PROJECTILES

Question 1 (**)

A particle is projected from a point O on level horizontal ground with speed of 28 ms^{-1} at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

The particle is moving freely under gravity, reaching a greatest height above the ground before it lands on the ground at a point P.

a) Find the time it takes the particle to reach the greatest height above the ground.

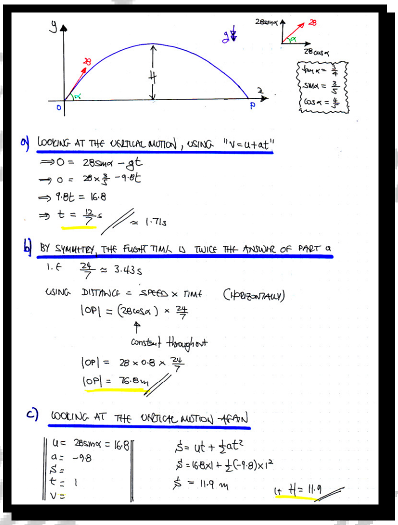
b) Hence determine the distance OP.

The particle reaches a height H m above ground, 1 second after leaving O.

c) Find the value of H.

d) Hence calculate the length of time for which the height of the particle above the ground is greater than H.

$\boxed{t = \frac{12}{7}}$, $\boxed{OP = 76.8 \text{ m}}$, $\boxed{H = 11.9}$, $\boxed{t = \frac{10}{7}}$



a) LOOKING AT THE VERTICAL MOTION, USING "v = u + at"

$\Rightarrow 0 = 28\sin\alpha - gt$

$\Rightarrow 0 = 28 \times \frac{3}{5} - 9.8t$

$\Rightarrow 9.8t = 16.8$

$\Rightarrow t = \frac{12}{7}\text{ s} \approx 1.71\text{ s}$

b) BY SYMMETRY, THE FLIGHT TIME IS TWICE THE ANSWER OF PART a

I.E $\frac{24}{7} \approx 3.43\text{ s}$

USING DISTANCE = SPEED × TIME (HORIZONTALLY)

$|OP| = (28\cos\alpha) \times \frac{24}{7}$

constant throughout

$|OP| = 28 \times 0.8 \times \frac{24}{7}$

$|OP| = 76.8\text{ m}$

c) LOOKING AT THE VERTICAL MOTION AGAIN

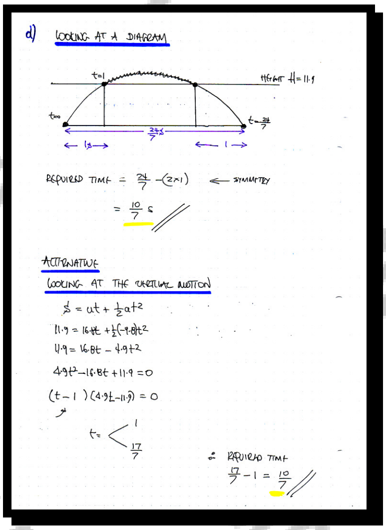
$u = 28\sin\alpha = 16.8$
$a = -9.8$
$s =$
$t = 1$
$v =$

$s = ut + \frac{1}{2}at^2$

$s = 16.8 \times 1 + \frac{1}{2}(-9.8) \times 1^2$

$s = 11.9\text{ m}$

i.e. $H = 11.9$



d) LOOKING AT A DIAGRAM

REQUIRED TIME $= \frac{24}{7} - (2 \times 1)$ ← SYMMETRY

$= \frac{10}{7}\text{ s}$

ALTERNATIVE

LOOKING AT THE VERTICAL MOTION

$s = ut + \frac{1}{2}at^2$

$11.9 = 16.8t + \frac{1}{2}(-9.8)t^2$

$11.9 = 16.8t - 4.9t^2$

$4.9t^2 - 16.8t + 11.9 = 0$

$(t-1)(4.9t - 11.9) = 0$

$t = 1$ or $t = \frac{17}{7}$

∴ REQUIRED TIME

$\frac{17}{7} - 1 = \frac{10}{7}$

Question 2 (**+)

A particle is projected from a point O on level horizontal ground with speed of 45.5 ms^{-1} at an angle β to the horizontal, where $\tan\beta = \frac{12}{5}$.

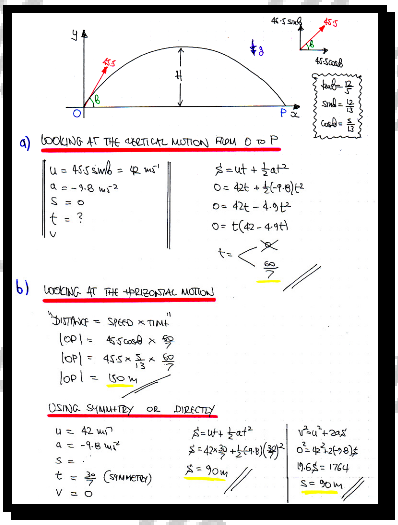
The particle is moving freely under gravity, reaching a greatest height of H m above the ground before it lands on the ground at a point P.

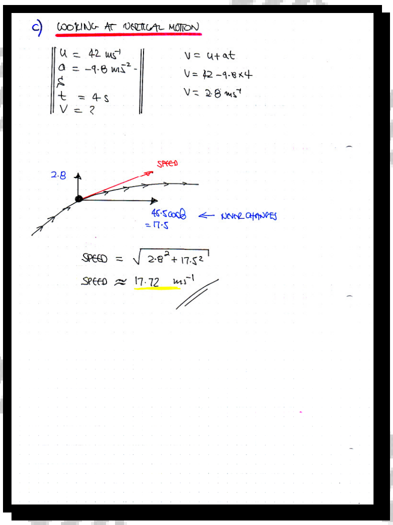
a) Find the flight time of the particle.

b) Hence determine the distance OP and the value of H.

c) Determine the speed of the particle 4 seconds after leaving O.

$\boxed{}$, $\boxed{t = \frac{60}{7}}$, $\boxed{OP = 150 \text{ m}}$, $\boxed{H = 90}$, $\boxed{v \approx 17.72 \text{ ms}^{-1}}$





Question 3 (**+)

A particle is projected from a point A on level horizontal ground with speed of 39.2 ms^{-1} at an angle θ to the horizontal.

The particle is travelling freely under gravity and after 3 s it reaches the point C, where C is at the greatest height of the particle above the ground.

a) Show that $\sin\theta = \frac{3}{4}$.

b) Find the height of C, above the ground.

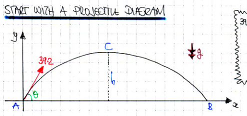
The particle eventually lands on the ground at a point B.

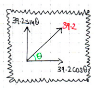
c) Calculate the distance AB.

d) State the least speed of the particle.

$\boxed{}$, $\boxed{h = 44.1 \text{ m}}$, $\boxed{|AB| \approx 155.57 \text{ m}}$, $\boxed{v_{\min} \approx 25.93 \text{ ms}^{-1}}$

START WITH A PROJECTILE DIAGRAM





LOOKING AT THE VERTICAL MOTION FROM A TO C

$u = 39.2\sin\theta \text{ ms}^{-1}$
$a = -9.8 \text{ ms}^{-2}$
$s = ?$
$t = 3 \text{ s}$
$v = 0 \text{ ms}^{-1}$

a) $v = u + at$

$\Rightarrow 0 = 39.2\sin\theta - 9.8 \times 3$

$\Rightarrow 29.4 = 39.2\sin\theta$

$\Rightarrow \sin\theta = \frac{3}{4}$ // AS REQUIRED

b) $s = ut + \frac{1}{2}at^2$

$\Rightarrow s = (39.2\sin\theta) \times 3 + \frac{1}{2}(-9.8) \times 3^2$

$\Rightarrow s = \left(39.2 \times \frac{3}{4} \times 3\right) - (4.9 \times 9)$

$\Rightarrow s = 88.2 - 44.1$

$\Rightarrow s = 44.1 \text{ m}$ //

c) BY SYMMETRY THE TOTAL FLIGHT TIME WILL BE $2 \times 3 = 6$ SECONDS

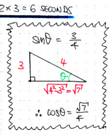
LOOKING AT THE HORIZONTAL DISPLACEMENT

DISTANCE = CONSTANT SPEED × TIME

DISTANCE = $39.2\cos\theta \times 6$

DISTANCE = $39.2 \times \frac{\sqrt{7}}{4} \times 6$

DISTANCE $\simeq 155.57$ m //



d) LEAST SPEED OCCURS AT THE HIGHEST POINT OF THE PATH, POINT C

AT THAT POINT (C) THERE IS ONLY HORIZONTAL VELOCITY

$\therefore$ LEAST SPEED $= 39.2\cos\theta$

$= 39.2 \times \frac{\sqrt{7}}{4}$

$\simeq 25.93 \text{ ms}^{-1}$ //

Question 4 (***)

A particle is projected from a point O on level horizontal ground with speed of 21 ms^{-1} at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$.

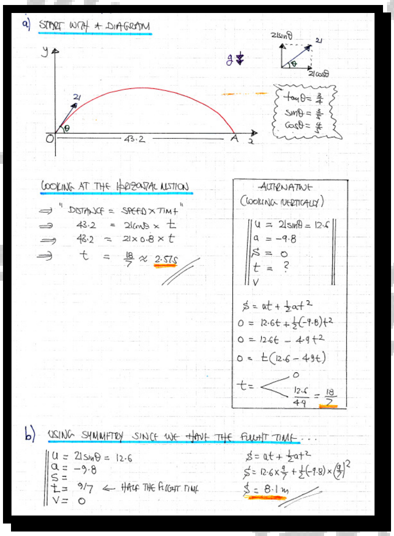
The particle is moving freely under gravity and lands at a point A on the ground, which is 43.2 m away from O.

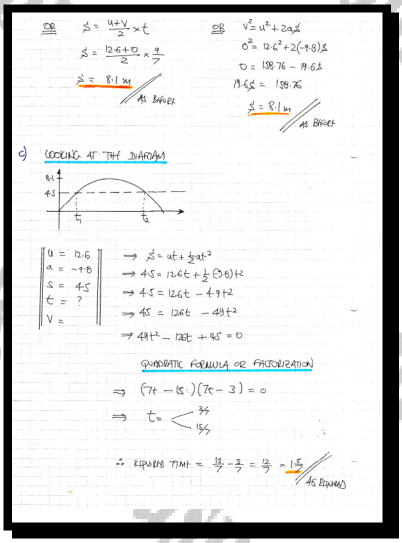
a) Find the time it takes the particle to travel from O to A.

b) Determine the greatest height above the ground reached by the particle during its flight.

c) Show that the particle remains at a height of at least 4.5 m above ground for exactly $1\frac{5}{7}$ seconds.

$\boxed{}$, $\boxed{t = \frac{18}{7}}$, $\boxed{h = 8.1 \text{ m}}$





Question 5 (***)

A golf ball is struck from a point O on level horizontal ground with a speed of 28 ms^{-1} at an angle of elevation of $30°$.

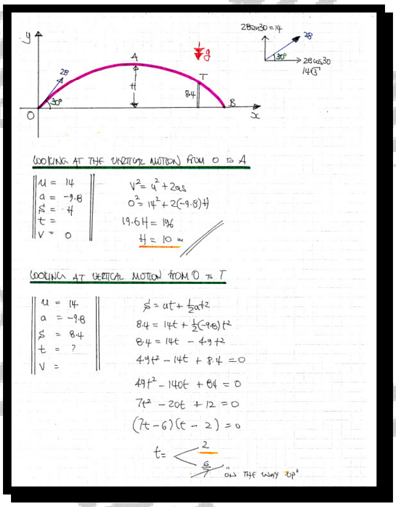
The ball is travelling freely under gravity and on its way down just clears the top of a tree T, whose height is 8.4 m. The ball is modelled as a particle.

a) Determine the greatest height achieved by the ball as it travels from O to T.

b) Calculate the time it takes the ball to travel from O to T.

c) Find the speed of the ball as it passes through T.

$\boxed{}$, $\boxed{10 \text{ m}}$, $\boxed{t = 2\text{s}}$, $\boxed{v \approx 24.9 \text{ ms}^{-1}}$



LOOKING AT THE VERTICAL MOTION FROM O TO A

$u = 14$, $a = -9.8$, $s = H$, $t =$, $v = 0$

$v^2 = u^2 + 2as$

$0^2 = 14^2 + 2(-9.8)H$

$19.6H = 196$

$H = 10 \text{ m}$

LOOKING AT VERTICAL MOTION FROM O TO T

$u = 14$, $a = -9.8$, $s = 8.4$, $t = ?$, $v =$

$s = ut + \frac{1}{2}at^2$

$8.4 = 14t + \frac{1}{2}(-9.8)t^2$

$8.4 = 14t - 4.9t^2$

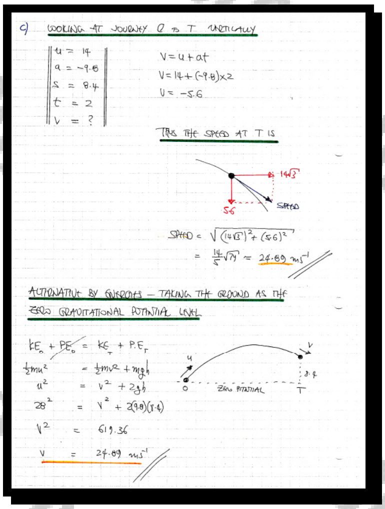
$4.9t^2 - 14t + 8.4 = 0$

$49t^2 - 140t + 84 = 0$

$7t^2 - 20t + 12 = 0$

$(7t - 6)(t - 2) = 0$

$t = 2$ or $\cancel{\frac{6}{7}}$ "ON THE WAY UP"



c) LOOKING AT JOURNEY O TO T VERTICALLY

$u = 14$, $a = -9.8$, $s = 8.4$, $t = 2$, $v = ?$

$v = u + at$

$v = 14 + (-9.8) \times 2$

$v = -5.6$

THUS THE SPEED AT T IS

$\text{SPEED} = \sqrt{(14\sqrt{3})^2 + (5.6)^2}$

$= \frac{14}{5}\sqrt{79} \approx 24.89 \text{ ms}^{-1}$

ALTERNATIVE BY ENERGIES – TAKING THE GROUND AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL

$KE_O + \cancel{PE_O} = KE_T + PE_T$

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh$

$u^2 = v^2 + 2gh$

$28^2 = v^2 + 2(9.8)(8.4)$

$v^2 = 619.36$

$v = 24.89 \text{ ms}^{-1}$

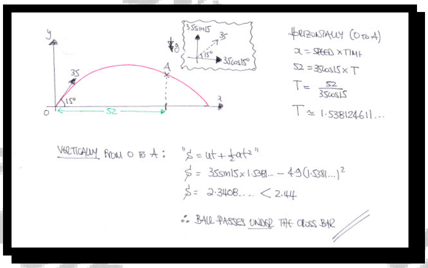
Question 6 (***)

A football is kicked with speed of 35 ms^{-1} from level horizontal ground at an angle of $15°$ to the horizontal. The ball travels towards the goalpost which stands at a height of 2.44 m, at a distance of 52 m from where it was kicked.

The ball is travelling freely under gravity, in a vertical plane perpendicular to the crossbar of the goalpost.

Determine by calculation whether the ball go over the cross bar or under it, stating any assumptions made.

$\boxed{\text{under}}$



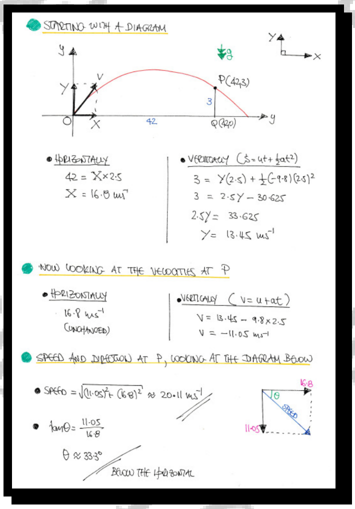
Question 7 (***)

A boy kicks a football from a point O on level horizontal ground. The football travels freely under gravity and $2\frac{1}{2}$ seconds later it just clears the top of a tree P.

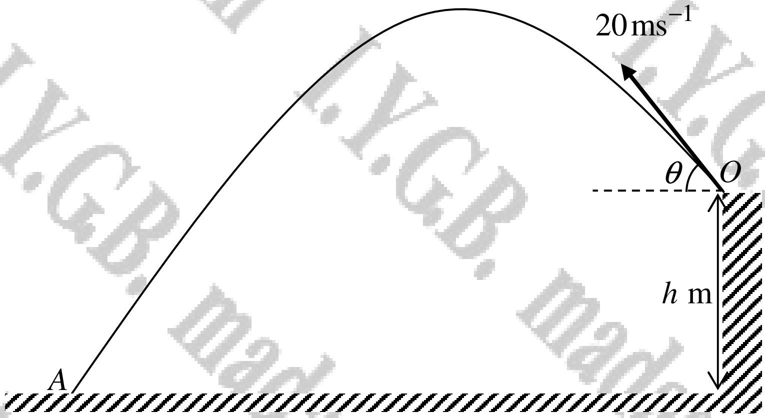
The **horizontal** distance from O to P is 42 m and the tree is 3 m tall.

Find the speed and the direction of the football as it passes through P.

$\boxed{}$, $\boxed{\text{speed} \approx 20.11 \text{ ms}^{-1}}$, $\boxed{\theta \approx 33.3° \text{ below the horizontal}}$



Question 8 (***+)



A particle P is projected with a speed of 20 ms^{-1}, at an angle of elevation θ, from a point O which is h m above level horizontal ground.

P moves freely under gravity and first strikes the ground at a point A as shown in the figure above. The speed of P just before it reaches A is 35 ms^{-1}.

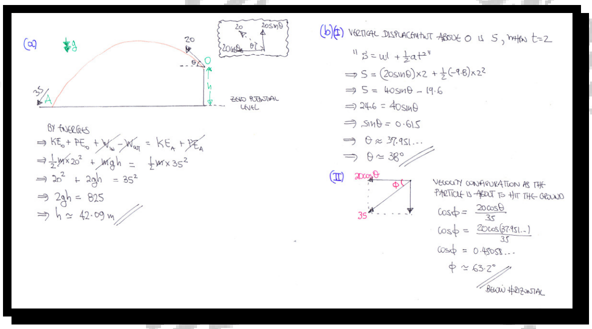
a) Use energy considerations to find the value of h.

b) Given that 2 s after it is projected from O, P is at a point 5 m above the level of O, calculate ...

i. … the value of θ.

ii. … the direction of motion of P just before it reaches A.

$\boxed{h \approx 41.09}$, $\boxed{\theta \approx 38°}$, $\boxed{\approx 63.2° \text{ below the horizontal}}$



Question 9 (***+)

A particle is projected from a point O on level horizontal ground with speed of 23.8 ms^{-1} at an angle ψ to the horizontal, where $\tan\psi = \frac{15}{8}$.

