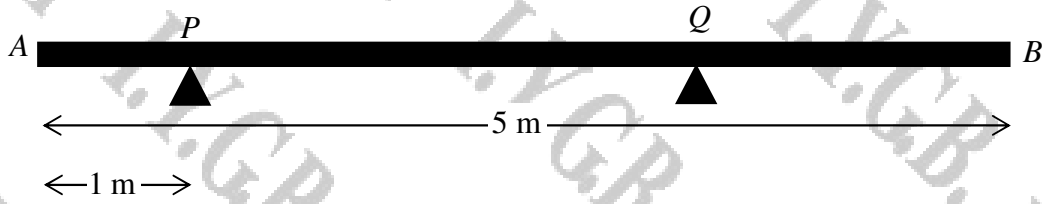


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MOMENTS

Created by T. Madas

Question 1 (**)



A uniform rod AB has length 5 m and weight 100 N.

The rod rests in a horizontal position on two smooth supports at P and Q , where $AP = 1$ m, as shown in the figure above.

The magnitude of the reaction force on the rod at P is 40 N.

- Determine the magnitude of the reaction force on the rod at Q .
- Calculate the distance AQ .

, $R_Q = 60$ N , $|AQ| = 3.5$ m

SUPPORTING WITH A DIAGRAM

BEFORE'S APPROACH

$$40 + R = 100$$

$$R = 60 \text{ N}$$

TAKING MOMENTS ABOUT A

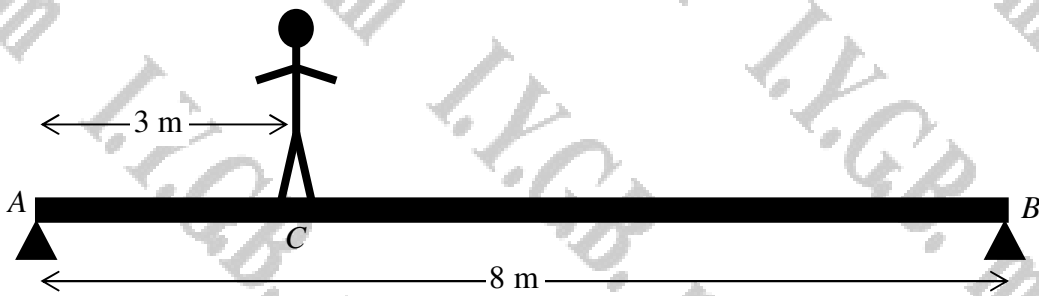
$$R_Q \times 4 = 100 \times 2.5$$

$$40 + 60d = 250$$

$$60d = 210$$

$$d = 3.5 \text{ m}$$

Question 2 (**)



A non uniform plank of wood AB has length 8 m and mass 100 kg.

The plank is smoothly supported at its two ends A and B . A boy of mass 60 kg stands on the plank at the point C , where $AC = 3$ m, as shown in the figure above.

The plank with the boy standing on the plank, remains in equilibrium with AB horizontal. The plank is modelled as a non uniform rod and the boy as a particle.

- a) Given that the reactions at the two supports are equal, determine the distance of the centre of mass of the plank from A .
- b) Explain in the context of this problem the model of
 - i. ... the plank is a rod
 - ii. ... the boy is a particle.

$\sqrt{16.5}$, $|AC| = 4.6$ m

a) SMOOTH SUPPORTS

REACTING WEIGHTS

$$R + R = 60g + 100g$$

$$2R = 160g$$

$$R = 80g$$

TAKING MOMENTS ABOUT A

$$\curvearrowright: (60g \times 2) + (100g \times 4) = R \times 8$$

$$120g + 400g = 8R$$

$$100g + 100g = 8R$$

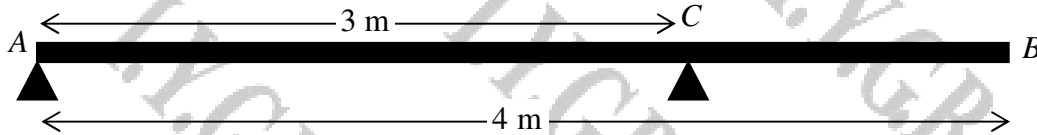
$$200g = 8R$$

$$R = 25g$$

b) "ROD" MODEL AND CENTER OF MASS OF ITS UNIFORMITY, COMPARED TO ITS LENGTH AND MASS, SO IT IS TREATED AS A POINT MASS DIMENSIONLESS OBJECT

"PARTICLE" MODEL CAN BE TREATED AS A "POINT MASS" SO ITS CENTER OF MASS CAN BE PLACED EXACTLY 3 METERS FROM A

Question 3 (**)



A plank of wood AB has length 4 m and mass 40 kg. The plank is smoothly supported at A and at C , where $AC = 3$ m, as shown in the figure above.

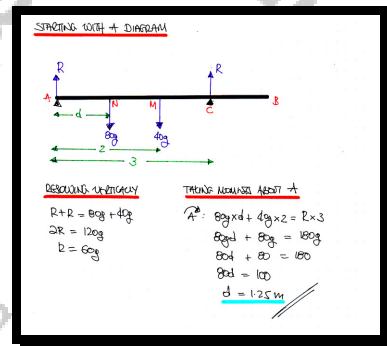
A man of mass 80 kg stands on the plank at a distance d m from A .

