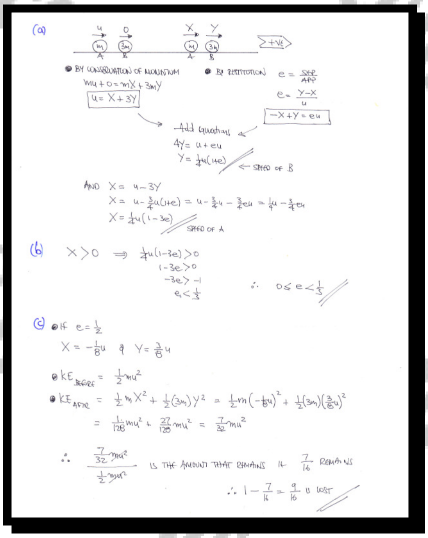
The two spheres are modelled as particles and the coefficient of restitution between them is e.

a) Find, in terms of e and u, the speeds of the two spheres after their collision.

b) Given that the direction of A is unchanged after the collision find the range of the possible values of e.

c) Given instead that $e = \frac{1}{2}$, show that $\frac{9}{16}$ of the kinetic energy is lost as a result of the collision.

$\boxed{V_A = \frac{1}{4}u(1-3e)}$, $\boxed{V_B = \frac{1}{4}u(1+e)}$, $\boxed{0 \le e < \frac{1}{3}}$



(a)

• BY CONSERVATION OF MOMENTUM

$mu + 0 = mX + 3mY$

$\boxed{u = X + 3Y}$

• BY RESTITUTION

$e = \frac{SEP}{APP}$

$e = \frac{Y-X}{u}$

$\boxed{-X + Y = eu}$

Add equations

$4Y = u + eu$

$Y = \frac{1}{4}u(1+e)$ ← SPEED OF B

AND $X = u - 3Y$

$X = u - \frac{3}{4}u(1+e) = u - \frac{3}{4}u - \frac{3}{4}eu = \frac{1}{4}u - \frac{3}{4}eu$

$X = \frac{1}{4}u(1-3e)$ SPEED OF A

(b) $X > 0 \Rightarrow \frac{1}{4}u(1-3e) > 0$

$1 - 3e > 0$

$-3e > -1$

$e < \frac{1}{3}$

$\therefore \; 0 \le e < \frac{1}{3}$

(c) • If $e = \frac{1}{2}$

$X = -\frac{1}{8}u$ & $Y = \frac{3}{8}u$

• $KE_{BEFORE} = \frac{1}{2}mu^2$

• $KE_{AFTER} = \frac{1}{2}mX^2 + \frac{1}{2}(3m)Y^2 = \frac{1}{2}m\left(-\frac{1}{8}u\right)^2 + \frac{1}{2}(3m)\left(\frac{3}{8}u\right)^2$

$= \frac{1}{128}mu^2 + \frac{27}{128}mu^2 = \frac{7}{32}mu^2$

$\therefore \; \dfrac{\frac{7}{32}mu^2}{\frac{1}{2}mu^2}$ IS THE AMOUNT THAT REMAINS ie $\frac{7}{16}$ REMAINS

$\therefore 1 - \frac{7}{16} = \frac{9}{16}$ is LOST

Question 19 (****)

A smooth sphere P of mass $4m$ is moving with speed $2u$ on a smooth horizontal plane. It collides directly with a smooth sphere Q of mass m moving with speed $5u$ in the opposite direction as P. The spheres are modelled as particles and the coefficient of restitution between P and Q is e.

a) Show that the speed of Q after the collision is $\frac{1}{5}u(3+28e)$.

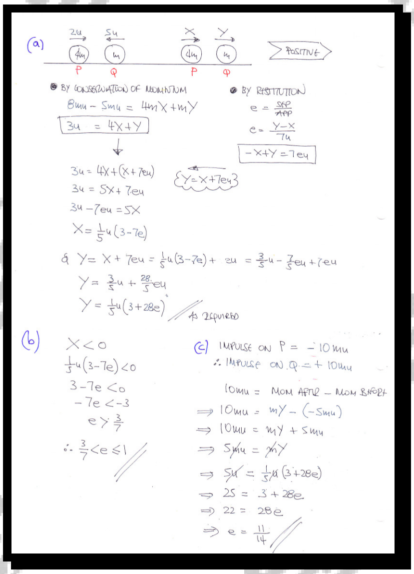
As a result of the impact the direction of motion of P is reversed after the collision.

b) Find the range of the possible values of e.

The magnitude of the impulse of Q on P is $10mu$.

c) Determine the value of e.

