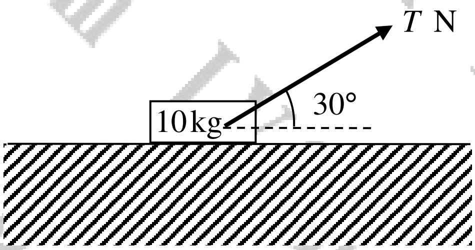
DYNAMICS

Question 1 (**)



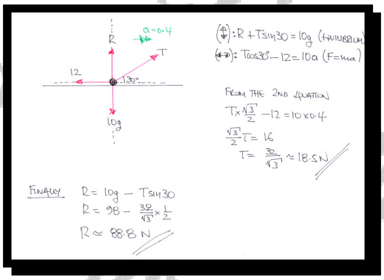
The figure above shows a small box of mass 10 kg, pulled by a rope inclined at 30° to the horizontal, along rough horizontal ground.

The tension in the rope is T N and the box is accelerating at 0.4 ms^{-2}.

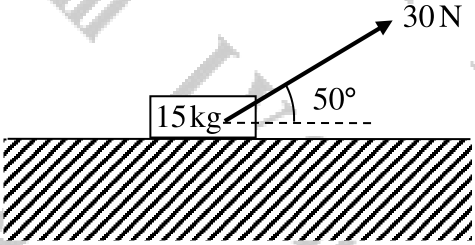
The box is modelled as a particle experiencing a frictional force of 12 N and a normal reaction of R N.

Determine the value of T and the value of R.

$\boxed{T \approx 18.5 \text{ N}}$, $\boxed{R \approx 88.8 \text{ N}}$



Question 2 **(**)**



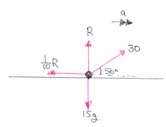
The figure above shows a small box of mass 15 kg, pulled by a rope inclined at 50° to the horizontal, along rough horizontal ground.

The tension in the rope is $30\,\text{N}$ and the particle is accelerating at a ms^{-2}.

The box is modelled as a particle experiencing a normal reaction of R N and a constant frictional force of magnitude $\frac{1}{10}R$ N.

Determine the value of a.

$\boxed{a \approx 0.459\ \text{ms}^{-2}}$



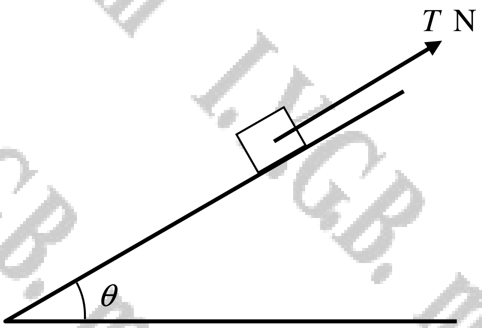
$(\uparrow)$: $R + 30\sin 50 = 15g$ (EQUILIBRIUM)

$(\rightarrow)$: $\underbrace{30\cos 50 - \frac{1}{10}R}_{F} = \underbrace{15}_{m} \times \underbrace{a}_{a}$

HENCE $R = 15g - 30\sin 50$
$R = 124.018...$

THUS $30\cos 50 - \frac{1}{10} \times 124.018... = 15a$
$15a = 6.88176..$
$a = 0.459\ \text{ms}^{-2}$

Question 3 (**)



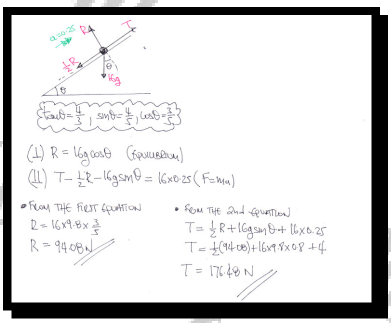
The figure above shows a box, of mass 16 kg, on a plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

The box is pulled up the plane by a rope whose tension is T N acts in the direction of the greatest slope, causing an acceleration of 0.25 ms^{-2}.

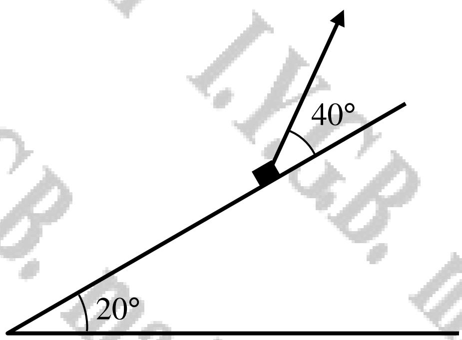
The box is also experiencing friction of magnitude $\frac{1}{2}R$ N **down** the plane, where R N is the normal reaction between the box and the plane.

By modelling the box as a particle, find the value of R and the value of T.

$\boxed{R = 94.08 \text{ N}}$, $\boxed{T = 176.48 \text{ N}}$



Question 4 (**)



The figure above shows a box, of mass 17 kg, on a plane inclined at an angle of 20° to the horizontal.

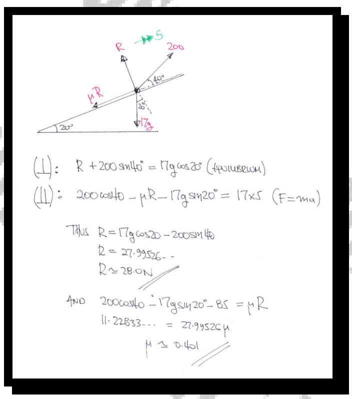
A force of 200 N, acting at an angle of 40° to the direction of the greatest slope of the plane, is pulling the box up the plane.

The box is also experiencing friction of magnitude μR N, where R is the normal reaction between the box and the plane and μ is a constant.

The box is accelerating up the plane at 5 ms^{-2}.

By modelling the box as a particle, find the value of R and the value of μ.

$\boxed{R \approx 28.0}$, $\boxed{\mu \approx 0.401}$



Question 5 (**+)

A lift, of mass M kg, is pulled up a vertical mineshaft by a cable attached to the top of the lift. A man of mass m kg is standing inside the lift.

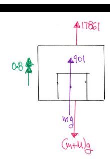
The lift is uniformly accelerating upwards at 0.8 ms^{-2}.

The man experiences a constant normal reaction of magnitude 901 N from the floor of the lift and there is a constant tension of 17861 N in the cable of the lift.

Determine the value of m and the value of M.

