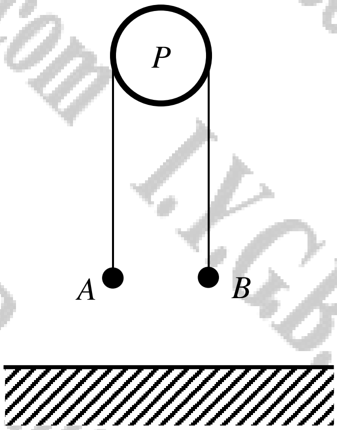
CONNECTED PARTICLES

Question 1 (**)



Two particles A and B of respective masses 5 kg and 9 kg are each attached to the two ends of a light inextensible string which passes over a smooth pulley P.

The two particles are both held at rest, 1.75 m above a horizontal floor with the portions of the strings, not in contact with the pulley, vertical.

The system is then released from rest.

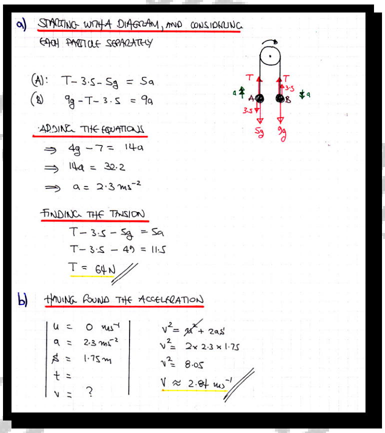
When in motion, each particle is subject to a constant air resistance of 3.5 N.

In the resulting motion B reaches the floor before A reaches P.

a) Calculate the tension in the string, for the period before B reaches the floor.

b) Determine the speed with which B strikes the floor.

$\boxed{}$, $\boxed{T = 64\text{ N}}$, $\boxed{v \approx 2.84 \text{ ms}^{-1}}$



a) STARTING WITH A DIAGRAM, AND CONSIDERING EACH PARTICLE SEPARATELY

(A): $T - 3.5 - 5g = 5a$
(B) $9g - T - 3.5 = 9a$

ADDING THE EQUATIONS

$\Rightarrow 4g - 7 = 14a$
$\Rightarrow 14a = 32.2$
$\Rightarrow a = 2.3 \text{ ms}^{-2}$

FINDING THE TENSION

$T - 3.5 - 5g = 5a$
$T - 3.5 - 49 = 11.5$
$T = 64\text{N}$

b) HAVING FOUND THE ACCELERATION

$u = 0 \text{ ms}^{-1}$
$a = 2.3 \text{ ms}^{-2}$
$s = 1.75\text{m}$
$t =$
$v = ?$

$v^2 = \cancel{u^2} + 2as$
$v^2 = 2 \times 2.3 \times 1.75$
$v^2 = 8.05$
$v \approx 2.84 \text{ ms}^{-1}$

Question 2 (**+)

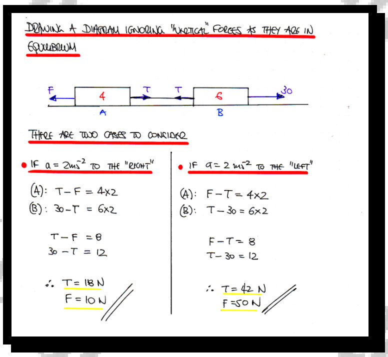
Two blocks A and B of respective masses 4 kg and 6 kg lie on a smooth horizontal surface and are connected by a light inextensible string.

Two collinear forces, of magnitudes F N and 30 N, act on each of the blocks, and in opposite directions, as shown in the figure above.

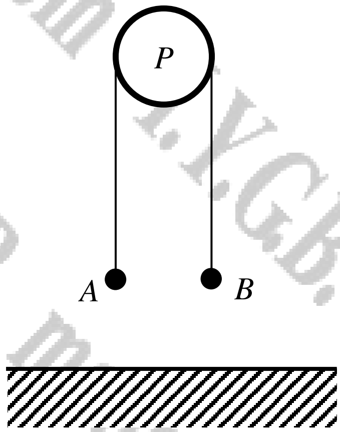
The system has constant acceleration of magnitude 2 ms^{-2}.