$\boxed{\frac{3}{7} < e \le 1}$, $\boxed{e = \frac{11}{14}}$



(a)

$2u \rightarrow$, $\leftarrow 5u$; $4m$ (P), m (Q); $X \rightarrow$, $Y \rightarrow$; $4m$ (P), m (Q); POSITIVE

• BY CONSERVATION OF MOMENTUM

$8mu - 5mu = 4mX + mY$

$\boxed{3u = 4X + Y}$

• BY RESTITUTION

$e = \frac{SEP}{APP}$

$e = \frac{Y-X}{7u}$

$\boxed{-X+Y = 7eu}$

$\{Y = X + 7eu\}$

$3u = 4X + (X + 7eu)$

$3u = 5X + 7eu$

$3u - 7eu = 5X$

$X = \frac{1}{5}u(3-7e)$

& $Y = X + 7eu = \frac{1}{5}u(3-7e) + 7eu = \frac{3}{5}u - \frac{7}{5}eu + 7eu$

$Y = \frac{3}{5}u + \frac{28}{5}eu$

$Y = \frac{1}{5}u(3+28e)$ // AS REQUIRED

(b)

$X < 0$

$\frac{1}{5}u(3-7e) < 0$

$3 - 7e < 0$

$-7e < -3$

$e > \frac{3}{7}$

$\therefore \frac{3}{7} < e \le 1$ //

(c) IMPULSE ON $P = -10mu$

$\therefore$ IMPULSE ON $Q = +10mu$

$10mu =$ MOM AFTER $-$ MOM BEFORE

$\Rightarrow 10mu = mY - (-5mu)$

$\Rightarrow 10mu = mY + 5mu$

$\Rightarrow 5\cancel{m}u = \cancel{m}Y$

$\Rightarrow 5\cancel{u} = \frac{1}{5}\cancel{u}(3+28e)$

$\Rightarrow 25 = 3 + 28e$

$\Rightarrow 22 = 28e$

$\Rightarrow e = \frac{11}{14}$ //

Question 20 (****)

A smooth sphere A of mass m is moving with speed $2u$ on a smooth horizontal plane when it collides directly with a smooth sphere B of mass $4m$ which is at rest. As a result of the collision the direction of motion of A is reversed. The two spheres are modelled as particles and the coefficient of restitution between them is e.

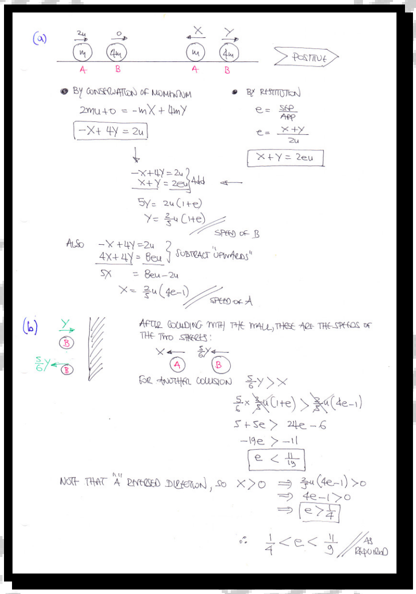
a) Find, in terms of e and u, the speeds of the two spheres after their collision.

After the collision, B strikes at right angles a fixed smooth vertical wall, and rebounds at right angles. The coefficient of restitution between B and the wall is $\frac{5}{6}$.

b) Given that A and B collide again show that

$$\frac{1}{4} < e < \frac{11}{19}.$$

$$\boxed{V_A = \frac{2}{5}u(4e-1)}, \quad \boxed{V_B = \frac{2}{5}u(1+e)}$$



(a)

$2u$, 0, X, Y

m $4m$ m $4m$

A B A B

POSITIVE

• BY CONSERVATION OF MOMENTUM

$2mu + 0 = -mX + 4mY$

$\boxed{-X + 4Y = 2u}$

• BY RESTITUTION

$e = \frac{SEP}{APP}$

$e = \frac{X+Y}{2u}$

$\boxed{X + Y = 2eu}$

$-X + 4Y = 2u$
$X + Y = 2eu$ } Add

$5Y = 2u(1+e)$

$Y = \frac{2}{5}u(1+e)$ // SPEED OF B

ALSO $-X + 4Y = 2u$
$4X + 4Y = 8eu$ } SUBTRACT "UPWARDS"

$5X = 8eu - 2u$

$X = \frac{2}{5}u(4e-1)$ // SPEED OF A

(b)

Y B

$\frac{5}{6}Y$ B

AFTER COLLIDING WITH THE WALL, THESE ARE THE SPEEDS OF THE TWO SPHERES:

X A $\frac{5}{6}Y$ B

FOR ANOTHER COLLISION $\frac{5}{6}Y > X$

$\frac{5}{6} \times \cancel{\frac{2}{5}u}(1+e) > \cancel{\frac{2}{5}u}(4e-1)$

$5 + 5e > 24e - 6$

$-19e > -11$

$\boxed{e < \frac{11}{19}}$

NOTE THAT "A" REVERSED DIRECTION, SO $X > 0 \Rightarrow \frac{2}{5}u(4e-1) > 0$

$\Rightarrow 4e - 1 > 0$

$\Rightarrow \boxed{e > \frac{1}{4}}$

$\therefore \frac{1}{4} < e < \frac{11}{19}$ // AS REQUIRED

Question 21 (****)

A smooth sphere A of mass $3m$ is moving with speed $3u$ on a smooth horizontal plane. It collides directly with a smooth sphere B of mass $4m$ moving with speed u in the same direction as A. The spheres are modelled as particles and the coefficient of restitution between A and B is $\frac{1}{2}$.

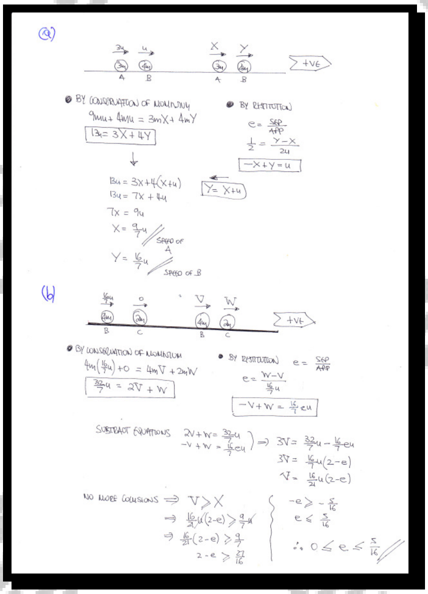
a) Find, in terms of u, the speeds of the two spheres after their collision.

After the collision between A and B, B collides directly with a third sphere C of mass $2m$ which was at rest on the same smooth horizontal plane as A and B.

The sphere C is also modelled as a particle and the coefficient of restitution between B and C is e.

b) Given that there are no more collisions between the three spheres find the range of possible values of e.

$\boxed{V_A = \frac{9}{7}u}$, $\boxed{V_B = \frac{16}{7}u}$, $\boxed{0 \le e \le \frac{5}{16}}$



(a)

- BY CONSERVATION OF MOMENTUM

$9mu + 4mu = 3mX + 4mY$

$\boxed{13u = 3X + 4Y}$

- BY RESTITUTION

$e = \frac{SEP}{APP}$

$\frac{1}{2} = \frac{Y - X}{2u}$

$\boxed{-X + Y = u}$

$\boxed{Y = X + u}$

$13u = 3X + 4(X + u)$

$13u = 7X + 4u$

$7X = 9u$

$X = \frac{9}{7}u$ // SPEED OF A

$Y = \frac{16}{7}u$ // SPEED OF B

(b)

- BY CONSERVATION OF MOMENTUM

$4m\left(\frac{16}{7}u\right) + 0 = 4mV + 2mW$

$\boxed{\frac{32}{7}u = 2V + W}$

- BY RESTITUTION $e = \frac{SEP}{APP}$

$e = \frac{W - V}{\frac{16}{7}u}$

$\boxed{-V + W = \frac{16}{7}eu}$

SUBTRACT EQUATIONS $\left.\begin{array}{l} 2V + W = \frac{32}{7}u \\ -V + W = \frac{16}{7}eu \end{array}\right\} \Rightarrow 3V = \frac{32}{7}u - \frac{16}{7}eu$

$3V = \frac{16}{7}u(2 - e)$

$V = \frac{16}{21}u(2 - e)$

NO MORE COLLISIONS $\Rightarrow V \geqslant X$

$\Rightarrow \frac{16}{21}u(2 - e) \geqslant \frac{9}{7}u$

$\Rightarrow \frac{16}{21}(2 - e) \geqslant \frac{9}{7}$

$2 - e \geqslant \frac{27}{16}$

$-e \geqslant -\frac{5}{16}$

$e \leqslant \frac{5}{16}$

$\therefore 0 \leqslant e \leqslant \frac{5}{16}$ //

Question 22 (****)

Three smooth spheres P, Q and R have respective masses $2m$, m and km, where k is a positive constant. The spheres lie at rest in a straight line, in that order, on a smooth horizontal plane. The spheres are modelled as particles. The coefficient of restitution between any pair of spheres is $\frac{1}{2}$.