$\boxed{}$, $\boxed{m = 85 \text{ kg}}$, $\boxed{M = 1600 \text{ kg}}$

WHEN THE LIFT IS ACCELERATING UPWARDS



LOOKING AT THE MAN

$\Rightarrow$ "$F = ma$"

$\Rightarrow 901 - mg = ma$

$\Rightarrow 901 - mg = 0.8m$

$\Rightarrow 901 = 0.8m + 9.8m$

$\Rightarrow 901 = 10.6m$

$\Rightarrow m = 85 \text{ kg}$

LOOKING AT THE LIFT + MAN AS A SYSTEM

$\Rightarrow$ "$F = ma$"

$\Rightarrow 17861 - (m+M)g = (m+M) \times 0.8$

$\Rightarrow 17861 = (m+M)(g+0.8)$

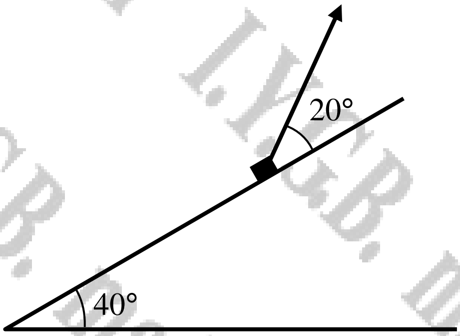
$\Rightarrow 17861 = (85+M)(9.8+0.8)$

$\Rightarrow 17861 = (85+M) \times 10.6$

$\Rightarrow 1685 = 85 + M$

$\Rightarrow M = 1600 \text{ kg}$

Question 6 **(***)**



The figure above shows a box, of mass 10 kg, on a plane inclined at an angle of $40°$ to the horizontal.

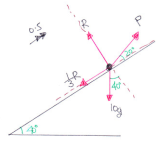
A force of magnitude P N, acting at an angle of $20°$ to the direction of the greatest slope of the plane, is pulling the box up the plane.

The box is also experiencing ground friction of magnitude $\frac{1}{3}R$ N, where R is the normal reaction between the box and the plane.

The box is accelerating up the plane at 0.5 ms^{-2}.

By modelling the box as a particle, find the value of P and the value of R.

$\boxed{P \approx 88.2768...}$, $\boxed{R \approx 44.8798...}$



(⊥) $R + P\sin 20 = 10g\cos 40$
(EQUILIBRIUM)

(∥) $\underbrace{P\cos 20 - \frac{1}{3}R - 10g\sin 40}_{F} = \underset{m}{10} \times \underset{a}{0.5}$

$\boxed{R = 10g\cos 40 - P\sin 20}$

SUB INTO THE OTHER

$\Rightarrow P\cos 20 - \frac{1}{3}\left[10g\cos 40 - P\sin 20\right] - 10g\sin 40 = 5$

$\Rightarrow P\cos 20 - \frac{10}{3}g\cos 40 + \frac{1}{3}P\sin 20 - 10g\sin 40 = 5$

$\Rightarrow P\left(\cos 20 + \frac{1}{3}\sin 20\right) = 5 + 10g\sin 40 + \frac{10}{3}g\cos 40$

$\Rightarrow P = \dfrac{5 + 10g\sin 40 + \frac{10}{3}g\cos 40}{\cos 20 + \frac{1}{3}\sin 20}$

$\Rightarrow P = 88.2768...$

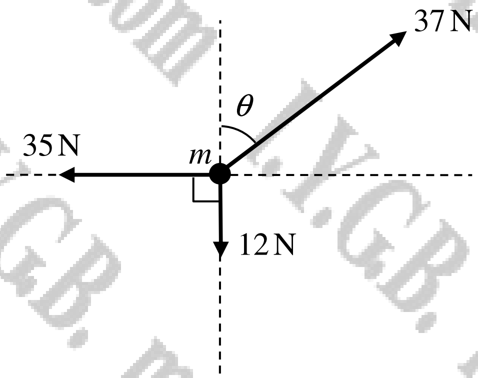
$\Rightarrow P \simeq 88.3 \text{ N}$

& $R = 10g\cos 40 - (88.2768...) \times \sin 20$

$R = 44.8798...$

$R \simeq 44.9 \text{ N}$

Question 7 (***)

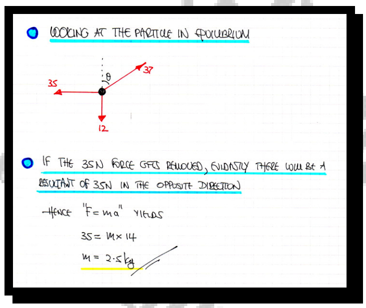


The figure above shows 3 forces, which all lie on the same plane, acting on a particle of mass m kg. The particle remains in equilibrium when the magnitudes of these 3 forces and their relative directions are those shown in the figure.

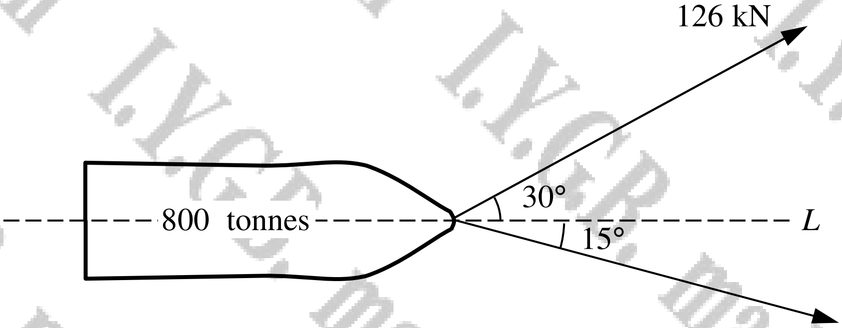
When the 35 N force is suddenly removed the particle begins to move with constant acceleration of magnitude 14 ms^{-2}.

Determine the value of m.

[illegible], $\boxed{m = 2.5}$



Question 8 (***+)



The figure above shows a ship, of mass 800 tonnes, being pulled by two tugs using horizontal cables, so that the ship is moving in a straight line L.

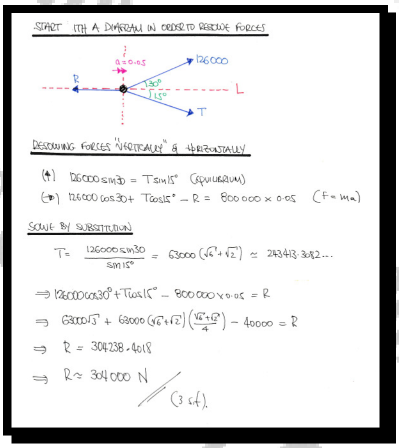
One tug exerts a force of 126 kN, at an angle of $30°$ to L.

The other tug exerts a force at an angle of $15°$ to L.

The ship is accelerating along L at 0.05 ms^{-2}.

Determine the magnitude of the resistance opposing the motion of the ship.