Determine the possible values of F, and in each case the corresponding value of the tension in the string.

$\boxed{F = 10\,\text{N}, \quad T = 18\,\text{N}}$, $\boxed{F = 50\,\text{N}, \quad T = 42\,\text{N}}$



Question 3 (***)



Two particles A and B of respective masses 3 kg and $m \text{ kg}$ are each attached to the two ends of a light inextensible string which passes over a smooth pulley P. The two particles are held at rest, both at a height of 1.28 m above a horizontal floor with the portions of the strings not in contact with the pulley vertical.

The system of the two particles is then released from rest with B accelerating towards the floor at 1.96 ms^{-2}, while A never reaches P.

a) For the period before B reaches the floor, calculate the tension in the string.

b) Determine the value of m.

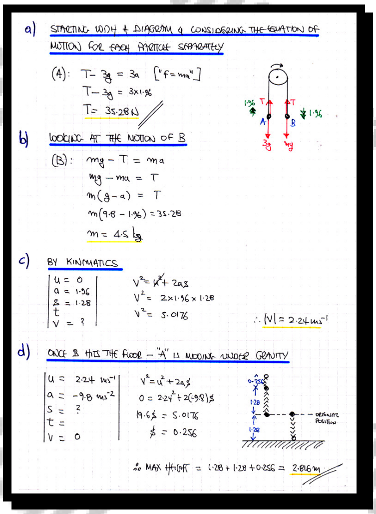
c) Calculate the speed with which B strikes the floor.

When B reaches the floor it remains at rest.

d) Determine the greatest height above the floor reached by A.

$\boxed{}$, $\boxed{T = 35.28 \text{ N}}$, $\boxed{m = 4.5}$, $\boxed{v = 2.24 \text{ ms}^{-1}}$, $\boxed{h_{\text{max}} = 2.816 \text{ m}}$

[solution overleaf]



a) STARTING WITH A DIAGRAM & CONSIDERING THE EQUATION OF MOTION FOR EACH PARTICLE SEPARATELY

(A): $T - 3g = 3a$ ["$F = ma$"]

$T - 3g = 3 \times 1.96$

$T = 35.28\text{ N}$

b) LOOKING AT THE MOTION OF B

(B): $mg - T = ma$

$mg - ma = T$

$m(g - a) = T$

$m(9.8 - 1.96) = 35.28$

$m = 4.5\text{ kg}$

c) BY KINEMATICS

$u = 0$	$v^2 = \cancel{u^2} + 2as$
$a = 1.96$	$v^2 = 2 \times 1.96 \times 1.28$
$s = 1.28$	$v^2 = 5.0176$
t	
$v = ?$	

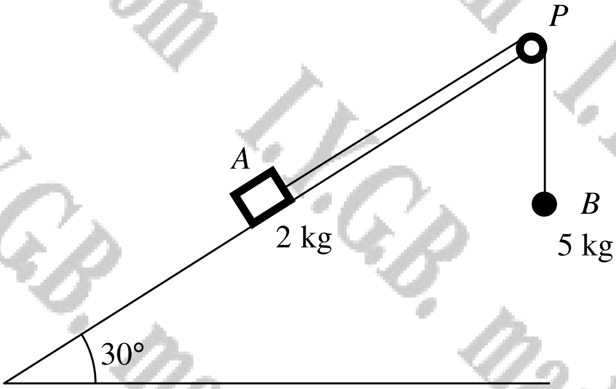
$\therefore |v| = 2.24\text{ ms}^{-1}$

d) ONCE B HITS THE FLOOR – "A" IS MOVING UNDER GRAVITY

$u = 2.24\text{ ms}^{-1}$	$v^2 = u^2 + 2as$
$a = -9.8\text{ ms}^{-2}$	$0 = 2.24^2 + 2(-9.8)s$
$s = ?$	$19.6s = 5.0176$
$t =$	$s = 0.256$
$v = 0$	