The plank, with the man standing on it, remains in equilibrium with AB horizontal, and the reactions on the plank at A and at C equal.

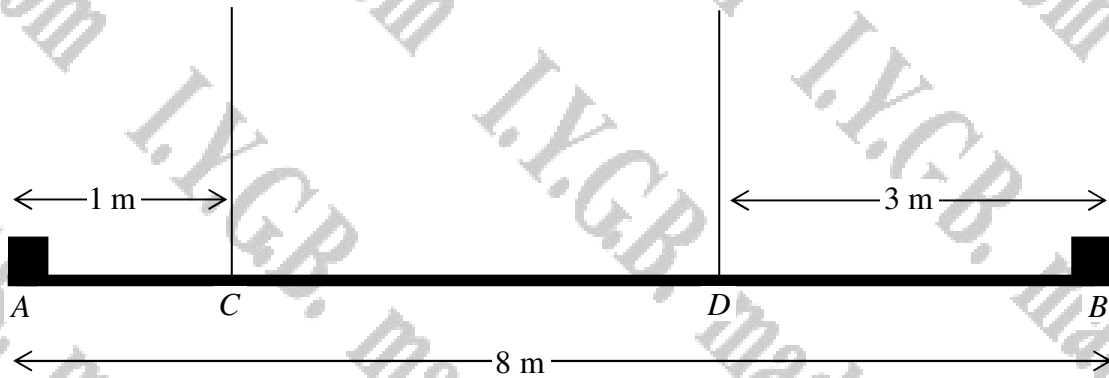
The plank is modelled as a uniform rod and the man as a particle.

Determine the value of d .

, $d = 1.25$ m



Question 4 (**)



A uniform iron girder AB has length 8 m and weight W N. A load of 250 N is attached to the girder at A and a load of 400 N is attached to the girder at B .

The loaded girder is suspended by two light vertical cables attached to the girder at points C and D , where $AC = 1$ m and $DB = 3$ m. When the loaded girder rests undisturbed in a horizontal position, the tension in the cable at D is four times the tension at the cable at C .

The girder is modelled as a uniform rod and the two loads as particles.

- Determine magnitude of the tension on the girder at C .
- Find the value of W .

, $T_C = 600$ N , $W = 2350$ N

START WITH A DIAGRAM

TAKE MOMENTS ABOUT THE WEIGHT

$$(250 \times 4) + (4T \times 1) = (T \times 3) + (400 \times 4)$$

$$1000 + 4T = 3T + 1600$$

$$T = 600 \text{ N}$$

RESOLVE VERTICALLY

$$T + 4T = 250 + W + 400$$

$$600 + 2400 = 650 + W$$

$$W = 2350 \text{ N}$$

Question 5 (**)

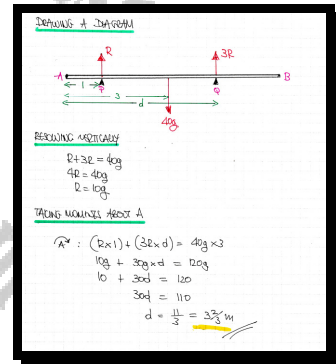
A uniform rod AB has length 6 m and weight 40 kg.

The rod rests in a horizontal position on two smooth supports at P and Q , where $AP = 1$ m and $AQ = d$ m.

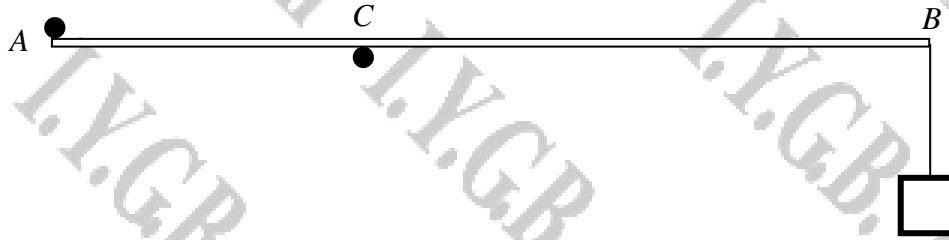
The magnitude of the reaction force on the rod at Q is 3 times as large as that at P .

Calculate the value of d .

$$\boxed{}, \quad \boxed{d = 3\frac{2}{3} \text{ m}}$$



Question 6 (***)



A box of mass 76 kg is attached by a string to one end B of a uniform rod AB of length 5 m and mass 24 kg.

The rod is held horizontally in equilibrium by two smooth cylindrical pegs, one at A and one at C , where $|AC| = 2$ m, as shown in the figure above.

Calculate the magnitude of the forces exerted by each of the pegs onto the rod.