$\boxed{}$, $\boxed{R = 304238.4018... \approx 304000 \text{ N}}$



START WITH A DIAGRAM IN ORDER TO RESOLVE FORCES

RESOLVING FORCES "VERTICALLY" & HORIZONTALLY

$(\uparrow)$ $126000 \sin 30 = T \sin 15°$ (EQUILIBRIUM)

$(\rightarrow)$ $126000 \cos 30 + T\cos 15° - R = 800000 \times 0.05$ $(F = ma)$

SOLVE BY SUBSTITUTION

$T = \frac{126000 \sin 30}{\sin 15°} = 63000(\sqrt{6}+\sqrt{2}) \simeq 243413.3082...$

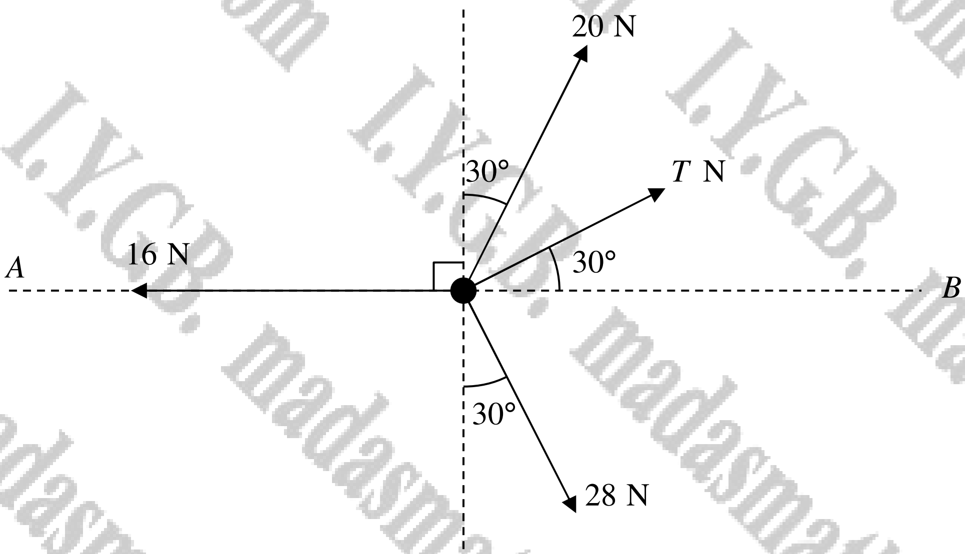
$\Rightarrow 126000\cos 30° + T\cos 15° - 800000 \times 0.05 = R$

$\Rightarrow 63000\sqrt{3} + 63000(\sqrt{6}+\sqrt{2})\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - 40000 = R$

$\Rightarrow R = 304238.4018$

$\Rightarrow R \simeq 304000$ N (3 s.f.)

Question 9 (***+)



A particle of mass 80 kg is accelerating in the direction AB.

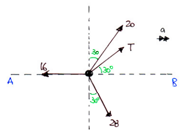
Four **horizontal** forces of different magnitudes are acting on the particle.

The magnitude and direction of these four forces, together with other important information is shown in the figure above.

Find the acceleration of the particle.

$\boxed{}$, $\boxed{0.25 \text{ ms}^{-2}}$

LOOKING AT THE DIAGRAM – IF THE PARTICLE IS ACCELERATING "ALONG AB" IT MUST BE IN EQUILIBRIUM "PERPENDICULAR" TO AB



$(\updownarrow)$: $20\cos 30 + T\sin 30 = 28\cos 30$ (EQUILIBRIUM)

$\Rightarrow 20 + T\tan 30 = 28$ DIVIDE THROUGH BY $\cos 30°$, TO CREATE $\tan 30°$

$\Rightarrow T\tan 30 = 8$

$\Rightarrow T = 8\sqrt{3}$ N

NOW IN THE DIRECTION AB THERE IS ACCELERATION, THEREFORE RESULTANT

$(\rightarrow)$: $20\sin 30 + T\cos 30 + 28\sin 30 - 16 = 80a$ $(F = ma)$

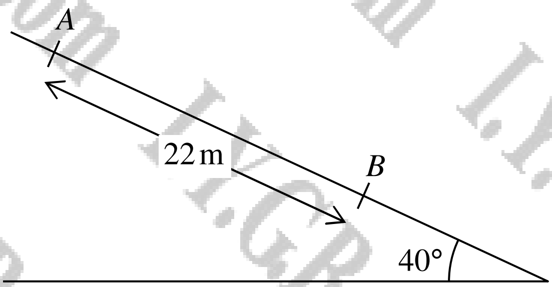
$\Rightarrow 20 \times \frac{1}{2} + 8\sqrt{3} \times \frac{\sqrt{3}}{2} + 28 \times \frac{1}{2} - 16 = 80a$

$\Rightarrow 10 + 12 + 14 - 16 = 80a$

$\Rightarrow 80a = 20$

$\Rightarrow a = 0.25 \text{ ms}^{-2}$

Question 10 (***+)



A particle P slides with constant acceleration down the line of greatest slope of a rough plane inclined at $40°$ to the horizontal.

The particle covers a distance AB, where $|AB| = 22$ m, in 4 s.

a) Given the speed of P at A is 3 ms^{-1}, calculate …

i. … the speed of P at B.

ii. … the acceleration of P.

b) Hence find the coefficient of friction between the particle and the plane.

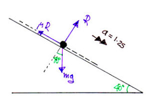
$\boxed{}$, $\boxed{v = 8 \text{ ms}^{-1}}$, $\boxed{a = 1.25 \text{ ms}^{-2}}$, $\boxed{\mu \approx 0.673}$

a) STARTING WITH KINEMATICS FOR THE JOURNEY A TO B

$u = 3 \text{ ms}^{-1}$, $a = ?$, $s = 22 \text{ m}$, $t = 4 \text{ s}$, $v = ?$

- $s = ut + \frac{1}{2}at^2$
  $22 = 3 \times 4 + \frac{1}{2} \times a \times 4^2$
  $22 = 12 + 8a$
  $8a = 10$
  $a = 1.25 \text{ ms}^{-2}$
- $v = u + at$
  $v = 3 + 1.25 \times 4$
  $v = 8 \text{ ms}^{-1}$

b) LOOKING AT THE DIAGRAM BELOW



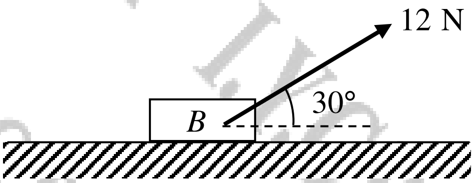
RESOLVING PARALLEL (∥) & PERPENDICULAR TO THE PLANE (⊥)

(⊥): $R = mg\cos 40$ [EQUILIBRIUM]
(∥): $mg\sin 40 - \mu R = ma$ [F = ma]