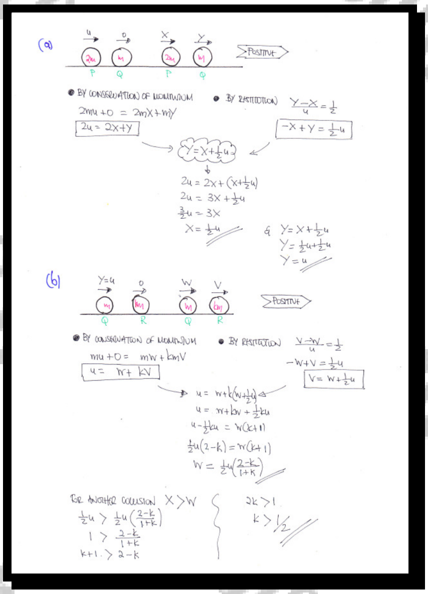
P is then projected towards Q with speed u so that the spheres collide directly.

a) Find, in terms of u, the speed of P and the speed of Q after the collision.

Consequently there is a collision between Q and R.

b) Given that there is a third collision between P and Q, show that $k > \frac{1}{2}$.

$\boxed{V_P = \frac{1}{2}u}$, $\boxed{V_Q = u}$



Question 23 (****)

A smooth sphere P of mass m is moving with speed u on a smooth horizontal plane. It collides directly with a smooth sphere Q of mass $4m$ which is initially at rest. The spheres are modelled as particles and the coefficient of restitution between P and Q is e, where $e > \frac{1}{4}$.

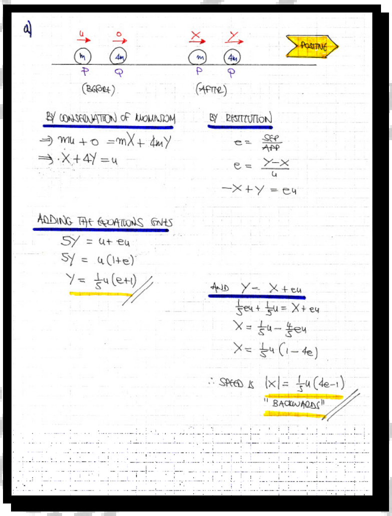
a) Show that the **speed** of P after the collision is $\frac{1}{5}u(4e-1)$ and find a similar expression for the speed of Q.

Three smooth spheres A, B and C lie in a straight line in that order on the same smooth horizontal plane. The masses of A and C are $4m$ each, while the mass of B is m. The three spheres are modelled as particles and the coefficient of restitution between any of these spheres is 0.75.

The spheres are initially at rest when B is projected towards C with speed u.

b) Show that after B and C collide, there will be another collision between A and B, and no more collisions between the spheres thereafter.

$\boxed{}$, $\boxed{v_Q = \frac{1}{5}u(1+e)}$



a)

(BEFORE) (AFTER) POSITIVE

BY CONSERVATION OF MOMENTUM

$\Rightarrow mu + 0 = mX + 4mY$

$\Rightarrow X + 4Y = u$

BY RESTITUTION

$e = \frac{\text{SEP}}{\text{APP}}$

$e = \frac{Y-X}{u}$

$-X + Y = eu$

ADDING THE EQUATIONS GIVES

$5Y = u + eu$

$5Y = u(1+e)$

$Y = \frac{1}{5}u(e+1)$

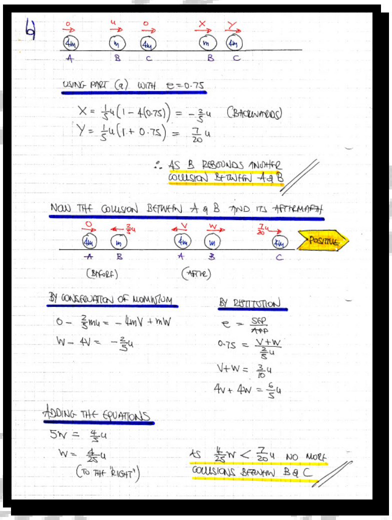
AND $Y = X + eu$

$\frac{1}{5}eu + \frac{1}{5}u = X + eu$

$X = \frac{1}{5}u - \frac{4}{5}eu$

$X = \frac{1}{5}u(1-4e)$

$\therefore$ SPEED IS $|X| = \frac{1}{5}u(4e-1)$ "BACKWARDS"



b)

USING PART (a) WITH $e = 0.75$

$X = \frac{1}{5}u(1-4(0.75)) = -\frac{2}{5}u$ (BACKWARDS)

