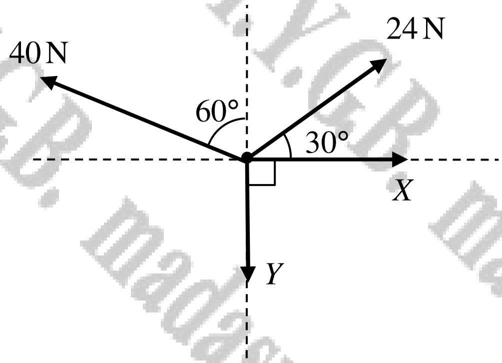
EQUILIBRIUM

EQUILIBRIUM INTRODUCTION

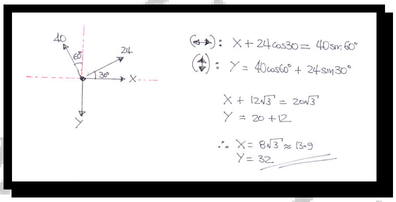
Question 1

The four coplanar forces, shown in the figure below, are in equilibrium.



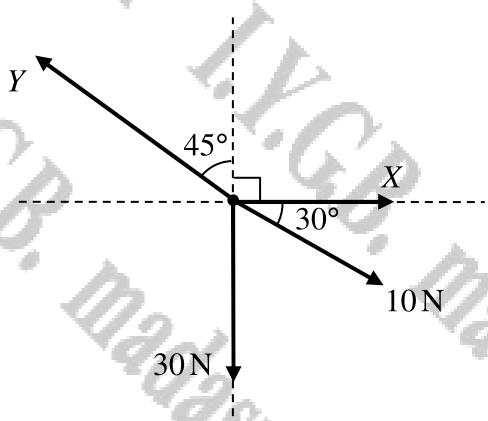
Determine the value of X and the value of Y.

$\boxed{X = 8\sqrt{3} \approx 13.9}$, $\boxed{Y = 32}$



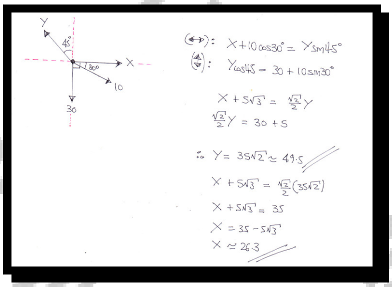
Question 2

The four coplanar forces, shown in the figure below, are in equilibrium.



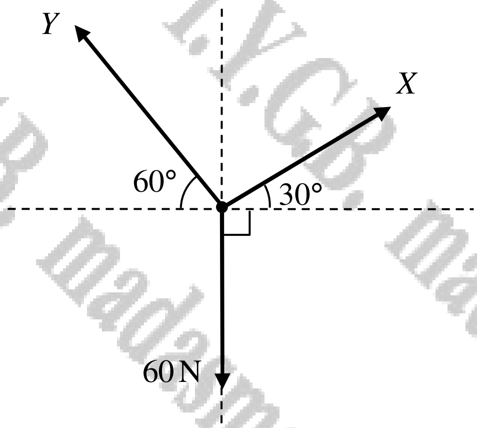
Determine the value of X and the value of Y.

$\boxed{X = 35 - 5\sqrt{3} \approx 26.3}$, $\boxed{Y = 35\sqrt{2} \approx 49.5}$



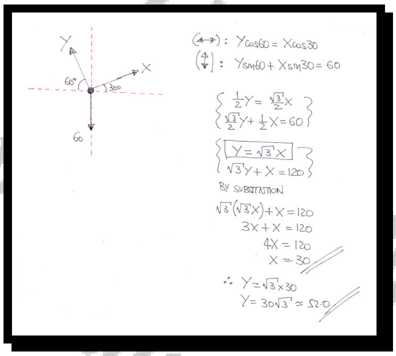
Question 3

The three coplanar forces, shown in the figure below, are in equilibrium.



Determine the value of X and the value of Y.

$\boxed{X = 30}$, $\boxed{Y = 30\sqrt{3} \approx 52.0}$



Question 4

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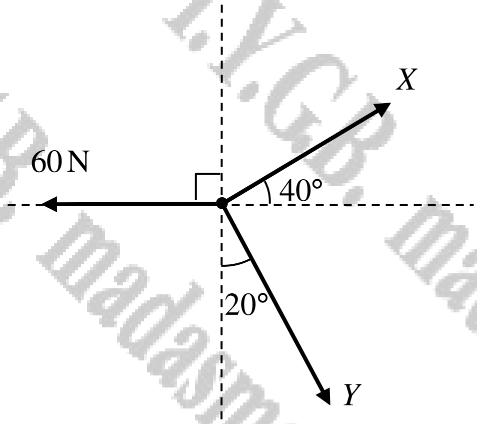
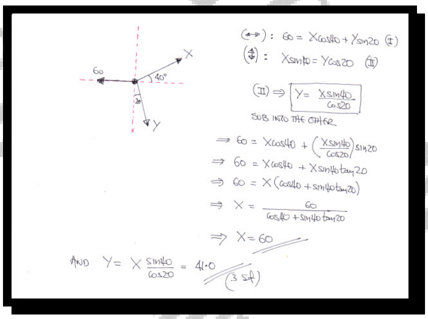


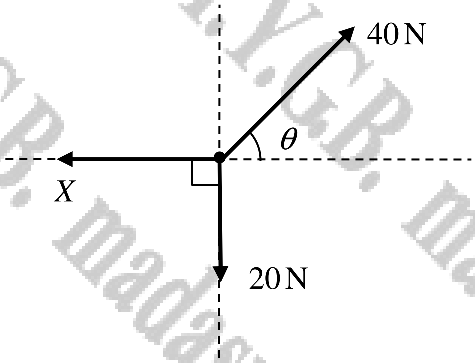
Determine the value of X and the value of Y.

$\boxed{X = 60}$, $\boxed{Y \approx 41.0}$



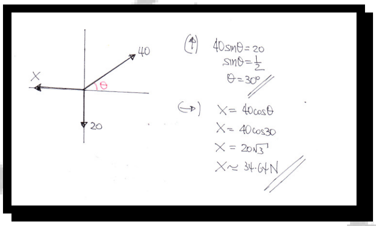
Question 5

The three coplanar forces, shown in the figure below, are in equilibrium.



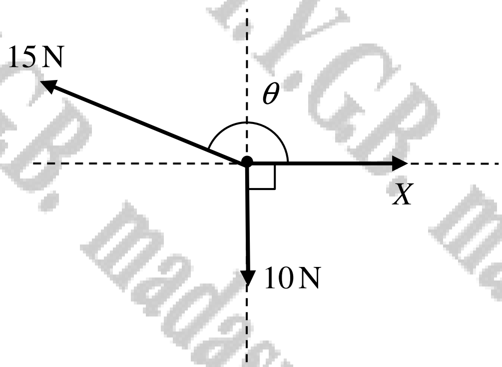
Determine the value of X and the value of θ.

$\boxed{X \approx 34.64}$, $\boxed{\theta = 30°}$



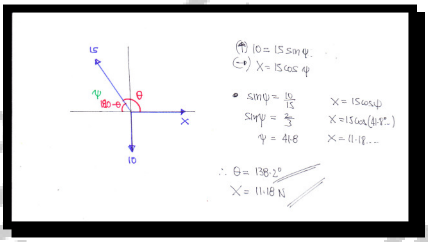
Question 6

The three coplanar forces, shown in the figure below, are in equilibrium.



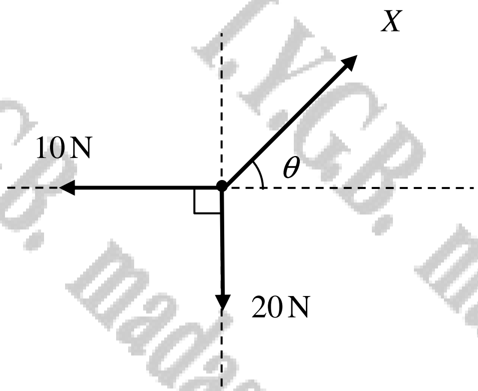
Determine the value of X and the value of θ.

$\boxed{X \approx 11.18}$, $\boxed{\theta = 138.2°}$



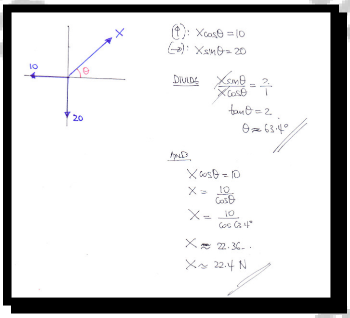
Question 7

The three coplanar forces, shown in the figure below, are in equilibrium.



Determine the value of X and the value of θ.

$\boxed{X \approx 22.4}$, $\boxed{\theta = 63.4°}$



Question 8

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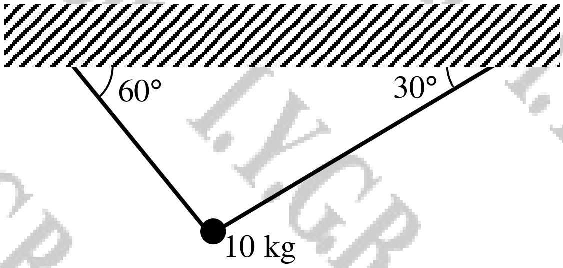
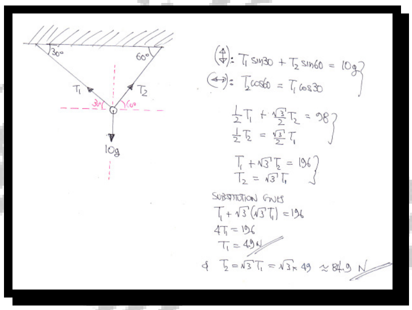


The figure above shows a particle of mass 10 kg suspended by two strings from a fixed horizontal ceiling. The particle hangs in equilibrium.

The strings are light and inextensible and are inclined at $30°$ and $60°$ to the ceiling.

Find the tension in each of the two strings.