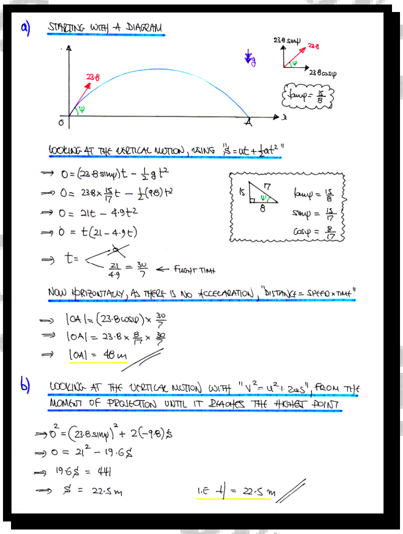
The particle is moving freely under gravity, reaching a greatest height of H m above the ground before it lands on the ground at a point A.

a) Determine the distance OA

b) Find the value of H.

c) Calculate, to three significant figures, the speed of the particle when it is at a height of 20 m above the ground.

$\boxed{}$, $\boxed{|OA| = 48 \text{ m}}$, $\boxed{H = 22.5}$, $\boxed{v \approx 13.2 \text{ ms}^{-1}}$



a) STARTING WITH A DIAGRAM

LOOKING AT THE VERTICAL MOTION, USING "$s = ut + \frac{1}{2}at^2$"

$\Rightarrow 0 = (23.8\sin\psi)t - \frac{1}{2}gt^2$

$\Rightarrow 0 = 23.8 \times \frac{15}{17}t - \frac{1}{2}(9.8)t^2$

$\Rightarrow 0 = 21t - 4.9t^2$

$\Rightarrow 0 = t(21 - 4.9t)$

$\Rightarrow t = \frac{21}{4.9} = \frac{30}{7}$ ← FLIGHT TIME

$\tan\psi = \frac{15}{8}$, $\sin\psi = \frac{15}{17}$, $\cos\psi = \frac{8}{17}$

NOW HORIZONTALLY, AS THERE IS NO ACCELERATION, "DISTANCE = SPEED × TIME"

$\Rightarrow |OA| = (23.8\cos\psi) \times \frac{30}{7}$

$\Rightarrow |OA| = 23.8 \times \frac{8}{17} \times \frac{30}{7}$

$\Rightarrow |OA| = 48 \text{ m}$

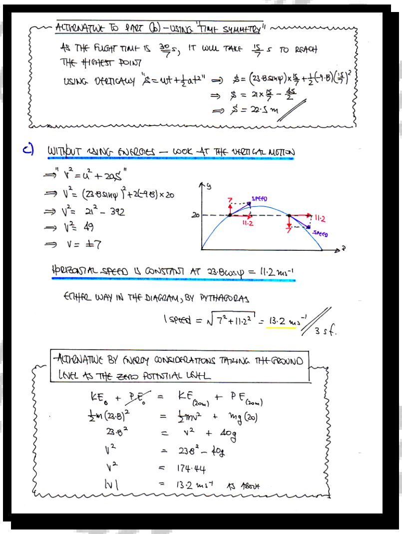
b) LOOKING AT THE VERTICAL MOTION WITH "$v^2 = u^2 + 2as$", FROM THE MOMENT OF PROJECTION UNTIL IT REACHES THE HIGHEST POINT

$\Rightarrow 0^2 = (23.8\sin\psi)^2 + 2(-9.8)s$

$\Rightarrow 0 = 21^2 - 19.6s$

$\Rightarrow 19.6s = 441$

$\Rightarrow s = 22.5 \text{ m}$ i.e. $H = 22.5$ m



ALTERNATIVE TO PART (b) – USING "TIME SYMMETRY"

AS THE FLIGHT TIME IS $\frac{30}{7}$ s, IT WILL TAKE $\frac{15}{7}$ s TO REACH THE HIGHEST POINT

USING VERTICALLY "$s = ut + \frac{1}{2}at^2$" $\Rightarrow s = (23.8\sin\psi) \times \frac{15}{7} + \frac{1}{2}(-9.8)\left(\frac{15}{7}\right)^2$

$\Rightarrow s = 21 \times \frac{15}{7} - \frac{45}{2}$

$\Rightarrow s = 22.5 \text{ m}$

c) WITHOUT USING ENERGIES – LOOK AT THE VERTICAL MOTION

$\Rightarrow$ "$v^2 = u^2 + 2as$"

$\Rightarrow v^2 = (23.8\sin\psi)^2 + 2(-9.8) \times 20$

$\Rightarrow v^2 = 21^2 - 392$

$\Rightarrow v^2 = 49$

$\Rightarrow v = \pm 7$

HORIZONTAL SPEED IS CONSTANT AT $23.8\cos\psi = 11.2 \text{ ms}^{-1}$

EITHER WAY IN THE DIAGRAM, BY PYTHAGORAS

$|\text{speed}| = \sqrt{7^2 + 11.2^2} = 13.2 \text{ ms}^{-1}$ 3 s.f.

ALTERNATIVE BY ENERGY CONSIDERATIONS TAKING THE GROUND LEVEL AS THE ZERO POTENTIAL LEVEL

$KE_0 + \cancel{PE_0} = KE_{(20m)} + PE_{(20m)}$

$\frac{1}{2}m(23.8)^2 = \frac{1}{2}mv^2 + mg(20)$

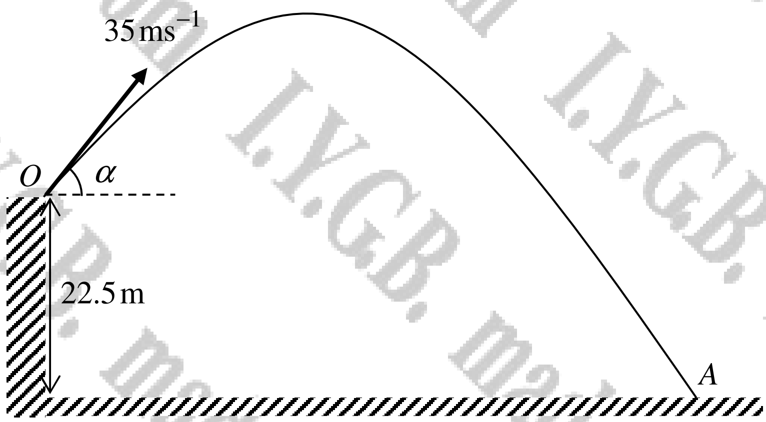
$23.8^2 = v^2 + 40g$

$v^2 = 23.8^2 - 40g$

$v^2 = 174.44$

$|v| = 13.2 \text{ ms}^{-1}$ AS ABOVE

Question 10 (*+)**



A particle is projected with a speed of 35 ms^{-1} at an angle of elevation α, where $\sin\alpha = \frac{4}{7}$ from a point O which is 22.5 m above level horizontal ground.

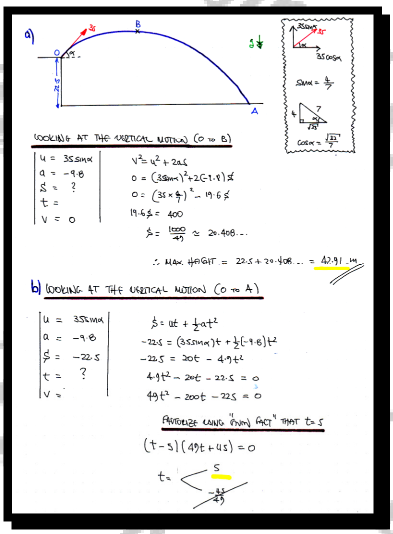
The particle is moving freely under gravity and first strikes the ground at a point A, as shown in the figure above.

a) Find the greatest height above ground, achieved by the particle.

b) Show that the flight time of the particle from O to A is 5 s.

c) Determine the speed and direction of motion of the particle as it reaches A.

$\boxed{}$, $\boxed{h_{max} \approx 42.91 \text{ m}}$, $\boxed{\text{speed} \approx 40.82 \text{ ms}^{-1}, \approx 45.3° \text{ to the ground}}$



a)

$\sin\alpha = \frac{4}{7}$, $\cos\alpha = \frac{\sqrt{33}}{7}$

LOOKING AT THE VERTICAL MOTION (O to B)

$u = 35\sin\alpha$, $a = -9.8$, $s = ?$, $t =$, $v = 0$

$v^2 = u^2 + 2as$

$0 = (35\sin\alpha)^2 + 2(-9.8)s$

$0 = (35 \times \frac{4}{7})^2 - 19.6s$

$19.6s = 400$

$s = \frac{1000}{49} \approx 20.408...$

$\therefore$ MAX HEIGHT $= 22.5 + 20.408... = 42.91$ m

b) LOOKING AT THE VERTICAL MOTION (O to A)

$u = 35\sin\alpha$, $a = -9.8$, $s = -22.5$, $t = ?$, $v =$

$s = ut + \frac{1}{2}at^2$

$-22.5 = (35\sin\alpha)t + \frac{1}{2}(-9.8)t^2$

$-22.5 = 20t - 4.9t^2$

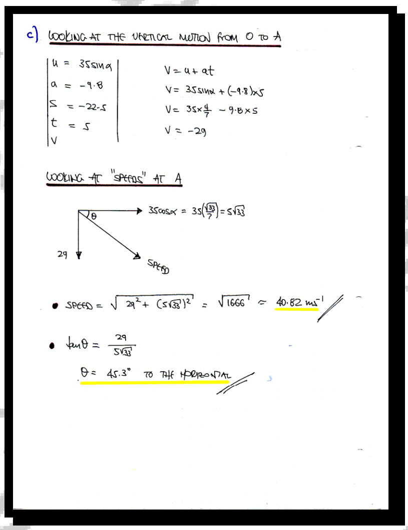
$4.9t^2 - 20t - 22.5 = 0$

$49t^2 - 200t - 225 = 0$

FACTORIZE USING "GIVEN FACT" THAT $t=5$

$(t-5)(49t+45) = 0$

$t = 5$ or $\cancel{-\frac{45}{49}}$



c) LOOKING AT THE VERTICAL MOTION FROM O TO A

$u = 35\sin\alpha$, $a = -9.8$, $s = -22.5$, $t = 5$, v

$v = u + at$

$v = 35\sin\alpha + (-9.8)\times 5$

$v = 35 \times \frac{4}{7} - 9.8 \times 5$

$v = -29$

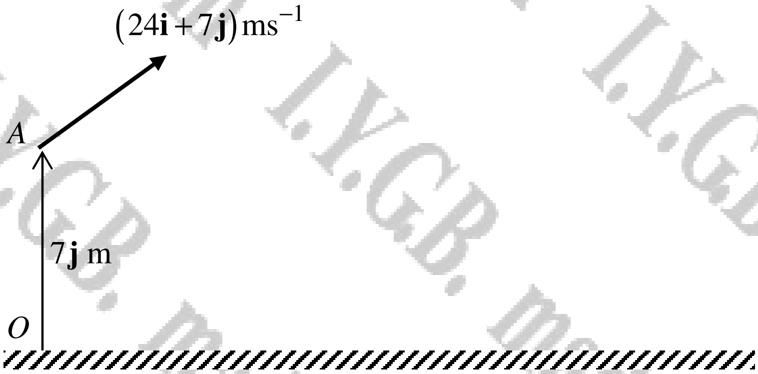
LOOKING AT "SPEEDS" AT A

$35\cos\alpha = 35\left(\frac{\sqrt{33}}{7}\right) = 5\sqrt{33}$

- SPEED $= \sqrt{29^2 + (5\sqrt{33})^2} = \sqrt{1666} \approx 40.82$ ms^{-1}
- $\tan\theta = \frac{29}{5\sqrt{33}}$

$\theta = 45.3°$ TO THE HORIZONTAL

Question 11 (***+)



A fixed origin O is located on level horizontal ground and the vectors $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing horizontally and vertically, respectively.

A particle P is projected from the point A with position vector $7\mathbf{j}$ m with velocity $(24\mathbf{i}+7\mathbf{j})$ ms^{-1}, as shown in the figure above.

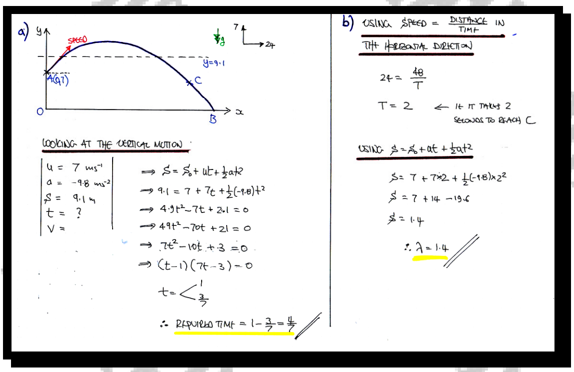
In the subsequent motion the particle is moving freely under gravity.

a) Find the length of time for which P is at least 9.1 m above the ground.

P is passing through the point with position vector $(48\mathbf{i}+\lambda\mathbf{j})$ m.

b) Determine the value of λ.

$\boxed{\quad}$, $\boxed{\frac{4}{7}\text{ s}}$, $\boxed{\lambda = 1.4}$



Question 12 (***+)

A cricket ball is struck from a point A which is 1 m above level horizontal ground with speed of 25 ms^{-1} at an angle $30°$ above the horizontal.

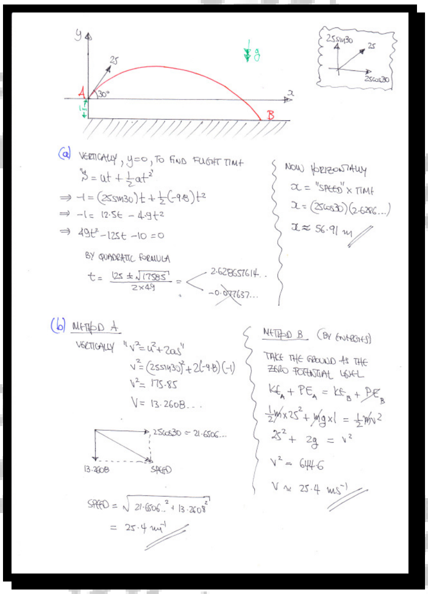
The ball first hits the ground at a point B.

The ball is modelled as particle moving through still air without any resistance.

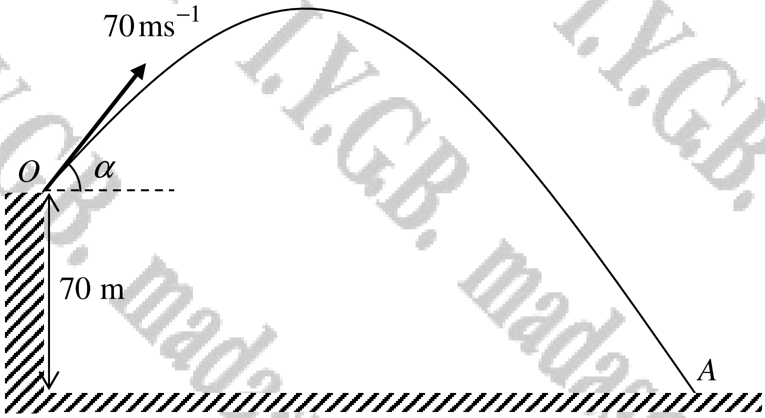
a) Determine the horizontal distance from A to B.

b) Calculate, to three significant figures, the speed of the ball as it reaches B.

$\boxed{|AB| \approx 56.91 \text{ m}}$, $\boxed{v \approx 25.39 \text{ ms}^{-1}}$



Question 13 (***+)



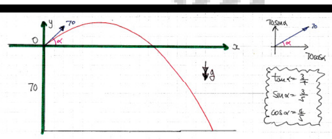
A particle is projected with a speed of $70\ \text{ms}^{-1}$ at an angle of elevation α, where $\tan\alpha = \frac{3}{4}$, from a point O which is 70 m above level horizontal ground.

The particle is moving freely under gravity and first strikes the ground at a point A as shown in the figure above.

a) Find the horizontal distance of A from O.

b) Show that as the particle reaches A, it hits the ground at $45°$

$\boxed{}$, $\boxed{560\text{ m}}$



$\tan\alpha = \frac{3}{4}$
$\sin\alpha = \frac{3}{5}$
$\cos\alpha = \frac{4}{5}$

a) LOOKING AT THE VERTICAL MOTION

$u = 70\sin\alpha = 42$
$a = -9.8$
$s = -70$
$t = ?$
$v = ?$ (FOR PART b)

$\Rightarrow s = ut + \frac{1}{2}at^2$
$\Rightarrow -70 = 42t + \frac{1}{2}(-9.8)t^2$
$\Rightarrow -70 = 42t - 4.9t^2$
$\Rightarrow 4.9t^2 - 42t - 70 = 0$
$\Rightarrow 49t^2 - 420t - 700 = 0$
$\Rightarrow 7t^2 - 60t - 100 = 0$
$\Rightarrow (7t + 10)(t - 10) = 0$
$\Rightarrow t = \begin{cases} 10 & \leftarrow \text{FLIGHT TIME} \\ \cancel{-\frac{10}{7}} \end{cases}$

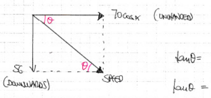
HORIZONTALLY NOW, DISTANCE = SPEED × TIME

$\Rightarrow x = 70\cos\alpha \times 10$
$\Rightarrow x = 70 \times \frac{4}{5} \times 10$
$\Rightarrow x = 560\text{ m}$

b) LOOKING AT THE VERTICAL MOTION IN PART (a)

$\Rightarrow v = u + at$
$\Rightarrow v = 42 + (-9.8)10$
$\Rightarrow v = -56$

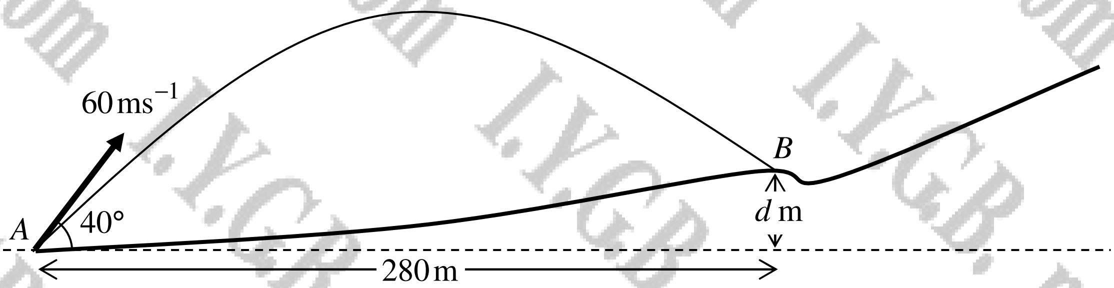
THUS AS THE PARTICLE REACHES A



$\tan\theta = \frac{56}{70\cos\alpha}$
$\tan\theta = 1$
$\theta = 45°$

AS REQUIRED

Question 14 (***+)



A golf ball is driven from a point A with speed 60 ms^{-1} at an angle of elevation $40°$ and lands at a point B. The point B lies in the same vertical plane as A at a vertical distance of d m above the level of A and a horizontal distance of 280 m from A, as shown in the figure above.

The ball is modelled as a particle moving freely under the action of its weight.