$Y = \frac{1}{5}u(1+0.75) = \frac{7}{20}u$

$\therefore$ AS B REBOUNDS ANOTHER COLLISION BETWEEN A & B

NOW THE COLLISION BETWEEN A & B AND ITS AFTERMATH

(BEFORE) (AFTER) POSITIVE

BY CONSERVATION OF MOMENTUM

$0 - \frac{2}{5}mu = -4mV + mW$

$W - 4V = -\frac{2}{5}u$

BY RESTITUTION

$e = \frac{\text{SEP}}{\text{APP}}$

$0.75 = \frac{V+W}{\frac{2}{5}u}$

$V + W = \frac{3}{10}u$

$4V + 4W = \frac{6}{5}u$

ADDING THE EQUATIONS

$5W = \frac{4}{5}u$

$W = \frac{4}{25}u$ (TO THE "RIGHT")

AS $\frac{4}{25}W < \frac{7}{20}u$ NO MORE COLLISIONS BETWEEN B & C

Question 24 (****)

A smooth sphere P of mass m is moving with speed u on a smooth horizontal plane when it collides directly with a smooth sphere Q of mass $3m$, which is initially at rest. The spheres are modelled as particles and the coefficient of restitution between the two spheres is e.

a) Find, in terms of e and u, the speeds of the two spheres after their collision.

It is now given that P reverses direction as a result of the collision.

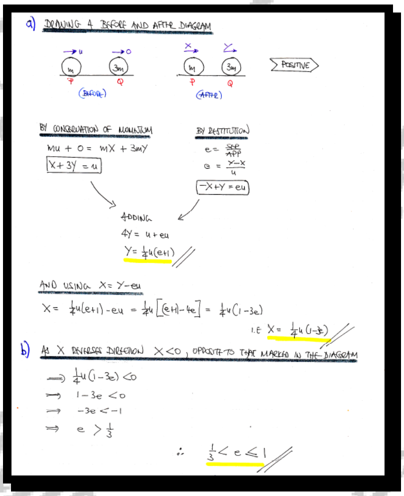
b) State the range of the possible values of e.

After the collision, Q strikes at right angles a fixed smooth vertical wall, and rebounds at right angles.

The coefficient of restitution between the Q and the wall is $2e$.

c) Show that there will always be another collision between P and Q.

FM1-K, $\boxed{V_P = \frac{1}{4}u(1-3e)}$, $\boxed{V_Q = \frac{1}{4}u(1+e)}$, $\boxed{\frac{1}{3} < e \le 1}$



a) DRAWING A BEFORE AND AFTER DIAGRAM

(BEFORE) (AFTER) POSITIVE

BY CONSERVATION OF MOMENTUM

$mu + 0 = mX + 3mY$

$\boxed{X + 3Y = u}$

BY RESTITUTION

$e = \frac{SEP}{APP}$

$e = \frac{Y-X}{u}$

$\boxed{-X+Y = eu}$

ADDING

$4Y = u + eu$

$Y = \frac{1}{4}u(e+1)$

AND USING $X = Y - eu$

$X = \frac{1}{4}u(e+1) - eu = \frac{1}{4}u\left[(e+1) - 4e\right] = \frac{1}{4}u(1-3e)$

I.E $X = \frac{1}{4}u(1-3e)$

b) AS X REVERSES DIRECTION $X<0$, OPPOSITE TO THAT MARKED IN THE DIAGRAM

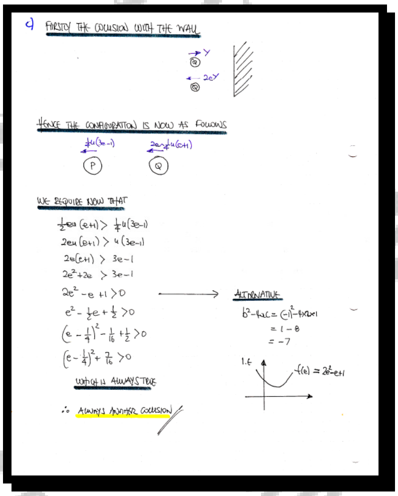
$\Rightarrow \frac{1}{4}u(1-3e) < 0$

$\Rightarrow 1 - 3e < 0$

$\Rightarrow -3e < -1$

$\Rightarrow e > \frac{1}{3}$

$\therefore \frac{1}{3} < e \le 1$



c) FIRSTLY THE COLLISION WITH THE WALL

Y, $2eY$

HENCE THE CONFIGURATION IS NOW AS FOLLOWS

$\frac{1}{4}u(3e-1)$ P, $2e \times \frac{1}{4}u(e+1)$ Q

WE REQUIRE NOW THAT

$\frac{1}{2}eu(e+1) > \frac{1}{4}u(3e-1)$

$2eu(e+1) > u(3e-1)$

$2e(e+1) > 3e-1$

$2e^2 + 2e > 3e - 1$

$2e^2 - e + 1 > 0$

$e^2 - \frac{1}{2}e + \frac{1}{2} > 0$

$\left(e - \frac{1}{4}\right)^2 - \frac{1}{16} + \frac{1}{2} > 0$

$\left(e - \frac{1}{4}\right)^2 + \frac{7}{16} > 0$

WHICH IS ALWAYS TRUE

$\therefore$ ALWAYS ANOTHER COLLISION

ALTERNATIVE

$b^2 - 4ac = (-1)^2 - 4 \times 2 \times 1$

$= 1 - 8$

$= -7$

I.E $f(e) = 2e^2 - e + 1$

Question 25 (****)