49N, 84.9N



Question 9

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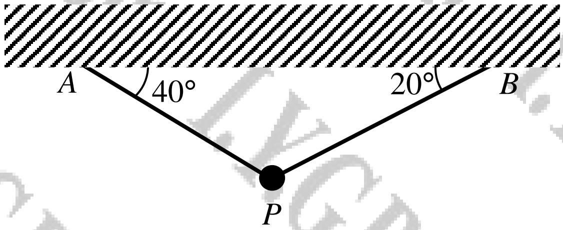
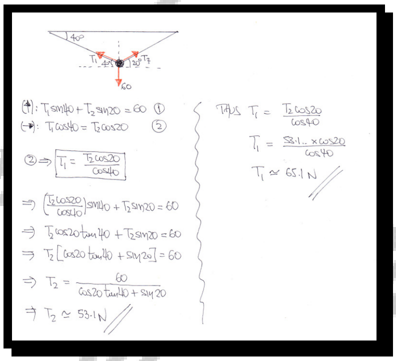


A particle P of weight 60 N is suspended by two strings from a fixed horizontal ceiling. The particle hangs in equilibrium.

The strings are light and inextensible and are inclined at 40° and 20° to the ceiling, as shown in the figure above.

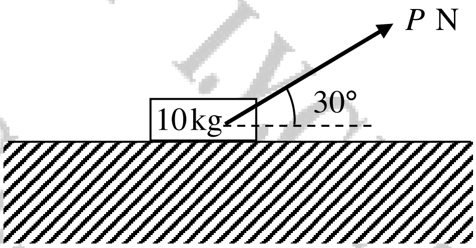
Find the tension in each of the two strings.

$\boxed{T_A \approx 65.1 \text{ N}}$, $\boxed{T_B \approx 53.1 \text{ N}}$



EQUILIBRIUM
FURTHER PROBLEMS

Question 1 **(**)**



The figure above shows a small box of mass 10 kg, pulled along rough horizontal ground by a light inextensible rope, which is inclined at $30°$ to the horizontal.

The force supplied by the rope is P N.

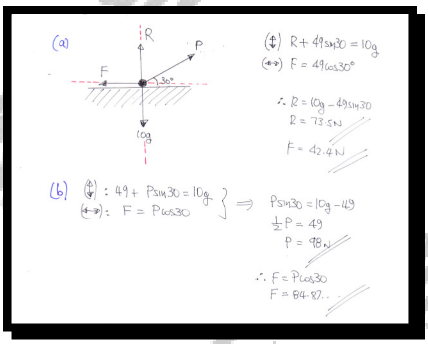
The box is modelled as a particle experiencing a constant frictional force of F N and a normal reaction of R N.

The box is in **equilibrium**.

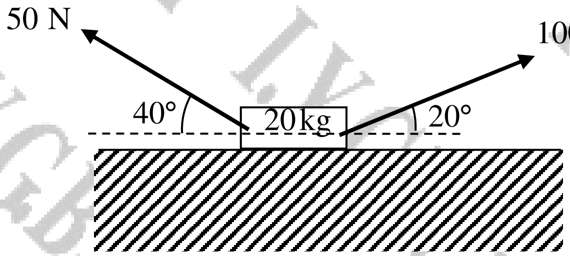
a) Given that $P = 49$ N, determine the value of R and the value of F.

b) Given instead that $R = 49$ N, determine the value of P and the value of F.

$\boxed{R \approx 73.5, \ F \approx 42.4}$, $\boxed{P = 98, \ F \approx 84.9}$



Question 2 (**)



The figure above shows a small box of mass 20 kg, pulled by two light inextensible strings along rough horizontal ground.

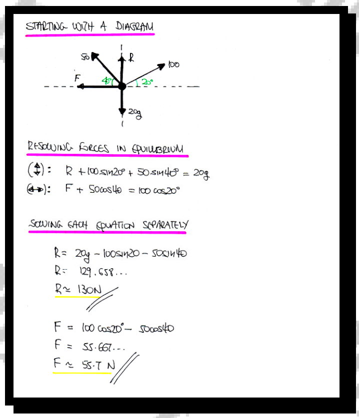
The tension in the rope inclined at $20°$ to the horizontal is 100 N.

The tension in the rope inclined at $40°$ to the horizontal is 50 N.

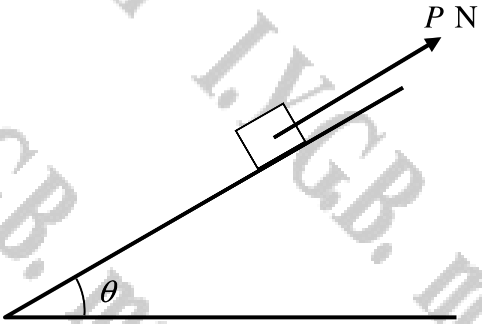
The box is modelled as a particle in equilibrium, experiencing a constant frictional force of F N and a normal reaction of R N.

Determine the value of R and the value of F.

$R \approx 129.66...,\quad F \approx 55.667...$



Question 3 (**)



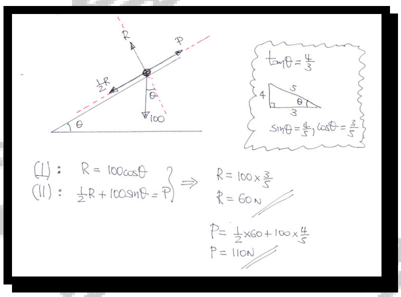
The figure above shows a box, of weight 100 N, on a plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

The box is kept in **equilibrium** by a force P N, acting up the plane, in the direction of the greatest slope.

The box is also experiencing a frictional force of magnitude $\frac{1}{2}R$ N **down** the plane, where R N is the normal reaction between the box and the plane.

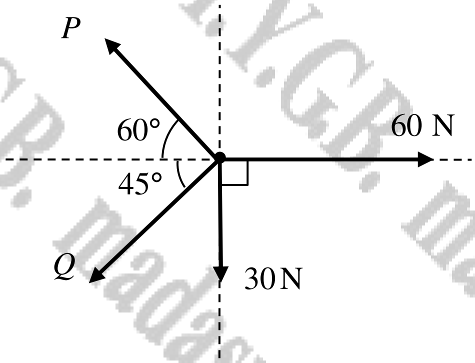
By modelling the box as a particle, find the value of R and the value of P.

$$\boxed{R = 60, \quad P = 110}$$



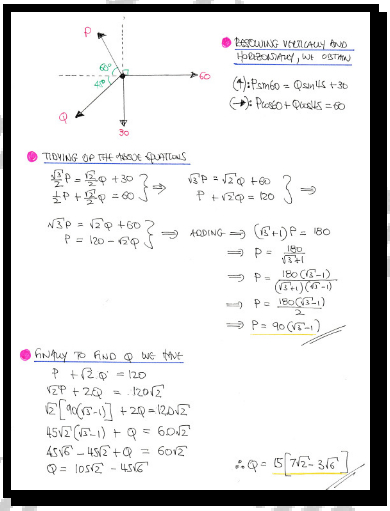
Question 4 (**)

The four coplanar forces, shown in the figure below, are in equilibrium.

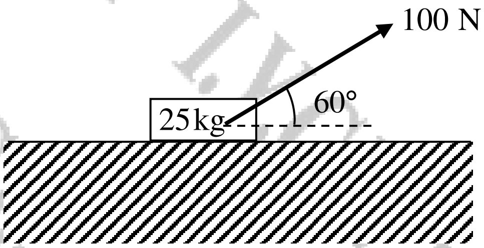


Determine in exact surd form the value of P and the value of Q.

$\boxed{}$, $\boxed{P = 90\left[\sqrt{3} - 1\right]}$, $\boxed{Q = 15\left[7\sqrt{2} - 3\sqrt{6}\right]}$



Question 5 (**)



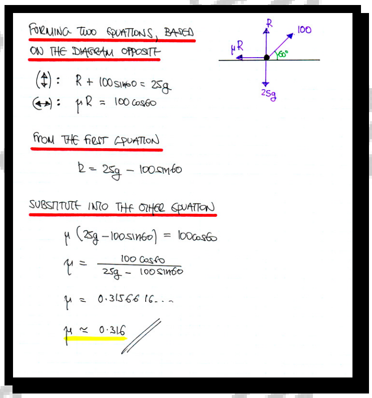
The figure above shows a small box of mass 25 kg, pulled along rough horizontal ground by a light inextensible rope, which is inclined at 60° to the horizontal.

The force supplied by the rope is 100 N.

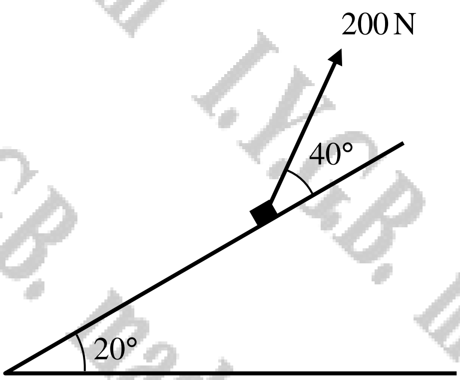
The box, which is modelled as a particle, is in limiting **equilibrium**.

Calculate the value of the coefficient of friction between the box and the ground.

$\mu \approx 0.316$



Question 6 (+)**

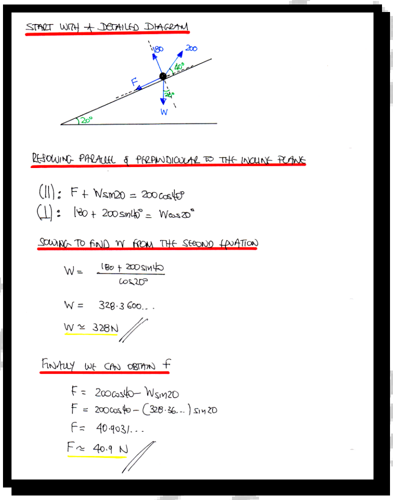


The figure above shows a box of weight W N, resting on a plane inclined at an angle of $20°$ to the horizontal. The box is kept in **equilibrium** by a force of 200 N, acting up the plane, at an angle of $40°$ to the direction of the greatest slope of the plane.

The box is experiencing a frictional force of magnitude F N **down** the plane. The normal reaction between the box and the plane has magnitude 180 N.

By modelling the box as a particle, find the value of W and the value of F.

