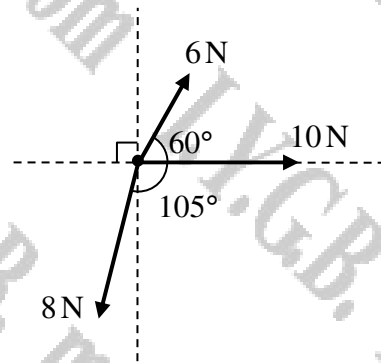


Created by T. Madas

RESULTANT FORCES

Created by T. Madas

Question 1 (**)



The figure above shows three forces which lie on the same plane, acting on a particle.

The magnitudes of these forces and their relative directions are shown in the figure.

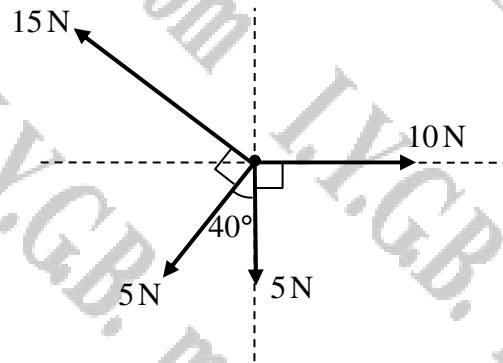
- Find the magnitude of the resultant of the above three forces.
- Give the direction of the resultant as a bearing.

$$R \approx 11.22 \text{ N}, \approx 103^\circ$$

$(\rightarrow) : 10 + 6\cos 60^\circ - 8\sin 15^\circ$
 $(\uparrow) : 6\sin 60^\circ - 8\cos 15^\circ$
 Horizontal component = 10.923...
 Vertical component = -2.531...
 "downwards"

$R = \sqrt{(10.923)^2 + (-2.531)^2}$
 $R \approx 11.22 \text{ N}$
 $\theta = 13.04^\circ$
 \therefore Bearing: $90 + 13.04 \approx 103^\circ$

Question 2 (**)



The figure above shows four forces which lie on the same plane, acting on a particle.

The magnitudes of these forces and their relative directions are shown in the figure.

- Find the magnitude of the resultant of the above four forces.
- Give the direction of the resultant as a bearing.

$$R \approx 4.77 \text{ N}, \approx 280^\circ$$

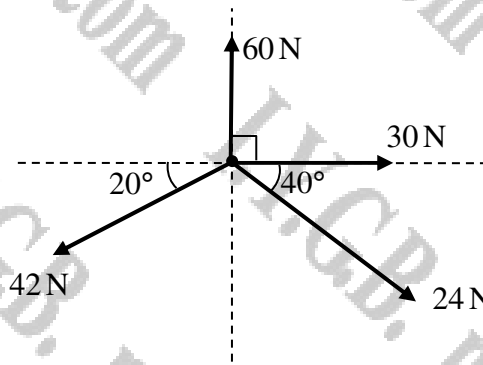
$(\rightarrow): 10 - 5 \sin 40 = 10 - 5 \sin 40 \approx 4.765 \dots$
 $(\uparrow): 5 \cos 40 = 5 - 5 \cos 40 \approx 0.612 \dots$

$R = \sqrt{(4.765)^2 + (0.612)^2}$
 $R \approx 4.77 \text{ N}$

$\tan \theta = \frac{0.612}{4.765}$
 $\theta \approx 80.2^\circ$

$\therefore \text{bearing} = 360 - 80.2 \approx 279.8^\circ$

Question 3 (**)



The figure above shows four forces which lie on the same plane, acting on a particle.

The magnitudes of these forces and their relative directions are shown in the figure.

- Find the magnitude of the resultant of the above four forces.
- Give the direction of the resultant as a bearing.

$$R \approx 31.5 \text{ N}, \approx 016.4^\circ$$

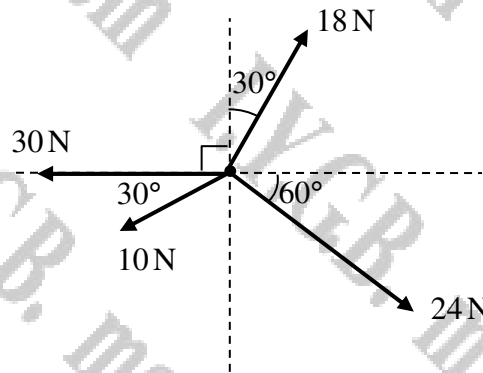
\rightarrow : $30 + 24 \cos 40^\circ = 42.572$
 \uparrow : $60 - 24 \sin 40^\circ = 42.572$

$X = 42.572$
 $Y = 42.572$

$R = \sqrt{X^2 + Y^2}$
 $R = \sqrt{42.572^2 + 42.572^2}$
 $R = 31.491$
 $R \approx 31.5 \text{ N}$

$\tan \theta = \frac{Y}{X}$
 $\tan \theta = \frac{42.572}{42.572}$
 $\theta \approx 16.4^\circ$

Question 4 (**)



The figure above shows four forces which lie on the same plane, acting on a particle.