A smooth sphere A of mass m is moving with speed u on a smooth horizontal plane when it collides directly with a smooth sphere B of mass $3m$ which is initially at rest. The direction of motion of A is reversed as a result of the collision.

The spheres are modelled as particles and the coefficient of restitution between the two spheres is e.

a) Find, in terms of e and u, the speeds of the two spheres after their collision.

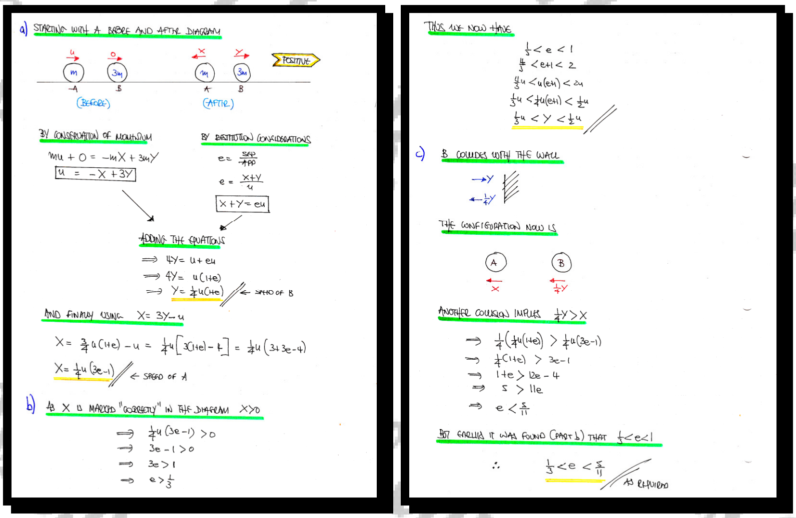
b) Find the range of the possible values of the speed of B.

Consequently sphere B strikes at right angles a fixed smooth vertical wall, and rebounds at right angles. The coefficient of restitution between B and the wall is $\frac{1}{4}$.

c) Given there is another collision between the spheres show clearly that

$$\frac{1}{3} < e < \frac{5}{11}.$$

$\boxed{}$, $\boxed{V_A = \frac{1}{4}u(3e-1)}$, $\boxed{V_B = \frac{1}{4}u(1+e)}$, $\boxed{\frac{1}{3}u < V_B < \frac{1}{2}u}$



Question 26 (****)

A particle, of mass m, lies on a smooth horizontal surface.

Initially the particle is at rest at a point O, which lies midway between a pair of fixed, smooth, parallel vertical walls, which are $2L$ apart. At time $t=0$ the particle is projected from O with speed u in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is e.

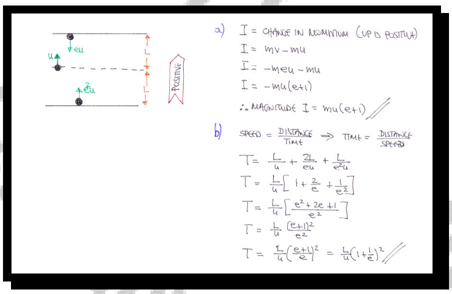
a) Find, in terms of m, u and e, the magnitude of the impulse on the particle due to the **first** impact with the wall.

The particle returns to O, having bounced off each wall once, at time $t=T$.

b) Show clearly that

$$T=\frac{L}{u}\left(1+\frac{1}{e}\right)^2.$$

$$\boxed{|I| = mu(e+1)}$$



Question 27 (****)

Two particles, A and B, of respective masses m and km, where k is a positive constant, lie on a smooth horizontal surface. Initially the particles are at rest at some point on the surface between a pair of fixed, smooth, parallel vertical walls.

A and B are simultaneously projected, with respective speeds u and $2u$, away from each other in directions perpendicular to the walls. After rebounding from the walls, A and B collide directly with each other.

The coefficient of restitution between **all** collisions in this question is taken to be e.