$W \approx 328, \quad F \approx 40.9$



Question 7 (+)**

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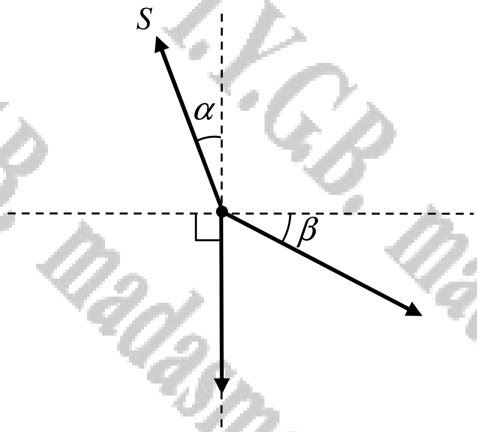
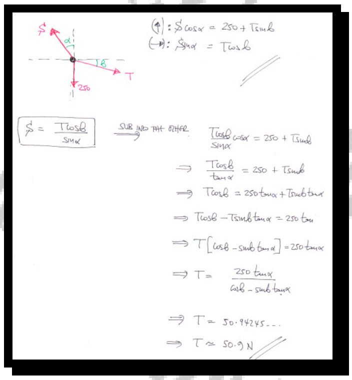
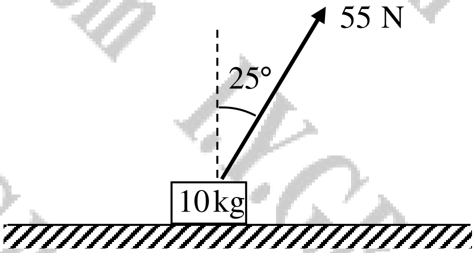


By writing two equations, or otherwise, determine a simplified expression for T, in terms of α and β.

$$T = \frac{250\tan\alpha}{\cos\beta - \sin\beta\tan\alpha}$$



Question 8 (***)



The figure above shows a small box of mass 10 kg pulled by a rope inclined at 25° to the vertical, along rough horizontal ground.

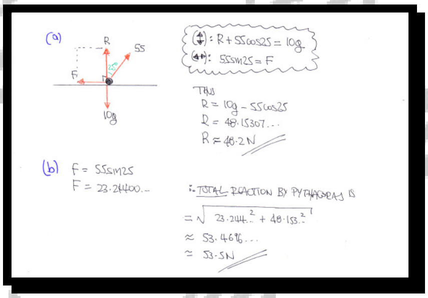
When the tension in the rope is 55 N the box rests in **equilibrium**, on the ground.

By modelling the box as a particle, calculate …

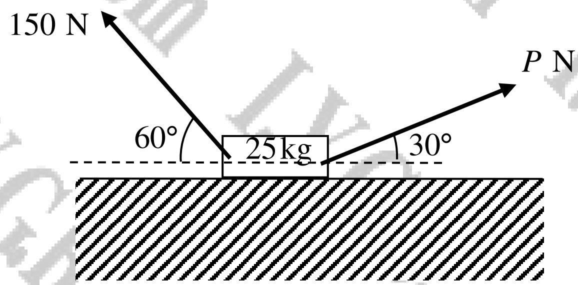
a) … the normal reaction between the box and the ground.

b) … the magnitude of the total force exerted by the ground on the box.

$\boxed{R \approx 48.153...\text{N}}$, $\boxed{R_{\text{total}} \approx 53.4696...\text{N}}$



Question 9 (***)



The figure above shows a small box of mass 25 kg, pulled by two light inextensible strings along rough horizontal ground.

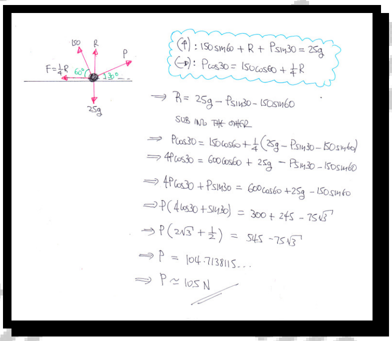
The tension in the rope inclined at 30° to the horizontal is P N.

The tension in the rope inclined at 60° to the horizontal is 150 N.

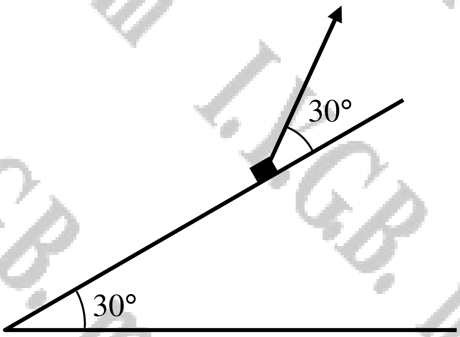
The box is modelled as a particle experiencing a constant frictional force of F N and a normal reaction of R N. The friction opposes potential motion by the action of P.

Given that when the box is in equilibrium $F = \frac{1}{4}R$, calculate the value of P.

$P \approx 104.7138...$



Question 10 (***)



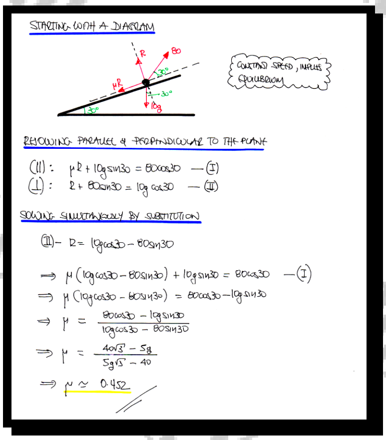
A box of mass 10 kg is pulled by a rope on a fixed rough inclined plane. The rope is modelled as a light inextensible string and the box is modelled as a particle. The plane is at an angle of $30°$ to the horizontal, as shown in the figure above.

The rope lies in a vertical plane containing a line of greatest slope of the incline plane and is inclined at $30°$ to the plane. When the tension in the rope is 80 N the box is travelling up the plane, at constant speed.

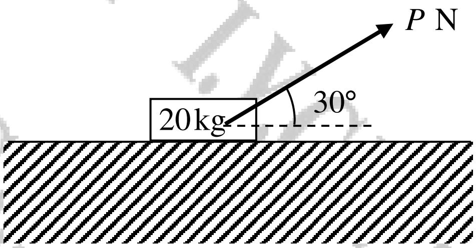
The normal reaction between the box and the plane is R N .

Given that the magnitude of the friction between the box and the plane is μR, where μ is a positive constant, determine the value of μ.

$\boxed{}$, $\boxed{\mu \approx 0.452}$



Question 11 **(***)**



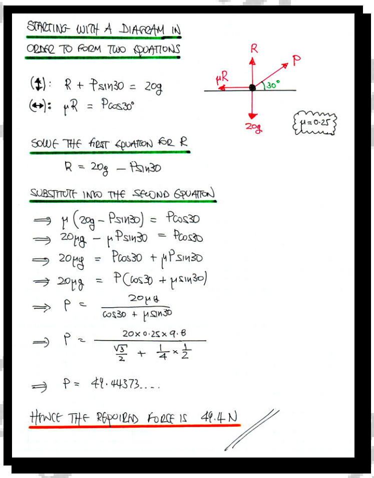
The figure above shows a small box of mass 20 kg, pulled along rough horizontal ground by a light inextensible rope, which is inclined at 30° to the horizontal.

The force supplied by the rope is P N.

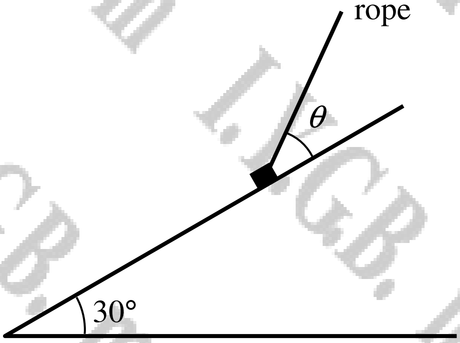
The box, which is modelled as a particle, is at the point of slipping.

Given further that the coefficient of friction between the box and the ground is 0.25, calculate the value of P.

$P \approx 49.4$



Question 12 **(***)**



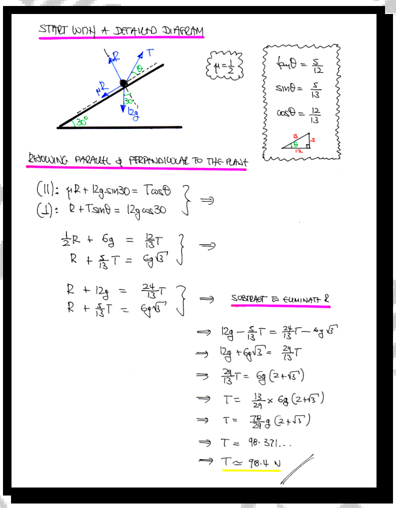
A box of mass 12 kg is held by a rope, in limiting equilibrium, on a fixed rough inclined plane. The plane is at an angle of 30° to the horizontal, as shown in the figure above.

The rope lies in a vertical plane containing a line of greatest slope of the incline plane and is inclined to the plane at an angle θ, where $\tan\theta = \frac{5}{12}$.

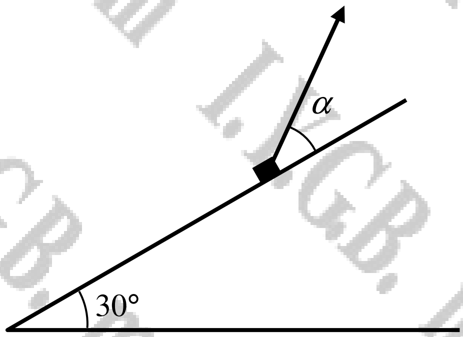
The rope is modelled as a light inextensible string and the box is modelled as a particle. The coefficient of friction between the box and the plane is $\frac{1}{2}$.

Determine the tension in the rope when the box is at the point of slipping up the plane.

$\boxed{}$, $\boxed{T \approx 98.3717\ldots \text{ N}}$



Question 13 (***)



A box of weight 20 N is held by a rope, in limiting equilibrium on a fixed rough inclined plane. The rope is modelled as a light inextensible string and the box is modelled as a particle.

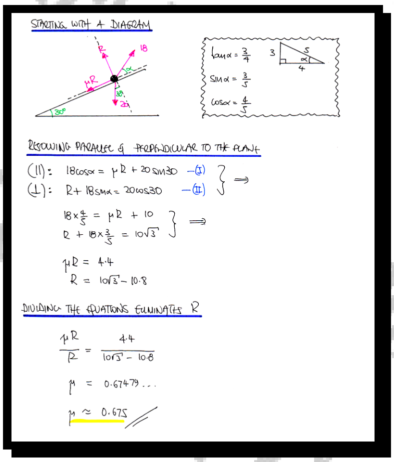
The rope lies in a vertical plane containing a line of greatest slope of the incline plane and is inclined to the plane at an angle α, where $\tan\alpha = \frac{3}{4}$.