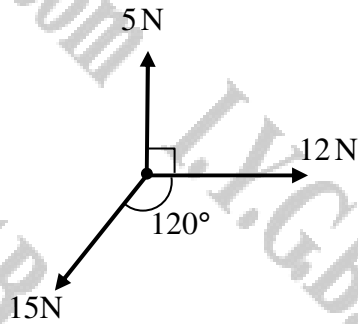
The magnitudes of these forces and their relative directions are shown in the figure.

- Find the magnitude of the resultant of the above four forces.
- Give the direction of the resultant as a bearing.

$$R \approx 20.4 \text{ N}, \quad \theta = 240^\circ$$

$\uparrow Y = 18 \cos 30 - 10 \sin 30 = 24.246$
 $\rightarrow X = 18 \sin 30 + 24 \cos 60 - 30 = -10$
 $X = -10$
 $Y = -17.66$
 $R = \sqrt{10^2 + 17.66^2}$
 $R = \sqrt{100 + 311.8756}$
 $R = \sqrt{411.8756}$
 $R = 20.3923$
 $R \approx 20.4 \text{ N}$
 $\tan \theta = \frac{|X|}{|Y|}$
 $\tan \theta = \frac{10}{17.66}$
 $\theta = 60.00$
 $\therefore \text{BEARING} = 180 + 60 = 240^\circ$

Question 5 (**)



The figure above shows three forces which lie on the same plane, acting on a particle.

The magnitudes of these forces and their relative directions are shown in the figure.

- Find the magnitude of the resultant of the above three forces.
- Find the angle the resultant makes with the 5 N force.

The direction in which these three forces act can be changed.

- State, with full justification, the least and the greatest magnitudes of the resultant force.

, $R \approx 9.17 \text{ N}$, $\approx 150.6^\circ$, $R_{\max} = 32 \text{ N}$, $R_{\min} = 0 \text{ N}$

a) REDUCE THE SYSTEM OF 3 FORCES INTO 2 FORCES

- NET FORCE TO THE "RIGHT" (\leftarrow)
 $12 - 15 \sin 30^\circ = 4.5$
- NET FORCE "DOWNWARDS" (\uparrow)
 $5 - 15 \cos 30^\circ = 5 - \frac{15\sqrt{3}}{2}$
 $\approx -7.9938 \dots$

RECOMBINING USING A NEW DIAGRAM

RESULTANT MAGNITUDE BY PYTHAGORAS

RESULTANT = $\sqrt{4.5^2 + 7.9938^2}$
 $\approx 9.17091 \dots$
 $\approx 9.17 \text{ N}$

b) BY SIMPLE TRIGONOMETRY (LOOKING AT THE PREVIOUS DIAGRAM)

$\tan \theta = \frac{7.9938}{4.5}$ $\therefore \theta = 40.412^\circ$
 \therefore REQUIRED ANGLE = $90 + \theta$
 $= 151^\circ$ (3 s.f.)

c) BY INSERION

- MAX MAGNITUDE = $12 + 5 + 15 = 32 \text{ N}$ (ALL THREE IN THE SAME DIRECTION)
- MIN MAGNITUDE = 0 N (WHEN THEY CANCEL A REMAINDER)

Question 6 (***)



Two forces, F_1 N and F_2 N, are acting on a particle at right angles to each other, as shown in the figure above. The resultant of the two forces has magnitude 41 N.

- a) Given that the magnitude of F_1 is 9 N, find the magnitude of F_2 .
- b) Determine the angle the resultant makes with F_2 .

A third force F_3 is added on the particle so that all three forces are in equilibrium.

- c) State the magnitude of F_3 .
- d) Calculate the angle F_3 makes with F_2 .

, $|F_1| = 40$, $\approx 12.68^\circ$, $|F_3| = 41$, $\approx 167.32^\circ$

a) LOOKING AT THE DIAGRAM

$\rightarrow |F_1|^2 + |F_2|^2 = 41^2$
 $\rightarrow |F_2|^2 + 9^2 = 1681$
 $\rightarrow |F_2|^2 = 1681 - 81$
 $\rightarrow |F_2|^2 = 1600$
 $\rightarrow |F_2| = 40$ //

b) BY SIMPLE TRIGONOMETRY

$\tan \theta = \frac{9}{40}$ or $\sin \theta = \frac{9}{41}$ or $\cos \theta = \frac{40}{41}$
 $\theta = 12.7^\circ$ // 3 c.f.

c) BY INSPECTION, $|F_3| = 41$, so THE RESULTANT CANCELS

d) LOOKING AT A NEW DIAGRAM

REFERENCE ANGLE IS ϕ
 $\phi = 180 - \theta$
 $\phi = 180 - 12.7$
 $\phi \approx 167.3^\circ$ //

Question 7 (***)

Three coplanar forces F_1 , F_2 and F_3 act on a particle.