$\therefore$ MAX HEIGHT $= 1.28 + 1.28 + 0.256 = 2.816\text{ m}$

Question 4 (***)



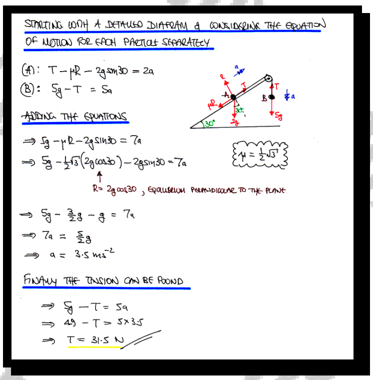
Two particles A and B, of mass 2 kg and 5 kg respectively, are attached to each of the ends of a light inextensible string. The string passes over a smooth pulley P, at the top of a fixed rough plane, inclined at $30°$ to the horizontal.

Particle A is placed at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P, as shown in the figure above. The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane.

The particles are released from rest with the string taut. Particle A begins to move up the incline plane, where the coefficient between A and the plane is $\frac{1}{2}\sqrt{3}$.

Ignoring air resistance, calculate the tension in the string immediately after the particles are released.

$\boxed{T = 31.5 \text{ N}}$


STARTING WITH A DETAILED DIAGRAM & CONSIDERING THE EQUATION OF MOTION FOR EACH PARTICLE SEPARATELY

(A): $T - \mu R - 2g\sin 30 = 2a$
(B): $5g - T = 5a$

ADDING THE EQUATIONS

$\Rightarrow 5g - \mu R - 2g\sin 30 = 7a$
$\Rightarrow 5g - \frac{1}{2}\sqrt{3}(2g\cos 30) - 2g\sin 30 = 7a$

$\mu = \frac{1}{2}\sqrt{3}$

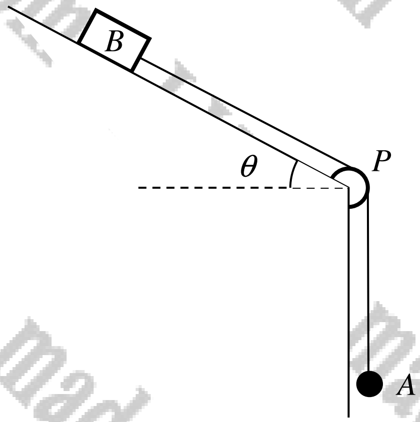
$R = 2g\cos 30$, EQUILIBRIUM PERPENDICULAR TO THE PLANE

$\Rightarrow 5g - \frac{3}{2}g - g = 7a$
$\Rightarrow 7a = \frac{5}{2}g$
$\Rightarrow a = 3.5 \text{ ms}^{-2}$

FINALLY THE TENSION CAN BE FOUND

$\Rightarrow 5g - T = 5a$
$\Rightarrow 49 - T = 5 \times 3.5$
$\Rightarrow T = 31.5 \text{ N}$

Question 5 (***)



A particle A and a small box B, with respective masses of 3 kg and 7 kg, are attached to the ends of a light inextensible string.

B is held at rest on a rough plane inclined at θ to the horizontal, where $\tan\theta = \frac{3}{4}$.

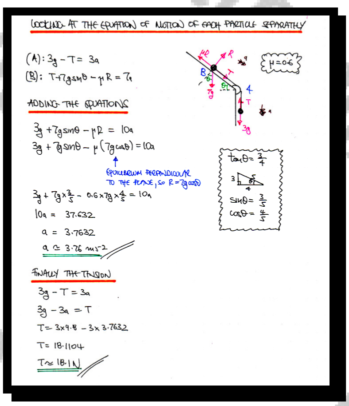
The coefficient of friction between the box and the plane is 0.6.