The plane is at an angle of $30°$ to the horizontal, as shown in the figure above.

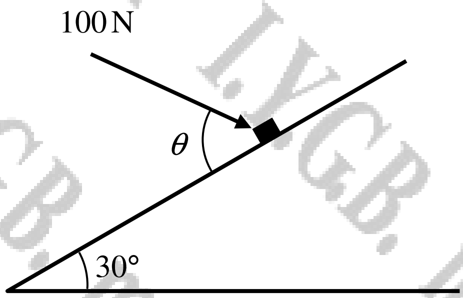
When the tension in the rope is 18 N the box is at the point of slipping up the plane.

Calculate the value of the coefficient of friction between the box and the plane.

$\boxed{}$, $\boxed{\mu \approx 0.675}$



Question 14 (***)

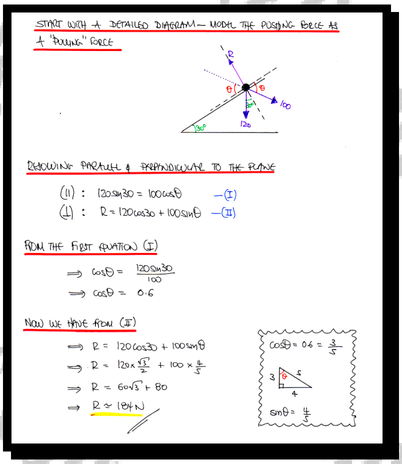


A particle of weight 120 N, rests on a **smooth** plane inclined at an angle of 30° to the horizontal.

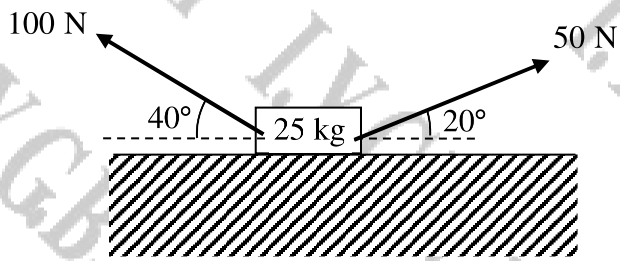
The box is kept in **equilibrium** by a force of magnitude 100 N, **pushing** at an angle of θ $(\theta > 30°)$ to the direction of the greatest slope of the plane, as shown in the above figure.

Calculate the normal reaction between the particle and the plane.

$R \approx 184$



Question 15 **(***)**



The figure above shows a small box of mass 25 kg, pulled by two light inextensible strings along rough horizontal ground.

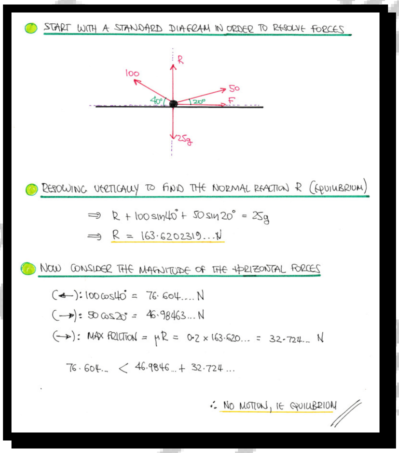
The tension in the rope inclined at 20° to the horizontal is 50 N.

The tension in the rope inclined at 40° to the horizontal is 100 N.

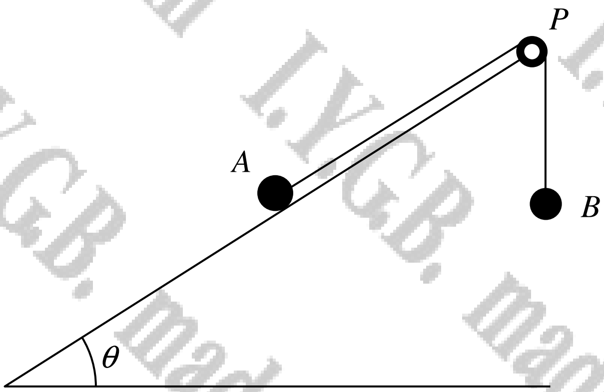
The box is modelled as a particle experiencing a constant frictional force, where the coefficient of fiction between the box and ground is 0.2.

Determine, by detailed calculations, whether the box remains in equilibrium.

, It remains in equilibrium



Question 16 (***)



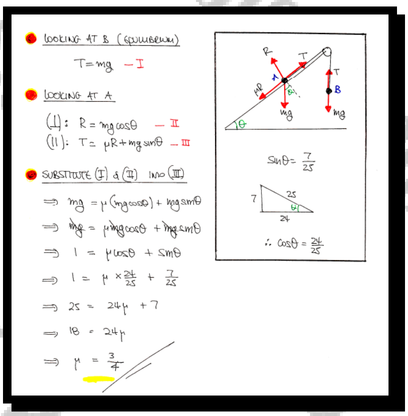
Two particles A and B, of equal mass are attached to each of the ends of a light inextensible string. The string passes over a smooth pulley P, at the top of a fixed rough plane, inclined at θ to the horizontal, where $\sin\theta = 0.28$.

Particle A is placed at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P, as shown in the figure above. The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane.

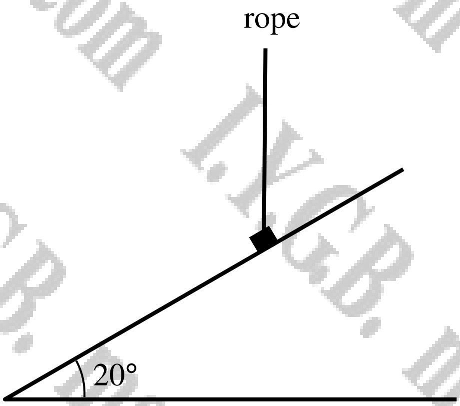
When the particles are released, A is at the point of slipping up the incline plane.

Find the value of the coefficient of friction between A and the plane.

$\boxed{\mu = \frac{3}{4}}$



Question 17 (*+)**



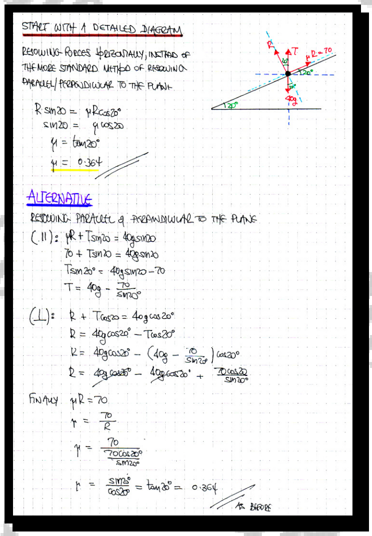
A box of mass 40 kg is held by a rope on a fixed rough inclined plane. The plane is at an angle of 20° to the horizontal. The rope is vertical and lies in a vertical plane containing a line of greatest slope of the incline plane, as shown in the figure above.

The rope is modelled as a light inextensible string and the box is modelled as a particle, at the point of slipping down the plane.

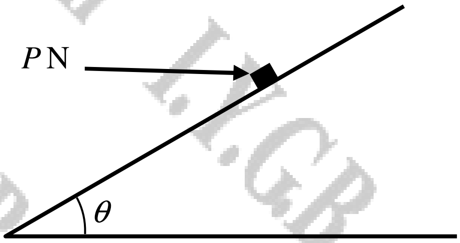
The coefficient of friction between the box and the plane is μ.

Given that the ground friction has magnitude 70 N, determine the value of μ.

$\boxed{}$, $\boxed{\mu \approx 0.364}$



Question 18 (*+)**



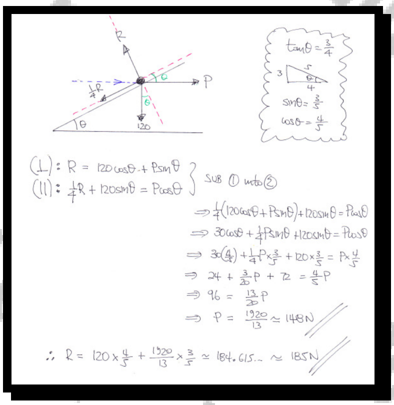
A box of weight 120 N, rests on a plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$.

The box is kept in **equilibrium** by a **horizontal** force P N, pushing the box up the plane, as shown in the above figure.

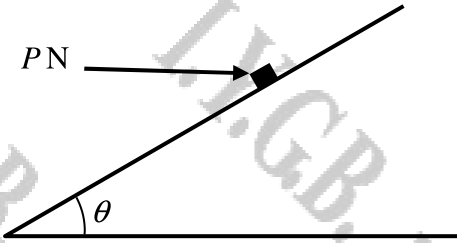
The box is also experiencing friction of magnitude $\frac{1}{4}R$ N **down** the plane, where R N is the normal reaction between the box and the plane.

By modelling the box as a particle, find the value of P and the value of R.

$$\boxed{P \approx 148, \quad R \approx 185}$$



Question 19 (***+)

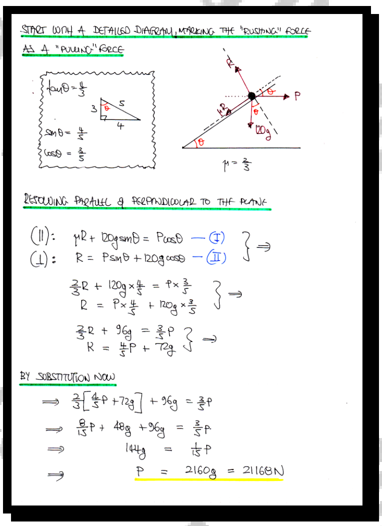


The figure above shows a box of mass 120 kg, resting in limiting equilibrium, on a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

A **horizontal** force P N, pushes the box so that the box is at the point of slipping up the plane. The coefficient of friction between the box and the plane is $\frac{2}{3}$.

By modelling the box as a particle, find the value of P.