Given further that the direction of motion of A is **not** reversed after colliding with B, show that

$$e < \frac{1-2k}{3k}.$$

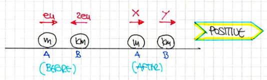
$\boxed{}$, $\boxed{\text{proof}}$

- STARTING WITH THE COLLISIONS WITH THE WALLS

AFTER REBOUNDING

$V_A = eu$ & $V_B = 2eu$

- NEXT THE COLLISION BETWEEN THEM



- BY CONSERVATION OF MOMENTUM

$\Rightarrow meu - 2kmeu = mX + kmY$

$\Rightarrow \boxed{eu - 2keu = X + kY}$

- BY THE RESTITUTION COEFFICIENT

$\Rightarrow \frac{Y-X}{eu+2eu} = e$

$\Rightarrow -X + Y = 3e^2u$

$\Rightarrow \boxed{kX - kY = -3ke^2u}$ $\times(-k)$

- ADDING THE EQUATIONS

$kX - kY = -3ke^2u$

$X + kY = eu - 2keu$

$(k+1)X = eu - 2keu - 3ke^2u$

$X = \frac{eu(1-2k-3ke)}{(k+1)}$

- A DOES NOT REVERSE ITS DIRECTION

$X > 0$

$\Rightarrow \frac{eu(1-2k-3ke)}{k+1} > 0$

$\Rightarrow 1-2k-3ke > 0 \quad [eu > 0,\ k+1 > 0]$

$\Rightarrow -3ke > 2k-1$

$\Rightarrow e < \frac{1-2k}{3k}$ // AS REQUIRED

Question 28 (****+)

A smooth sphere A of mass $3m$ is moving with speed $4u$ on a smooth horizontal plane when it collides directly with a smooth sphere B of mass $2m$ which is moving with speed u in the same direction as A. The direction of motion of A is **not** reversed as a result of the collision.

The spheres are modelled as particles and the coefficient of restitution between the two spheres is e.

a) Show that the speed of B after the collision is $\frac{1}{5}u(14+9e)$.

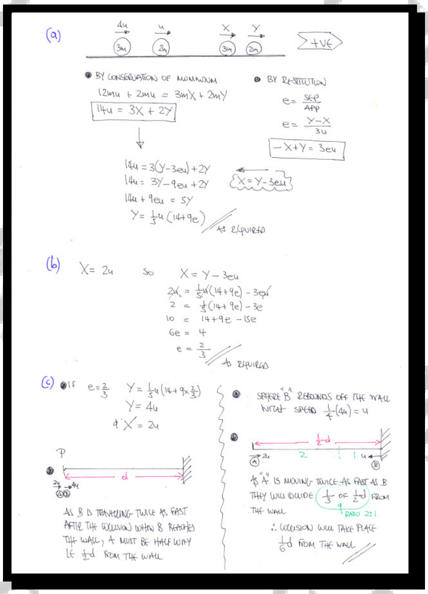
b) Given that the speed of A after the collision is $2u$ show that $e=\frac{2}{3}$.

Consequently sphere B strikes at right angles a fixed smooth vertical wall, and rebounds at right angles. The coefficient of restitution between B and the wall is $\frac{1}{4}$.

The initial collision between A and B takes place at the point P, which is at a distance d from the wall. A second collision between A and B takes place at the point Q.

c) Find, in terms of d, the distance of Q from the wall.

$\boxed{\frac{1}{6}d}$



Question 29 (****+)

A small bouncy ball is held at a height of 1.225 m above a smooth horizontal surface. The ball is released from rest and impacts the horizontal surface with speed V ms^{-1}.

a) Determine the value of V.

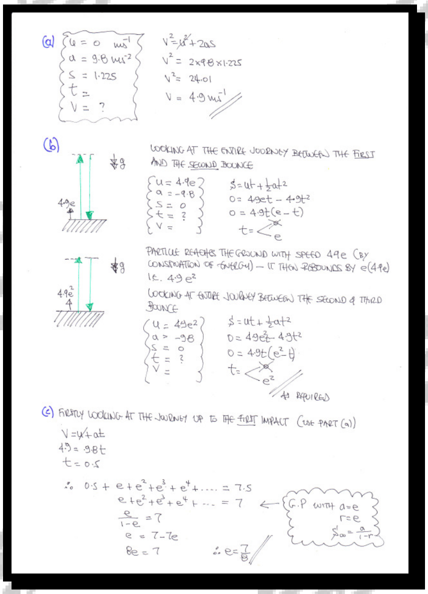
The coefficient of restitution the ball and surface is e.

b) Show that the time between the second impact and the third impact of the ball and the surface is e^2.

The bouncy ball takes 7.5 s from the instant it was first released until the instant it comes to rest.

c) Find the value of e.