The string lies along the plane and passes over a small smooth pulley P which is fixed at the bottom end of the plane.

A is hanging vertically below the end of the plane. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in the figure above. B is released from rest with the string taut.

After release, determine the acceleration of the system and the tension in the string.

$\boxed{}$, $\boxed{a = 3.7632 \text{ ms}^{-2}}$, $\boxed{T = 18.1104 \text{ N}}$


LOOKING AT THE EQUATION OF MOTION OF EACH PARTICLE SEPARATELY

(A): $3g - T = 3a$

(B): $T + 7g\sin\theta - \mu R = 7a$

ADDING THE EQUATIONS

$3g + 7g\sin\theta - \mu R = 10a$

$3g + 7g\sin\theta - \mu(7g\cos\theta) = 10a$

EQUILIBRIUM PERPENDICULAR TO THE PLANE, SO $R = 7g\cos\theta$

$3g + 7g \times \frac{3}{5} - 0.6 \times 7g \times \frac{4}{5} = 10a$

$10a = 37.632$

$a = 3.7632$

$a \simeq 3.76 \text{ ms}^{-2}$

FINALLY THE TENSION

$3g - T = 3a$

$3g - 3a = T$

$T = 3 \times 9.8 - 3 \times 3.7632$

$T = 18.1104$

$T \approx 18.1 \text{ N}$

$\mu = 0.6$

$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

Question 6 (***)

A car of mass 1500 kg is towing a trailer of mass 1000 kg by means of a light inextensible rope. The car is experiencing a constant air resistance of 200 N, while the corresponding constant air resistance on the trailer is 300 N.

The car and trailer are modelled as particles, with the tow rope remaining taut and horizontal throughout the motion.

a) Given that the driving force acting on the car is 750 N, determine …

i. … the acceleration of the system.

ii. … the tension in the tow rope.

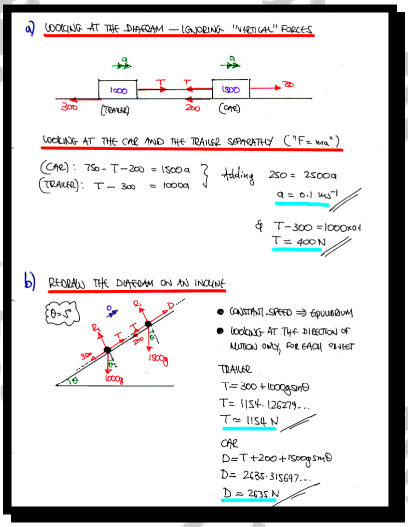
Later in the journey, the car and the trailer are ascending on a road which inclined at $5°$ to the horizontal. The air resistance on the car and trailer are unchanged.

b) Assuming that the system now moves with constant speed, calculate …

i. … a new figure for the tension in the tow rope.

ii. … a new figure for the driving force of the car.

$\boxed{}$, $\boxed{a = 0.1 \text{ ms}^{-2}}$, $\boxed{T = 400 \text{ N}}$, $\boxed{T \approx 1154 \text{ N}}$, $\boxed{D \approx 2635 \text{ N}}$



Question 7 (***)

A car of mass 1500 kg is towing a trailer of mass 500 kg by means of a light rigid horizontal towbar. The car is experiencing a constant air resistance of 300 N, while the corresponding constant air resistance on the trailer is 100 N.

The car and trailer are modelled as particles.

a) Given the tension in the towbar is 200 N, calculate …

i. … the acceleration of the system.

ii. … the driving force of the car.

Later in the journey, the car's driving force is removed and the car's brakes are applied, providing a constant breaking force of 400 N, on the car only.