SOLVING BY SUBSTITUTION

$\Rightarrow \cancel{m}g\sin 40 - \mu(\cancel{m}g\cos 40) = \cancel{m}a$
$\Rightarrow g\sin 40 - \mu g\cos 40 = a$
$\Rightarrow g\sin 40 - a = \mu g\cos 40$
$\Rightarrow \mu = \dfrac{g\sin 40 - a}{g\cos 40} = \dfrac{(9.8 \times \sin 40) - 1.25}{9.8 \times \cos 40} \simeq 0.673$

Question 11 (***+)



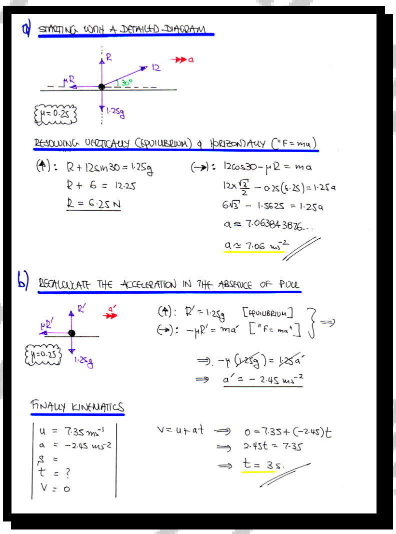
A box B of mass 1.25 kg is pulled along rough horizontal ground by a force of magnitude 12 N inclined at 30° to the horizontal, as shown in the figure above. The box is modelled as a particle moving on a rough horizontal plane where coefficient of friction between the particle and the plane is 0.25.

a) Determine the acceleration of the box.

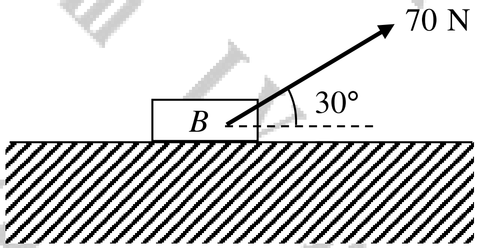
The pulling force is suddenly removed when the box has a speed of 7.35 ms^{-1}.

b) Find the time it takes the box to come to rest from the instant the pulling force was removed.

$\boxed{}$, $\boxed{a \approx 7.06 \text{ ms}^{-2}}$, $\boxed{t = 3 \text{ s}}$



Question 12 (***+)



A box B of weight 147 N is pulled at **constant** speed on rough horizontal ground by a pulling force of magnitude 70 N inclined at 30° to the horizontal, as shown in the figure above. The box is modelled as a particle moving on a rough horizontal plane where the coefficient of friction between the particle and the plane is μ.

a) Determine the value of μ.

The pulling force is suddenly removed and the box decelerates uniformly coming to rest after covering a further 12.25 m.

b) Find the speed of the box at the instant when the pulling force was removed.

$\boxed{}$, $\boxed{\mu = \frac{5}{16}\sqrt{3} \approx 0.541}$, $\boxed{u \approx 11.39990952\ldots \text{ ms}^{-1}}$

a) STARTING WITH A DIAGRAM & NOTING THAT CONSTANT SPEED IMPLIES EQUILIBRIUM

($\updownarrow$): $R + 70\sin 30 = 147$
$R + 35 = 147$
$R = 112$

($\leftrightarrow$): $70\cos 30 = \mu R$
$70 \times \frac{\sqrt{3}}{2} = \mu \times 112$
$\mu = \frac{5}{16}\sqrt{3} \simeq 0.541$

b) CALCULATE THE ACCELERATION (DECELERATION) ONCE "PULL" IS REMOVED

($\updownarrow$): $R' = 147$ [EQUILIBRIUM]
($\leftrightarrow$): $-\mu R' = ma$ ["F = ma"]

$\Rightarrow -\left(\frac{5}{16}\sqrt{3}\right)(147) = \frac{147}{g}a$
$\Rightarrow -\frac{5}{16}\sqrt{3} = \frac{a}{g}$
$\Rightarrow a = -\frac{5}{16}\sqrt{3}g \simeq -5.3044\ldots$

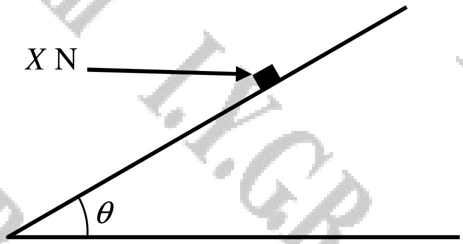
!! $mg = 147$, $m = \frac{147}{g}$!!

FINALLY KINEMATICS

$u = ?$
$a = -5.3044 \text{ ms}^{-2}$
$s = 12.25 \text{ m}$
$t = -$
$v = 0$

$v^2 = u^2 + 2as \Rightarrow 0 = u^2 + 2(-5.3044\ldots)(12.25)$
$\Rightarrow u^2 = 129.957\ldots$
$\Rightarrow |u| = 11.3999\ldots$
$\Rightarrow u \simeq 11.4 \text{ ms}^{-1}$

Question 13 (***+)



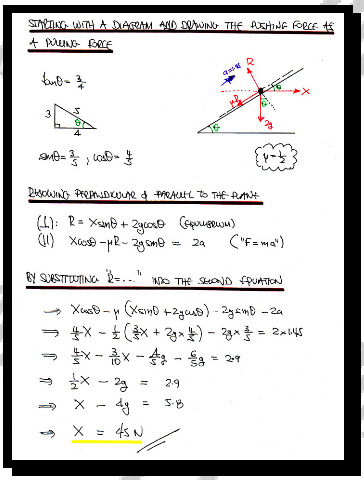
A box of mass 2 kg, is pushed up a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$, by a horizontal force X N, as shown in the above figure.

The force acts in a vertical plane, which contains the box and a line of greatest slope of the plane. The coefficient of friction between the box and the plane is $\frac{1}{2}$.

The box is accelerating up the plane at 1.45 ms^{-2}.

By modelling the box as a particle, find the value of X.