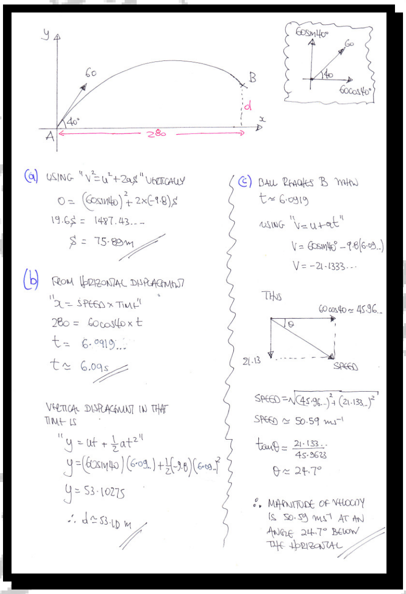
a) Determine the maximum height of the ball above the level of A.

b) Calculate the time it takes the ball to travel from A to B and hence deduce the value of d.

c) Find the magnitude and direction of the velocity of the ball as it reaches B.

$\boxed{\approx 75.89 \text{ m}}$, $\boxed{t \approx 6.09}$, $\boxed{d \approx 53.10 \text{ m}}$,

$\boxed{\text{magnitude} \approx 50.59 \text{ms}^{-1}, \text{ at } 24.7° \text{ below the horizontal}}$



(a) USING "$v^2 = u^2 + 2as$" VERTICALLY

$0 = (60\sin 40)^2 + 2 \times (-9.8)s$

$19.6s = 1487.43...$

$s = 75.89 \text{ m}$

(b) FROM HORIZONTAL DISPLACEMENT

"x = SPEED × TIME"

$280 = 60\cos 40 \times t$

$t = 6.0919...$

$t \simeq 6.09 \text{ s}$

VERTICAL DISPLACEMENT IN THAT TIME IS

"$y = ut + \frac{1}{2}at^2$"

$y = (60\sin 40)(6.09..) + \frac{1}{2}(-9.8)(6.09..)^2$

$y = 53.10275$

$\therefore d \simeq 53.10 \text{ m}$

(c) BALL REACHES B WHEN $t \approx 6.0919$

USING "$v = u + at$"

$V = 60\sin 40° - 9.8(6.09..)$

$V = -21.1333...$

THUS

$60\cos 40 \simeq 45.96..$

21.13

SPEED

SPEED $= \sqrt{(45.96..)^2 + (21.133..)^2}$

SPEED $\simeq 50.59 \text{ ms}^{-1}$

$\tan\theta = \frac{21.133..}{45.9623}$

$\theta \simeq 24.7°$

$\therefore$ MAGNITUDE OF VELOCITY IS 50.59 ms^{-1} AT AN ANGLE 24.7° BELOW THE HORIZONTAL

Question 15 (***+)

A particle is projected from a point O on level horizontal ground with a speed of 30 ms^{-1} at an angle of elevation of $50°$. The particle moves freely under the action of its own weight and after T s it hits the ground at a point A, where $|OA| = X$ m.

a) Find, to two decimal places, the value of T and the value of X.

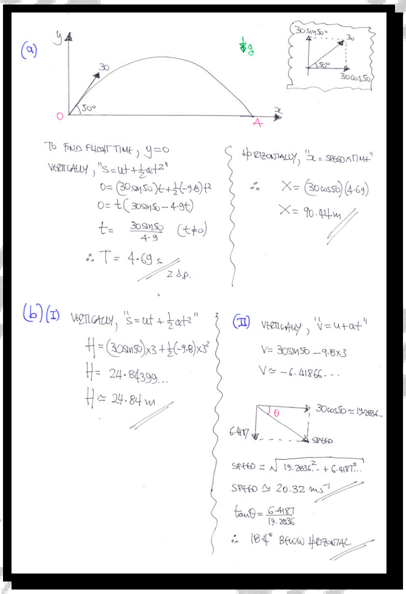
At time 3 s after leaving O, the particle is at a height H m above level horizontal ground, moving with velocity V ms^{-1}.

b) Determine …

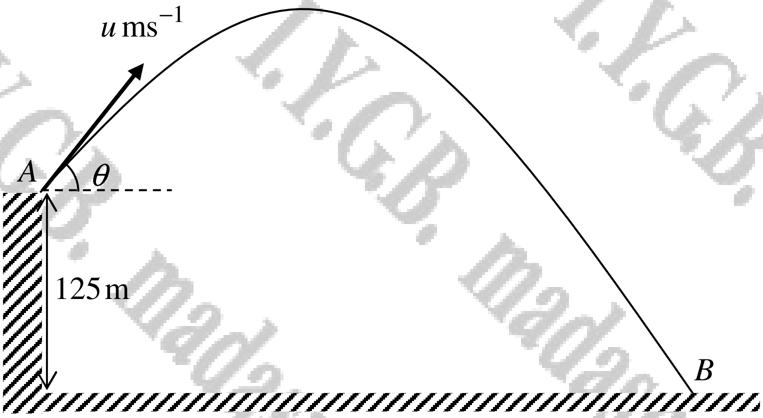
i. … the value of H.

ii. … the magnitude and direction of V.

$\boxed{T \approx 4.69}$, $\boxed{X \approx 90.44}$, $\boxed{H \approx 24.84}$, $\boxed{|V| \approx 20.32 \text{ ms}^{-1}\text{, at} \approx 18.4° \text{ below horizontal}}$



Question 16 (***+)



A particle P is projected with a speed of u ms^{-1} at an angle of elevation θ, from a point A which is 125 m above level horizontal ground. The particle is moving freely under gravity and first strikes the ground at a point B, as shown in the figure above.

It took T s for P to travel from A to B, and the speed of the particle at B is $3u$ ms^{-1}.

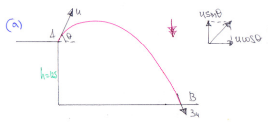
a) Use energy conservation to find the value of u.

b) Given that $T = 6$, calculate the value of θ.

c) Determine the minimum speed of P during the journey from A to B.

$\boxed{u = 17.5}$, $\boxed{\theta \approx 29.31°}$, $\boxed{v_{\text{min}} \approx 15.26 \text{ ms}^{-1}}$

(a)



TAKING THE LEVEL OF A, AS THE ZERO POTENTIAL LEVEL

$\Rightarrow KE_A + PE_A = KE_B + \cancel{PE_B}$

$\Rightarrow \frac{1}{2}mu^2 + mgh = \frac{1}{2}m(3u)^2$

$\Rightarrow u^2 + 2gh = 9u^2$

$\Rightarrow u^2 + 2g \times 125 = 9u^2$

$\Rightarrow 2450 = 8u^2$

$\Rightarrow u^2 = 306.25$

$\Rightarrow u = 17.5 \text{ ms}^{-1}$

(b) VERTICALLY: "$s = ut + \frac{1}{2}at^2$"

$-125 = (17.5\sin\theta)\times 6 + \frac{1}{2}(-9.8)\times 6^2$

$-125 = 105\sin\theta - 176.4$

$51.4 = 105\sin\theta$

$\sin\theta = \frac{257}{525}$

$\theta \simeq 29.31°$

(c) BY ENERGIES MINIMUM SPEED WILL OCCUR AT THE HIGHEST POINT OF THE PATH, IE AT THE POINT OF MAXIMUM POTENTIAL ENERGY. BUT AT THE HIGHEST POINT THERE IS ONLY HORIZONTAL SPEED

So $V_{MIN} = 17.5\cos(29.31)$

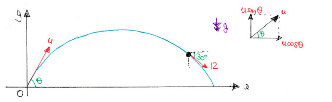
$V_{MIN} \simeq 15.26 \text{ ms}^{-1}$

Question 17 (****)

A particle is projected from horizontal ground, at some angle of elevation, and 3 s later is observed moving with speed 12 ms^{-1}, at an angle of $30°$ **below** the horizontal.

Determine the time for which the particle has a speed less than 12 ms^{-1}.

$\boxed{}$, $\boxed{t = \frac{12}{g} = \frac{60}{49} \approx 1.22 \text{ s}}$



- LOOKING AT VERTICAL, AND CONSIDERING VELOCITY IN THAT DIRECTION

"$v = u + at$"

$-12\sin 30 = u\sin\theta - g \times 3$

$-6 = u\sin\theta - 3g$

$u\sin\theta = 3g - 6$

- REPEAT, LOOKING FOR VERTICAL VELOCITY +6

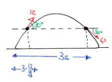
"$v = u + at$"

$6 = u\sin\theta - gt$

$6 = 3g - 6 - gt$

$gt = 3g - 12$

$t = 3 - \frac{12}{g}$



- HENCE THE REQUIRED TIME IS

$3 - \left(3 - \frac{12}{g}\right) = \frac{12}{g} = \frac{60}{49}$

$\approx 1.22\text{s}$

Question 18 (****)

A man in a park throws a small ball and his dog catches it with his mouth.

The man throws the ball from a height of 1.6 m and the dog catches the ball at a height of 0.7 m.

The man throws the ball with speed 15 ms^{-1} at an angle of $30°$ above the horizontal.

The ball is modelled as a particle and the park grounds as a level horizontal plane.

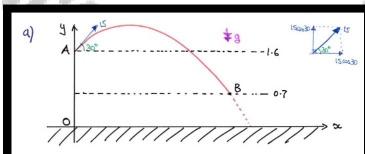
a) Calculate the horizontal distance covered by the ball during its flight from the man's hand to the dog's mouth.

b) Find the greatest speed of the ball during its flight.

You may not use direct energy calculations for this part.

$\boxed{d \approx 21.34 \text{ m}}$, $\boxed{V_{max} \approx 15.58 \text{ ms}^{-1}}$

a)



- LOOKING AT THE JOURNEY FROM A TO B, IN THE VERTICAL

$\Rightarrow s = s_0 + ut + \frac{1}{2}at^2$

$\Rightarrow 0.7 = 1.6 + (15\sin 30°)t + \frac{1}{2}(-9.8)t^2$

$\Rightarrow 0.7 = 1.6 + 7.5t - 4.9t^2$

$\Rightarrow 4.9t^2 - 7.5t - 0.9 = 0$

$\Rightarrow 49t^2 - 75t - 9 = 0$

- BY THE QUADRATIC FORMULA WE OBTAIN

$\Rightarrow t = \frac{75 \pm \sqrt{7389}}{2 \times 49} = \begin{cases} \cancel{-0.11\ldots} \\ 1.64244\ldots \leftarrow \text{FLIGHT TIME} \end{cases}$

- LOOKING AT THE HORIZONTAL MOTION WE HAVE

$\Rightarrow$ "DISTANCE = "SPEED × "TIME"

$\Rightarrow x = (15\cos 30°) \times 1.64244\ldots$

$\Rightarrow x = 21.3359\ldots \approx 21.34 \text{ m}$ (2 dp)

b)

- NOW USING "$v = u + at$" WE CAN FIND THE VERTICAL VELOCITY OF THE BALL AT B

$\Rightarrow V = (15\sin 30) + (-9.8)(1.6424\ldots)$

$\Rightarrow V = -8.5959\ldots \text{ ms}^{-1}$

- THE MAXIMUM SPEED WILL OCCUR AT B, IE AT THE LOWEST POINT OF THE PATH

AS THE HORIZONTAL SPEED IS CONSTANT, WE HAVE

$\Rightarrow |V_{MAX}| = \sqrt{(-8.5959\ldots)^2 + (15\cos 30)^2}$

$\Rightarrow |V_{MAX}| = 15.58 \text{ ms}^{-1}$ (2 d.p.)

ALTERNATIVE BY ENERGIES (NOT ALLOWED HERE)

- TAKING THE GROUND AS THE ZERO POTENTIAL LEVEL

$\Rightarrow KE_A + P.E_A = KE_B + PE_B$

$\Rightarrow \frac{1}{2}\cancel{m}u^2 + \cancel{m}gH = \frac{1}{2}\cancel{m}v^2 + \cancel{m}gh$

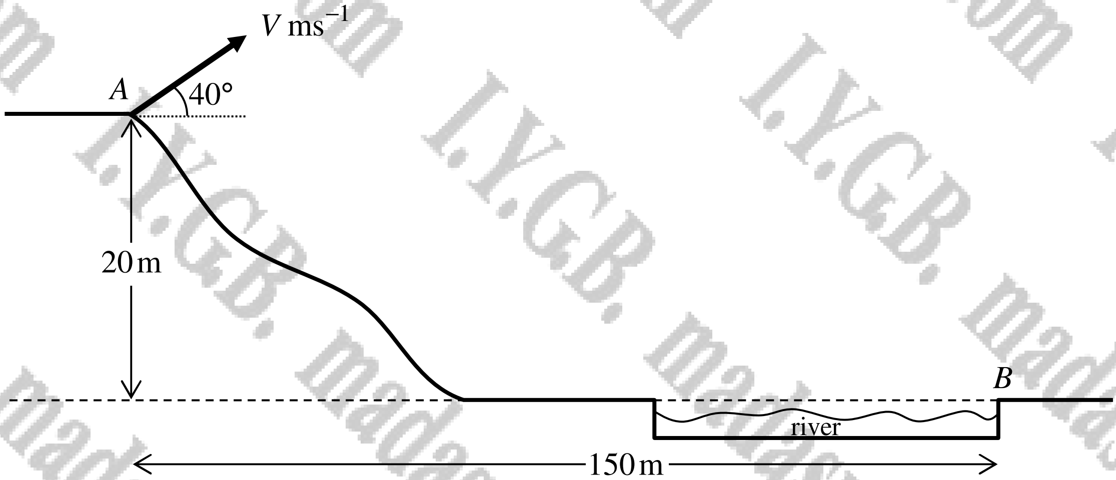
$\Rightarrow u^2 + 2gH = v^2 + 2gh$

$\Rightarrow 15^2 + 2g(1.6) = v^2 + 2g(0.7)$

$\Rightarrow v^2 = 242.64$

$\Rightarrow |v| = 15.58 \text{ ms}^{-1}$ (2 d.p)

Question 19 (****)



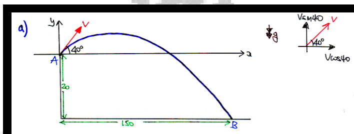
A golf ball is struck from a point A with a speed V ms^{-1} at an angle of elevation $40°$, so it can clear a river. The ball is modelled as a particle moving freely under the action of its weight. In order to clear the river the ball must land further than a point B on the opposite river bank. The point B lies a vertical distance of 20 m below the level of A and a horizontal distance of 150 m from A, as shown in the figure above.

a) Find, to two decimal places, the minimum value of V.

b) Given instead that $V = 50$, determine the magnitude and direction of the velocity of the ball 5 s after it was struck.

$\boxed{}$, $\boxed{V_{\text{min}} \approx 35.89}$, $\boxed{\text{speed} \approx 41.85 \text{ ms}^{-1}, \approx 23.8° \text{ below horizontal}}$

a)



WE CONSIDER THE LIMITING CASE, IE THE VALUE OF V, SO THAT IT LANDS ON B – LET THE FLIGHT TIME BE T

- VERTICALLY FROM A TO B

  "$s = ut + \frac{1}{2}at^2$"

  $-20 = (V\sin40)T + \frac{1}{2}(-9.8)T^2$

  $-20 = VT\sin40 - 4.9T^2$

- HORIZONTALLY FROM A TO B

  "x = SPEED × TIME"

  $150 = V\cos40 \times T$

  $150 = VT\cos40$

SOLVING SIMULTANEOUSLY

$VT = \frac{150}{\cos40} \implies -20 = \left(\frac{150}{\cos40}\right)\sin40 - 4.9T^2$

$\implies -20 = 150\tan40 - 4.9T^2$

$\implies 4.9T^2 = 150\tan40 + 20$

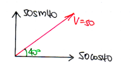
$\implies T^2 = 29.768....$

$\implies T = 5.456038...$

$\therefore V = \frac{150}{T\cos40} = 35.888...$

$\therefore V_{\text{MIN}} = 35.89$ ms^{-1}

b) IF $V = 50$ – LOOKING AT THE VELOCITY COMPONENTS



VERTICALLY "$v = u + at$"

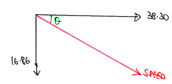
$v = 50\sin40 - 9.8\times5$

$v = -16.86...$

HORIZONTALLY UNCHANGED

$50\cos40 \simeq 38.30...$

LOOKING AT THE SPEEDS WHEN $t = 5$

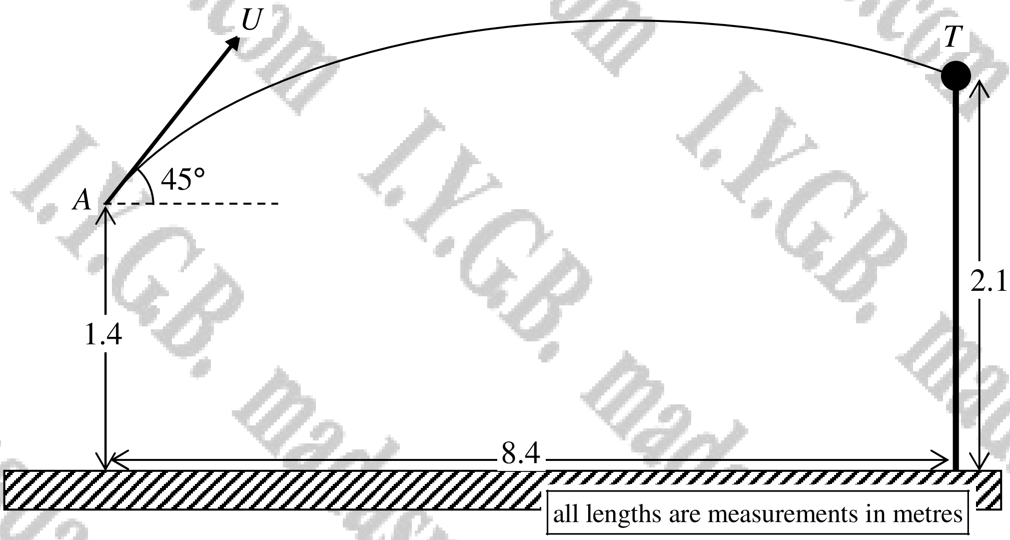


- SPEED $= \sqrt{(16.86...)^2 + (38.30...)^2} \simeq 41.85$ ms^{-1}
- $\tan\theta = \frac{16.86...}{38.30...}$

  $\theta = 23.8°$

  BELOW THE HORIZONTAL

Question 20 (****)



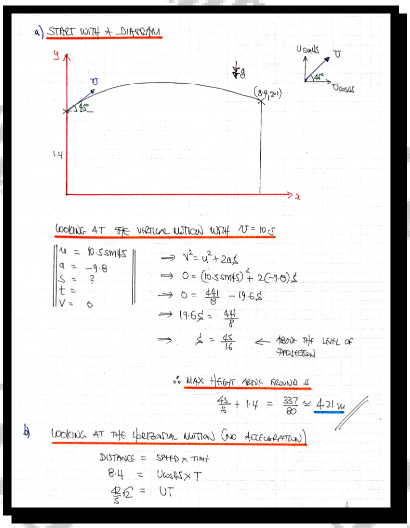
In a fun fair a child throws a small ball from a point A which is at a height of 1.4 m above level horizontal ground aiming at a small target T. The target is at the top of a vertical pole of height 2.1 m. The horizontal distance of the child from the pole is 8.4 m as shown in the figure above.

The initial speed of the ball is U ms^{-1} at an angle of elevation $45°$. The ball is modelled as a particle moving freely under gravity.

a) If $U = 10.5$ find the greatest height of the ball above the ground.

b) Given instead that the ball hits the target, determine the value of U.