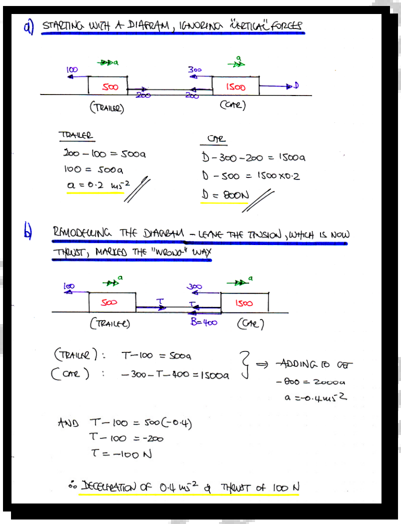
The air resistance on the car and trailer are unchanged.

b) Determine …

i. … the deceleration of the system.

ii. … the thrust in the towbar.

$\boxed{}$, $\boxed{a = 0.2\,\text{ms}^{-2}}$, $\boxed{D = 800\text{ N}}$, $\boxed{|a| = 0.4\text{ ms}^{-2}}$, $\boxed{T = 100\text{ N}}$


a) STARTING WITH A DIAGRAM, IGNORING "VERTICAL" FORCES

TRAILER

$200 - 100 = 500a$
$100 = 500a$
$a = 0.2 \text{ ms}^{-2}$

CAR

$D - 300 - 200 = 1500a$
$D - 500 = 1500 \times 0.2$
$D = 800\text{N}$

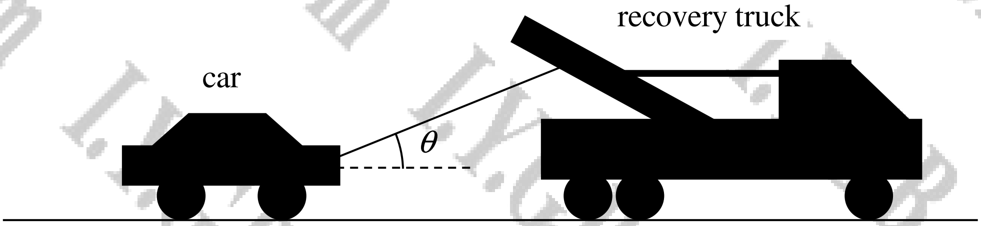
b) REMODELLING THE DIAGRAM – LEAVE THE TENSION, WHICH IS NOW THRUST, MARKED THE "WRONG" WAY

(TRAILER) : $T - 100 = 500a$
(CAR) : $-300 - T - 400 = 1500a$
$\Rightarrow$ ADDING TO GET
$-800 = 2000a$
$a = -0.4 \text{ ms}^{-2}$

AND $T - 100 = 500(-0.4)$
$T - 100 = -200$
$T = -100\text{ N}$

$\therefore$ DECELERATION OF 0.4 ms^{-2} & THRUST OF 100 N

Question 8 (***)



A recovery truck of mass 2800 kg is towing a car of mass 1200 kg along a straight horizontal road. The tow cable is inclined at an angle θ to the horizontal, where $\cos\theta = 0.75$, as shown in the figure above. The tow cable is modelled a light inextensible string and the two vehicles as particles.

The two vehicles were travelling at constant speed 12 ms^{-1} with the tow cable taut as they were travelling in an urban area. On leaving this urban area, the truck begins to accelerate uniformly bringing their speed to 27 ms^{-1} over a distance of 2.34 km.

a) Calculate the acceleration of the truck and the car.

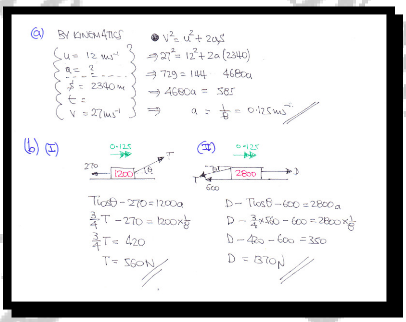
There is a constant resistance to the motion of the truck of 600 N, and a constant resistance to the motion of the car of 270 N.

b) For the part of the journey during which the two vehicles accelerate, determine …