, $R_A = 1176$ N , $R_C = 2156$ N

SENDING WITH A DIAGRAM & NOTING THAT THE ROD IS UNIFORM

BEFORE: UNUSUALLY

$$R_2 = R_1 + 24g + 76g$$

$$R_2 = R_1 + 100g$$

TAKING MOMENTS ABOUT C

$$R_1 \times 2 = 24g \times 0.5 + 76g \times 3$$

$$2R_1 = 12g + 228g$$

$$2R_1 = 240g$$

$$R_1 = 120g$$

$$R_1 = 1176 \text{ N} \quad \text{✓ (ACTION AT A)}$$

HENCE THE R_2

$$R_2 = R_1 + 100g$$

$$R_2 = 120g + 100g$$

$$R_2 = 220g$$

$$R_2 = 2156 \text{ N} \quad \text{✓ (ACTION AT C)}$$

Question 7 (*)**

A beam AB has length 5.5 m and mass 20 kg .

The beam is smoothly supported at the point P , where $AP = 2\text{ m}$.

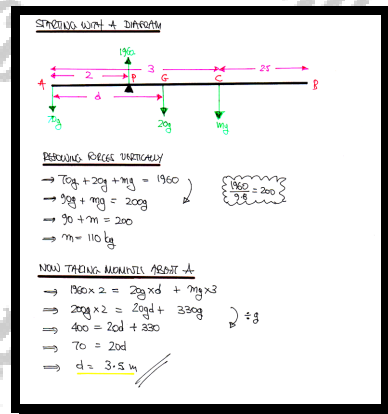
A man of mass 70 kg stands on the beam at A and another man stands on the beam at a distance of 2.5 m from B .

The beam is modelled as a non-uniform rod and the men are modelled as particles.

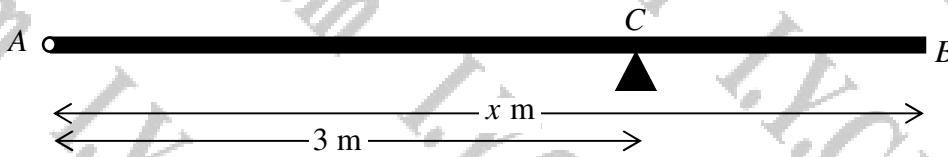
The beam is in equilibrium in a horizontal position with the reaction on the beam at P having a magnitude of 1960 N .

Calculate the distance of the centre of mass of the beam from A .

,



Question 8 (***)



The figure above shows a uniform wooden beam AB , of length x m and weight 80 N. The beam is smoothly hinged at A and rests in a horizontal position on a smooth support at C , where $AC = 3$ m.

When a rock of weight 70 N is placed on the beam at B the magnitude of the reaction force on the beam at C is 165 N.

The beam is modelled as a uniform rod and the rock as a particle.

- a) Calculate the value of x .
- b) Explain briefly the model ...
 - i. ... the beam is a uniform rod.
 - ii. ... the rock is a particle.

The rock is next moved to a new position D on the beam, so that the beam with the rock at D remains in equilibrium in a horizontal position. The magnitude of reaction force at the support at C is now twenty times as large as the reaction force at the hinge at A .

- c) Calculate the distance AD .

, $x = 4.5$ m , $|AD| \approx 3.55$ m or ≈ 420 m

a) LOOKING AT THE DIAGRAM

$\sum \tau = 80 \times \frac{x}{2} + 70 \times x = 165 \times 3$
 $40x + 70x = 495$
 $110x = 495$
 $x = 4.5$ m

b) i) "UNIFORM ROD" ... A ROD OF UNIFORM MASS, WHERE MASS ACTS AT ITS CENTRE "PARTICLE" ... ROCK CAN BE PLACED EXACTLY AT THE POINT B

c) LOOKING AT A NEW DIAGRAM

IF THE REACTION AT A IS "DROUNTS"
 $\bullet \sum \tau = 80 + 70 = 150$
 $R = \frac{150}{2}$
 $\bullet \sum \tau = 80 \times 2.25 + 70 \times d = 20R \times 3$
 $180 + 70d = 60R$
 $180 + 70d = 60 \times \frac{150}{2}$
 $70d = 1750$
 $d \approx 3.55$ m

LOOK THAT IF THE REACTION AT A IS POINTING DOWNWARDS

$\bullet R + 70 + 80 = 20R$
 $150 = 19R$
 $R = \frac{150}{19}$
 $\bullet 80 \times 2.25 + 70d = 20R \times 3$
 $180 + 70d = 60R$
 $180 + 70d = 60 \times \frac{150}{19}$
 $70d = \frac{580}{19}$
 $d = 4.20$ m

Question 9 (***)



A mechanical lever consists of a uniform steel rigid rod AB , of length 2 m and weight 100 N, placed over a smooth pivot at C .

A box of weight 2400 N is suspended by a light inextensible string at B . When a vertical force is applied at A , as shown in the figure above, the lever remains in equilibrium, with AB horizontal.

- a) Given that $CB = 0.3$ m, determine the magnitude of the force applied at A .

The position of the pivot is changed so that lever remains in equilibrium when the vertical force applied at A has magnitude 200 N.

- b) Calculate the new distance of the pivot from B .