$\boxed{}$, $\boxed{\approx 4.21 \text{ m}}$, $\boxed{U \approx 9.48}$


a) START WITH A DIAGRAM

LOOKING AT THE VERTICAL MOTION WITH $U = 10.5$

$u = 10.5\sin 45$, $a = -9.8$, $s = ?$, $t =$, $v = 0$

$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow 0 = (10.5\sin 45)^2 + 2(-9.8)s$

$\Rightarrow 0 = \frac{441}{8} - 19.6s$

$\Rightarrow 19.6s = \frac{441}{8}$

$\Rightarrow s = \frac{45}{16}$ ← ABOVE THE LEVEL OF PROJECTION

∴ MAX HEIGHT ABOVE GROUND IS

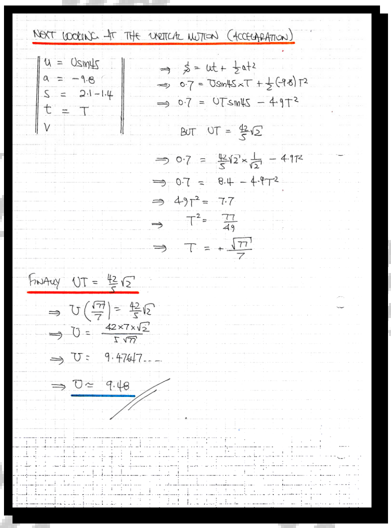
$\frac{45}{16} + 1.4 = \frac{337}{80} \approx 4.21$ m

b) LOOKING AT THE HORIZONTAL MOTION (NO ACCELERATION)

DISTANCE = SPEED × TIME

$8.4 = U\cos 45 \times T$

$\frac{42}{5}\sqrt{2} = UT$


NEXT LOOKING AT THE VERTICAL MOTION (ACCELERATION)

$u = U\sin 45$, $a = -9.8$, $s = 2.1 - 1.4$, $t = T$, v

$\Rightarrow s = ut + \frac{1}{2}at^2$

$\Rightarrow 0.7 = U\sin 45 \times T + \frac{1}{2}(-9.8)T^2$

$\Rightarrow 0.7 = UT\sin 45 - 4.9T^2$

BUT $UT = \frac{42}{5}\sqrt{2}$

$\Rightarrow 0.7 = \frac{42}{5}\sqrt{2} \times \frac{1}{\sqrt{2}} - 4.9T^2$

$\Rightarrow 0.7 = 8.4 - 4.9T^2$

$\Rightarrow 4.9T^2 = 7.7$

$\Rightarrow T^2 = \frac{77}{49}$

$\Rightarrow T = +\frac{\sqrt{77}}{7}$

FINALLY $UT = \frac{42}{5}\sqrt{2}$

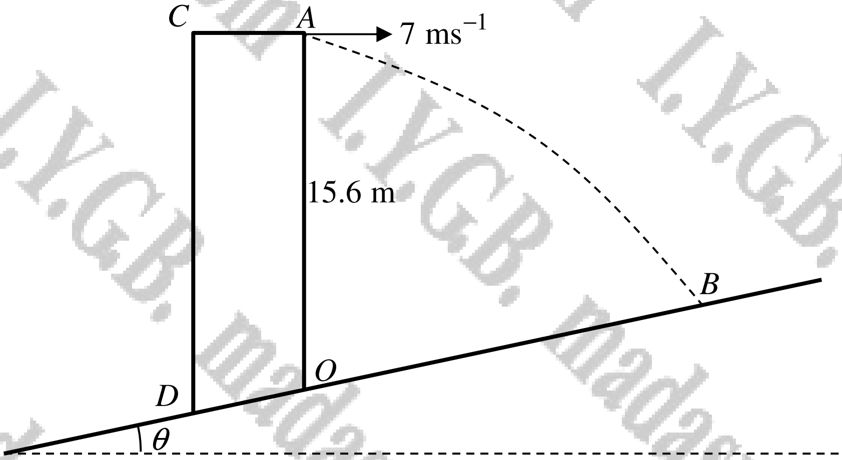
$\Rightarrow U\left(\frac{\sqrt{77}}{7}\right) = \frac{42}{5}\sqrt{2}$

$\Rightarrow U = \frac{42 \times 7 \times \sqrt{2}}{5\sqrt{77}}$

$\Rightarrow U = 9.47647...$

$\Rightarrow U \approx 9.48$

Question 21 (****)



The figure above shows the cross section of a vertical tower $OACD$ standing on a plane inclined at an angle θ to the horizontal, where $\tan\theta = 0.1$.

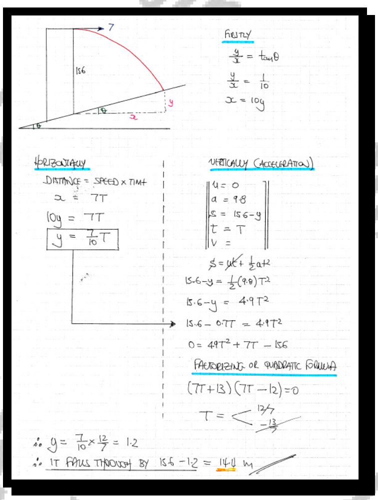
A particle is projected horizontally from A hitting the incline plane at the point B.

The journey of the particle is in a vertical plane containing O, A and B.

Given that $|OA| = 15.6$ m determine the vertical distance through which the particle falls as it travels from A to B.

You may assume that the only force acting on the particle is its weight.

$\boxed{}$, $\boxed{14.4 \text{ m}}$



Question 22 (****)

A particle is projected from a point A on level horizontal ground with speed of U ms^{-1} at an angle of elevation θ.

The particle moves through still air without any resistance, reaching a maximum height H above ground, before it first hits the ground at a point which is R m away from A.

a) Show clearly, in any order, that …

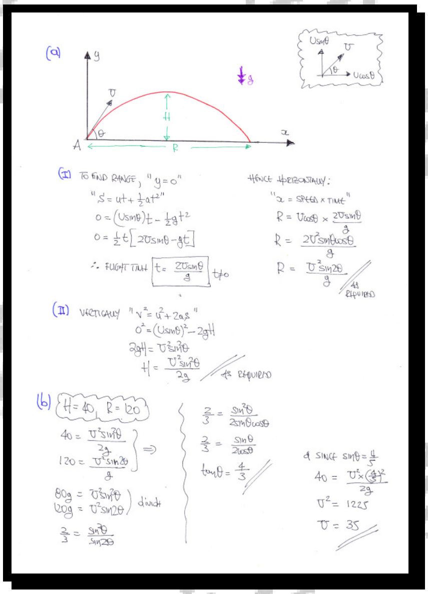
i. … $R = \dfrac{U^2 \sin 2\theta}{g}$.

ii. … $H = \dfrac{U^2 \sin^2 \theta}{2g}$.

It is now given that the particle reaches a greatest height above the ground of 40 m, after travelling a horizontal distance of 60 m from A.

b) Determine in any order the value of U and the value of $\tan\theta$.

$\boxed{U = 35}$, $\boxed{\tan\theta = \dfrac{4}{3}}$



Question 23 (****)

A particle P is projected from the point A on level horizontal ground with a speed of 21 ms^{-1} at an angle θ to the horizontal.

In the subsequent motion P is moving under gravity, without any air resistance.

The particle passes through the point B, t s later. The horizontal and vertical displacement of B from A are 12 m and 2 m, respectively.

a) By considering the horizontal component of the motion of P show

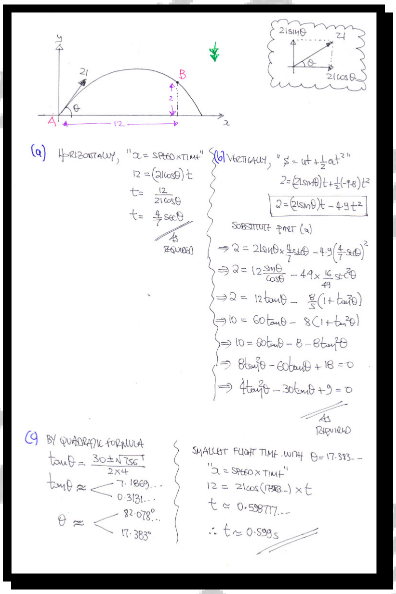
$$t = \frac{4}{7}\sec\theta .$$

b) By considering the vertical component of the motion of P show

$$4\tan^2\theta - 30\tan\theta + 9 = 0 .$$

c) Determine, to three significant figures, the smallest possible flight time of P from A to B.

$\boxed{t \approx 0.599}$



Question 24 (****)

A particle is projected from a point O on level horizontal ground with speed of $u \text{ ms}^{-1}$ at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

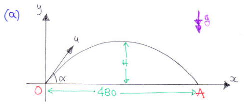
The particle is moving freely under gravity, reaching a greatest height of H m above the ground before it lands on the ground at a point A.

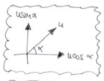
a) Given that the distance OA is 480 m, calculate the length of time it takes for the particle to travel from O to A, and hence show that $u = 70$.

b) Find the value of H.

$\boxed{t = \frac{60}{7}}$, $\boxed{H = 90 \text{ m}}$

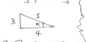
(a)





$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$



HORIZONTALLY, "x = SPEED × TIME", FROM O TO A

$480 = u\cos\alpha \times T$

$480 = \frac{4}{5}uT$

$\boxed{600 = uT}$

VERTICALLY, "$s = ut + \frac{1}{2}at^2$", FROM O TO A

$0 = (u\sin\alpha)T + \frac{1}{2}(-9.8)T^2$

$0 = u \times \frac{3}{5}T - 4.9T^2$

$\boxed{0 = 3uT - 24.5T^2}$

BY SUBSTITUTION NOW: $0 = 3 \times (\underset{uT}{600}) - 24.5T^2$

$24.5T^2 = 1800$

$T^2 = \frac{3600}{49}$

$T = \frac{60}{7}$ ← FLIGHT TIME

FINALLY $uT = 600$

$u \times \frac{60}{7} = 600$

$u = 70$

AS REQUIRED

(b) WE HAVE FOUND THE FLIGHT TIME IN PART (a)

BY SYMMETRY IT REACHES MAX HEIGHT WHEN $t = \frac{1}{2} \times \frac{60}{7} = \frac{30}{7}$

USING "$s = ut + \frac{1}{2}at^2$"

$H = (u\sin\alpha) \times \frac{30}{7} + \frac{1}{2}(-9.8)\left(\frac{30}{7}\right)^2$

$H = 70 \times \frac{3}{5} \times \frac{30}{7} - 4.9 \times \left(\frac{30}{7}\right)^2$

$H = 90$ m

ALTERNATIVE, USE VERTICALLY "$v^2 = u^2 + 2as$"

$0 = (u\sin\alpha)^2 + 2(-9.8)H$

$0 = \left(70 \times \frac{3}{5}\right)^2 - 19.6H$

$19.6H = 1764$

$H = 90$ m

AS BEFORE

Question 25 (****)

In a golf driving range, a golf ball is struck with a speed of 49 ms^{-1} at an angle of elevation α from a point A, which lies 4.9 m above level horizontal ground.

The ball first strikes the ground at the point B which lies at a horizontal distance of 98 m from A.

The ball is modelled as a particle moving under gravity, without any air resistance.

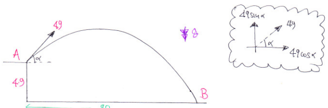
a) Show clearly that

$$4\tan^2 a - 20\tan\alpha + 3 = 0.$$

b) Hence find, to three significant figures, the two possible values of α.

c) Determine, to three significant figures, the smallest possible flight time of the ball from A to B.

$\boxed{\alpha \approx 8.80°, 78.34°}$, $\boxed{t \approx 2.02 \text{ s}}$



$x = 49\cos\alpha \times t$
$y = 49\sin\alpha \times t - \frac{1}{2}gt^2$ $\Rightarrow$ $98 = 49\cos\alpha\, T$, $-4.9 = 49\sin\alpha\, T - \frac{1}{2}gT^2$

HENCE $\boxed{T = \frac{2}{\cos\alpha}}$

SO $-4.9 = 49\sin\alpha\left(\frac{2}{\cos\alpha}\right) - \frac{1}{2}g\left(\frac{2}{\cos\alpha}\right)^2$

$\Rightarrow -4.9 = 98\tan\alpha - \frac{2g}{\cos^2\alpha}$

$\Rightarrow -4.9 = 98\tan\alpha - 2g\sec^2\alpha$

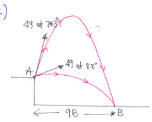
$\Rightarrow -4.9 = 98\tan\alpha - 19.6(1+\tan^2\alpha)$

$\Rightarrow -1 = 20\tan\alpha - 4(1+\tan^2\alpha)$

$\Rightarrow -1 = 20\tan\alpha - 4 - 4\tan^2\alpha$

$\Rightarrow 4\tan^2\alpha - 20\tan\alpha + 3 = 0$

AS REQUIRED



(b) BY QUADRATIC FORMULA

$\tan\alpha = \frac{20 \pm \sqrt{352}}{2\times 4} = \begin{cases} 4.8452\ldots \\ 0.15479\ldots \end{cases}$

$\alpha \simeq \begin{cases} 78.34° \\ 8.8° \end{cases}$

(c)

$x = 49\cos\alpha \times t$
$98 = 49\cos(8.8) \times t$
$t \simeq 2.02 \text{ s}$

Question 26 (****)

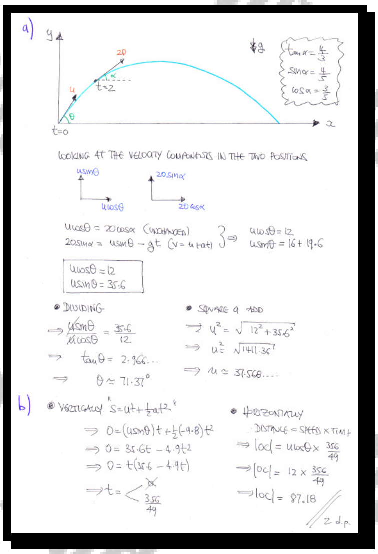
At time $t = 0$, a particle is projected from a fixed point O on level horizontal ground with speed u ms^{-1} at an angle $\theta°$ to the horizontal.

The particle moves freely under gravity and passes through the point A when $t = 2$ s. As it passes through A, the particle is moving upwards with speed 20 ms^{-1}, at an angle $\alpha°$ to the horizontal, where $\tan\alpha = \frac{4}{3}$. The particle finally reaches the ground at the point C.

a) Find the value of u and the value of θ.

b) Calculate the distance OC.

$\boxed{u \approx 37.6 \quad \theta \approx 71.4°}$, $\boxed{|OC| \approx 87.18 \text{ m}}$



a)

LOOKING AT THE VELOCITY COMPONENTS IN THE TWO POSITIONS

$u\sin\theta$, $u\cos\theta$ — $20\sin\alpha$, $20\cos\alpha$

$u\cos\theta = 20\cos\alpha$ (UNCHANGED)
$20\sin\alpha = u\sin\theta - gt$ ($v = u + at$)
$\Rightarrow u\cos\theta = 12$, $u\sin\theta = 16 + 19.6$

$\boxed{u\cos\theta = 12 \quad u\sin\theta = 35.6}$

• DIVIDING

$\Rightarrow \frac{\cancel{u}\sin\theta}{\cancel{u}\cos\theta} = \frac{35.6}{12}$

$\Rightarrow \tan\theta = 2.966\ldots$

$\Rightarrow \theta \simeq 71.37°$

• SQUARE & ADD

$\Rightarrow u^2 = \sqrt{12^2 + 35.6^2}$

$\Rightarrow u^2 = \sqrt{1411.36}$

$\Rightarrow u \simeq 37.568\ldots$

b) • VERTICALLY "$s = ut + \frac{1}{2}at^2$"

$\Rightarrow 0 = (u\sin\theta)t + \frac{1}{2}(-9.8)t^2$

$\Rightarrow 0 = 35.6t - 4.9t^2$

$\Rightarrow 0 = t(35.6 - 4.9t)$

$\Rightarrow t = \begin{cases} \cancel{0} \\ \frac{356}{49} \end{cases}$

• HORIZONTALLY

DISTANCE = SPEED × TIME

$\Rightarrow |OC| = u\cos\theta \times \frac{356}{49}$

$\Rightarrow |OC| = 12 \times \frac{356}{49}$

$\Rightarrow |OC| = 87.18$ // 2 d.p.

Question 27 (****)

A cannon fires a shell with a speed of u ms^{-1}, at an angle of elevation θ, from a point O on level horizontal ground. The shell has horizontal and vertical displacements of x m and y m from O at time t s. The shell is modelled as a particle moving under gravity, without any air resistance.

a) Show clearly that

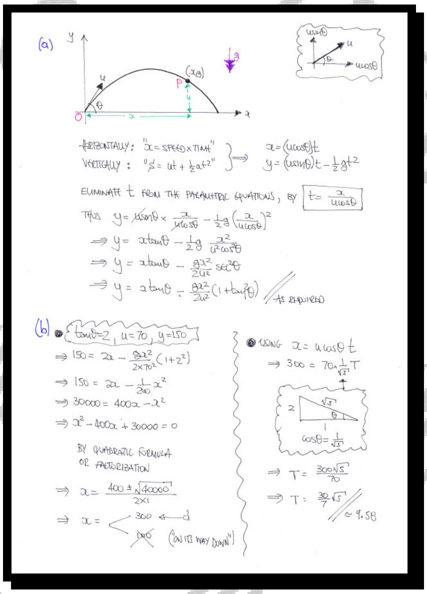
$$y = x\tan\theta - \frac{gx^2}{2u^2}\left(1+\tan^2\theta\right).$$

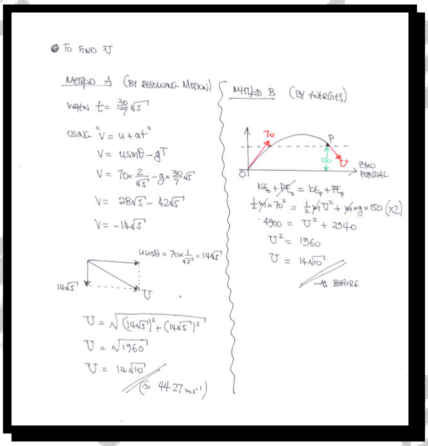
The cannon is aimed at the gate of a fortress which is on a hill at a height of 150 m above the level of the cannon, and a horizontal distance d m from the cannon.

A shell is fired at 70 ms^{-1} at an angle of elevation $\arctan 2$, which hits the gate of the fortress on its way **down**, with a speed U ms^{-1}, T s after it was fired.

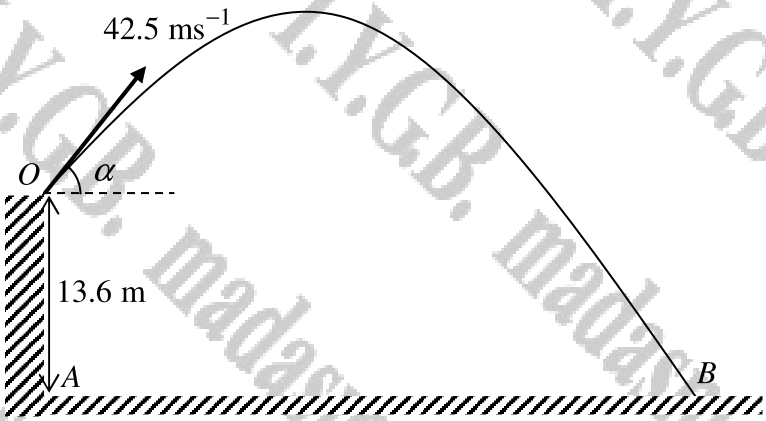
b) Determine in any order the value of d, the value of T and the value of U.