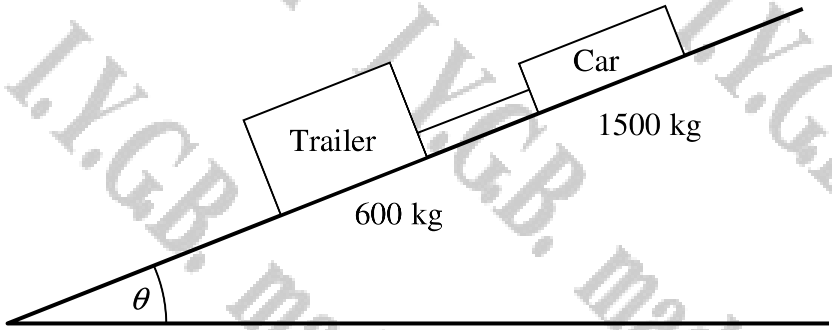
i. … the force in the tow cable.

ii. … the driving force of the truck

$\boxed{a = 0.125 \text{ ms}^{-2}}$, $\boxed{T = 560 \text{ N}}$, $\boxed{D = 1370 \text{ N}}$



Question 9 (***+)

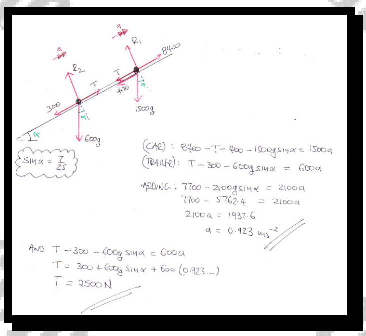


A trailer of mass 600 kg is connected to a car of mass 1500 kg by means of a light rigid tow bar. The car is moving up a line of greatest slope of a plane inclined at θ to the horizontal, where $\sin\theta = \frac{7}{25}$, as shown in the figure above.

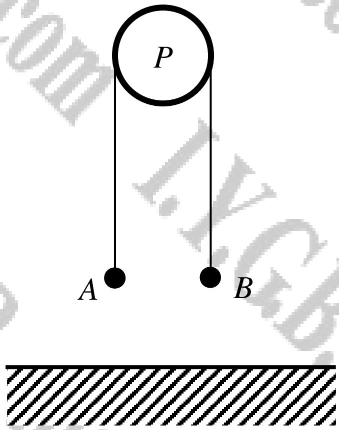
A constant resistance of magnitude 400 N acts on the car, and a constant resistance of magnitude 300 N acts on the trailer. The engine of the car produces a constant forward driving force of 8400 N.

Determine the acceleration of the car and the tension in the tow bar.

$\boxed{a \approx 0.923\text{ ms}^{-2}}$, $\boxed{T = 2500\text{ N}}$



Question 10 (***+)



Two particles A and B of respective masses 2 kg and 5 kg are attached to the ends of a light inextensible string which passes over a smooth pulley P. The two particles are held at rest, at the same level above a horizontal floor with the portions of the strings not in contact with the pulley vertical. The system is then released from rest.

a) For the period before B reaches the floor, calculate …

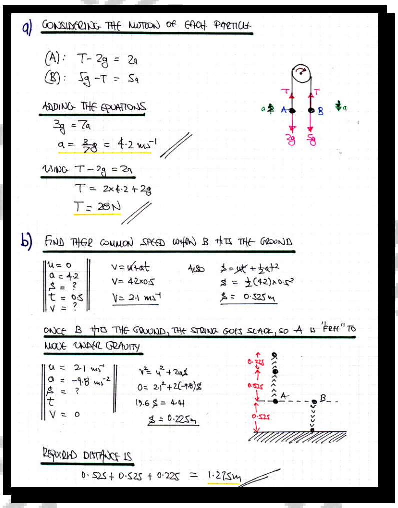
  i. … the acceleration of the system.

  ii. … the tension in the string.

Eventually B reaches the floor 0.5 s after release and **does not** rebound. In the ensuing motion A does not reach P.

b) Determine the greatest height of A above the floor.