, $F = \frac{6500}{17} \approx 382$ N, $d = \frac{5}{27} \approx 0.185$ m

a) STATE WITH DIAGRAM

Diagram for part a: A horizontal rod AB of length 2 m. A pivot is at C, 0.3 m from B. A force F is applied downwards at A. The weight of the rod (100 N) acts downwards at its center (1 m from A). A reaction force R acts upwards at C. A weight of 2400 N acts downwards at B.

TAKING MOMENTS ABOUT C

$$\rightarrow (F \times 1.7) + (100 \times 1) = 2400 \times 0.3$$

$$\Rightarrow 1.7F + 100 = 720$$

$$\Rightarrow 1.7F = 620$$

$$\Rightarrow F \approx 365 \text{ N}$$

b) REDRAW THE DIAGRAM

Diagram for part b: A horizontal rod AB of length 2 m. A pivot is at C, a distance d from B. A force of 200 N is applied downwards at A. The weight of the rod (100 N) acts downwards at its center (1 m from A). A reaction force R acts upwards at C. A weight of 2400 N acts downwards at B.

TAKING MOMENTS ABOUT B & NOTING THAT $R = 2700$ N

$$\Rightarrow (200 \times 2) + (100 \times 1) = R \times d$$

$$\Rightarrow 500 = 2700d$$

$$\Rightarrow d = \frac{5}{27}$$

$$\Rightarrow d \approx 0.185 \text{ m}$$

Question 10 (***)

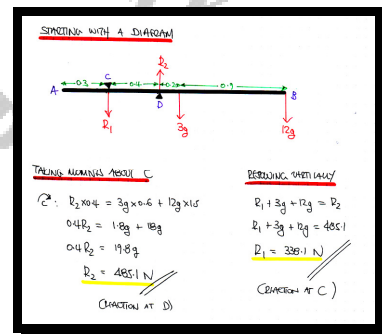


The figure above shows a uniform rod AB of length 1.8 m and mass 3 kg, held in a horizontal position by two small smooth pegs C and D .

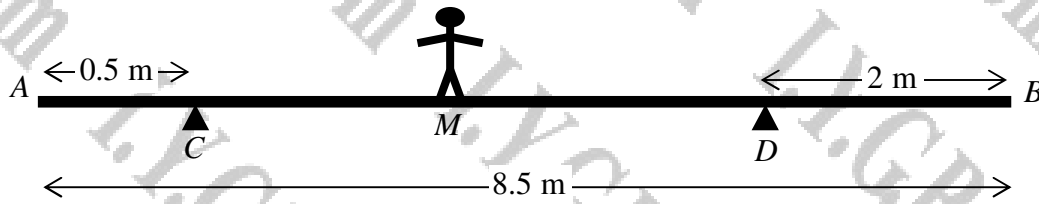
A particle of mass 12 kg, is placed at B .

Given that $|AC| = 0.3$ m and $|CD| = 0.4$ m, determine the magnitude of each of the forces exerted on the rod by the pegs.

, $R_C = 338.1$ N , $R_D = 485.1$ N



Question 11 (***)



A non uniform plank of wood AB has length 8.5 m and mass 20 kg. The centre of mass of the plank is 3.75 m from B . The plank is smoothly supported at C and D , where $AC = 0.5$ m and $DB = 2$ m, as shown in the figure above.

A boy of mass 40 kg stands on the plank at the point M , where M is the midpoint of CD . The plank with the boy standing on the plank, remains in equilibrium with AB horizontal.

The plank is modelled as a non uniform rod and the boy as a particle.

- a) Calculate the magnitude of each of the reaction forces acting on the rod at C and D .

The boy next moves and stands at the point E on the plank, so that the plank is at the point of tilting about D .

- b) Determine the distance DE .

, $|R_C| \approx 253.16... \text{ N}$, $|R_D| \approx 334.83... \text{ N}$, $|DE| = 0.875 \text{ m}$

a) THINK ABOUT IT - DIAGRAM

Diagram showing plank AB with supports at C and D. Distances: AC = 0.5, CD = 3.25, DE = 1.25, EB = 2. Masses: 40g at M, 20g at G. Reaction forces R_1 at C and R_2 at D.

THINK ABOUT IT - ABOUT C

$$(40g \times 3) + (20g \times 4.25) = R_2 \times 6$$

$$120g + 85g = 6R_2$$

$$205g = 6R_2$$

$$R_2 = 334.833...$$

THINK ABOUT IT - ABOUT D

$$R_1 + R_2 = 40g + 20g$$

$$R_1 + 334.833 = 304.833$$

$$R_1 = 253.166...$$

∴ REACTION AT C IS 253.166 N AND REACTION AT D IS 334.833 N

b) THINK ABOUT IT ⇒ $R_1 = 0$ & $R_2 = 60g$

Diagram showing plank AB with support at D. Distances: DE = x, DG = 1.25. Masses: 40g at E, 20g at G.

$$60g \times x = 20g \times 1.25 = 40g \times 2.5$$

$$30x = 40 \times 2.5$$

$$3x = 40$$

$$x = 13.33...$$

∴ $x = 0.875 \text{ m}$

Question 12 (***)



The figure above shows a uniform rod AB of length 4 m and mass 100 kg.