$\boxed{d = 300}$, $\boxed{T = \frac{30}{7}\sqrt{5} \approx 9.58}$, $\boxed{U = 14\sqrt{10} \approx 44.27}$





Question 28 (****)



A particle is projected from a point O with speed 42.5 ms^{-1}, at an angle of elevation α, where $\sin\alpha = \frac{15}{17}$. The point O is 13.6 m vertically above the point A, which is on level horizontal ground, as shown in the figure above.

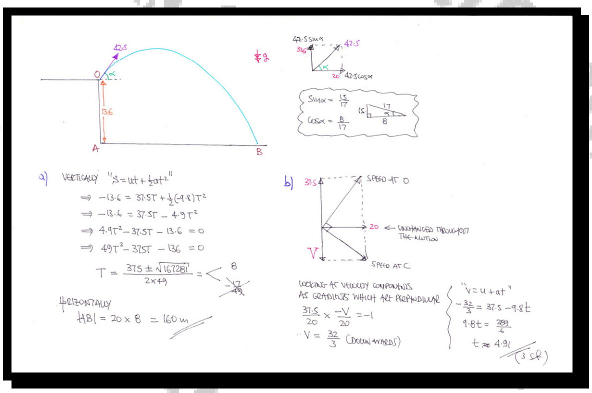
The particle moves freely under gravity and reaches the ground at the point B.

a) Find the distance AB.

The point C lies on the path of the particle. The direction of motion of the particle at C is perpendicular to the direction of motion of the particle at O.

b) Calculate the time taken by the particle to travel from O to C.

$\boxed{|AB| = 160 \text{ m}}$, $\boxed{\text{time} = \frac{1445}{294} \approx 4.91 \text{ s}}$



Question 29 (****)

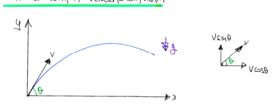
A particle P, which is subject to no other external forces except its weight, is projected from a fixed point O with speed 42 ms^{-1} at an angle of elevation $\arctan\frac{4}{3}$.

3 s later another particle Q, which is also subject to no other external forces except its weight, is projected from O with speed $U \text{ ms}^{-1}$ at an angle of elevation ψ.

If the two particles collide 2 s after Q is projected, calculate the value of ψ and the value of U.

$\boxed{\psi = 27.3°}$, $\boxed{U \approx 70.9}$

STARTING WITH A GENERAL DIAGRAM



FOR THE PARTICLE P,

$x = V\cos\theta \times t$
$y = V\sin\theta \times t + \frac{1}{2}(-g)t^2$ $\Big\} \Rightarrow$ $x = Vt\cos\theta$, $y = Vt\sin\theta - 4.9t^2$

COLLISION TAKES PLACE WHEN $t=5$ AND NOTE $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $V = 42$, $g = 9.8$

$x = 42 \times 5 \times 0.6$
$y = 42 \times 5 \times 0.8 - 4.9 \times 5^2$ $\quad \therefore (x,y) = (126, 45.5)$

NOW PARTICLE Q IS AT $(126, 45.5)$ WHEN $t=2$ WITH $\theta = \psi$ AND $v = U$

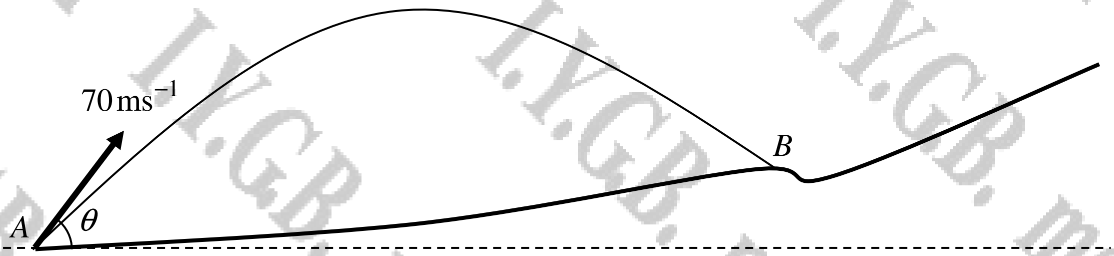
$x = U\cos\psi \times t$
$y = U\sin\psi \times t + \frac{1}{2}(-g)t^2$ $\Big\} \Rightarrow$ $126 = U\cos\psi \times 2$, $45.5 = 2U\sin\psi - 4.9 \times 2^2$ $\Big\} \Rightarrow$

$\boxed{\begin{aligned} U\cos\psi &= 63 \\ U\sin\psi &= 32.55 \end{aligned}}$

- DIVIDING $\tan\psi = \frac{31}{60}$
- SQUARING $U = \sqrt{63^2 + 32.55^2}$

$\therefore \psi \approx 27.3°$ & $U \approx 70.9 \text{ ms}^{-1}$

Question 30 (****)

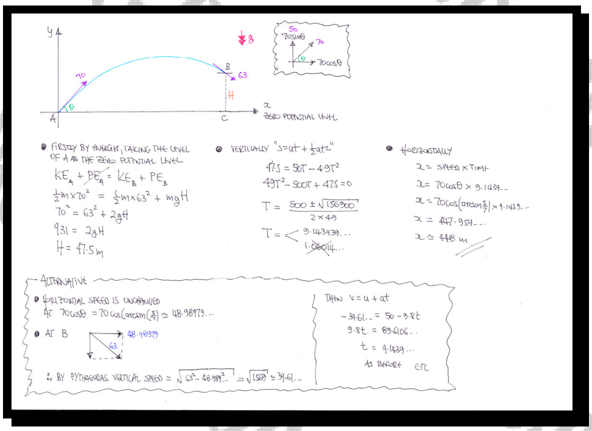


A golf ball is driven from a point A with speed 70 ms^{-1} at an angle of elevation θ, where $\sin\theta = \frac{5}{7}$, and lands at a point B. The point B lies in the same vertical plane as A and vertically higher than the level of A, as shown in the figure above.

The ball is modelled as a particle moving freely under the action of its weight.

Given that the golf ball has a speed of 63 ms^{-1} as it lands at B, determine the horizontal distance AB.

$\boxed{\approx 448 \text{ m}}$



Question 31 (****)

A particle is projected from a point O with speed $36\ \text{ms}^{-1}$ at an angle of elevation β.

It reaches a point P which is at the same vertical level as O and at a horizontal distance of 60 m from O.

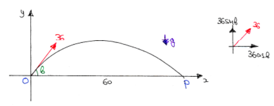
The particle is subject to no other external forces except its weight.

a) Find the two possible values of β.

b) Determine the shortest possible flight time for the journey.

$\boxed{\beta = 13.5° \cup \beta = 76.5°}$, $\boxed{t_{\text{min}} \approx 1.71}$

a) STARTING WITH A STANDARD DIAGRAM



DETERMINE THE FLIGHT TIME, FROM THE VERTICAL MOTION

$u = 36\sin\beta$, $a = -9.8$, $s = 0$, $t = ?$, $v =$

"$s = ut + \frac{1}{2}at^2$"

$0 = (36\sin\beta)t + \frac{1}{2}(-9.8)t^2$

$0 = 36\sin\beta - 4.9t \quad (t \neq 0)$

$t = \frac{36\sin\beta}{4.9}$

LOOKING AT THE HORIZONTAL MOTION

$\Rightarrow$ "DISTANCE = SPEED × TIME"

$\Rightarrow 60 = 36\cos\beta \times \frac{36\sin\beta}{4.9}$

$\Rightarrow 294 = 1296\cos\beta\sin\beta$

$\Rightarrow 294 = 648(2\sin\beta\cos\beta)$

$\Rightarrow 294 = 648\sin 2\beta$

$\Rightarrow \sin 2\beta = \frac{49}{108}$

$2\beta = 26.98... \pm 360n$
$2\beta = 153.02... \pm 360n$ $\quad n = 0,1,2,3,...$

$\beta = 13.5° \pm 180n$
$\beta = 76.5° \pm 180n$

$\therefore \beta = 13.5°, 76.5°$

b) USING THE ANSWER FROM EARLIER WITH $\beta = 13.5$, AS THIS GIVES THE LEAST SIN

$t = \frac{36\sin\beta}{4.9} = \frac{36\sin(13.5°)}{4.9} \simeq 1.71\ \text{s}$

Question 32 (****)

A particle is projected with speed u ms^{-1} at an angle θ **above** the horizontal, from a point O **above** level horizontal ground. The particle's horizontal and vertical distances from O at time t s after projection, are x m and y m, respectively. The particle is moving under gravity, without any air resistance.

a) Show clearly that

$$y = x\tan\theta + \frac{gx^2}{2u^2}\sec^2\theta .$$

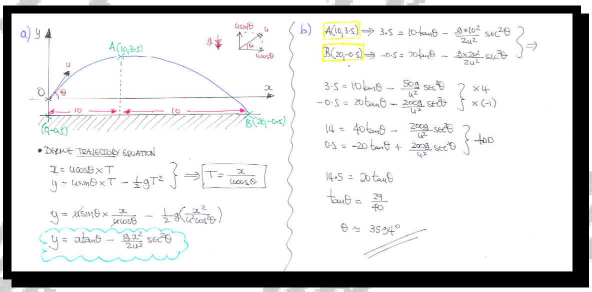
The tennis ball is struck from a height of 0.5 m above level horizontal ground.

The ball passes at a height of 4 m above ground, over the opposing player which was standing 10 m away fro where the ball was struck.

The ball first hits the ground at a distance of 10 m from where it was struck.

b) Determine the angle at which the ball was projected.

$\boxed{\approx 35.94°}$



Question 33 (****)

A particle is projected with speed u ms^{-1} at an angle θ **below** the horizontal, from a point O **above** level horizontal ground. The particle's horizontal and vertical distances from O at time t s after projection, are x m and y m, respectively. The particle is moving under gravity, without any air resistance.

a) Show clearly that

$$y = x\tan\theta + \frac{gx^2}{2u^2\cos^2\theta}.$$

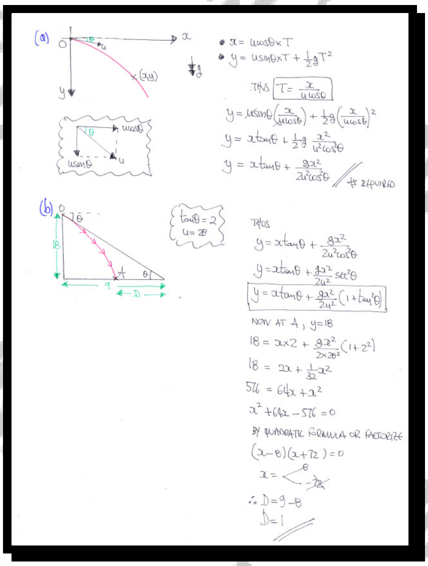
A child is throwing a tennis ball from tower block aiming at a target on the ground.

The tennis ball is thrown from a height of 18 m, with a speed of 28 ms^{-1}, aiming **directly** at the target which is at a horizontal distance of 9 m, from the foot of the tower block.

The tennis ball lands D m short of the target because of the effect of gravity.

b) Determine the value of D.

$\boxed{D = 1}$



Question 34 (****)

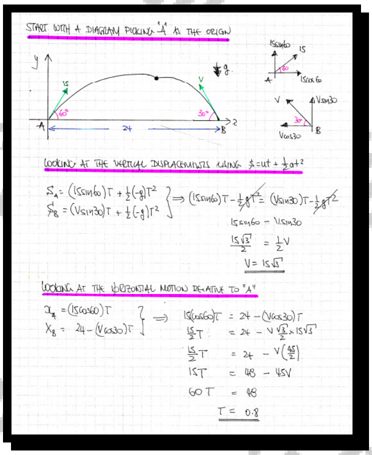
The points A and B lie on level horizontal ground, 24 m apart.

A particle is projected from A towards B, with a speed of 15 ms^{-1} at an angle of elevation $60°$, and at the same time another particle is projected from B towards A, at an angle of elevation $30°$.

The particles collide in mid air.

Determine the speed of the particle projected from B, just before the impact with the other particle.

$\boxed{\text{MMS-Z}}$, $\boxed{\approx 23.08 \text{ ms}^{-1}}$



START WITH A DIAGRAM PICKING "A" AS THE ORIGIN

LOOKING AT THE VERTICAL DISPLACEMENTS USING $s = ut + \frac{1}{2}at^2$

$$\left.\begin{aligned} s_A &= (15\sin60)T + \tfrac{1}{2}(-g)T^2 \\ s_B &= (V\sin30)T + \tfrac{1}{2}(-g)T^2 \end{aligned}\right\} \Rightarrow (15\sin60)T - \cancel{\tfrac{1}{2}gT^2} = (V\sin30)T - \cancel{\tfrac{1}{2}gT^2}$$

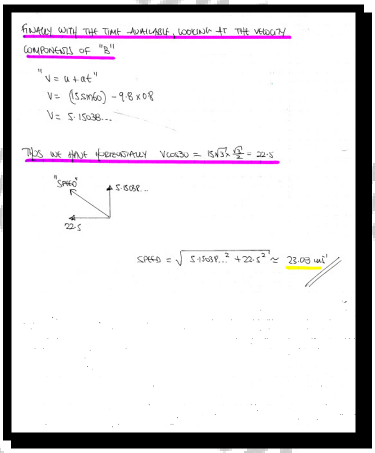
$$15\sin60 = V\sin30$$

$$\frac{15\sqrt{3}}{2} = \frac{1}{2}V$$

$$V = 15\sqrt{3}$$

LOOKING AT THE HORIZONTAL MOTION RELATIVE TO "A"

$$\left.\begin{aligned} x_A &= (15\cos60)T \\ X_B &= 24 - (V\cos30)T \end{aligned}\right\} \Rightarrow \begin{aligned} 15(\cos60)T &= 24 - (V\cos30)T \\ \tfrac{15}{2}T &= 24 - V\tfrac{\sqrt{3}}{2} \times 15\sqrt{3} \\ \tfrac{15}{2}T &= 24 - V\left(\tfrac{45}{2}\right) \\ 15T &= 48 - 45V \\ 60T &= 48 \\ T &= 0.8 \end{aligned}$$



FINALLY WITH THE TIME AVAILABLE, LOOKING AT THE VELOCITY COMPONENTS OF "B"

"$v = u + at$"

$V = (15\sin60) - 9.8 \times 0.8$

$V = 5.15038\ldots$

THUS WE HAVE HORIZONTALLY $V\cos30 = 15\sqrt{3} \times \frac{\sqrt{3}}{2} = 22.5$

$$\text{SPEED} = \sqrt{5.15038\ldots^2 + 22.5^2} \simeq 23.08 \text{ ms}^{-1}$$

Question 35 (****+)

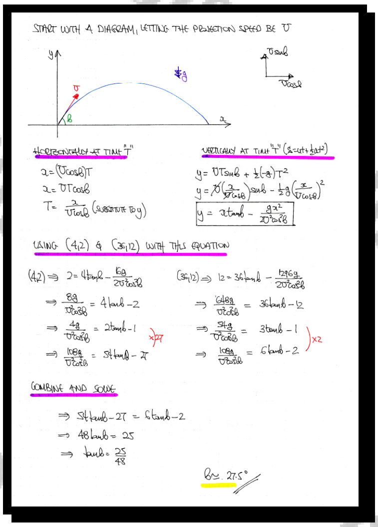
A particle is projected from a point O on level horizontal ground at an angle of elevation β, and continues to move freely under gravity without any air resistance.

The particle just clears a vertical wall of height 2 m, which is at a horizontal distance of 4 m away from O.

In the subsequent motion the particle just clears the top of a vertical transmitter of height 12 m, which is at a horizontal distance of 36 m away from O.

Calculate the value of β.

$\boxed{\beta \approx 27.5°}$


START WITH A DIAGRAM, LETTING THE PROJECTION SPEED BE U

HORIZONTALLY AT TIME "T"

$x = (U\cos\beta)T$

$x = UT\cos\beta$

$T = \frac{x}{U\cos\beta}$ (SUBSTITUTE TO y)

VERTICALLY AT TIME "T" ($s = ut + \frac{1}{2}at^2$)

$y = UT\sin\beta + \frac{1}{2}(-g)T^2$

$y = \cancel{U}\left(\frac{x}{\cancel{U}\cos\beta}\right)\sin\beta - \frac{1}{2}g\left(\frac{x}{U\cos\beta}\right)^2$

$\boxed{y = x\tan\beta - \frac{gx^2}{2U^2\cos^2\beta}}$

USING $(4,2)$ & $(36,12)$ WITH THIS EQUATION

$(4,2) \Rightarrow 2 = 4\tan\beta - \frac{16g}{2U^2\cos^2\beta}$

$\Rightarrow \frac{8g}{U^2\cos^2\beta} = 4\tan\beta - 2$

$\Rightarrow \frac{4g}{U^2\cos^2\beta} = 2\tan\beta - 1$) ×27

$\Rightarrow \frac{108g}{U^2\cos^2\beta} = 54\tan\beta - 27$

$(36,12) \Rightarrow 12 = 36\tan\beta - \frac{1296g}{2U^2\cos^2\beta}$

$\Rightarrow \frac{648g}{U^2\cos^2\beta} = 36\tan\beta - 12$

$\Rightarrow \frac{54g}{U^2\cos^2\beta} = 3\tan\beta - 1$) ×2

$\Rightarrow \frac{108g}{U^2\cos^2\beta} = 6\tan\beta - 2$

COMBINE AND SOLVE

$\Rightarrow 54\tan\beta - 27 = 6\tan\beta - 2$

$\Rightarrow 48\tan\beta = 25$

$\Rightarrow \tan\beta = \frac{25}{48}$

$\beta \simeq 27.5°$

Question 36 (****+)

A fixed origin O is located on level horizontal ground and the vectors $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing horizontally and vertically, respectively.

A mortar shell is fired from O with velocity $(U\mathbf{i}+V\mathbf{j})\ \text{ms}^{-1}$, where U and V are positive constants. The shell lands on the enemy target which is located on the same level horizontal ground as O. The highest point on the path of the shell has position vector $(300\mathbf{i}+122.5\mathbf{j})$ m.

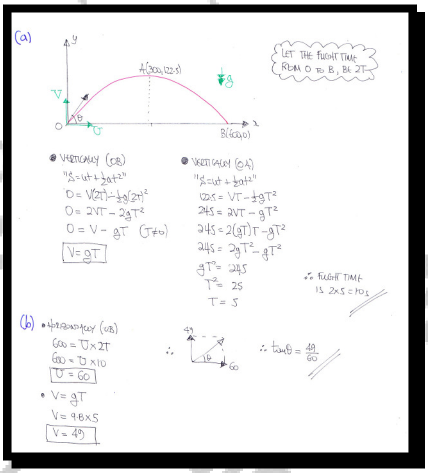
The shell is modelled as a particle moving freely under gravity.

a) Show that the time it takes the shell to hit the target is 10 s.