$\boxed{a = 4.2\,\text{ms}^{-2}}$, $\boxed{T = 28\,\text{N}}$, $\boxed{h_{\text{max}} = 1.275\text{ m}}$


a) CONSIDERING THE MOTION OF EACH PARTICLE

(A): $T - 2g = 2a$
(B): $5g - T = 5a$

ADDING THE EQUATIONS
$3g = 7a$
$a = \frac{3}{7}g = 4.2\text{ ms}^{-1}$

USING $T - 2g = 2a$
$T = 2 \times 4.2 + 2g$
$T = 28\text{N}$

b) FIND THEIR COMMON SPEED WHEN B HITS THE GROUND

$u = 0$, $a = 4.2$, $s = ?$, $t = 0.5$, $v = ?$

$v = u + at$
$v = 4.2 \times 0.5$
$v = 2.1\text{ ms}^{-1}$

ALSO $s = ut + \frac{1}{2}at^2$
$s = \frac{1}{2}(4.2) \times 0.5^2$
$s = 0.525\text{ m}$

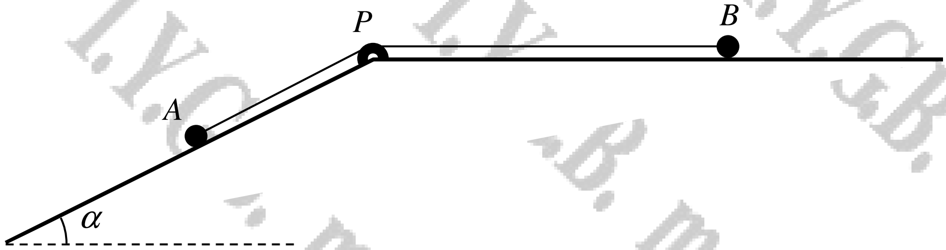
ONCE B HITS THE GROUND, THE STRING GOES SLACK, SO A IS "FREE" TO MOVE UNDER GRAVITY

$u = 2.1\text{ ms}^{-1}$, $a = -9.8\text{ ms}^{-2}$, $s = ?$, t, $v = 0$

$v^2 = u^2 + 2as$
$0 = 2.1^2 + 2(-9.8)s$
$19.6s = 4.41$
$s = 0.225\text{m}$

REQUIRED DISTANCE IS
$0.525 + 0.525 + 0.225 = 1.275\text{m}$

Question 11 (***+)



Two particles A and B have masses 0.5 kg and 0.2 kg, respectively. The particles are attached to the ends of a light inextensible string. Particle B is held at rest on a rough horizontal table. The string lies along the table and passes over a small smooth pulley P which is fixed to the edge of the table. Particle A is at rest on a smooth plane which is inclined to the horizontal at an angle α, where $\tan\alpha = 0.75$. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in the figure above.

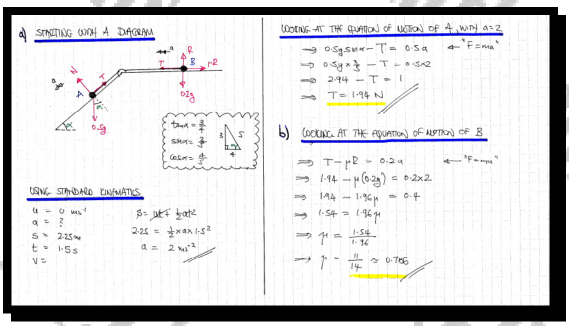
Particle B is released from rest with the string taut.

During the first 1.5 s of the motion B does not reach the pulley and A moves 2.25 m down the plane.

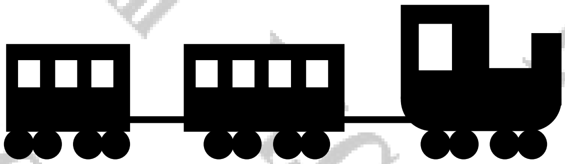
a) Find the tension in the string during the first 1.5 s of the motion.

b) Calculate the coefficient of friction between B and the table.