F_1 has magnitude 25 N, acting in a bearing of 270° .

F_2 has magnitude X N, acting in a bearing of 180° .

F_3 has magnitude $(X + 2)$ N, acting in a bearing of 90° .

The resultant of these three forces has magnitude 37 N.

Determine, as a bearing, the angle at which the resultant of these three forces is acting.

, $\theta \approx 161^\circ$

SPLIT INTO A TRIANGLE AND RESOLVE IT TO TWO FORCES

BY PYTHAGORAS USE 90°

$$\Rightarrow X^2 + (X - 25)^2 = 37^2$$

$$\Rightarrow X^2 + X^2 - 40X + 625 = 1369$$

$$\Rightarrow 2X^2 - 40X - 744 = 0$$

$$\Rightarrow X^2 - 20X - 372 = 0$$

$$\Rightarrow (X - 35)(X + 12) = 0$$

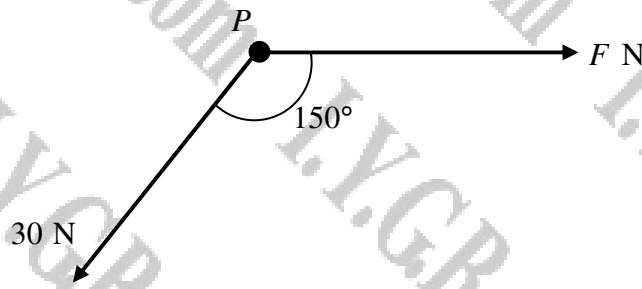
$$\Rightarrow X = 35$$

CHECK THE BEARING OF F_3 YOU GET 161°

FINALLY USE HADLEY

With $\frac{25}{12}$
 $\theta = 71.07^\circ$
 \therefore BEARING OF RESULTANT IS
 $90 + 71.07 = 161.07^\circ$
 $\approx 161^\circ$

Question 8 (****)



Two forces, act on a particle P so that the angle between the two forces is 150° .

The magnitude of one of these forces is 30 N and the magnitude of the other force is $F\text{ N}$, as shown in the figure above.

The resultant of these two forces has magnitude $R\text{ N}$, and acts at 60° to the force with magnitude $F\text{ N}$.

Calculate in any order the value of R and the value of F .

, ,

METHOD A - BY RESOLVING
 (LOOKING AT THE DIAGRAM BELOW)

- BALANCE TO THE RIGHT (\rightarrow) $F - 30 \cos 30 = R \cos 60$
- BALANCE UP/DOWNWARDS (\uparrow) $30 \sin 30 = R \sin 60$

THIS WE OBTAIN

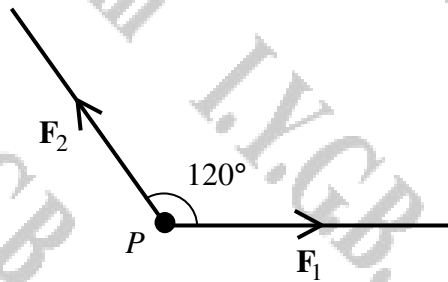
$\Rightarrow 30 \sin 30 = R \sin 60$ AND $F - 30 \cos 30 = R \cos 60$
 $\Rightarrow 30 \times \frac{1}{2} = R \times \frac{\sqrt{3}}{2}$ AND $F - 30 \times \frac{\sqrt{3}}{2} = R \times \frac{1}{2}$
 $\Rightarrow 30 = R \sqrt{3}$ AND $F - 15\sqrt{3} = \frac{R}{2}$
 $\Rightarrow 30 \sqrt{3} = 3R$ AND $F = 20 \sqrt{3}$
 $\Rightarrow R = 10 \sqrt{3} \approx 17.3\text{ N}$ AND $F = 20 \sqrt{3} \approx 34.6\text{ N}$

METHOD B - BY TRIANGLE OF FORCES
 (LOOKING AT THE DIAGRAM OPPOSITE)

- $\sin 30 = \frac{R}{30}$ AND $\cos 30 = \frac{F}{R}$
- $R = 30 \sin 30$ AND $F = \frac{30}{\cos 30}$
- $R = 10 \sqrt{3}$ AND $F = 20 \sqrt{3}$

AS ABOVE

Question 9 (***)



The figure above shows two forces F_1 and F_2 , of magnitude 24 N and x N respectively, acting on a particle P .

The angle between the lines of action of F_1 and F_2 is 120° .