The shell was projected at an angle of elevation θ.

b) Determine the value of $\tan\theta$.

$$\boxed{\tan\theta = \frac{49}{60}}$$



Question 37 (****+)

A footballer sees the goalkeeper off his line and kicks the ball from level horizontal ground with speed U ms^{-1}, at an angle of elevation θ, where $\tan\theta = \frac{5}{12}$.

When the ball was kicked, it was at horizontal distance of 52.8 m from the goal line and perpendicular to it. Consequently a goal is scored as the ball passes just under the horizontal cross bar which stands 2.40 m in vertical height.

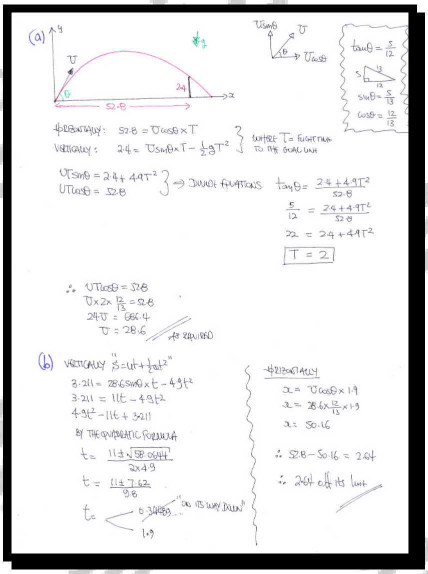
The ball is modelled as a particle moving freely under gravity, whose path lies in a vertical plane perpendicular to the goal line and the cross bar.

a) By considering the horizontal and vertical displacements of the ball, show clearly that $U = 28.6$.

The goalkeeper whose vertical reach is 3.211 m could not prevent the goal.

b) Given that the keeper jumped to save the goal when the ball was on its way down determine the distance of the goalkeeper from his goal line when he jumped for the ball.

$\boxed{h = 2.64 \text{ m}}$



(a)

$U\sin\theta$, U, $U\cos\theta$

$\tan\theta = \frac{5}{12}$, $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

HORIZONTALLY: $52.8 = U\cos\theta \times T$

VERTICALLY: $2.4 = U\sin\theta \times T - \frac{1}{2}gT^2$

WHERE T = FLIGHT TIME TO THE GOAL LINE

$UT\sin\theta = 2.4 + 4.9T^2$

$UT\cos\theta = 52.8$

$\Rightarrow$ DIVIDE EQUATIONS $\tan\theta = \frac{2.4 + 4.9T^2}{52.8}$

$\frac{5}{12} = \frac{2.4 + 4.9T^2}{52.8}$

$22 = 2.4 + 4.9T^2$

$\boxed{T = 2}$

$\therefore UT\cos\theta = 52.8$

$U \times 2 \times \frac{12}{13} = 52.8$

$24U = 686.4$

$U = 28.6$ AS REQUIRED

(b) VERTICALLY "$s = ut + \frac{1}{2}at^2$"

$3.211 = 28.6\sin\theta \times t - 4.9t^2$

$3.211 = 11t - 4.9t^2$

$4.9t^2 - 11t + 3.211$

BY THE QUADRATIC FORMULA

$t = \frac{11 \pm \sqrt{58.0644}}{2 \times 4.9}$

$t = \frac{11 \pm 7.62}{9.8}$

$t = \cancel{0.34489\ldots}$ or 1.9 "ON ITS WAY DOWN"

HORIZONTALLY

$x = U\cos\theta \times 1.9$

$x = 28.6 \times \frac{12}{13} \times 1.9$

$x = 50.16$

$\therefore 52.8 - 50.16 = 2.64$

$\therefore$ 2.64 off its line

Question 38 (****+)

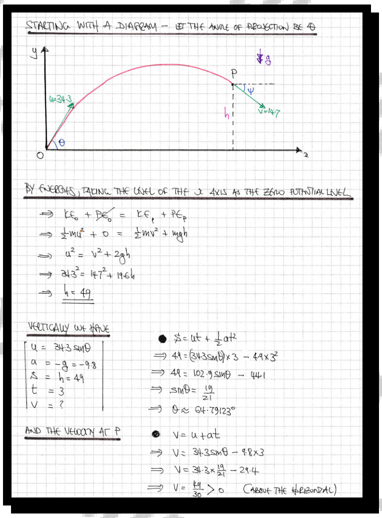
A particle is projected from a point O on level horizontal ground with a speed of 34.3 ms^{-1} at some angle of elevation.

The particle is moving freely under gravity, reaching a greatest height above the ground before it passes through the point P, 3 s after it was projected.

When the particle passes through P it has a speed of 14.7 ms^{-1}, at an angle ψ to the horizontal.

Show that $\psi = \arcsin\left(\frac{1}{9}\right)$, stating further whether this angle is above the horizontal or below the horizontal.

$\boxed{}$, $\boxed{\text{proof}}$


STARTING WITH A DIAGRAM – LET THE ANGLE OF PROJECTION BE θ

BY ENERGIES, TAKING THE LEVEL OF THE x AXIS AS THE ZERO POTENTIAL LEVEL

$\Rightarrow KE_0 + \cancel{PE_0} = KE_P + PE_P$

$\Rightarrow \frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mgh$

$\Rightarrow u^2 = v^2 + 2gh$

$\Rightarrow 34.3^2 = 14.7^2 + 19.6h$

$\Rightarrow \underline{\underline{h = 49}}$

VERTICALLY WE HAVE

$u = 34.3\sin\theta$, $a = -g = -9.8$, $s = h = 49$, $t = 3$, $v = ?$

- $s = ut + \frac{1}{2}at^2$

$\Rightarrow 49 = (34.3\sin\theta)\times 3 - 4.9\times 3^2$

$\Rightarrow 49 = 102.9\sin\theta - 44.1$

$\Rightarrow \sin\theta = \frac{19}{21}$

$\Rightarrow \theta \approx 64.79123°$

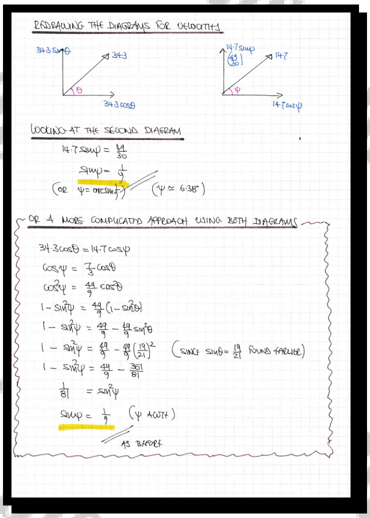
AND THE VELOCITY AT P

- $v = u + at$

$\Rightarrow v = 34.3\sin\theta - 9.8\times 3$

$\Rightarrow v = 34.3\times\frac{19}{21} - 29.4$

$\Rightarrow v = \frac{49}{30} > 0$ (ABOVE THE HORIZONTAL)


REDRAWING THE DIAGRAMS FOR VELOCITIES

LOOKING AT THE SECOND DIAGRAM

$14.7\sin\psi = \frac{49}{30}$

$\sin\psi = \frac{1}{9}$

(OR $\psi = \arcsin\frac{1}{9}$) ($\psi \simeq 6.38°$)

OR A MORE COMPLICATED APPROACH USING BOTH DIAGRAMS

$34.3\cos\theta = 14.7\cos\psi$

$\cos\psi = \frac{7}{3}\cos\theta$

$\cos^2\psi = \frac{49}{9}\cos^2\theta$

$1 - \sin^2\psi = \frac{49}{9}(1 - \sin^2\theta)$

$1 - \sin^2\psi = \frac{49}{9} - \frac{49}{9}\sin^2\theta$

$1 - \sin^2\psi = \frac{49}{9} - \frac{49}{9}\left(\frac{19}{21}\right)^2$ (SINCE $\sin\theta = \frac{19}{21}$ FOUND EARLIER)

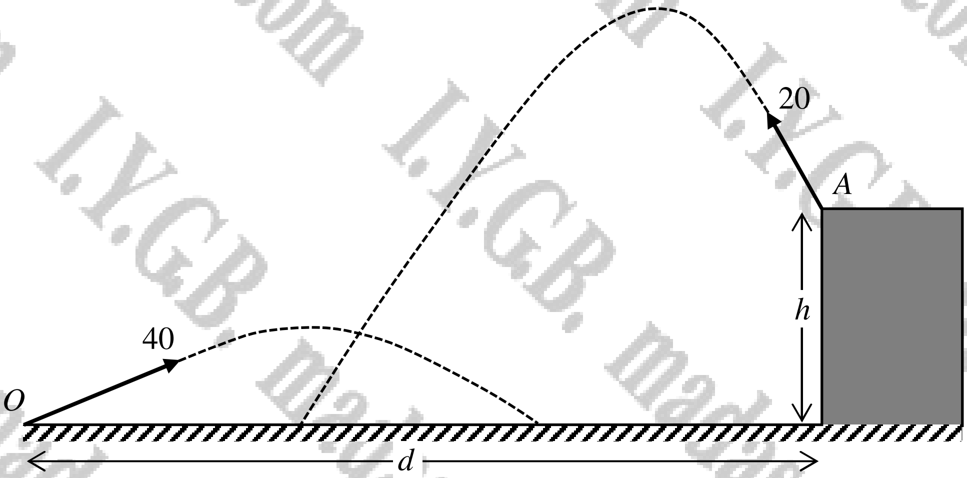
$1 - \sin^2\psi = \frac{49}{9} - \frac{361}{81}$

$\frac{1}{81} = \sin^2\psi$

$\sin\psi = \frac{1}{9}$ (ψ ACUTE)

AS BEFORE

Question 39 (****+)



The point O lies on level horizontal ground and the point A is at a horizontal distance d m away from O and at a height d m above the ground.

A particle is projected from O with speed 40 ms^{-1} at an angle of elevation $\arctan\left(\frac{3}{4}\right)$.

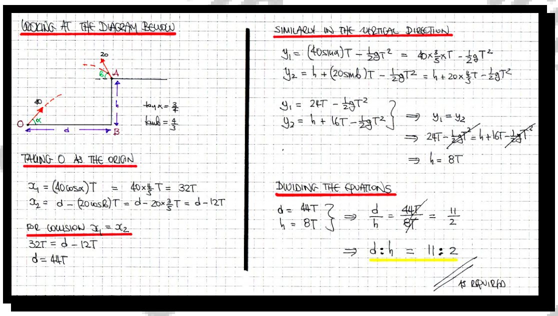
At the same time another particle is projected from A with speed 20 ms^{-1} at an angle of elevation $\arctan\left(\frac{4}{3}\right)$, as shown in the figure above.

The motion of the two particles takes place in the same vertical plane.

Assuming that there is no air resistance present, show that, if the two particles collide during their flights, then

$$d : h = 11 : 2.$$

[ ], [proof]



LOOKING AT THE DIAGRAM BELOW

$\tan\alpha = \frac{3}{4}$

$\tan\beta = \frac{4}{3}$

TAKING O AS THE ORIGIN

$x_1 = (40\cos\alpha)T = 40 \times \frac{4}{5}T = 32T$

$x_2 = d - (20\cos\beta)T = d - 20 \times \frac{3}{5}T = d - 12T$

FOR COLLISION $x_1 = x_2$

$32T = d - 12T$

$d = 44T$

SIMILARLY IN THE VERTICAL DIRECTION

$y_1 = (40\sin\alpha)T - \frac{1}{2}gT^2 = 40 \times \frac{3}{5} \times T - \frac{1}{2}gT^2$

$y_2 = h + (20\sin\beta)T - \frac{1}{2}gT^2 = h + 20 \times \frac{4}{5}T - \frac{1}{2}gT^2$

$\left.\begin{array}{l} y_1 = 24T - \frac{1}{2}gT^2 \\ y_2 = h + 16T - \frac{1}{2}gT^2 \end{array}\right\} \Rightarrow y_1 = y_2$

$\Rightarrow 24T - \cancel{\frac{1}{2}gT^2} = h + 16T - \cancel{\frac{1}{2}gT^2}$

$\Rightarrow h = 8T$

DIVIDING THE EQUATIONS

$\left.\begin{array}{l} d = 44T \\ h = 8T \end{array}\right\} \Rightarrow \frac{d}{h} = \frac{44\cancel{T}}{8\cancel{T}} = \frac{11}{2}$

$\Rightarrow d : h = 11 : 2$

AS REQUIRED

Question 40 (****+)

In this question take $g = 10 \text{ ms}^{-2}$.

Two particles, A and B, are projected from the same fixed point O, with the same speed $u \text{ ms}^{-1}$, at angles of elevation θ and 2θ respectively.

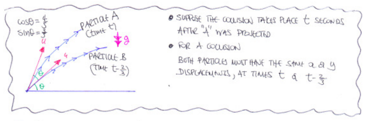
It is further given that …

… B is projected $\frac{2}{3}$ s after A

... $\tan\theta = \frac{3}{4}$.

If A and B collide in the subsequent motion determine the value of u.

$\boxed{u = 12.5}$



- SUPPOSE THE COLLISION TAKES PLACE t SECONDS AFTER "A" WAS PROJECTED
- FOR A COLLISION BOTH PARTICLES MUST HAVE THE SAME x & y DISPLACEMENTS, AT TIMES t & $t-\frac{2}{3}$

HORIZONTALLY: $u\cos\theta\left(t-\frac{2}{3}\right) = (u\cos 2\theta)t$

$\left(t-\frac{2}{3}\right)\cos\theta = t\cos 2\theta$

$t\cos\theta - \frac{2}{3}\cos\theta = t\cos 2\theta$

$t\cos\theta - t\cos 2\theta = \frac{2}{3}$

$t(\cos\theta - \cos 2\theta) = \frac{2}{3}$

$t(\cos\theta - (2\cos^2\theta - 1)) = \frac{2}{3}$

$t\left[\cos\theta + 1 - 2\cos^2\theta\right] = \frac{2}{3}$

$t\left[\frac{4}{5} + 1 - 2\left(\frac{4}{5}\right)\right] = \frac{2}{3}$

$\frac{1}{5}t = \frac{2}{3}$

$\boxed{t = \frac{10}{3}}$

VERTICALLY NOW: $(u\sin 2\theta)t - \frac{1}{2}gt^2 = (u\sin\theta)\left(t-\frac{2}{3}\right) - \frac{1}{2}g\left(t-\frac{2}{3}\right)^2$

$2u t\sin\theta\cos\theta - \frac{1}{2}gt^2 = \left(t-\frac{2}{3}\right)u\sin\theta - \frac{1}{2}g\left(t-\frac{2}{3}\right)^2$

SUB IN ALL THE VALUES — NOTE $g=10$

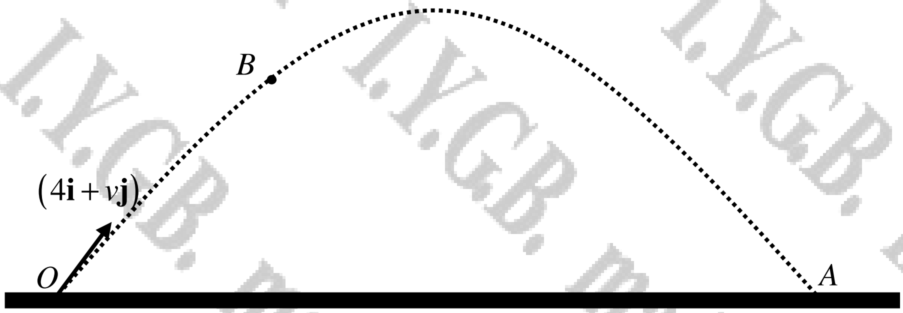
$2u \times \frac{10}{3} \times \frac{3}{5} \times \frac{4}{5} - \frac{1}{2}\times 10 \times \left(\frac{10}{3}\right)^2 = \frac{8}{3}u \times \frac{3}{5} - \frac{1}{2}\times 10 \times \left(\frac{8}{3}\right)^2$

$\frac{16}{5}u - \frac{500}{9} = \frac{8}{5}u - \frac{320}{9}$

$\frac{8}{5}u = 20$

$u = 12.5 \text{ ms}^{-1}$

Question 41 (****+)



The unit vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are in a vertical plane,* $\mathbf{i}$ *being horizontal and* $\mathbf{j}$ *being vertically upwards.*

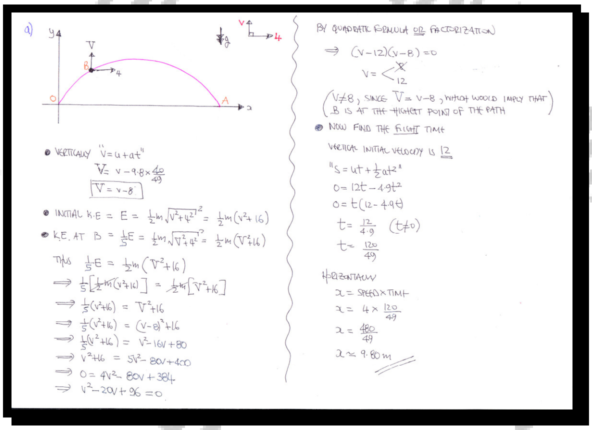
A particle is projected from a point O on level horizontal ground with velocity $(4\mathbf{i}+v\mathbf{j})$ ms^{-1}, where v is a positive scalar constant.

The particle is moving freely under gravity, passing through a point B, T s after projection, before it reaches its greatest height above the ground, finally lands on the ground at the point A.

The kinetic energy of the particle at O is E J and at B is $\frac{1}{5}E$ J.

Given further that $T=\frac{20}{49}$, determine the distance OA.

$$\boxed{|OA|=\frac{480}{49}\approx 9.80 \text{ m}}$$



a)

- VERTICALLY "$V=u+at$"
  $V = v - 9.8\times\frac{40}{49}$
  $\boxed{V = v-8}$
- INITIAL K.E $= E = \frac{1}{2}m\sqrt{v^2+4^2}^2 = \frac{1}{2}m(v^2+16)$
- K.E. AT B $= \frac{1}{5}E = \frac{1}{2}m\sqrt{V^2+4^2}^2 = \frac{1}{2}m(V^2+16)$

THUS $\frac{1}{5}E = \frac{1}{2}m(V^2+16)$

$\Rightarrow \frac{1}{5}\left[\cancel{\tfrac{1}{2}m}(v^2+16)\right] = \cancel{\tfrac{1}{2}m}\left[V^2+16\right]$

$\Rightarrow \frac{1}{5}(v^2+16) = V^2+16$

$\Rightarrow \frac{1}{5}(v^2+16) = (v-8)^2+16$

$\Rightarrow \frac{1}{5}(v^2+16) = v^2-16v+80$

$\Rightarrow v^2+16 = 5v^2-80v+400$

$\Rightarrow 0 = 4v^2-80v+384$

$\Rightarrow v^2-20v+96=0$

BY QUADRATIC FORMULA OR FACTORIZATION

$\Rightarrow (v-12)(v-8)=0$

$v = \cancel{8}$ or 12

($v\neq 8$, SINCE $V=v-8$, WHICH WOULD IMPLY THAT B IS AT THE HIGHEST POINT OF THE PATH)

- NOW FIND THE FLIGHT TIME

VERTICAL INITIAL VELOCITY IS $\underline{12}$

"$s=ut+\frac{1}{2}at^2$"

$0=12t-4.9t^2$

$0=t(12-4.9t)$

$t=\frac{12}{4.9}$ $(t\neq 0)$

$t=\frac{120}{49}$

HORIZONTALLY

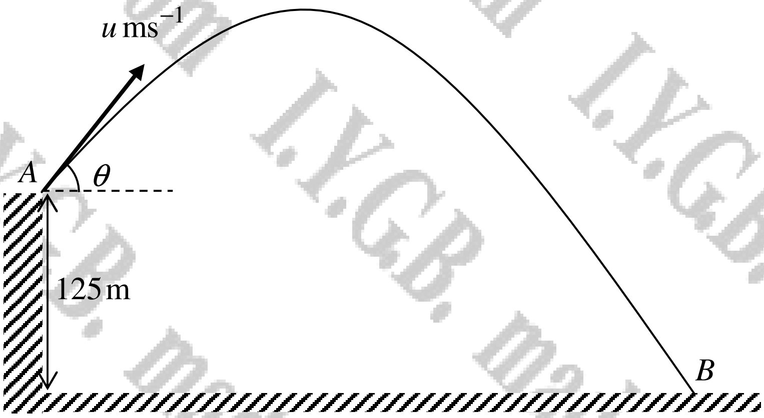
$x=$ SPEED $\times$ TIME

$x = 4\times\frac{120}{49}$

$x=\frac{480}{49}$

$x\simeq 9.80$ m

Question 42 (****+)



A particle P is projected with a speed of u ms^{-1} at an angle of elevation θ, from a point A which is 125 m above level horizontal ground. The particle is moving freely under gravity and first strikes the ground at a point B, as shown in the figure above.