$\boxed{}$, $\boxed{P = 2160g = 21168\text{ N}}$



START WITH A DETAILED DIAGRAM, MARKING THE "PUSHING" FORCE AS A "PULLING" FORCE

$\tan\theta = \frac{4}{3}$, $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$

$\mu = \frac{2}{3}$

RESOLVING PARALLEL & PERPENDICULAR TO THE PLANE

$(\parallel)$: $\mu R + 120g\sin\theta = P\cos\theta$ — (I)
$(\perp)$: $R = P\sin\theta + 120g\cos\theta$ — (II)

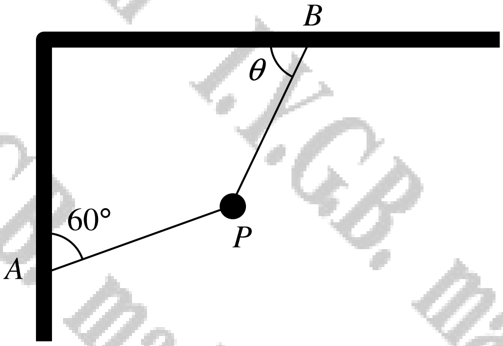
$\frac{2}{3}R + 120g \times \frac{4}{5} = P \times \frac{3}{5}$
$R = P \times \frac{4}{5} + 120g \times \frac{3}{5}$

$\frac{2}{3}R + 96g = \frac{3}{5}P$
$R = \frac{4}{5}P + 72g$

BY SUBSTITUTION NOW

$\Rightarrow \frac{2}{3}\left[\frac{4}{5}P + 72g\right] + 96g = \frac{3}{5}P$
$\Rightarrow \frac{8}{15}P + 48g + 96g = \frac{3}{5}P$
$\Rightarrow 144g = \frac{1}{15}P$
$\Rightarrow P = 2160g = 21168\text{N}$

Question 20 (***+)

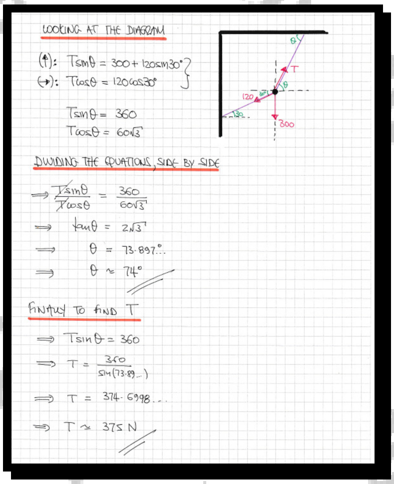


A particle P, of weight 300 N, is hanging in a equilibrium by two light inextensible strings, AP and BP, which lie in the same vertical plane.

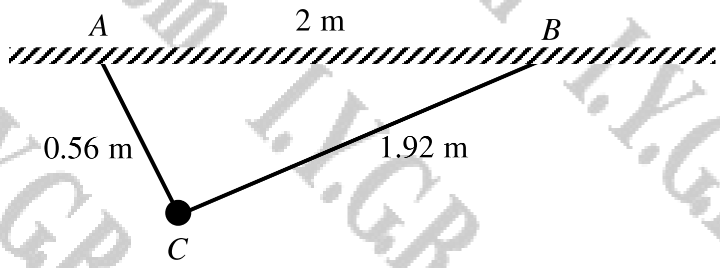
It is further given that AP is forms an angle of $60°$ with a vertical wall and BP forms an angle θ with a horizontal ceiling.

Calculate the value of θ and the tension in BP, if the tension in AP is 120 N.

$\boxed{}$, $\boxed{\theta \approx 74°}$, $\boxed{T_{BP} \approx 375 \text{ N}}$



Question 21 (****)



The points A and B are 2 m apart and lie on a fixed horizontal ceiling. A particle C, of weight 100 N, is suspended by two strings from A and B, so that $AC = 0.56 \text{ m}$ and $BC = 1.92 \text{ m}$ as shown in the figure above. The particle hangs in equilibrium.

a) Show clearly that $\measuredangle ACB = 90°$.

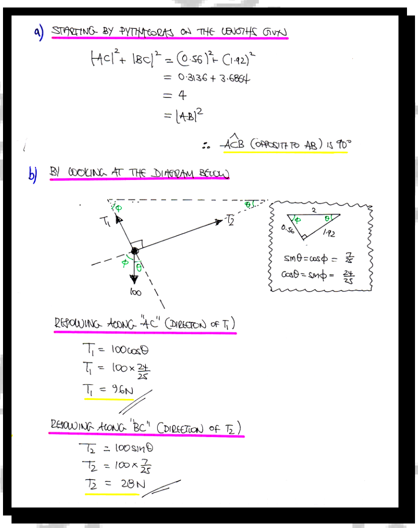
b) Calculate the tension in each of the two strings.

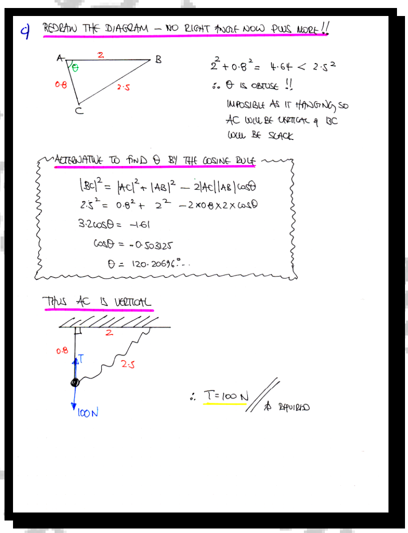
The particle is next suspended by two different strings from A and B, so that $AC = 0.8 \text{ m}$ and $BC = 2.5 \text{ m}$.

The particle still hangs in equilibrium.

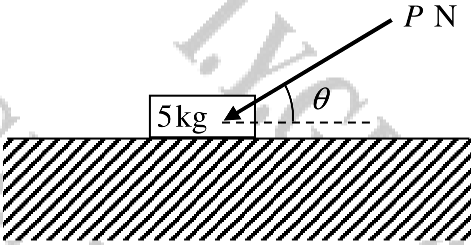
c) Show further that the tension in the string AC is 100 N.

$\boxed{}$, $\boxed{T_{BC} = 28 \text{ N}}$, $\boxed{T_{AC} = 96 \text{ N}}$





Question 22 (****)

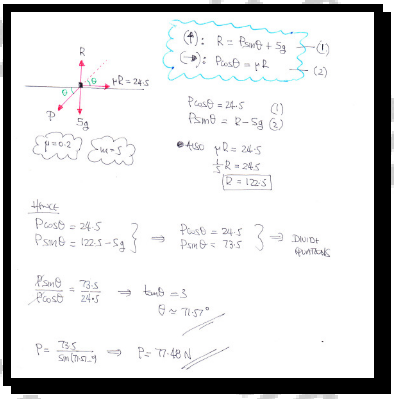


The figure above shows a small box of mass 5 kg, pushed by a constant force P.

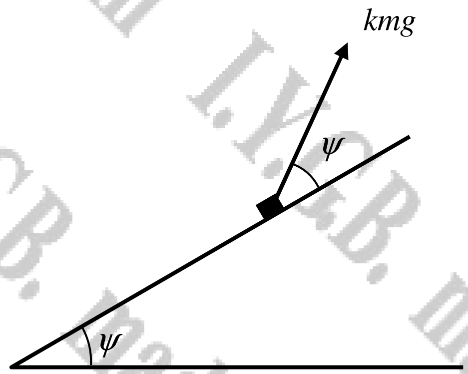
The force pushing the box has magnitude P N and is inclined at θ to the horizontal. The box is modelled as a particle experiencing a constant **ground** frictional force of 24.5 N, where the coefficient of friction is 0.2.

Given further that the box is in limiting equilibrium, determine the value of P and the value of θ.

$\boxed{P = 77.48 \text{ N}}$, $\boxed{\theta \approx 71.57°}$



Question 23 (****)



A fixed rough plane is inclined to the horizontal at an angle ψ, where $\tan\psi = \frac{1}{2}$.

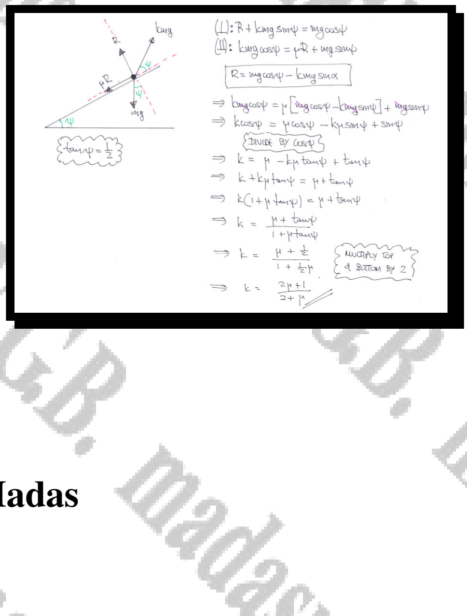
A small box of mass m is at rest on the plane. A force of magnitude kmg, where k is a positive constant, is applied to the box. The line of action of the force is at angle ψ to the line of greatest slope of the plane through the box, as shown in the above figure, and lies in the same vertical plane as this line of greatest slope.

The coefficient of friction between the box and the plane is μ.

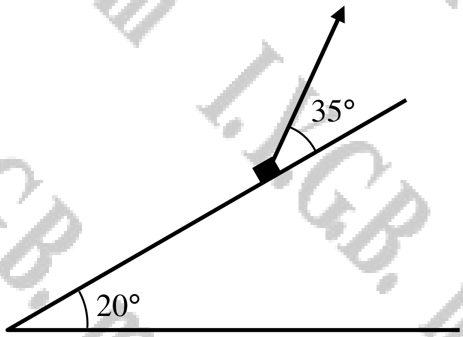
The box is on the point of slipping up the plane.

By modelling the box as a particle, find a simplified expression for k in terms of μ.

$$\boxed{k = \frac{2\mu + 1}{\mu + 2}}$$



Question 24 (****)



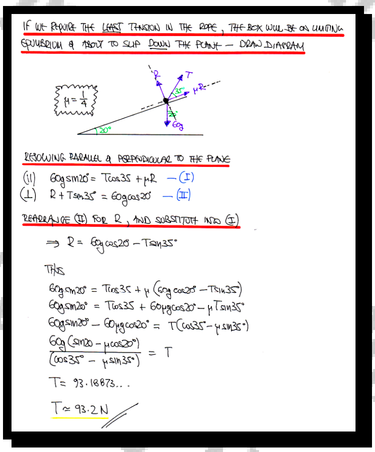
A box of mass 60 kg is held in limiting equilibrium, on a fixed rough inclined plane, by a rope. The plane is at an angle of $20°$ to the horizontal, as shown in the figure above.