The rod rests in equilibrium in a horizontal position, on two supports at C and D , where $AC = 0.5$ m and $DB = x$ m.

- a) Given that the reaction force at the support at D is three times as large as the reaction force at the support at C , determine the value of x .

The support at D is next moved to a new position E , where $EB = 0.75$ m and an additional mass of m kg is placed at B . The rod remains in equilibrium in a horizontal position and the reaction force at the support at E is now twice as large as the reaction force at the support at C .

- b) Calculate the value of m .

, $x = 1.5$, $m = 20$

a)

Diagram: A horizontal rod AB of length 4 m. Support C is 0.5 m from A, and support D is 2 m from C (so 1.5 m from A). The weight of the rod is 100g acting downwards at the center (2 m from A). Reaction forces are R at C and 3R at D.

RESOLVING VERTICALLY
 $R + 3R = 100g$
 $4R = 100g$
 $R = 25g$

TAKING MOMENTS ABOUT A
 $R \times 0.5 = 3R \times (2-2)$
 $1.5R = 3R(2-2)$
 $0.5 = 2-2$
 $2 = 1.5m$

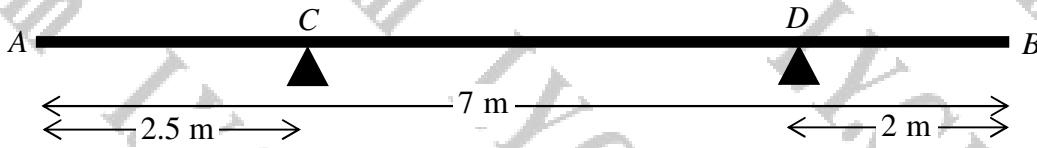
b)

Diagram: A horizontal rod AB of length 4 m. Support C is 0.5 m from A, and support E is 1.25 m from C (so 1.75 m from A). The weight of the rod is 100g acting downwards at the center (2 m from A). An additional mass m is placed at B, with weight mg acting downwards. Reaction forces are R at C and 2R at E.

TAKING MOMENTS ABOUT B
 $(2R \times 0.75) + (100 \times 3.5) = 100g \times 2$
 $1.5R + 350 = 200g$
 $1.5R = 200g - 350$
 $R = 40g$

RESOLVING VERTICALLY
 $R + 2R = 100g + mg$
 $3R = 100g + mg$
 $120g = 100g + mg$
 $20g = mg$
 $m = 20$

Question 13 (***)



A non uniform rod AB has length 7 m and weight 300 N. The centre of mass of the rod is x m from A.

The rod is placed on two smooth supports at C and D, where $AC = 2.5$ m and $DB = 2$ m. The supports at C and D are at the same horizontal level, as shown in the figure above.

When a particle of weight W N is placed on the rod at A the reaction force on the rod at C is 200 N. The rod and the particle rest in equilibrium, with AB in a horizontal position.

- a) Show clearly that

$$200 = 60x - W$$

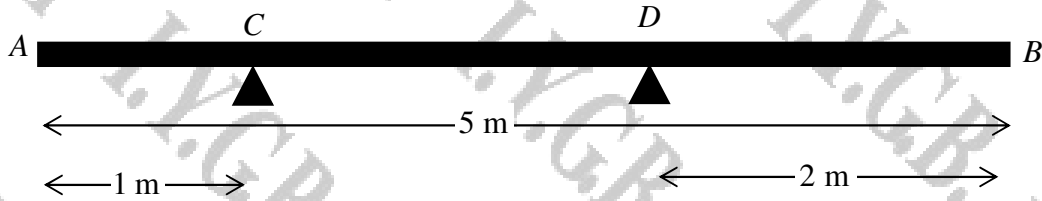
The particle is then removed from A and placed on the rod at B. The rod and the particle remain in equilibrium, with AB in a horizontal position and the reaction force on the rod at C is now 80 N.

- b) Calculate the value of W and the value of x .

SIMS, $W = \frac{300}{7} \approx 42.86$, $x = \frac{85}{21} \approx 4.05$

The handwritten solution shows two parts: (a) and (b). Part (a) shows a free-body diagram of the rod with a weight W at A, a reaction force of 200 N at C, and a reaction force at D. It then takes moments about D to derive the equation $200 = 60x - W$. Part (b) shows a free-body diagram with the weight of 300 N at the center of mass, a reaction force of 80 N at C, and a reaction force at D. It takes moments about D to find x and then uses the sum of forces to find W .

Question 14 (***)



A uniform rod AB has length 5 m and weight 300 N.

The rod rests in a horizontal position on two smooth supports at C and D , where $AC = 1$ m and $DB = 2$ m, as shown in the figure above.

A particle of weight W N is placed on the rod at the point E , where $AE = x$ m.

The magnitude of the reaction on the rod at C is twice the magnitude of the reaction on the rod at D .

- a) Show clearly that

$$W = \frac{750}{5-3x}$$

- b) Determine the range of possible values of x .