$\boxed{}$, $\boxed{X = 45}$


STARTING WITH A DIAGRAM AND DRAWING THE PUSHING FORCE AS A PULLING FORCE

$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

$\mu = \frac{1}{2}$

RESOLVING PERPENDICULAR & PARALLEL TO THE PLANE

$(\perp)$: $R = X\sin\theta + 2g\cos\theta$ (EQUILIBRIUM)
$(\parallel)$ $X\cos\theta - \mu R - 2g\sin\theta = 2a$ ("F = ma")

BY SUBSTITUTING "$R = \dots$" INTO THE SECOND EQUATION

$\Rightarrow X\cos\theta - \mu(X\sin\theta + 2g\cos\theta) - 2g\sin\theta = 2a$
$\Rightarrow \frac{4}{5}X - \frac{1}{2}\left(\frac{3}{5}X + 2g \times \frac{4}{5}\right) - 2g \times \frac{3}{5} = 2 \times 1.45$
$\Rightarrow \frac{4}{5}X - \frac{3}{10}X - \frac{4}{5}g - \frac{6}{5}g = 2.9$
$\Rightarrow \frac{1}{2}X - 2g = 2.9$
$\Rightarrow X - 4g = 5.8$
$\Rightarrow X = 45 \text{ N}$

Question 14 (***+)

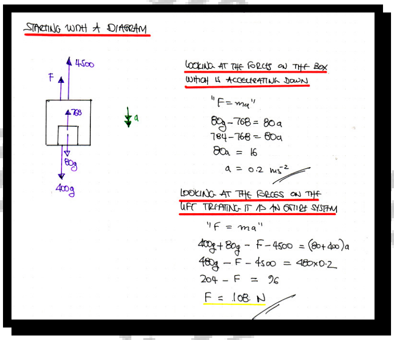
A lift, of mass 400 kg, is being lowered into a vertical mineshaft by a cable attached to the top of the lift. A load of mass 80 kg is sitting firm on the floor inside the lift. The lift is lowered with constant downward acceleration.

There is a constant resistance of magnitude F N opposing the motion of the lift.

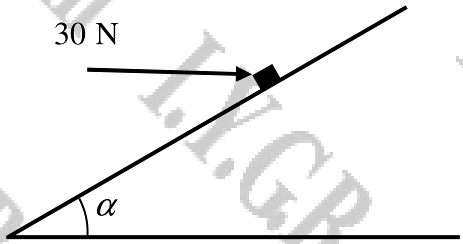
The load experiences a constant normal reaction of magnitude 768 N from the floor of the lift and there is a constant tension of 4500 N in the cable of the lift.

Determine the value of F.

$\boxed{F = 108 \text{ N}}$



Question 15 (***+)

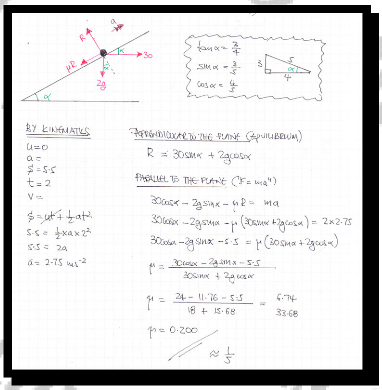


A small box, modelled as a particle of mass 2 kg, is pushed up a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a horizontal force 30 N, as shown in the above figure.

The force acts in a vertical plane, which contains the box and a line of greatest slope of the plane. The box starts from rest and accelerates a distance of 5.5 m up the plane, in 2 seconds.

Determine the value of the coefficient of friction between the box and the plane.

$\boxed{\mu \approx 0.200}$



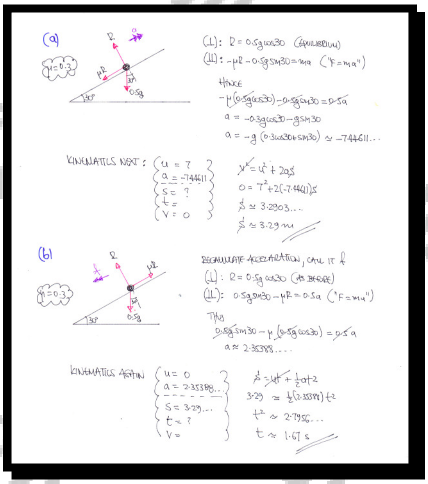
Question 16 (***+)

A particle of mass 0.5 kg, is projected with speed of 7 ms^{-1} up the line of greatest slope of a rough plane inclined at an angle 30° to the horizontal. The particle experiences no other resistance except ground friction. The coefficient of friction between the particle and the plane is 0.3.

a) Given the particle comes to instantaneous rest before it reaches the end of the plane, find the distance it moves up the plane.

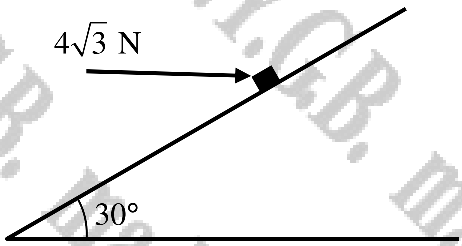
b) Determine the time it takes the particle to return from its highest position on the plane to its original starting position.

$\boxed{d \approx 3.29 \text{ m}}$, $\boxed{t \approx 1.67 \text{ s}}$



Question 17 (***+)

When a particle is gently placed on a rough plane inclined at an angle of 30° to the horizontal, it is at the point of slipping down a line of greatest slope of the plane.

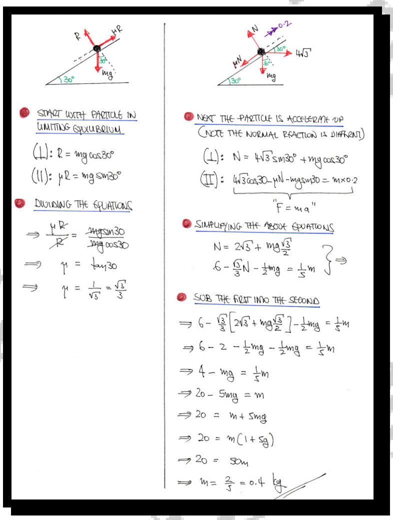


When a horizontal force of magnitude $4\sqrt{3}$ N is acting on the same particle and on the same incline plane, as shown in the above figure, the particle is accelerating up a line of greatest slope of the plane with constant acceleration 0.2 ms^{-2}.

This horizontal force acts in a vertical plane, which contains the box and a line of greatest slope of the plane.

Determine the mass of the particle.

$\boxed{}$, $\boxed{m = 0.4 \text{ kg}}$



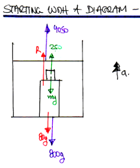
Question 18 (****)

A child of mass m kg, is sitting over the shoulders of his father whose mass is 80 kg. The father, with the child over his shoulders, is standing in a lift of mass 800 kg. When the tension in the cable of the lift is 9050 N, the lift and its occupants is accelerating, with the shoulders of the father exerting a force of 250 N on the child.

Determine the value of m.