The rope lies in a vertical plane containing a line of greatest slope of the incline plane and is inclined to the plane at an angle $35°$.

The rope is modelled as a light inextensible string and the box is modelled as a particle. The coefficient of friction between the box and the plane is $\frac{1}{4}$.

Determine the **least** possible tension in the rope.

$T \approx 93.1887...$ N


IF WE REQUIRE THE LEAST TENSION IN THE ROPE, THE BOX WILL BE ON LIMITING EQUILIBRIUM & ABOUT TO SLIP DOWN THE PLANE – DRAW DIAGRAM

$\mu = \frac{1}{4}$

RESOLVING PARALLEL & PERPENDICULAR TO THE PLANE

$(\parallel)$ $60g\sin20° = T\cos35 + \mu R$ — (I)

$(\perp)$ $R + T\sin35° = 60g\cos20°$ — (II)

REARRANGE (II) FOR R, AND SUBSTITUTE INTO (I)

$\Rightarrow R = 60g\cos20° - T\sin35°$

THUS

$60g\sin20° = T\cos35 + \mu(60g\cos20° - T\sin35°)$

$60g\sin20° = T\cos35 + 60\mu g\cos20° - \mu T\sin35°$

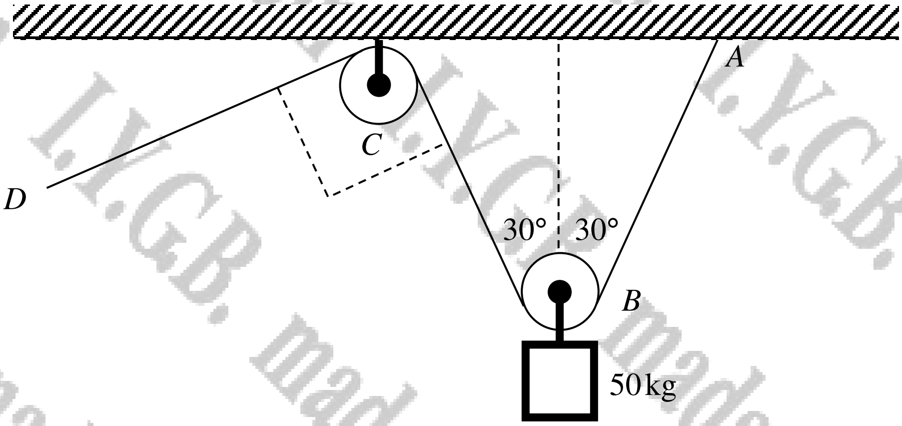
$60g\sin20° - 60\mu g\cos20° = T(\cos35° - \mu\sin35°)$

$\frac{60g(\sin20 - \mu\cos20°)}{(\cos35° - \mu\sin35°)} = T$

$T = 93.18873...$

$T \simeq 93.2$ N

Question 25 **(****)**



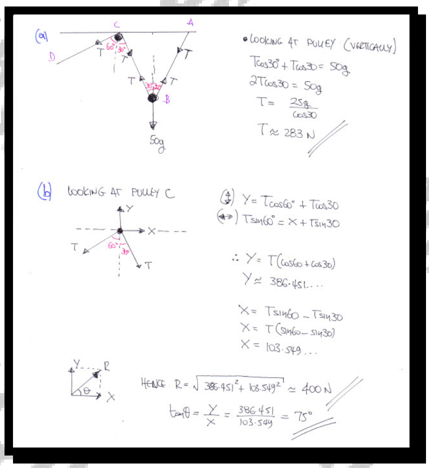
A light inextensible string passes through two smooth light pulleys at C and B, and the other end is tied at a point A on a fixed horizontal ceiling. A box of mass 50 kg, modelled as a particle, is attached to the pulley at B.

The string remains taut at all times by a force acting at D in the direction CD. The sections AB and BC are both inclined at 30° to the vertical and $\measuredangle DCB = 90°$. The system is in equilibrium with the points A, B, C and D lying on a vertical plane which is perpendicular to the ceiling.

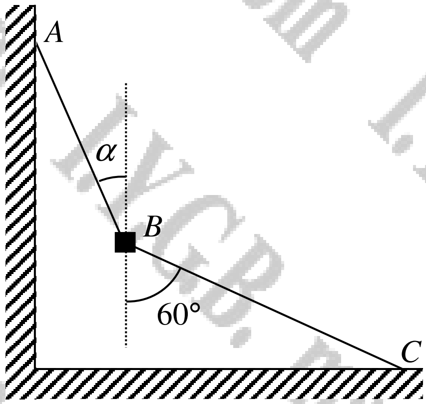
a) By considering the forces acting at B, find the tension in the string.

b) Determine the magnitude and direction of the force exerted by the string on the pulley at C.

$\boxed{T \approx 282.9016...}$, $\boxed{F \approx 400\,\text{N},\ \theta \approx 75° \text{ above the horizontal}}$



Question 26 (****)



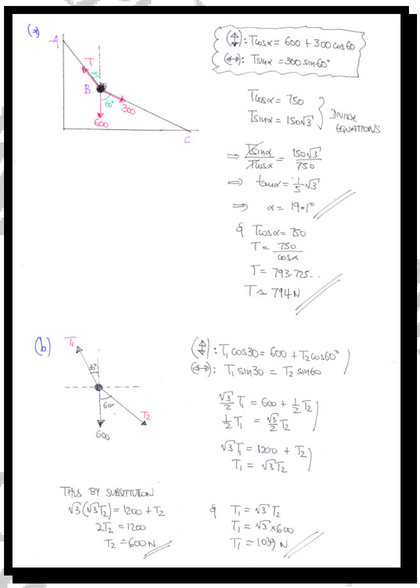
A box B of weight 600 N, modelled as a particle, is held in equilibrium by two light inextensible strings AB and BC. The end A of the string AB is tied on a fixed vertical wall while the end C of the string BC is tied on fixed horizontal ground. The strings BC and AB are inclined at an angle $60°$ and $\alpha°$ to the vertical, respectively, as shown in the figure above.

The points A, B and C lie on a vertical plane which is perpendicular to the wall and perpendicular to the ground.

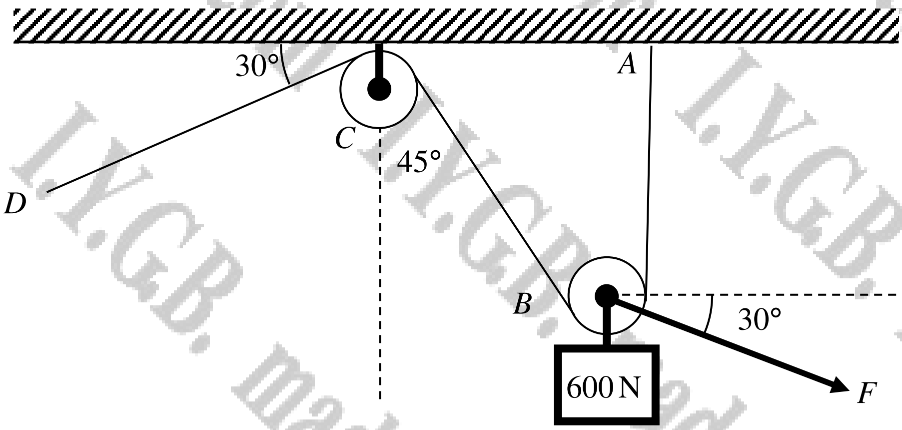
a) Given the tension in the string BC is 300 N, determine the tension in the string AB and the value of α.

b) Given **instead** that $\alpha = 30°$ determine the tension in the strings AB and BC.

$\boxed{\alpha \approx 19.1°, \quad T_{AB} \approx 794\,\text{N}}$, $\boxed{T_{AB} \approx 1039\,\text{N}, T_{BC} = 600\,\text{N}}$



Question 27 (****)

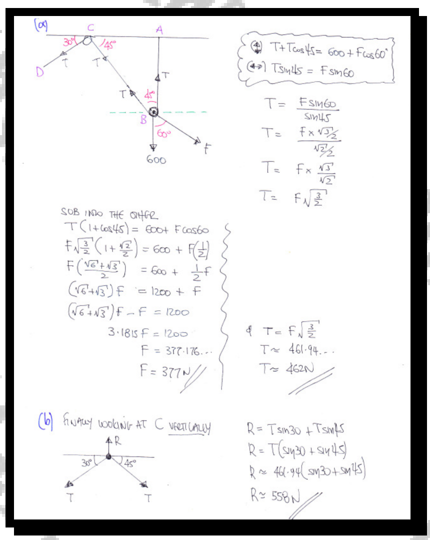


A light inextensible string passes through two smooth light pulleys at C and B, and the other end is tied at a point A on a fixed horizontal ceiling. A box of weight 600 N, modelled as a particle, is attached to the pulley at B. The string remains taut at all times by a force acting at D in the direction CD. The section CD is inclined at $30°$ to the horizontal, the section BC is inclined at $45°$ to the vertical and the section AB is vertical. A force F acting at B inclined at $30°$ below the horizontal keeps the system in equilibrium with the points A, B, C and D lying on a vertical plane which is perpendicular to the ceiling.

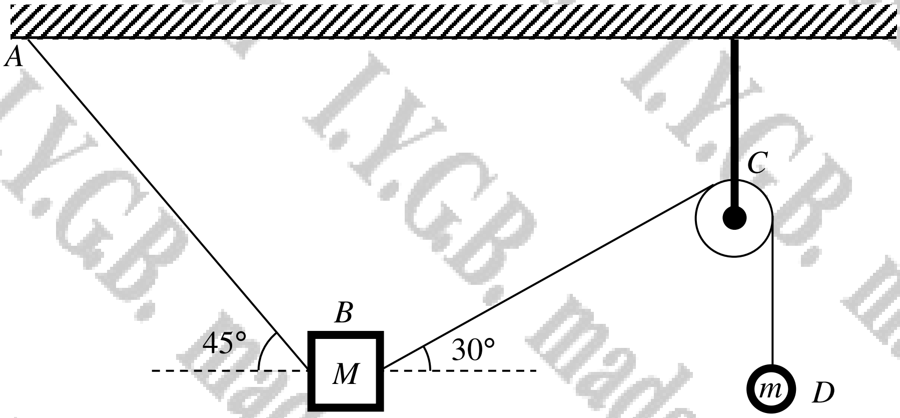
a) Find the tension in the string and the magnitude of F.

b) Determine the magnitude of the vertical component of the force exerted by the string on the pulley at C.