, $0 < x < \frac{5}{3}$

a) START WITH A DIAGRAM

RESOLVE VERTICALLY

$$R_1 + R_2 = W + 300$$

$$R_1 = W + 300$$

$$R_2 = \frac{1}{2}W + 150$$

TAKING MOMENTS ABOUT A

$$(2R_2 \times 1) + (2 \times 5) = Wx + 300 \times 1.5$$

$$2R_2 + 10 = Wx + 450$$

$$2R_2 = Wx + 440$$

$$5\left(\frac{1}{2}W + 150\right) = Wx + 440$$

$$\frac{5W}{2} + 750 = Wx + 440$$

$$5W + 1500 = 2Wx + 880$$

$$5W - 2Wx = 880 - 1500$$

$$5W - 2Wx = -620$$

$$W(5 - 2x) = -620$$

$$W = \frac{-620}{5 - 2x}$$

TWO CONSTRAINTS TO BE SATISFIED

$$0 < 2 < 5 \quad \text{AND} \quad W > 0$$

$$\frac{750}{5-3x} > 0$$

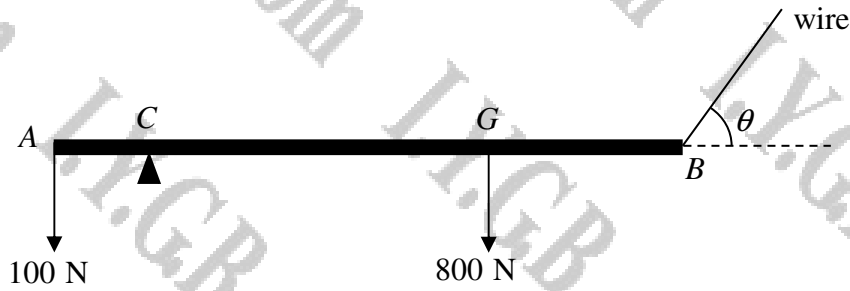
$$5-3x > 0$$

$$-3x > -5$$

$$x < \frac{5}{3}$$

COMBINING $0 < x < \frac{5}{3}$

Question 15 (****)



A thin rigid non uniform beam AB of length 6 m and weight 800 N has its centre of mass at G , where $AG = 4$ m. An additional weight of 100 N is fixed at A .

The beam lies in a horizontal position supported by a rough peg at C , where $AC = 1$ m, and a light inextensible wire attached at B .

When the wire is inclined at an angle θ to the horizontal, where $\sin \theta = 0.8$, the beam remains horizontal, in **limiting** equilibrium.

Calculate the tension in the wire and the value of the coefficient of friction between the peg and the beam.

, $T = 575$ N , $\mu = \frac{69}{88} \approx 0.784$

SIMILAR WITH A JAWAB

PROVING: VERTICAL IS HORIZONTAL

(+) $R + T \sin \theta = 100 + 800$
 (-) $\mu R = T \cos \theta$

TAKING MOMENTS ABOUT C

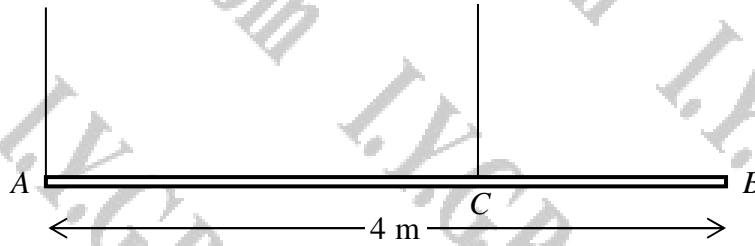
$100 \times 1 + T \sin \theta \times 5 = 800 \times 3$
 $100 + 5T \sin \theta = 2400$
 $5T \sin \theta = 2300$
 $5T \times 0.8 = 2300$
 $T = 575$ N

THE OTHER TWO EQUATIONS NOW YIELD

$R + 575 \times 0.8 = 900$ $R = 440$
 $\mu R = 575 \times 0.6$ $\mu R = 345$

DIVIDING OVS $\mu = \frac{345}{440} = \frac{69}{88}$
 $\mu \leq 0.784$

Question 16 (****)



A rod AB has mass m kg and length 4 m.

The rod is hanging in equilibrium in a horizontal position by two vertical strings attached to the rod. The rod is uniform and the strings are light and inextensible. One string is attached to A and the other string is attached to the point C on the rod as shown in the figure above.

The tension in the string attached at C is twice as large as the tension in the string attached at A .

Then a particle of mass λm kg is attached to the rod at B .

The rod remains in equilibrium in a horizontal position. The tension in the string attached at C is now four times as large as the tension in the string attached at A .

Determine the value of λ .