$\boxed{}$, $\boxed{m = 25}$

STARTING WITH A DIAGRAM – MARK THE ACCELERATION UPWARDS



LOOKING AT BOY ON THE SHOULDERS & USING "F = ma"

$250 - mg = ma$

LOOKING AT THE ENTIRE SYSTEM

$\Rightarrow 9050 - (mg + 80g + 800g) = (m + 80 + 800)a$

$\Rightarrow 9050 - mg - 80g - 800g = (m + 880)a$

$\Rightarrow 9050 - mg - 784 - 7840 = (m + 880)a$

$\Rightarrow 426 - mg = (m + 880)a$

$\Rightarrow 426 - mg = ma + 880a$

$\Rightarrow 426 - \cancel{mg} = (250 - \cancel{mg}) + 880a$ (FROM THE FIRST EQUATION)

$\Rightarrow 176 = 880a$

$a = 0.2 \text{ ms}^{-2}$

$\therefore$ FINALLY WE HAVE

$250 - mg = ma$

$250 = ma + mg$

$250 = m(a + g)$

$250 = 10m$

$\therefore m = 25 \text{ kg}$

Question 19 (****)

A particle is projected from a point A, **down** the line of greatest slope of a smooth incline plane and moves along a straight line with constant acceleration of 7.84 ms^{-2}.

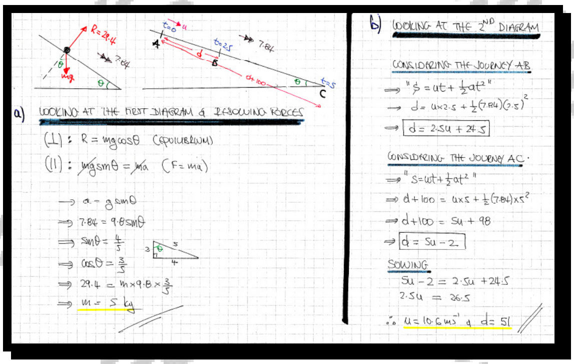
The particle experiences a constant normal reaction force of magnitude 29.4 N, throughout its motion.

a) Determine the mass of the particle.

The particle reaches the point B, 2.5 s after being projected and the point C, 1.5 s after passing through B.

b) Given further that the distance AC is 100 m greater than the distance AB, find the projection speed and the distance AB.

$\boxed{}$, $\boxed{m = 5 \text{ kg}}$, $\boxed{u = 10.6 \text{ ms}^{-1}}$, $\boxed{|AB| = 51 \text{ m}}$



a) LOOKING AT THE FIRST DIAGRAM & RESOLVING FORCES

$(\perp)$: $R = mg\cos\theta$ (EQUILIBRIUM)

$(\parallel)$: $\cancel{m}g\sin\theta = \cancel{m}a$ $(F = ma)$

$\Rightarrow a = g\sin\theta$

$\Rightarrow 7.84 = 9.8\sin\theta$

$\Rightarrow \sin\theta = \frac{4}{5}$

$\Rightarrow \cos\theta = \frac{3}{5}$

$\Rightarrow 29.4 = m \times 9.8 \times \frac{3}{5}$

$\Rightarrow m = 5 \text{ kg}$

b) LOOKING AT THE 2ND DIAGRAM

CONSIDERING THE JOURNEY AB

$\Rightarrow$ "$s = ut + \frac{1}{2}at^2$"

$\Rightarrow d = u \times 2.5 + \frac{1}{2}(7.84)(2.5)^2$

$\Rightarrow \boxed{d = 2.5u + 24.5}$

CONSIDERING THE JOURNEY AC.

$\Rightarrow$ "$s = ut + \frac{1}{2}at^2$"

$\Rightarrow d + 100 = u \times 5 + \frac{1}{2}(7.84) \times 5^2$

$\Rightarrow d + 100 = 5u + 98$

$\Rightarrow \boxed{d = 5u - 2}$

SOLVING

$5u - 2 = 2.5u + 24.5$

$2.5u = 26.5$

$\therefore u = 10.6 \text{ ms}^{-1}$ & $d = 51$

Question 20 **(****+)**

A particle is projected **down** the line of greatest slope of a rough incline plane and moves along a straight line with constant acceleration a ms^{-2}.

The particle achieves a speed of 24 ms^{-1}, 7 s after projection and covers 180 m in the first 10 seconds of its motion.

a) Assuming that the above described motion takes place entirely on the slope of the plane, determine the value of a.

The plane is inclined at $\arctan\frac{4}{3}$ to the horizontal.

b) Calculate the coefficient of friction between the particle and the plane.

$\boxed{}$, $\boxed{a=3}$, $\boxed{\mu = \frac{121}{147} \approx 0.823}$

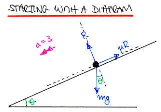
a) STANDARD KINEMATICS – TWO SEPARATE JOURNEYS WITH COMMON u & a

$u = U$	$u = U$
$a = ?$	$a = ?$
$s = -$	$s = 180$ m
$t = 7$ s	$t = 10$
$v = 24$ ms^{-1}	$v = -$
"$v = u + at$"	"$s = ut + \frac{1}{2}at^2$"
$24 = U + 7a$	$180 = 10U + \frac{1}{2} \times a \times 10^2$
$240 = 10U + 70a$ ($\times 10$)	$180 = 10U + 50a$

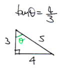
SUBTRACTING

$$10U + 70a = 240$$
$$10U + 50a = 180$$
$$20a = 60$$
$$a = 3 \text{ ms}^{-2}$$

b) STARTING WITH A DIAGRAM



$\tan\theta = \frac{4}{3}$



$\sin\theta = \frac{4}{5}$ & $\cos\theta = \frac{3}{5}$

RESOLVING PARALLEL AND PERPENDICULAR TO THE PLANE

$(\perp)$: $R = mg\cos\theta$ [EQUILIBRIUM]

$(\parallel)$: $mg\sin\theta - \mu R = ma$ ["F = ma"]

$$\Rightarrow mg\sin\theta - \mu(mg\cos\theta) = ma$$
$$\Rightarrow g\sin\theta - \mu g\cos\theta = a$$
$$\Rightarrow g \times \frac{4}{5} - \mu g \times \frac{3}{5} = 3$$
$$\Rightarrow 4g - 3\mu g = 15$$
$$\Rightarrow 4g - 15 = 3\mu g$$
$$\Rightarrow \mu = \frac{4g-15}{3g}$$
$$\Rightarrow \mu = \frac{121}{147} \approx 0.823$$

Question 21 (****+)

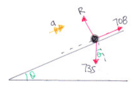
A box of weight 735 N is pulled up a line of greatest slope of a smooth incline plane by a constant force F, of magnitude 708 N, which acts in a direction parallel to the line of greatest slope which the box is moving on.

The plane is inclined at $\arcsin\frac{4}{5}$ to the horizontal.

The box starts from rest from point A on the plane and when it achieves a speed of 8 ms^{-1}, F is removed.

Show that the box returns to A, approximately $3\frac{1}{2}$ seconds after F is removed.