$\boxed{T \approx 462 \text{ N}}$, $\boxed{F \approx 377 \text{ N}}$, $\boxed{R \approx 558 \text{ N}}$



Question 28 (****)



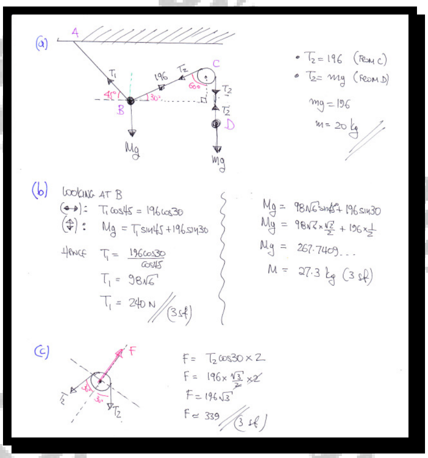
A box B of mass M, modelled as a particle, is held in equilibrium by two light inextensible strings. The end A of the string AB is tied at the point A on a fixed horizontal ceiling. The string BD passes over a fixed smooth pulley C and a particle of mass m is hanging vertically at D, as shown in the figure above. The sections AB and BC are inclined at $45°$ and $30°$ to the horizontal, respectively. The points A, B, C and D lie on a vertical plane which is perpendicular to the ceiling

a) Given the tension in the string through B, C and D is 196 N, find the value of m.

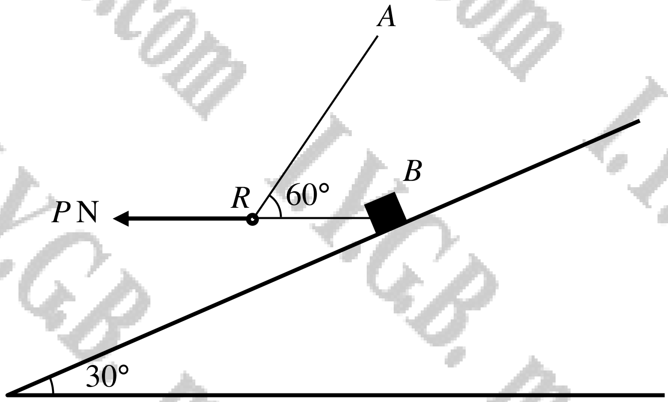
b) Determine the tension in the string AB and the value of M.

c) Find the magnitude of the force exerted by the string on the pulley at C.

$\boxed{m = 20}$, $\boxed{T_{AB} \approx 240\text{ N}, \ M \approx 27.3\text{ kg}}$, $\boxed{F \approx 339\text{ N}}$



Question 29 (****)



A box B of weight 44.1 N lies on a rough plane inclined at 30° to the horizontal. The box is attached to one end of a light inextensible string which passes through a small smooth ring R of mass m kg and the other end A is pulled taut so that BR is horizontal and $\measuredangle BRA = 60°$. The tension in the string is 7.84 N. A horizontal force P N acts away from the box as shown in the figure above.

The box B, the ring R, the string BRA and the force P all lie in the same vertical plane which contains the line of greatest slope of the incline plane. The box and the ring are modelled as particles, which are both in equilibrium.

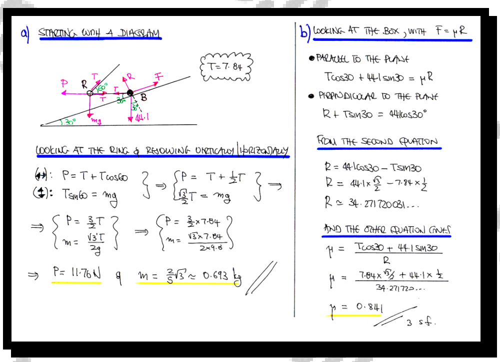
a) Calculate in any order the value of m and the value of P.

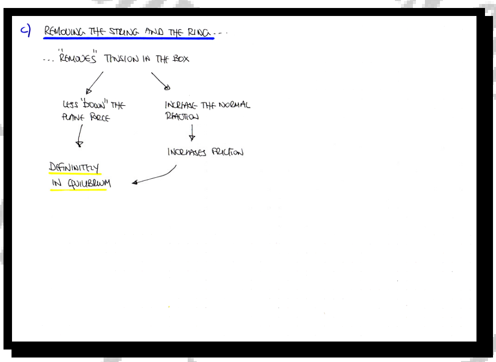
b) Given that the box is in limiting equilibrium, show that the coefficient of friction between the box and the plane is approximately 0.841.

The string and ring are suddenly removed.

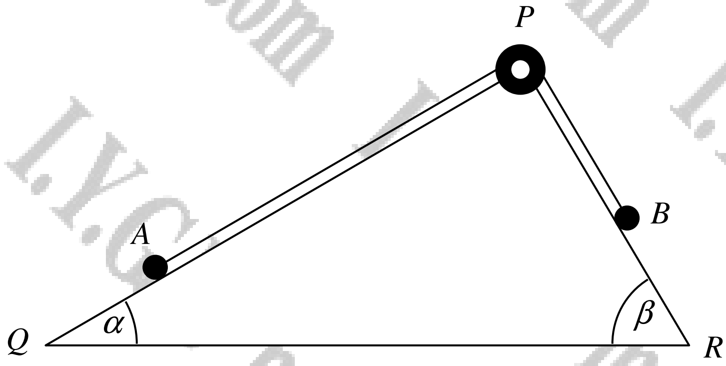
c) Determine, without any further calculations but with full justification, whether the box remains in equilibrium.

$\boxed{}$, $\boxed{m = \frac{2}{5}\sqrt{3} \approx 0.693}$, $\boxed{P = 11.76}$





Question 30 (****)



Two particles A and B have masses M and m, respectively.

The particles are attached to the ends of a light inextensible string.

The string passes over a small smooth pulley P which is fixed at the top of the cross section of a triangular prism QPR, where $\measuredangle PQR = \arctan\frac{1}{2}$ and $\measuredangle PRQ = \arctan 2$.

The string lies in the vertical plane which contains the pulley and lines of greatest slope of the inclined planes, PR and PQ, as shown in the figure above.

The system is in equilibrium, with the string taut, A on PQ and B on PR.

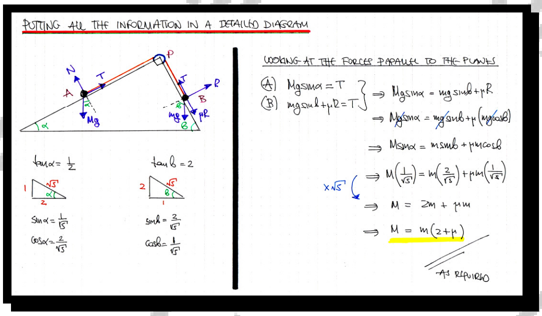
If the equilibrium is limiting with A about to slip down PQ, show that

$$M = m(2+\mu),$$

where μ is the coefficient of friction between B and PR.

You may assume that QP is smooth.

proof



Question 32 (****)

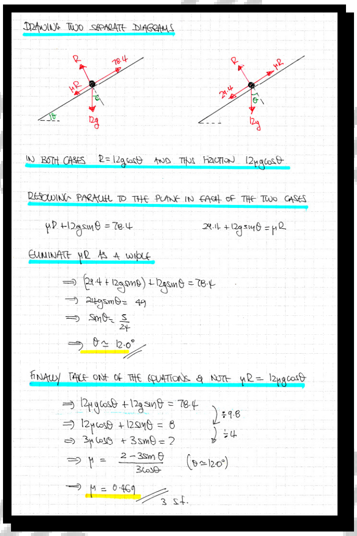
A particle of mass 12 kg is placed on a rough plane, inclined at an angle θ to the horizontal. The coefficient of friction between the particle and the plane is μ.

When a force of magnitude 78.4 N acts on the particle in the direction of a line of greatest slope, the particle is at the point of moving up the plane.

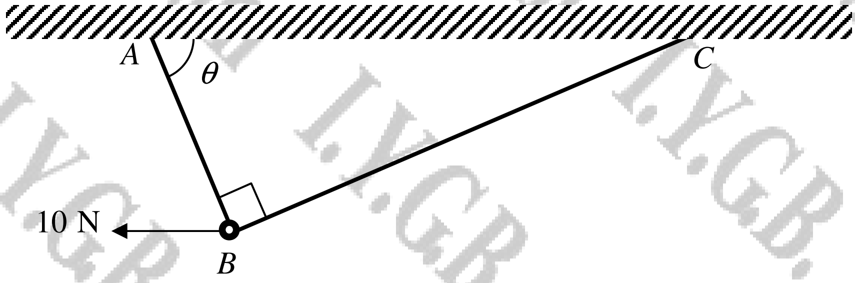
When a force of magnitude 29.4 N acts on the particle in the direction of a line of greatest slope and in a downward direction, the particle is at the point of moving down the plane.

Determine the value of θ and the value of μ.

$\boxed{\theta = \arcsin\left(\frac{5}{24}\right) \approx 12.0°}$, $\boxed{\mu \approx 0.469}$



Question 33 (****+)



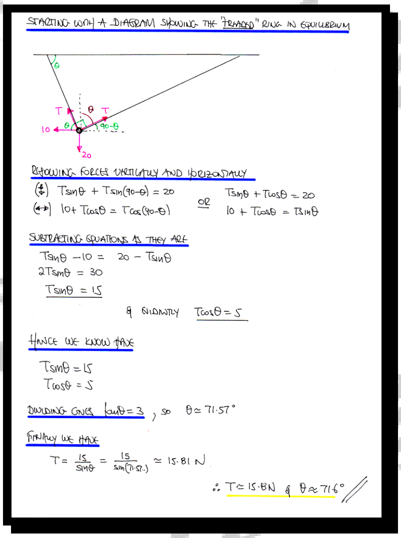
The figure above shows a smooth ring B, of weight 20 N, threaded by a string ABC whose end A and C are attached to a fixed horizontal ceiling.

The ring is modelled as a particle, and kept in equilibrium by a horizontal force 10 N as shown in figure. The points A, B, C and the horizontal force lie in the same vertical plane, which is perpendicular to the plane of the ceiling.