, $\lambda = \frac{1}{4}$

DRAW A DIAGRAM OF EACH OF THE TWO DESCRIBED SITUATIONS

Then, WORKING AGAIN A IN EACH OF THE ABOVE SITUATIONS

Case 1:
 $\sum \text{moments about } A = 0$
 $mg \times 2 = 2T \times 2$
 $\Rightarrow mg = 2T$
 $3T = mg$
 $\Rightarrow 3T = 2T$
 $\Rightarrow 2 = 3$

Case 2:
 PUTTING 2=3 INTO THE 2ND CASE
 $\sum \text{moments about } A = 0$
 $mg \times 2 + \lambda mg \times 4 = 4T \times 2$
 $\Rightarrow (2+4\lambda)mg = 8T$
 $3T = mg + \lambda mg$
 $ST = (2+\lambda)mg$
 $mg = \frac{ST}{\lambda+1}$
 $\Rightarrow (2+4\lambda)\left(\frac{ST}{\lambda+1}\right) = 8T$
 $\Rightarrow \frac{S(2+4\lambda)}{\lambda+1} = 8$
 $\Rightarrow 10 + 20\lambda = 8\lambda + 8$
 $\Rightarrow 8\lambda = 2$
 $\Rightarrow \lambda = \frac{1}{4}$

Question 17 (***)

The standard unit vectors \mathbf{i} and \mathbf{j} are oriented in the positive x direction and positive y direction, respectively.

Three forces $\mathbf{F}_1 = 4\mathbf{i} + b\mathbf{j}$, $\mathbf{F}_2 = 3a\mathbf{i} + 2b\mathbf{j}$ and $\mathbf{F}_3 = 10b\mathbf{i} + 3\mathbf{j}$, where a and b are scalar constants, are acting at the points $A(1,2)$, $B(4,-2)$ and $C(-3,-5)$, respectively.

- a) Given that the resultant of the three forces is zero, determine the magnitude and direction of the total moment of these three forces about O .
- b) Find, by direct calculation, the magnitude and direction of the total moment of these three forces about C .

, $|\mathbf{G}_O| = 64 \text{ Nm, clockwise}$, $|\mathbf{G}_C| = 64 \text{ Nm, clockwise}$

a)

FORCE	$4\mathbf{j} + b\mathbf{j}$	$3a\mathbf{i} + 2b\mathbf{j}$	$10b\mathbf{i} + 3\mathbf{j}$
POINT	$(1,2)$	$(4,-2)$	$(-3,-5)$

Firstly total force is zero

$$(4 + b)\mathbf{j} + (3a\mathbf{i} + 2b\mathbf{j}) + (10b\mathbf{i} + 3\mathbf{j}) = \mathbf{0}$$

$$(4 + 3a + 10b)\mathbf{i} + (3b + 3)\mathbf{j} = \mathbf{0}$$

$$3b + 3 = 0 \quad 4 + 3a + 10b = 0$$

$$3b = -3 \quad 4 + 3a - 10 = 0$$

$$b = -1 \quad 3a = 6$$

$$a = 2$$

Next draw a diagram - take moments about O

$-(4 \times 2) \left. \begin{array}{l} - (1 \times 1) \\ + (6 \times 2) \\ - (2 \times 4) \end{array} \right\} \begin{array}{l} \mathbf{F}_1 \text{ at } A \\ \mathbf{F}_2 \text{ at } B \\ \mathbf{F}_3 \text{ at } C \end{array}$
 $-(6 \times 5) \left. \begin{array}{l} - (3 \times 3) \\ - (6 \times 5) \end{array} \right\} \begin{array}{l} \mathbf{F}_2 \text{ at } B \\ \mathbf{F}_3 \text{ at } C \end{array}$

\therefore TOTAL MOMENT IS
 $= -8 - 12 - 8 - 9 - 30$
 $= -64$
 $= 64 \text{ Nm clockwise}$

b) MOMENT ABOUT C NOW

$$-(1 \times 4) - (4 \times 7) - (6 \times 3) - (2 \times 7) = -4 - 28 - 18 - 14$$

$$= -64$$

$= 64 \text{ Nm clockwise}$

ALTERNATIVE BY CROSS PRODUCT

a) $\mathbf{G}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 0 \\ 4 & -2 & 0 \\ -3 & -5 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -2 & 0 \\ 6 & -2 & 0 \\ -3 & -5 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -2 & 0 \\ 6 & -2 & 0 \\ -3 & -5 & 0 \end{vmatrix}$

$\mathbf{G}_O = (9, 0, -9) + (9, 0, 4) + (9, 0, -9)$
 $\mathbf{G}_O = (9, 0, -64)$
 i.e. $|\mathbf{G}_O| = 64 \text{ clockwise}$

b) $\vec{CA} = a - c = (1, 2) - (-3, -5) = (4, 7)$
 $\vec{CB} = b - c = (4, -2) - (-3, -5) = (7, 3)$

$\mathbf{G}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 7 & 0 \\ 7 & 3 & 0 \\ 6 & -2 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 0 \\ 4 & -2 & 0 \\ 6 & -2 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -2 & 0 \\ 6 & -2 & 0 \\ -3 & -5 & 0 \end{vmatrix}$

$\mathbf{G}_C = (9, 0, -32) + (9, 0, -32) + (9, 0, 0)$
 $\mathbf{G}_C = (9, 0, -64)$
 i.e. $|\mathbf{G}_C| = 64 \text{ clockwise}$

Question 18 (****)

A non uniform plank AB has length 12 m and mass M kg.

A smooth support is placed under the plank at the point C , where $|AC| = 3$ m. When a child of mass 30 kg stands at A , the plank rest horizontally in equilibrium.