$\boxed{V = 4.9 \text{ ms}^{-1}}$, $\boxed{e = \frac{7}{8}}$



(a) $u = 0$ ms^{-1}, $a = 9.8$ ms^{-2}, $s = 1.225$, $t =$, $V = ?$

$V^2 = \cancel{u^2} + 2as$

$V^2 = 2 \times 9.8 \times 1.225$

$V^2 = 24.01$

$V = 4.9$ ms^{-1}

(b) LOOKING AT THE ENTIRE JOURNEY BETWEEN THE FIRST AND THE SECOND BOUNCE

$u = 4.9e$, $a = -9.8$, $s = 0$, $t = ?$, $V =$

$s = ut + \frac{1}{2}at^2$

$0 = 4.9et - 4.9t^2$

$0 = 4.9t(e - t)$

$t = \cancel{0}$ or e

PARTICLE REACHES THE GROUND WITH SPEED $4.9e$ (BY CONSERVATION OF ENERGY) — IT THEN REBOUNDS BY $e(4.9e)$ i.e. $4.9e^2$

LOOKING AT ENTIRE JOURNEY BETWEEN THE SECOND & THIRD BOUNCE

$u = 4.9e^2$, $a = -9.8$, $s = 0$, $t = ?$, $V =$

$s = ut + \frac{1}{2}at^2$

$0 = 4.9e^2t - 4.9t^2$

$0 = 4.9t(e^2 - t)$

$t = \cancel{0}$ or e^2

AS REQUIRED

(c) FIRSTLY LOOKING AT THE JOURNEY UP TO THE FIRST IMPACT (USE PART (a))

$V = \cancel{u} + at$

$4.9 = 9.8t$

$t = 0.5$

$\therefore 0.5 + e + e^2 + e^3 + e^4 + \dots = 7.5$

$e + e^2 + e^3 + e^4 + \dots = 7$ ← G.P WITH $a = e$, $r = e$, $S_\infty = \frac{a}{1-r}$

$\frac{e}{1-e} = 7$

$e = 7 - 7e$

$8e = 7$

$\therefore e = \frac{7}{8}$

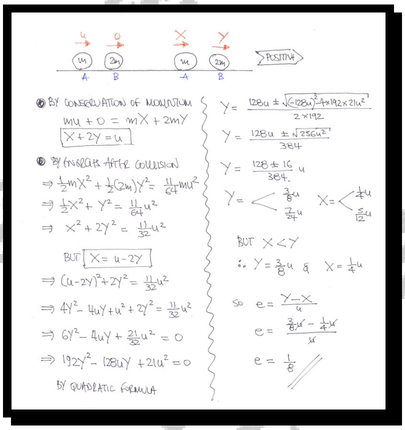
Question 30 (****+)

A smooth sphere A of mass m is moving with speed u on a smooth horizontal surface when it collides directly with a smooth sphere B of mass $2m$ which is at rest. The spheres are modelled as particles and the coefficient of restitution between the two spheres is e.

The total amount of kinetic energy after the collision is $\frac{11}{64}mu^2$

Determine the value of e.

$$\boxed{e = \frac{1}{8}}$$



BY CONSERVATION OF MOMENTUM

$mu + 0 = mX + 2mY$

$\boxed{X + 2Y = u}$

BY ENERGIES AFTER COLLISION

$\Rightarrow \frac{1}{2}mX^2 + \frac{1}{2}(2m)Y^2 = \frac{11}{64}mu^2$

$\Rightarrow \frac{1}{2}X^2 + Y^2 = \frac{11}{64}u^2$

$\Rightarrow X^2 + 2Y^2 = \frac{11}{32}u^2$

BUT $\boxed{X = u - 2Y}$

$\Rightarrow (u-2Y)^2 + 2Y^2 = \frac{11}{32}u^2$

$\Rightarrow 4Y^2 - 4uY + u^2 + 2Y^2 = \frac{11}{32}u^2$

$\Rightarrow 6Y^2 - 4uY + \frac{21}{32}u^2 = 0$

$\Rightarrow 192Y^2 - 128uY + 21u^2 = 0$

BY QUADRATIC FORMULA

$Y = \frac{128u \pm \sqrt{(-128u)^2 - 4 \times 192 \times 21u^2}}{2 \times 192}$

$Y = \frac{128u \pm \sqrt{256u^2}}{384}$

$Y = \frac{128 \pm 16}{384}u$

$Y = \frac{3}{8}u$ or $\frac{7}{24}u$ $\quad X = \frac{1}{4}u$ or $\frac{5}{12}u$

BUT $X < Y$

$\therefore Y = \frac{3}{8}u$ & $X = \frac{1}{4}u$

SO $e = \frac{Y - X}{u}$

$e = \frac{\frac{3}{8}u - \frac{1}{4}u}{u}$

$e = \frac{1}{8}$