The string is light and inextensible, and the section AB forms an angle θ with the horizontal ceiling. The tension in the string is T N.

By forming two equations in $T\sin\theta$ and $T\cos\theta$, or otherwise, find the value of T and the value of θ.

$T \approx 15.8$ N, $\theta \approx 71.6°$



Question 34 (****+)

A man is trying to push a box of weight W up the line of greatest slope of a rough plane inclined at an angle α to the horizontal, by applying a force parallel to a line of greatest slope.

The coefficient of friction between the man's feet and the plane is μ, $\mu > \tan\alpha$.

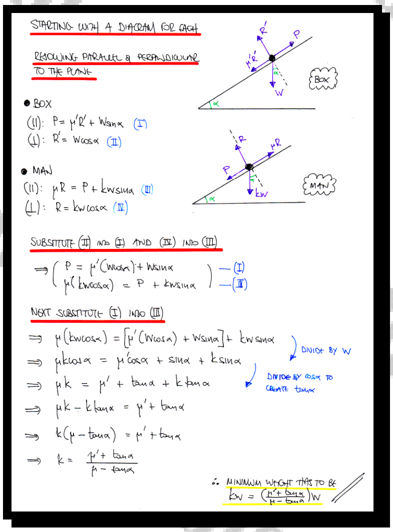
The coefficient of friction between the box and the plane is μ'.

The man and the box are both modelled as particles and air resistance is ignored.

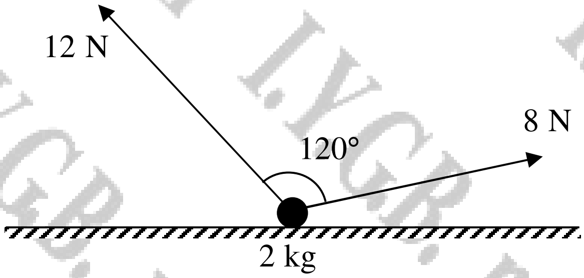
Show, with detailed workings, that if the man is to succeed in pushing the box up this plane his weight must exceed

$$\left(\frac{\mu' + \tan\alpha}{\mu - \tan\alpha}\right)W.$$

[ ], proof



Question 35 (****+)

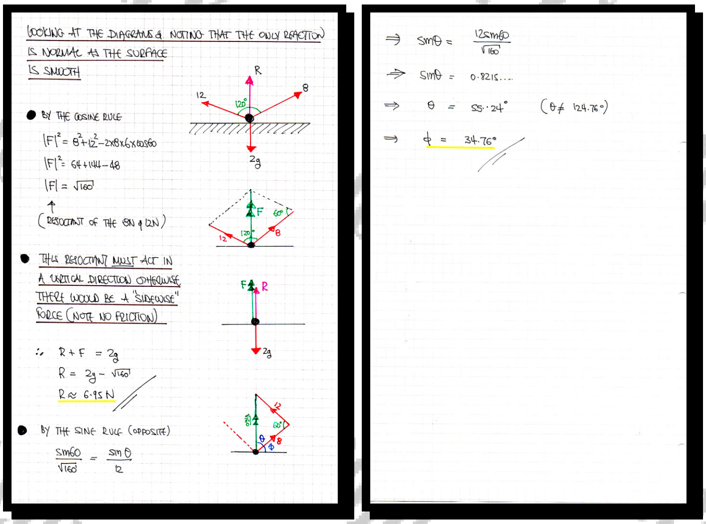


The figure above shows a particle, of mass 2 kg, resting in equilibrium on a smooth horizontal surface, under the action of two forces, of magnitudes of 8 N and 12 N.

The forces act in the same vertical plane and the angle between them is 120°.

Calculate, in any order, the magnitude of the force exerted on the particle by the surface and the acute angle between the 8 N force and the surface.

$\boxed{}$, $\boxed{R \approx 6.95 \text{ N}}$, $\boxed{\approx 34.76°}$



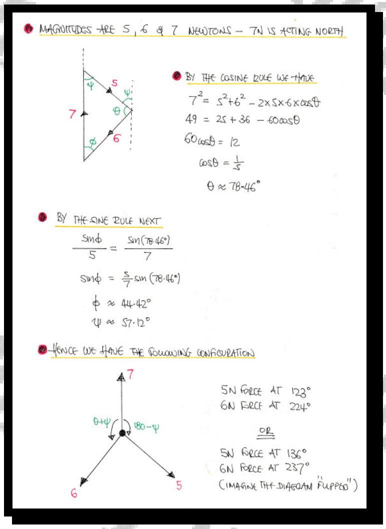
Question 36 (*****)

Three forces of magnitudes 5 N, 6 N and 7 N, are acting on a particle so that the three forces are in equilibrium.

It is further given that the 7 N acts due North.

Determine the possible bearing of the other two forces.

5N at $123°$ and 6N at $224°$ **OR** 5N at $136°$ and 6N at $237°$



Question 37 (*****)

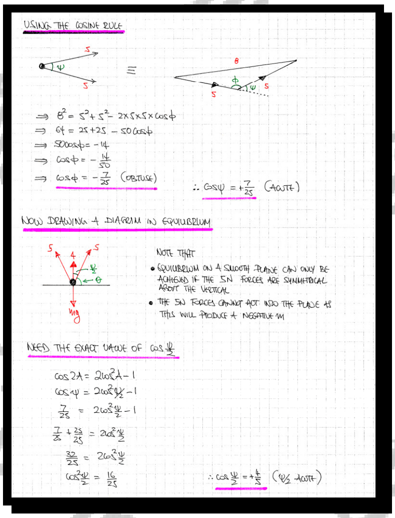
Two forces, both of magnitude 5 N each, have a resultant of magnitude 8 N.

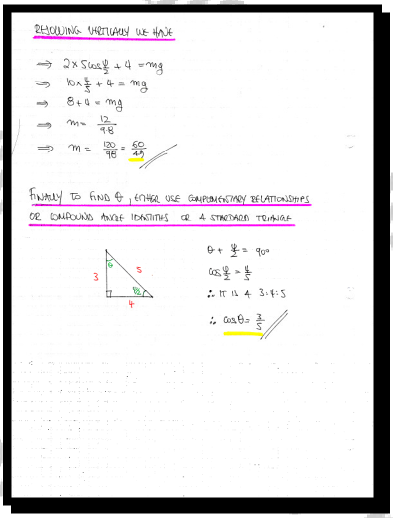
These two forces act on a particle, of mass m kg, which remains at rest on a smooth horizontal surface. The surface makes an acute angle θ with one of the 5 N forces.

Given that the surface exerts a force of 4 N to the particle determine the exact value of m and the exact value of $\cos\theta$.

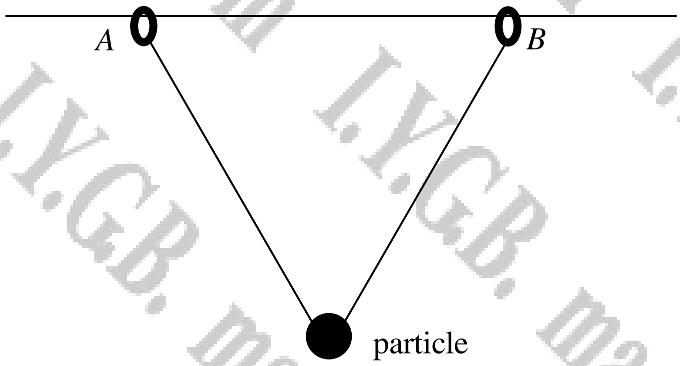
*You **may not** use any calculating aid in this question.*

$m = \frac{60}{49}$, $\cos\theta = \frac{3}{5}$





Question 38 (*****)

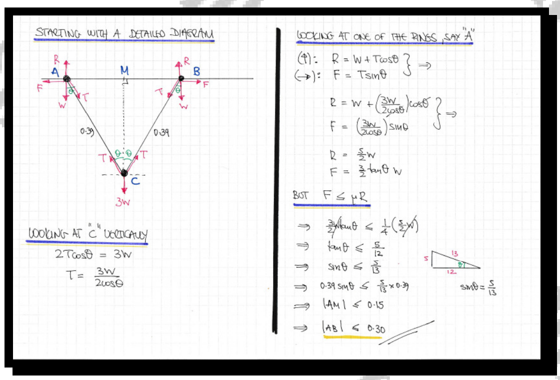


Two identical small rings, A and B, are threaded on a fixed, rough horizontal wire. A light inextensible string of length 78 cm connects the two rings and a particle is attached to the midpoint of the string. The particle is hanging in equilibrium as shown in the figure above.

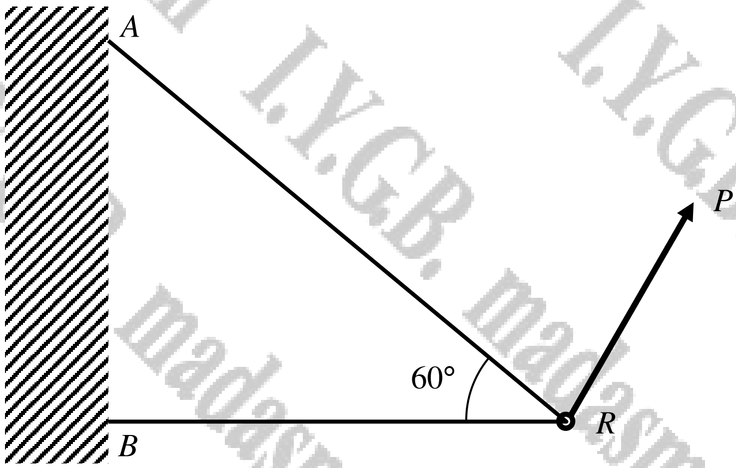
It is further given that the mass of each ring is $\frac{1}{3}$ of the mass of the particle, and the coefficient of friction between each ring and the wire is $\frac{1}{4}$.

Show that the length of AB is at most 30 cm.

[ ], [proof]



Question 39 (*****)



A light inextensible string is threaded through a ring R, of weight W. The two ends of the string, A and B, are attached on a wall with A vertically above B. The ring is in equilibrium by a force P acting on R, so that BR is horizontal with $\measuredangle BRA = 60°$, as shown in the figure above.

Determine, in terms of W, the magnitude of P, when the tension in the string is least.

$\boxed{}$, $\boxed{P_{\text{min}} = \frac{\sqrt{3}}{2}W}$