It took T s for P to travel from A to B, and the speed of the particle at B is $3u$ ms^{-1}.

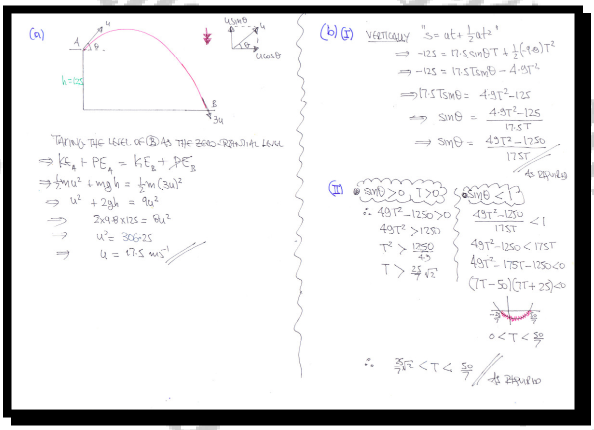
a) Find the value of u.

b) Show clearly that …

i. … $\sin\theta = \dfrac{49T^2 - 1250}{175T}$.

ii. … $\dfrac{25}{7}\sqrt{2} < T < \dfrac{50}{7}$.

$\boxed{u = 17.5}$



Question 43 (****+)

In this question take $g = 10 \text{ ms}^{-2}$.

A projectile is fired from a fixed point O with speed u ms^{-1} at an angle of elevation α so that it passes through a point P.

Relative to a Cartesian coordinate system with origin at O the point P has coordinates $\left(10\sqrt{5}, 5\sqrt{5}\right)$.

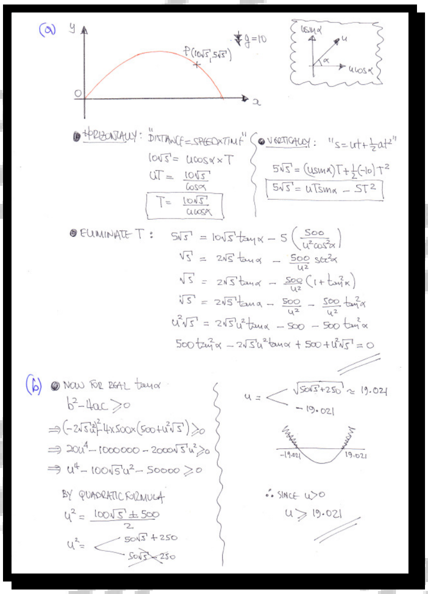
It is assumed that O and P lie in the same vertical plane, and the projectile can be modelled as a particle moving freely under gravity.

a) Show clearly that

$$500\tan^2\alpha - 2\sqrt{5}u^2\tan\alpha + 500 + \sqrt{5}u^2 = 0.$$

b) Hence determine the minimum value of u.

$\boxed{u \geq 19.021...}$



(a)

• HORIZONTALLY: "DISTANCE = SPEED × TIME"

$10\sqrt{5} = u\cos\alpha \times T$

$uT = \frac{10\sqrt{5}}{\cos\alpha}$

$\boxed{T = \frac{10\sqrt{5}}{u\cos\alpha}}$

• VERTICALLY: "$s = ut + \frac{1}{2}at^2$"

$5\sqrt{5} = (u\sin\alpha)T + \frac{1}{2}(-10)T^2$

$\boxed{5\sqrt{5} = uT\sin\alpha - 5T^2}$

• ELIMINATE T:

$5\sqrt{5} = 10\sqrt{5}\tan\alpha - 5\left(\frac{500}{u^2\cos^2\alpha}\right)$

$\sqrt{5} = 2\sqrt{5}\tan\alpha - \frac{500}{u^2}\sec^2\alpha$

$\sqrt{5} = 2\sqrt{5}\tan\alpha - \frac{500}{u^2}(1+\tan^2\alpha)$

$\sqrt{5} = 2\sqrt{5}\tan\alpha - \frac{500}{u^2} - \frac{500}{u^2}\tan^2\alpha$

$u^2\sqrt{5} = 2\sqrt{5}u^2\tan\alpha - 500 - 500\tan^2\alpha$

$500\tan^2\alpha - 2\sqrt{5}u^2\tan\alpha + 500 + u^2\sqrt{5} = 0$

(b)

• NOW FOR REAL $\tan\alpha$

$b^2 - 4ac \geq 0$

$\Rightarrow (-2\sqrt{5}u^2)^2 - 4 \times 500 \times (500 + u^2\sqrt{5}) \geq 0$

$\Rightarrow 20u^4 - 1000000 - 2000\sqrt{5}u^2 \geq 0$

$\Rightarrow u^4 - 100\sqrt{5}u^2 - 50000 \geq 0$

BY QUADRATIC FORMULA

$u^2 = \frac{100\sqrt{5} \pm 500}{2}$

$u^2 = 50\sqrt{5} + 250$ or $\cancel{50\sqrt{5} - 250}$

$u = \sqrt{50\sqrt{5}+250} \approx 19.021$ or -19.021

∴ SINCE $u > 0$

$u \geq 19.021$

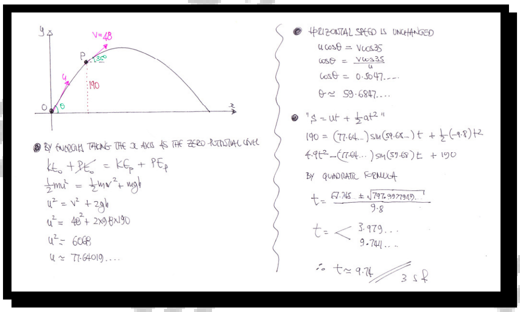
Question 44 (****+)

At time $t = 0$, a particle is projected from a point O on level horizontal ground in a non vertical direction.

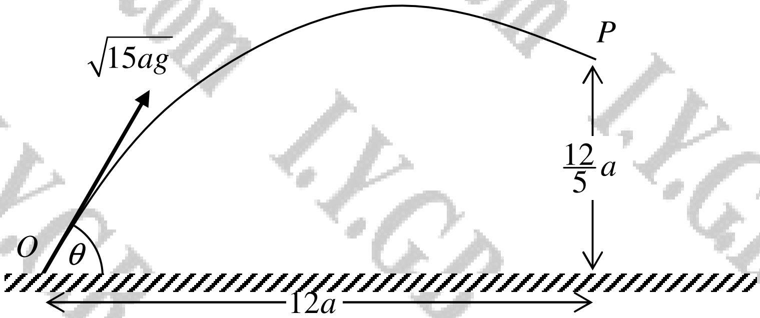
At some time later the particle is passing through a point P with speed 48 ms^{-1}, at an angle of $35°$ above the horizontal.

Given that P is at a height of 190 m above the ground, determine the time when the particle is **again** at a height of 190 m above the ground

$\boxed{t \approx 9.744... \text{ s}}$



Question 45 (****+)



A projectile is fired from a fixed point O with speed $\sqrt{15ag}$ at an angle of elevation θ so that it passes through a point P. Relative to a Cartesian coordinate system with origin at O the point P has coordinates $\left(12a, \frac{12}{5}a\right)$.

It is assumed that O and P lie in the same vertical plane, and the projectile can be modelled as a particle moving freely under gravity.

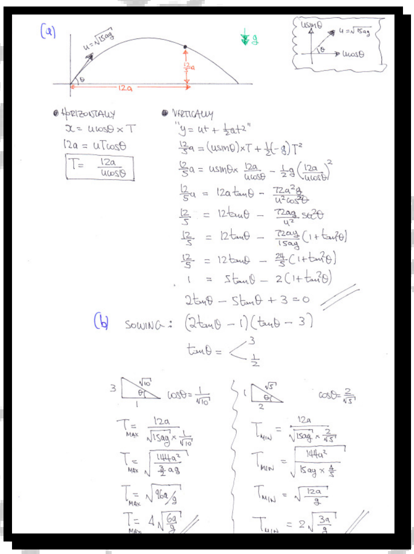
a) Show clearly that

$$2\tan^2\theta - 5\tan\theta + 3 = 0 .$$

b) Show further, that the respective minimum and maximum flight times of the projectile from O to P are

$$4\sqrt{\frac{6a}{g}} \quad \text{and} \quad 2\sqrt{\frac{3a}{g}} .$$

$\boxed{\text{proof}}$



Question 46 (****+)

Relative to a fixed origin O the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are oriented horizontally and vertically upwards, respectively. The origin lies on level horizontal ground.

A particle is projected with velocity $(7\mathbf{i}+14\mathbf{j})\,\text{ms}^{-1}$ from O and moves freely under gravity passing through the point P with position vector $(x\mathbf{i}+y\mathbf{j})$ m, in time t s.

a) Show clearly that

$$y=\frac{1}{10}x(20-x).$$

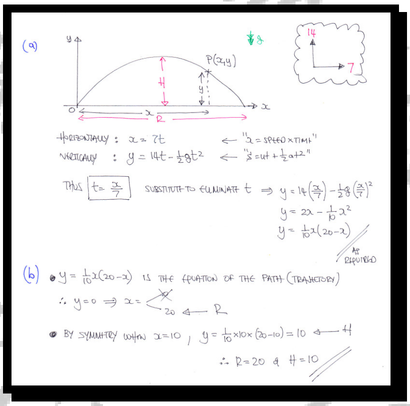
The particle reaches a maximum height of H m above the ground and has a horizontal range of R m.

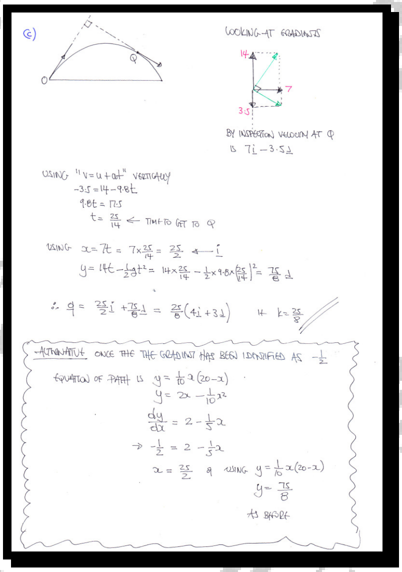
b) Find the value of R and the value of H.

The point Q lies on the particle's trajectory so that its velocity at P is perpendicular to its projection velocity.

c) Show that the position vector of Q is $k(4\mathbf{i}+3\mathbf{j})$ m, where k is an exact constant to be found.

$\boxed{R=20,\ H=10}$, $\boxed{k=\frac{25}{8}}$





Question 47 (****+)

A particle P is projected from a point O on level horizontal ground with speed 26 ms^{-1}, at an angle θ to the horizontal.

At the same time, another particle Q is projected horizontally with speed 10 ms^{-1}, from a point A, which lies 78.4 m vertically above O.

The motion of both particles takes place at the same vertical plane with both particles moving through still air without any resistance.

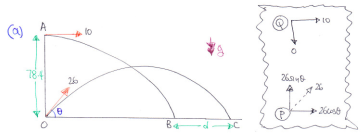
The particles hit the ground at the same time at two points which are d m apart.

a) Calculate the value of d.

b) Given instead that the particles collide before they reach the ground, determine by detailed calculations whether P is rising or falling immediately before the collision.

$\boxed{d \approx 28.33...}$

(a)



• LOOKING AT Q VERTICALLY

$u = 0$, $a = 9.8$, $s = 78.4$, $t = ?$, $V =$

"$s = \cancel{ut} + \frac{1}{2}at^2$"

$78.4 = \frac{1}{2}(9.8)t^2$

$t^2 = 16$

$\boxed{t = 4}$

• LOOKING AT Q HORIZONTALLY

"DISTANCE = SPEED × TIME"

$|OB| = 10 \times 4$

$|OB| = 40$

• LOOKING AT P VERTICALLY

$u = 26\sin\theta$, $a = -9.8$, $s = 0$, $t = 4$, $V =$

"$s = ut + \frac{1}{2}at^2$"

$0 = (26\sin\theta) \times 4 + \frac{1}{2}(-9.8) \times 4^2$

$0 = 104\sin\theta - 78.4$

$78.4 = 104\sin\theta$

$\sin\theta = \frac{49}{65}$

$(\theta \simeq 48.92°)$

• LOOKING AT P, HORIZONTALLY

"DISTANCE = SPEED × TIME"

$|OC| = 26\cos(48.92) \times 4$

$|OC| = 68.33...$

$\therefore d = 28.33 \text{ m}$

(b) FOR A COLLISION HORIZONTAL SPEEDS MUST BE IDENTICAL

• THUS $26\cos\theta = 10$

$\cos\theta = \frac{5}{13}$ & $\sin\theta = \frac{12}{13}$

• NEXT LOOKING AT VERTICAL DISPLACEMENTS RELATIVE TO O, USING "$s = s_0 + ut + \frac{1}{2}at^2$"

$y_P = (26\sin\theta)T - \frac{1}{2}gT^2$

$y_Q = 78.4 - \frac{1}{2}gT^2$

$\Rightarrow y_P = y_Q$

$\Rightarrow (26\sin\theta)T - \cancel{\frac{1}{2}gT^2} = 78.4 - \cancel{\frac{1}{2}gT^2}$

$\Rightarrow 26 \times \frac{12}{13}T = 78.4$

$\Rightarrow 24T = 78.4$

$\Rightarrow T = \frac{49}{15} \simeq 3.26...$

• FINALLY VERTICALLY FOR P

"$V = u + at$"

$V = 26\sin\theta - 9.8t$

$V = 26 \times \frac{12}{13} - \frac{49}{5} \times \frac{49}{15}$

$V = 24 - \frac{2401}{75}$

$V = -\frac{601}{75} \simeq -8.01 \text{ ms}^{-1}$ ⟸ MINUS IMPLIES THAT IT IS FALLING

Question 48 (****+)

Relative to a fixed origin O the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are oriented horizontally and vertically upwards, respectively.

The gravitational acceleration constant g is taken to be $-10\mathbf{j}$ ms^{-2} in this question.

A particle is projected with velocity $(u\mathbf{i}+v\mathbf{j})$ ms^{-1}, where u and v are positive constants, from a point P with position vector $105\mathbf{j}$ m.

The particle moves freely under gravity passing through the point Q with position vector $210\mathbf{i}$ m.

a) Show clearly that

$$u^2+2uv=2100.$$

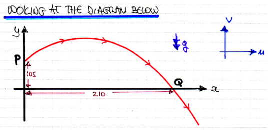
b) Given that when $t=2$ the particle is moving parallel to $\mathbf{i}$, determine the time it takes the particle to travel from P to Q.

The particle passes through the point R with a speed of $10\sqrt{29}$ ms^{-1}.

c) Show R is 80 m below the level of P.

$\boxed{}$, $\boxed{u=30,\ v=20}$, $\boxed{\text{flight time}=7\text{ s}}$

a) LOOKING AT THE DIAGRAM BELOW



HORIZONTALLY (DISTANCE = SPEED × TIME)

$210 = uT$

$T = \frac{210}{u}$

VERTICALLY $\left(s = s_0 + ut + \frac{1}{2}at^2\right)$

$0 = 105 + vT + \frac{1}{2}(-10)T^2$

$5T^2 - vT - 105 = 0$

$\Rightarrow 5\left(\frac{210}{u}\right)^2 - v\left(\frac{210}{u}\right) - 105 = 0$

$\Rightarrow \frac{220500}{u^2} - \frac{210v}{u} - 105 = 0$ $\div 105$

$\Rightarrow \frac{2100}{u^2} - \frac{2v}{u} - 1 = 0$ $\times u^2$

$\Rightarrow 2100 - 2uv - u^2 = 0$

$\Rightarrow u^2 + 2uv = 2100$ // AS REQUIRED

b) "MOVING PARALLEL TO $\mathbf{i}$ $\Rightarrow$ NO VERTICAL SPEED WHEN $t=2$

"$v = u + at$" $\Rightarrow 0 = v - 10\times 2$

$\Rightarrow v = 20$

AND USING $u^2 + 2uv = 2100$

$\Rightarrow u^2 + 40u = 2100$

$\Rightarrow u^2 + 40u - 2100$

$\Rightarrow (u-30)(u+70) = 0$

$\Rightarrow u = 30$ $(u > 0)$

FINALLY USING $210 = uT$

$\Rightarrow 210 = 30T$

$\Rightarrow T = 7$ //

c) BY CONSIDERING ENERGY & TAKING THE LEVEL OF P AS THE ZERO POTENTIAL LEVEL

$\Rightarrow KE_P + \cancel{PE_P} = KE_Q + PE_Q$

$\Rightarrow \frac{1}{2}mU^2 = \frac{1}{2}mV^2 + mgh$

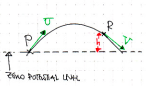
$\Rightarrow U^2 = V^2 + 2gh$

$\Rightarrow \left[\sqrt{\underset{u^2}{30^2} + \underset{v^2}{20^2}}\right]^2 = (10\sqrt{29})^2 + 2gh$

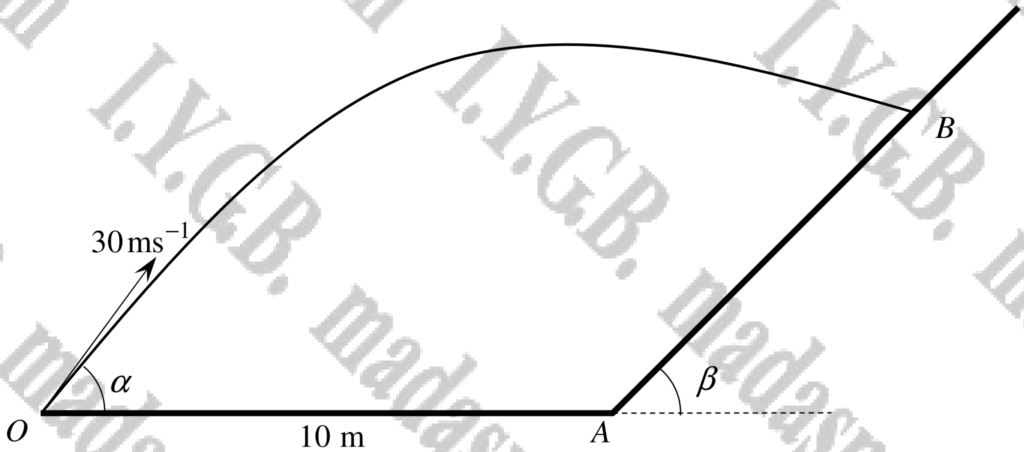
$\Rightarrow 1300 = 2900 + 2\times 10\times h$

$\Rightarrow h = -80$

// IT 80 m BELOW THE LEVEL OF PROJECTION



Question 49 **(****+)**



A particle is projected from a point O on level horizontal ground with speed 30 ms^{-1} at an angle of elevation α. The particle is freely moving under gravity, heading towards a plane, inclined at an angle β to the horizontal.