$\boxed{\text{proof}}$



$\sin\theta = \frac{4}{5}$

- FIRSTLY $W = mg$
  $735 = mg$
  $\boxed{m = 75}$
- NOW LOOKING UP THE PLANE
  $708 - 735\sin\theta = 75a$
  $708 - 735 \times \frac{4}{5} = 75a$
  $75a = 120$
  $\boxed{a = 1.6 \text{ ms}^{-2}}$
- $u = 0$, $a = 1.6 \text{ ms}^{-2}$, $s = ?$, t, $v = 8 \text{ ms}^{-1}$
  $v^2 = \cancel{u^2} + 2as$
  $8^2 = 2(1.6)s$
  $\boxed{s = 20 \text{ m}}$

NOW THE FORCE IS REMOVED, SO WE NEED TO RECALCULATE THE ACCELERATION, WHICH IS NOW ACTING DOWN THE PLANE

$-735\sin\theta = 75a'$
$-735 \times \frac{4}{5} = 75a'$
$\boxed{a' = -7.84 \text{ ms}^{-2}}$

NOW LOOKING AT THE ENTIRE JOURNEY AS ONE & USING DISPLACEMENTS

$u = 8 \text{ ms}^{-1}$, $a = -7.84 \text{ ms}^{-2}$, $s = -20$, v, $t = ?$

$s = ut + \frac{1}{2}at^2$
$-20 = 8t + \frac{1}{2}(-7.84)t^2$
$\frac{98}{25}t^2 - 8t - 20 = 0$
$\frac{49}{25}t^2 - 4t - 10 = 0$
$49t^2 - 100t - 250 = 0$

BY QUADRATIC FORMULA & IGNORING THE NEGATIVE SOLUTION

$t = \frac{100 + \sqrt{59000}}{2 \times 49} \simeq 3.4989798$

IE APPROX $3\frac{1}{2}$ SECONDS

Question 22 (****+)

A puck is struck and given an initial speed of 14 ms^{-1} along an ice rink.

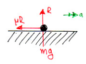
The puck travels in a straight line for 50 m until it hits the padding at the end of the rink, rebounding at half the speed with which it had before the impact with the padding.

After rebounding the puck travels in a straight line coming to a stop at the exact point at which it had originally been struck from.

Be modelling the puck as a particle and ignoring air resistance, determine the coefficient of friction between the puck and the rink.

$\boxed{\mu = 0.04}$

STARTING WITH DYNAMICS



$\Rightarrow$ "$F = ma$"

$\Rightarrow -\mu R = ma$

$\Rightarrow -\mu mg = ma$

$\Rightarrow a = -\mu g$

KINEMATICS UNTIL IT HITS THE PADDING

$u = 14$, $a = -\mu g$, $s = 50$, $t = /$, $v = ?$

"$v^2 = u^2 + 2as$"

$v^2 = 14^2 + 2(-\mu g) \times 50$

$v^2 = 196 - 100\mu g$

$v^2 = 196 - 980\mu$ I.E $v = \sqrt{196 - 980\mu}$

IT REBOUNDS WITH HALF THE SPEED, I.E $\frac{1}{2} \times \sqrt{196 - 980\mu}$

THE ACCELERATION (ACTUAL DECELERATION) IS THE SAME

$u = \frac{1}{2}\sqrt{196 - 980\mu}$, $a = -\mu g$, $s = 50$, $t = /$, $v = 0$

"$v^2 = u^2 + 2as$"

$0 = \frac{1}{4}(196 - 980\mu) + 2 \times (-\mu g) \times 50$

$0 = 49 - 245\mu - 100\mu g$

$0 = 49 - 245\mu - 980\mu$

$1225\mu = 49$

$\mu = 0.04$

Question 23 (*****)

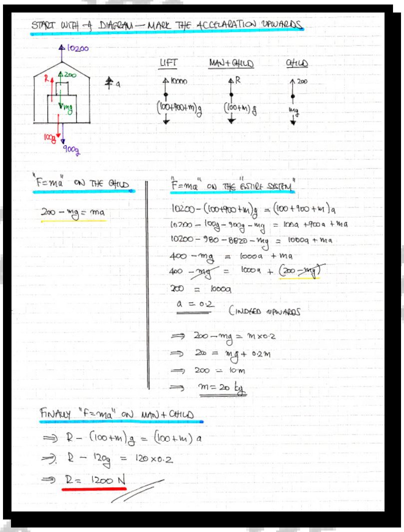
A child is sitting over the shoulders of his father whose mass is 100 kg.

The father, with the child over his shoulders, is standing in a lift of mass 900 kg.

When the tension in the cable of the lift is 10200 N, the lift and its occupants is accelerating, with the child exerting a force of 200 N on the shoulders of his father.

Find the magnitude of the force exerted by floor of the lift to the feet of the father.

$\boxed{}$, $\boxed{R = 1200 \text{ N}}$



START WITH A DIAGRAM — MARK THE ACCELARATION UPWARDS

"F = ma" ON THE CHILD

$200 - mg = ma$

"F = ma" ON THE "ENTIRE SYSTEM"

$10200 - (100+900+m)g = (100+900+m)a$

$10200 - 100g - 900g - mg = 100a + 900a + ma$

$10200 - 980 - 8820 - mg = 1000a + ma$

$400 - mg = 1000a + ma$

$400 - \cancel{mg} = 1000a + (200 - \cancel{mg})$

$200 = 1000a$

$a = 0.2$ (INDEED UPWARDS

$\Rightarrow 200 - mg = m \times 0.2$

$\Rightarrow 200 = mg + 0.2m$

$\Rightarrow 200 = 10m$

$\Rightarrow m = 20 \text{ kg}$

FINALLY "F = ma" ON MAN + CHILD

$\Rightarrow R - (100+m)g = (100+m)a$

$\Rightarrow R - 120g = 120 \times 0.2$

$\Rightarrow R = 1200 \text{ N}$

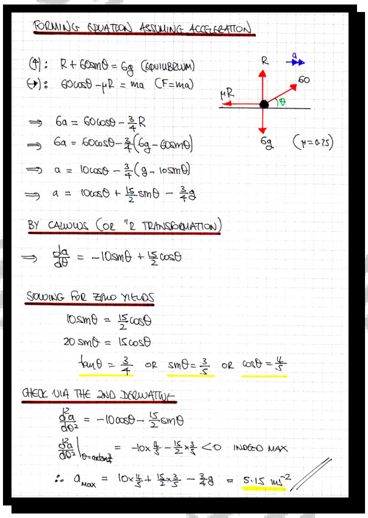
Question 24 (*****)

A particle of mass 6 kg, is pulled by a light inextensible string along a rough horizontal surface, where the coefficient of friction between the particle and the surface is 0.75.

The string is inclined at an acute angle to the horizontal.