The resultant of F_1 and F_2 is the force R , whose magnitude is $2x$ N.

a) Show clearly that $x = -4 + 4\sqrt{13}$.

b) Calculate the value of $|F_2 - F_1|$, correct to three significant figures.

, $|F_2 - F_1| \approx 30.6$ N

a) SIMPLY BY THE COSINE RULE ON $\triangle ABC$

$\Rightarrow |a|^2 = |b|^2 + |c|^2 - 2|b||c|\cos\theta$
 $\Rightarrow (2x)^2 = 24^2 + x^2 - 2(24)(x)\cos 120^\circ$
 $\Rightarrow 4x^2 = 576 + x^2 - 2(24)(x)(-\frac{1}{2})$
 $\Rightarrow 3x^2 + 24x - 576 = 0$
 $\Rightarrow x^2 + 8x - 192 = 0$

BY THE QUADRATIC FORMULA OR COMPLETING THE SQUARE

$\Rightarrow (x+4)^2 - 6 - 192 = 0$
 $\Rightarrow (x+4)^2 = 208$
 $\Rightarrow x+4 = \pm\sqrt{208}$
 $\Rightarrow x = -4 \pm \sqrt{208} = -4 \pm 4\sqrt{13}$ // As $x > 0$

b) LOOKING AT THE DIAGRAM BELOW

$|F_2 - F_1|^2 = |F_2|^2 + |F_1|^2 - 2|F_2||F_1|\cos 120^\circ$
 $|F_2 - F_1|^2 = x^2 + 24^2 - 2(x)(24)(-\frac{1}{2})$
 $|F_2 - F_1|^2 = x^2 + 576 + 24x$
 $|F_2 - F_1|^2 = 934 - 752016 \dots$

$\therefore |F_2 - F_1| = 30.6$ (3 s.f.)

Question 10 (****+)

Two forces, F_1 N and F_2 N, are acting on a particle so that the resultant of the two forces has magnitude 120 N and acts on a bearing of 120° .

It is further given that the F_1 acts due North and has magnitude 80 N.

Calculate in any order ...

- a) ... the magnitude of F_2
- b) ... the direction in which F_2 acts, giving the answer as a bearing.

, $|F_2| = 40\sqrt{19} \approx 174$ N , $\approx 143.4^\circ$

LOOKING AT TWO SEPARATE DIAGRAM

EQUATING 'DOWNWARD' FORCES IN THE TWO DIAGRAMS

$$2 \cos \theta = 80 = 120 \sin 30$$

EQUATING FORCES 'TO THE RIGHT' IN THE TWO DIAGRAMS

$$2 \sin \theta = 120 \cos 30$$

SOLVING SIMULTANEOUSLY

$$\begin{cases} 2 \cos \theta = 80 + 120 \sin 30 \\ 2 \sin \theta = 120 \cos 30 \end{cases} \Rightarrow \begin{cases} 2 \cos \theta = 140 \\ 2 \sin \theta = 60\sqrt{3} \end{cases}$$

SQUARING AND ADDING

$$\begin{aligned} 4 \cos^2 \theta &= 19600 \\ 4 \sin^2 \theta &= 10800 \\ \Rightarrow 4(\cos^2 \theta + \sin^2 \theta) &= 30400 \\ \Rightarrow 4 &= 30400 \\ \alpha &= 40(\sqrt{19}) \approx 174 \end{aligned}$$

DIVIDING SIDE BY SIDE

$$\begin{aligned} \frac{2 \sin \theta}{2 \cos \theta} &= \frac{60\sqrt{3}}{140} \\ \tan \theta &= 0.742367... \\ \theta &= 36.6^\circ \\ \therefore \text{BEARING} &= 180 - 36.6 \\ &\approx 143.4^\circ \end{aligned}$$

ALTERNATIVE METHOD BY GEOMETRY

LOOKING AT A TRIANGLE OF FORCES

BY THE COSINE RULE

$$\begin{aligned} 174^2 &= 80^2 + 120^2 - 2(80)(120)\cos 30 \\ 174^2 &= 6400 + 14400 - 19200 \cos 30 \\ 174^2 &= 30400 \\ 174 &= 40(\sqrt{19}) \approx 174 \end{aligned}$$

FINALLY BY THE SINE RULE

$$\begin{aligned} \frac{\sin \theta}{120} &= \frac{\sin 30}{174} \\ \Rightarrow \sin \theta &= \frac{120 \sin 30}{174} \\ \Rightarrow \sin \theta &= \frac{120 \times \frac{\sqrt{3}}{2}}{40\sqrt{19}} \\ \Rightarrow \sin \theta &= 0.53693... \\ \Rightarrow \theta &= 36.58^\circ \\ \therefore \text{BEARING} &= 180 - \theta = 143.413... \\ &\approx 143^\circ \end{aligned}$$