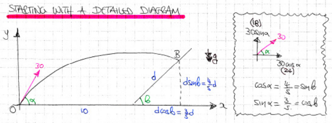
The foot, A, of this incline plane is located at a horizontal distance of 10 m from O, as shown in the figure above.

The particle strikes the incline plane at the point B, so that AB is a line of greatest slope in the same vertical plane which contains O.

Determine the distance AB, given further that $\alpha = \arctan\frac{3}{4}$ and $\beta = \arctan\frac{4}{3}$

M1456- , $\boxed{|AB| = \frac{-1550+400\sqrt{21}}{21} \approx 13.48\text{ m}}$

STARTING WITH A DETAILED DIAGRAM



FORMING TWO EQUATIONS FOR HORIZONTAL & VERTICAL DISPLACEMENTS

"DISTANCE = SPEED × TIME"

$\Rightarrow 10 + \frac{3}{5}d = 24 \times T$

$\Rightarrow 10 + \frac{3}{5}d = 24T$

$\Rightarrow 50 + 3d = 120T$

$200 + 12d = 480T$

"$s = ut + \frac{1}{2}at^2$"

$\Rightarrow \frac{4}{5}d = 18T - \frac{1}{2}gT^2$

$\Rightarrow \frac{4}{5}d = 18T - 4.9T^2$

$\Rightarrow 4d = 90T - \frac{49}{2}T^2$

$\Rightarrow 12d = 270T - \frac{147}{2}T^2$

EASIER TO FIND THE FLIGHT TIME FIRST

$\Rightarrow 200 + \left(270T - \frac{147}{2}T^2\right) = 480T$

$\Rightarrow 200 + 270T - \frac{147}{2}T^2 = 480T$

$\Rightarrow 400 + 540T - 147T^2 = 960T$

$\Rightarrow 0 = 147T^2 + 420T - 400$

QUADRATIC FORMULA & IGNORING THE NEGATIVE SOLUTION

$T = \frac{-420 \pm \sqrt{411600}}{294} = \frac{-420 \pm 140\sqrt{21}}{294} = \frac{-30 \pm 10\sqrt{21}}{21}$

$\therefore T = \frac{-30 + 10\sqrt{21}}{21}$

FINALLY TO FIND d

$\Rightarrow 200 + 12d = 480T$

$\Rightarrow 200 + 12d = 480\left(\frac{-30 + 10\sqrt{21}}{21}\right)$

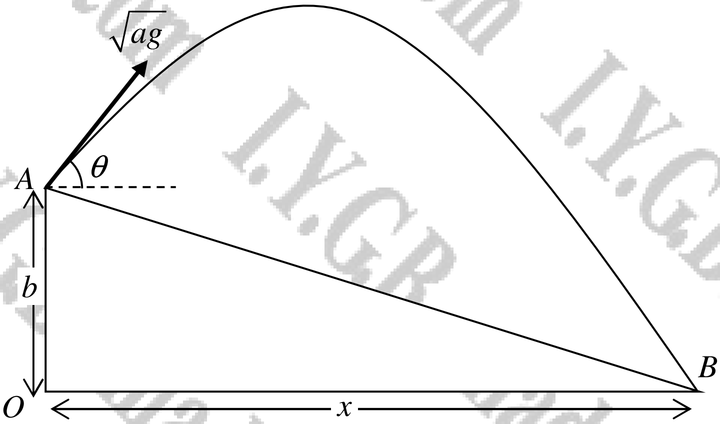
$\Rightarrow 200 + 12d = -\frac{4800}{7} + \frac{1600}{7}\sqrt{21}$

$\Rightarrow 50 + 3d = -\frac{1200}{7} + \frac{400}{7}\sqrt{21}$

$\Rightarrow 3d = -\frac{1550}{7} + \frac{400}{7}\sqrt{21}$

$\Rightarrow d = \frac{-1550 + 400\sqrt{21}}{21}$ // $\simeq 13.48\text{m}$

Question 50 (*****)



A particle is projected from a point B down an incline plane with a speed of $\sqrt{ag}$, where a is a positive constant, at an angle of elevation θ.

The particle is moving freely under gravity and first strikes the ground at a point B. The point O lies vertically below A and at the same horizontal level as B, as shown in the figure above. The plane has constant inclination and the particle moves in a vertical plane which contains the angle of greatest slope of the plane.

a) Show that

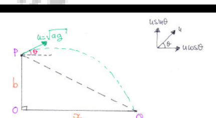
$$x^2 \tan^2\theta - 2ax\tan\theta + x^2 - 2ab = 0,$$

where $OA = b$ and $|OB| = x$.

b) Hence show that the maximum value of x and the corresponding angle of projection θ satisfy

$$x = \sqrt{a(a+2b)} \quad \text{and} \quad \tan\theta = \sqrt{\frac{a}{a+2b}}$$

proof



• HORIZONTALLY

$x = u\cos\theta \times T$

$$\boxed{T = \frac{x}{u\cos\theta}}$$

• VERTICALLY

$y = b + (u\sin\theta)T - \frac{1}{2}gT^2$

$$y = b + (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = b + x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}\sec^2\theta$$

$$y = b + x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}(1+\tan^2\theta)$$

$$(y-b) = x\tan\theta - \frac{gx^2}{2u^2} - \frac{gx^2}{2u^2}\tan^2\theta$$

WHEN AT Q $y = 0$

$$-b = x\tan\theta - \frac{gx^2}{2u^2} - \frac{gx^2}{2u^2}\tan^2\theta$$

$$\Rightarrow \frac{gx^2}{2u^2}\tan^2\theta - x\tan\theta + \frac{gx^2}{2u^2} - b$$

$$\Rightarrow gx^2\tan^2\theta - 2u^2x\tan\theta + gx^2 - 2bu^2 = 0$$

$$\Rightarrow gx^2\tan^2\theta - 2agx\tan\theta + gx^2 - 2bag = 0$$

$$\Rightarrow x^2\tan^2\theta - 2ax\tan\theta + x^2 - 2ab = 0$$

AS REQUIRED

Diff w.r.t θ

$$2x\frac{dx}{d\theta}\tan^2\theta + x^2(2\sec^2\theta\tan\theta) - 2a\frac{dx}{d\theta}\tan\theta - 2ax\sec^2\theta + 2x\frac{dx}{d\theta} = 0$$

FOR MIN/MAX $\frac{dx}{d\theta} = 0$

$$2x^2\sec^2\theta\tan\theta - 2ax\sec^2\theta = 0$$

$$2x^2\tan\theta - 2ax = 0 \qquad \sec^2\theta \neq 0$$

$$2x(x\tan\theta - a) = 0 \qquad x \neq 0 \text{ FOR MAX}$$

$$\tan\theta = \frac{a}{x}$$

OR

$$x\tan\theta = a$$

SUB INTO THE EQUATION

$$(x\tan\theta)^2 - 2a(x\tan\theta) + x^2 - 2ab = 0$$

$$a^2 - 2a^2 + x^2 - 2ab = 0$$

$$x^2 = 2ab + a^2$$

$$x = +\sqrt{a(2b+a)}$$

AS REQUIRED

& $\tan\theta = \frac{a}{x}$

$$\tan\theta = \frac{a}{\sqrt{2ab+a^2}}$$

$$\tan\theta = \sqrt{\frac{a^2}{2ab+a^2}}$$

$$\tan\theta = \sqrt{\frac{a}{a+2b}}$$

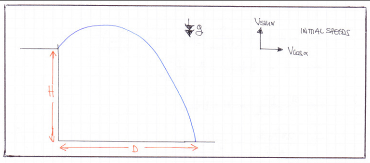
Question 51 (*****)

A particle is projected at an angle α above the horizontal, from a vertical cliff face of height H above level horizontal ground. It first hits the ground at a horizontal distance D, from the bottom of the cliff edge.

Assuming that air resistance can be ignored, show that the greatest height achieved by the particle from the level horizontal ground is

$$H + \frac{D\tan^2\alpha}{4(H + D\tan\alpha)}.$$

proof



- LET T BE THE TIME TAKEN TO REACH THE MAXIMUM HEIGHT

$$\text{"}v = u + at\text{"} \Rightarrow 0 = V\sin\alpha - gT$$
$$\Rightarrow T = \frac{V\sin\alpha}{g}$$

- HENCE THE MAXIMUM HEIGHT ABOVE GROUND IS GIVEN BY

$$H + \text{"}ut + \tfrac{1}{2}at^2\text{"} = H + VT\sin\alpha - \tfrac{1}{2}gT^2$$
$$= H + V\left(\frac{V\sin\alpha}{g}\right)\sin\alpha - \tfrac{1}{2}g\left(\frac{V\sin\alpha}{g}\right)^2$$
$$= H + \frac{V^2\sin^2\alpha}{g} - \frac{1}{2}\,\frac{V^2\sin^2\alpha}{g}$$
$$= H + \frac{V^2\sin^2\alpha}{2g}$$

- WE NEED TO GET RID OF V FROM THE ABOVE EXPRESSION — LET τ (TAU) BE THE FLIGHT TIME

HORIZONTALLY

$$D = (V\cos\alpha)\tau$$
$$\tau = \frac{D}{V\cos\alpha}$$

VERTICALLY ($s = ut + \tfrac{1}{2}at^2$)

$$-H = (V\sin\alpha)\tau - \tfrac{1}{2}g\tau^2$$
$$-H = V\tau\sin\alpha - \tfrac{1}{2}g\tau^2$$

SUBSTITUTE

$$\Rightarrow -H = V\left(\frac{D}{V\cos\alpha}\right)\sin\alpha - \tfrac{1}{2}g\left(\frac{D}{V\cos\alpha}\right)^2$$
$$\Rightarrow -H = D\tan\alpha - \frac{g}{2}\,\frac{D^2}{V^2\cos^2\alpha}$$
$$\Rightarrow \frac{g}{2}\,\frac{D^2}{V^2\cos^2\alpha} = H + D\tan\alpha$$
$$\Rightarrow \frac{2V^2\cos^2\alpha}{gD^2} = \frac{1}{H + D\tan\alpha}$$
$$\Rightarrow V^2\left(\frac{2\cos^2\alpha}{gD^2}\right) = \frac{1}{H + D\tan\alpha}$$
$$\Rightarrow V^2 = \frac{gD^2}{2\cos^2\alpha} \times \frac{1}{H + D\tan\alpha}$$

FINALLY SUBSTITUTE INTO $H + \dfrac{V^2\sin^2\alpha}{2g}$

$$= H + \frac{\cancel{g}D^2}{2\cos^2\alpha} \times \frac{1}{H + D\tan\alpha} \times \frac{\sin^2\alpha}{2\cancel{g}}$$
$$= H + \frac{D\sin^2\alpha}{4\cos^2\alpha\,(H + D\tan\alpha)}$$
$$= H + \frac{D\tan^2\alpha}{4(H + D\tan\alpha)}$$

Question 52 (*****)

A tennis player standing on a level horizontal court serves the ball from a height of 2.25 m above the court. The ball reaches a maximum height of 2.4 m above the court and first hits the court at a horizontal distance of 20 m from the point where the player served the ball. The ball rises for T_1 s and falls for T_2 s .

The ball is modelled as a particle moving through still air without any resistance.

a) Show clearly that

$$\frac{T_2}{T_1} = 4 .$$

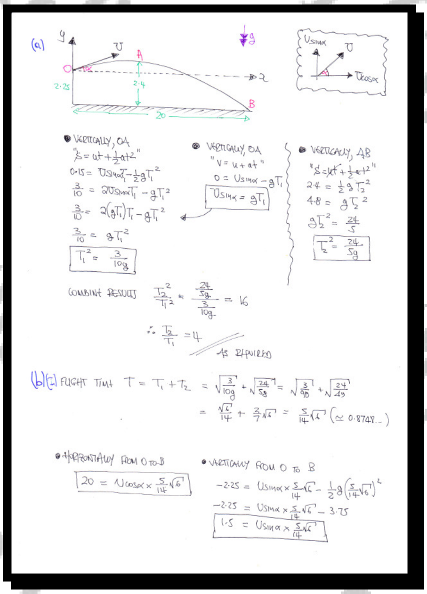
b) Determine the magnitude and direction of the velocity of the ball …

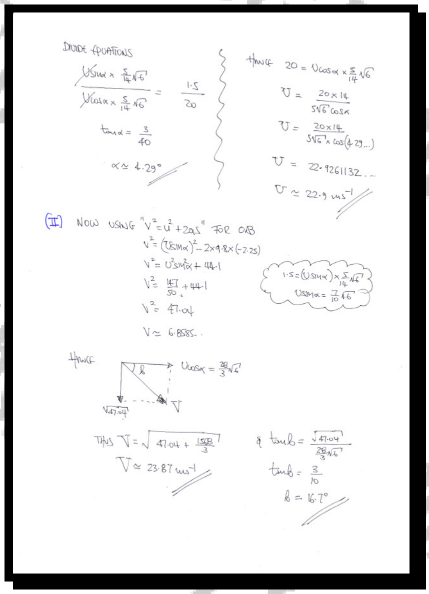
i. … when it was first served.

ii. … as it lands on the court.

$$\boxed{U \approx 22.926...\ \text{ms}^{-1},\ \tan\alpha = \frac{3}{40},\ \alpha \approx 4.289°...},$$

$$\boxed{V \approx 23.8685...\ \text{ms}^{-1},\ \tan\beta = \frac{3}{10},\ \beta \approx 16.699°...}$$





Question 53 (*****)

A particle P is projected from a fixed point O with speed v and at an angle of elevation $\theta°$.

It passes through a point Q which is at a horizontal distance a from O, and a vertical distance h below the level of O.

P is then projected from O with speed v at an angle of depression $(90-\theta)°$ and passes through Q again.

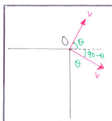
a) Show that

$$v^2 + ag\cot(2\theta) = 0$$

b) Deduce that

$$h + a\tan(2\theta) = 0\,.$$

$\boxed{\text{proof}}$



BOTH TRAJECTORIES PASS THROUGH THE POINT $(a,-h)$ RELATIVE TO THE "ORIGIN"

FOR PROJECTION AT ANGLE OF ELEVATION θ

$a = (v\cos\theta)t$

$-h = (v\sin\theta)t - \frac{1}{2}gt^2$

SIMILARLY FOR ANGLE OF DEPRESSION $(90-\theta)$

$a = (v\sin\theta)T$

$h = (v\cos\theta)T + \frac{1}{2}gT^2$

a) MANIPULATE EACH PAIR SEPARATELY BY ELIMINATING THE TIMES

$t = \dfrac{a}{v\cos\theta}$ $\qquad$ $T = \dfrac{a}{v\sin\theta}$

$-h = (v\sin\theta)\left(\dfrac{a}{v\cos\theta}\right) - \frac{1}{2}g\left(\dfrac{a^2}{v^2\cos^2\theta}\right)$ $\qquad$ $h = (v\cos\theta)\dfrac{a}{v\sin\theta} + \frac{1}{2}g\left(\dfrac{a^2}{v^2\sin^2\theta}\right)$

$-h = a\tan\theta - \dfrac{ga^2}{2v^2}\sec^2\theta$ $\qquad$ $h = a\cot\theta + \dfrac{ga^2}{2v^2}\operatorname{cosec}^2\theta$

$-h = a\tan\theta - \dfrac{ga^2}{2v^2}(1+\tan^2\theta)$ $\qquad$ $h = a\cot\theta + \dfrac{ga^2}{2v^2}(1+\cot^2\theta)$

ELIMINATE h

$\Rightarrow a\cot\theta + \dfrac{ga^2}{2v^2}(1+\cot^2\theta) = \dfrac{ga^2}{2v^2}(1+\tan^2\theta) - a\tan\theta$

$\Rightarrow 2av^2\cot\theta + ga^2(1+\cot^2\theta) = ga^2(1+\tan^2\theta) - 2av^2\tan\theta$

$\Rightarrow 2\cancel{a}v^2(\cot\theta+\tan\theta) + ga^{\cancel{2}}(1+\cot^2\theta-1-\tan^2\theta) = 0$

$\Rightarrow 2v^2(\cot\theta+\tan\theta) + ag(\cot^2\theta-\tan^2\theta) = 0$

$\Rightarrow 2v^2\cancel{(\cot\theta+\tan\theta)} + ag(\cot\theta-\tan\theta)\cancel{(\cot\theta+\tan\theta)} = 0$

$\Rightarrow 2v^2 + ag(\cot\theta-\tan\theta) = 0$

$\Rightarrow 2v^2 + ag\left(\dfrac{1}{\tan\theta}-\tan\theta\right) = 0$

$\Rightarrow 2v^2 + ag\left(\dfrac{1-\tan^2\theta}{\tan\theta}\right) = 0$

$\Rightarrow \cancel{2}v^2 + \cancel{2}ag\left(\dfrac{1-\tan^2\theta}{2\tan\theta}\right) = 0$

$\Rightarrow v^2 + ag\left(\dfrac{1}{\tan 2\theta}\right) = 0$

$\Rightarrow v^2 + ag\cot 2\theta = 0$

b) NOW $\boxed{-h = a\tan\theta - \dfrac{ga^2}{2v^2}(1+\tan^2\theta)}$ FROM EARLIER

$-h = a\tan\theta - \dfrac{ga^2}{2(-ag\cot2\theta)}(1+\tan^2\theta)$

$0 = h + a\tan\theta + \dfrac{ga^2(1+\tan^2\theta)}{2ag\cot2\theta}$

$0 = h + a\left[\tan\theta + \frac{1}{2}(1+\tan^2\theta)\tan2\theta\right]$

$0 = h + a\left[\tan\theta + \frac{1}{2}(1+\tan^2\theta)\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)\right]$

$0 = h + a\left[\tan\theta + \dfrac{(1+\tan^2\theta)\tan\theta}{1-\tan^2\theta}\right]$

$0 = h + a\left[\dfrac{\tan\theta(1-\tan^2\theta) + (1+\tan^2\theta)\tan\theta}{1-\tan^2\theta}\right]$

$0 = h + a\left[\dfrac{\tan\theta - \tan^3\theta + \tan\theta + \tan^3\theta}{1-\tan^2\theta}\right]$

$0 = h + a\left[\dfrac{2\tan\theta}{1-\tan^2\theta}\right]$

$0 = h + a\tan2\theta$