If the tension in the string remains constant at 60 N, but the angle in the string can vary, determine the greatest magnitude of the acceleration of the particle.

$\boxed{a_{\max} = 5.15 \text{ ms}^{-2}}$



FORMING EQUATION ASSUMING ACCELERATION

$(\uparrow)$: $R + 60\sin\theta = 6g$ (EQUILIBRIUM)

$(\rightarrow)$: $60\cos\theta - \mu R = ma$ ($F=ma$)

$\Rightarrow 6a = 60\cos\theta - \frac{3}{4}R$

$\Rightarrow 6a = 60\cos\theta - \frac{3}{4}(6g - 60\sin\theta)$

$\Rightarrow a = 10\cos\theta - \frac{3}{4}(g - 10\sin\theta)$

$\Rightarrow a = 10\cos\theta + \frac{15}{2}\sin\theta - \frac{3}{4}g$

BY CALCULUS (OR "R TRANSFORMATION)

$\Rightarrow \frac{da}{d\theta} = -10\sin\theta + \frac{15}{2}\cos\theta$

SOLVING FOR ZERO YIELDS

$10\sin\theta = \frac{15}{2}\cos\theta$

$20\sin\theta = 15\cos\theta$

$\tan\theta = \frac{3}{4}$ OR $\sin\theta = \frac{3}{5}$ OR $\cos\theta = \frac{4}{5}$

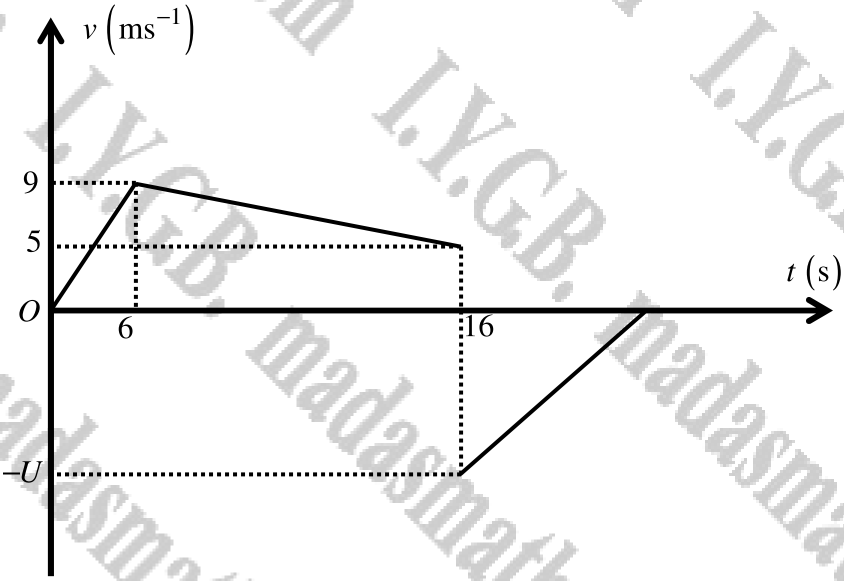
CHECK VIA THE 2ND DERIVATIVE

$\frac{d^2a}{d\theta^2} = -10\cos\theta - \frac{15}{2}\sin\theta$

$\left.\frac{d^2a}{d\theta^2}\right|_{\theta=\arctan\frac{3}{4}} = -10 \times \frac{4}{5} - \frac{15}{2} \times \frac{3}{5} < 0$ INDEED MAX

$\therefore a_{\max} = 10 \times \frac{4}{5} + \frac{15}{2} \times \frac{3}{5} - \frac{3}{4}g = 5.15 \text{ ms}^{-2}$

Question 25 (*****)



A particle of mass 2 kg is released from rest from a point A on an incline plane and begins to move down a line of greatest slope of the plane.

The plane has a different coefficient of friction at different sections so the resistance to the motion of the particle has different values at different sections of the plane, as the particle slides down.

The particle accelerates uniformly to a speed of 9 ms^{-1} in 6 s as it reaches point B.

The coefficient of friction increases at B so the particle continues to slide down with constant deceleration for 10 s achieving a speed of 5 ms^{-1} as it reaches point C.

At C the particle is instantaneously projected with speed $U \text{ ms}^{-1}$, **up** a line of greatest slope of the plane, coming to rest at B.

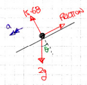
If the **normal** reaction between the plane and the particle has a magnitude of 15.68 N, determine the value of U, correct to 2 decimal places.

$\boxed{}$, $\boxed{U = \sqrt{1702.4} \approx 41.26}$

[solution overleaf]

IN THIS QUESTION IT TAKES A WHILE TO SEE WHAT CALCULATIONS ARE RELEVANT SO IN A FIRST ATTEMPT IT IS TYPICAL TO OBTAIN "ITEMS" NOT REQUIRED (NOT SHOWN HERE)

LOOKING AT THE JOURNEY A TO B, I.E. THE ACCELERATING SECTION



$15.68 = 2g\cos\theta$

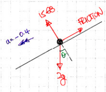
$15.68 = 19.6\cos\theta$

$\cos\theta = 0.8$

& CONSEQUENTLY $\sin\theta = 0.6$

LOOKING AT THE DECELERATING SECTION FROM B TO C

$a = \frac{\Delta v}{\Delta t} = \frac{5-9}{16-6} = \frac{-4}{10} = -0.4$



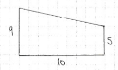
$\Rightarrow$ "$F = ma$"

$\Rightarrow 2g\sin\theta - \text{FRICTION} = 2(-0.4)$

$\Rightarrow 2(9.8)(0.6) - \text{FRICTION} = -0.8$

$\Rightarrow \text{FRICTION} = 12.56$ ← FRICTION IN THE BC SECTION

NEXT LOOKING AT THE SPEED TIME GRAPH

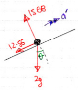


$\text{AREA} = \frac{9+5}{2} \times 10$

$\text{AREA} = 7 \times 10$

$\therefore$ DISTANCE B TO C IS 70

NOW THE JOURNEY BACK UP FROM C TO B



$\Rightarrow$ "$F = ma$"

$\Rightarrow -12.56 - 2g\sin\theta = 2a'$

$\Rightarrow -12.56 - 2g(0.6) = 2a'$

$\Rightarrow -24.32 = 2a'$

$\Rightarrow a' = -12.16 \text{ ms}^{-2}$

FINALLY KINEMATICS FOR THE JOURNEY B TO C

u =	?
a =	-12.16
s =	70
t =	
v =	0

"$v^2 = u^2 + 2as$"

$0 = u^2 + 2(-12.16) \times 70$

$u^2 = 1702.4$

$u = 41.26 \text{ ms}^{-1}$

I.E. $U = 41.26$