The smooth support is next placed under the plank at the point D , where $|BD| = 5$ m. When the same child stands at B , the plank again rest horizontally in equilibrium.

The plank is modelled as a non uniform rod whose centre of mass is at the point G , and the child is modelled as a particle.

- Determine the value of M .
- Calculate the distance AG .

Two smooth supports are next placed under the plank at the points C and D , and when the same child stands at E , the plank rest horizontally in equilibrium with the reactions at the two supports being equal.

- Find the distance AE .

, $M = 60$, $|AG| = 4.5$ m , $|AE| = 6$ m

(a)(i) SMIT WITH TWO SEPARATE DIAGRAMS SHOWING EACH OF THE TWO CASES

NOTE THAT $R = 30g$ IN BOTH CASES AND $|AG| = 3+x$

THE ONLY UNKNOWN ABOUT C & ABOUT D IS IN EACH CASE TO ELIMINATE R

$\sum \tau^A : 30g \times 3 = Mg \times x$
 $Ng = 90$

$\sum \tau^B : Mg \times (4-x) = 30g \times 5$
 $M(4-x) = 150$
 $4M - Nx = 150$

$\Rightarrow 4M - 90 = 150$
 $\Rightarrow 4M = 240$
 $\Rightarrow M = 60$

NOTE: YOU CAN FIND THE VALUE OF x & SUBSEQUENTLY THE DISTANCE $|AG|$

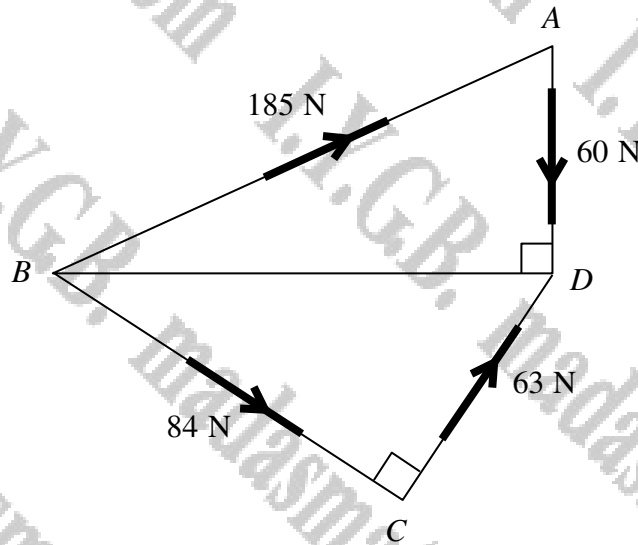
$\Rightarrow Nx = 90$
 $\Rightarrow 60x = 90$
 $\Rightarrow x = 1.5$
 $\therefore |AG| = 3 + x$
 $|AG| = 4.5$ m

(c) SMIT WITH A NEW DIAGRAM

$\bullet \sum \tau^A = 60g + 30g$
 $\sum \tau^B = 90g$
 $R' = 45g$

$\bullet \sum \tau^A : (60g \times 4.5) + (30g \times y) = (R' \times 3) + (R' \times 7)$
 $270g + 30gy = 32g + 70g$
 $270g + 30gy = 102g$
 $270g + 30gy = 10 \times 45g$
 $270 + 30y = 450$
 $27 + 3y = 45$
 $9 + y = 15$
 $y = 6$ m

Question 19 (*****)



The figure above shows a light rigid framework $ABCD$, where $\angle BDA = 90^\circ$ and $\angle BCD = 90^\circ$.

It is also given that $|AB| = 0.37$ m, $|BC| = 0.28$ m, $|CD| = 0.21$ m and $|AD| = 0.12$ m.

Forces of magnitude 185 N, 84 N, 63 N and 60 N are acting along AB , BC , CD and AD , in the directions indicated by the arrows in the figure.

The 4 forces reduce to a single force acting at some point P on AD , and at right angles to AD .

Determine the distance AP .

, $|AP| = 0.108$ m

• START WITH A GOOD DIAGRAM

• FIRSTLY LET US NOTE THAT
 $|BD| = \sqrt{0.37^2 - 0.12^2} = \sqrt{0.28^2 + 0.21^2} = 0.35$

• ON THE TOP HALF OF THE FRAMEWORK THE FORCES ARE IN PROPORTION TO THE PYTHAGOREAN TRIPLE OF THE LENGTHS BY A SCALE FACTOR OF 500.

• SIMILARLY IN THE BOTTOM HALF THE SCALE FACTOR IS 300.

• NEXT WE NEED TO FIND $|DE|$ (USING AREA)

$$\frac{1}{2}|BC||CD| = \frac{1}{2}|BD||DE|$$

$$0.28 \times 0.21 = 0.35|DE|$$

$$|DE| = 0.168$$

• HAVE THE PROBLEM BEEN REDUCED TO THE FOLLOWING:

• THIS BY INSPECTION (RATIO)

11 ← 500
 $|AP| = \frac{3}{8} \times (0.12 + 0.168)$
 $|AP| = 0.108$

175 : 105
5 : 3