$\boxed{}$, $\boxed{T = 1.94\text{ N}}$, $\boxed{\mu = \frac{11}{14} \approx 0.786}$



Question 12 (***+)



In a fun fair ride, a miniature electric train for small children consists of an engine with two carriages. The engine has mass 500 kg towing a larger carriage of mass 300 kg, which in turn tows a smaller carriage of mass 200 kg. The above masses include the driver and the children.

The engine and the carriages are modelled as particles and the couplings between the engine and the carriages are modelled as light rigid rods. When in motion, the engine and each of the carriages experiences a constant resistance of 100 N.

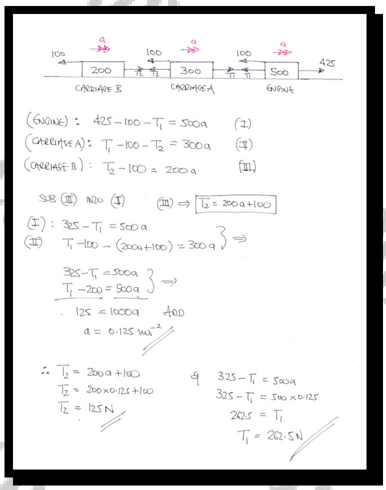
Given the engine provides a maximum driving force of 425 N, calculate …

a) … the maximum acceleration of the system.

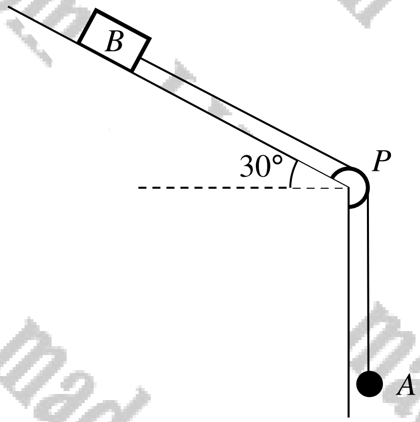
b) … the tension in the coupling between the engine and the first carriage when the train has maximum acceleration.

c) … the tension in the coupling between the first carriage and the second carriage when the train has maximum acceleration.

$\boxed{a = 0.125 \text{ ms}^{-2}}$, $\boxed{T_1 = 262.5 \text{ N}}$, $\boxed{T_2 = 125 \text{ N}}$



Question 13 (***+)



A particle A and a small box B, with respective masses of 2 kg and 5 kg, are attached to the ends of a light inextensible string.

B is held at rest on a rough plane inclined at $30°$ to the horizontal. The string lies along the plane and passes over a small smooth pulley P which is fixed at the bottom end of the plane. The coefficient of friction between the box and the plane is 0.8.

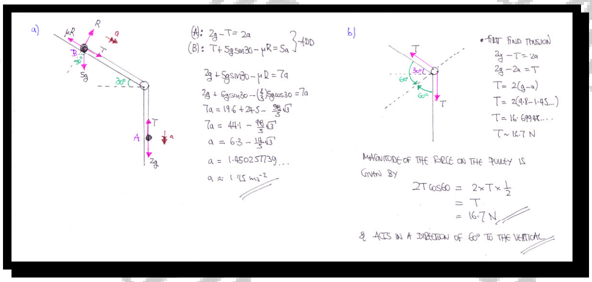
A is hanging vertically below the end of the plane. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in the figure above.

B is released from rest with the string taut.

a) Determine the acceleration of B immediately after B is released.

b) Calculate the magnitude and direction of force exerted on the pulley by the string immediately after B is released.

$\boxed{a \approx 1.45 \text{ ms}^{-2}}$, $\boxed{T \approx 16.7 \text{ N, } 60° \text{ to the vertical}}$



Question 14 (***+)

A car of mass 1400 kg is towing a caravan of mass 600 kg by means of a light rigid horizontal towbar. The car is experiencing a constant air resistance of 200 N, while the corresponding constant air resistance on the trailer is 300 N.

The car and caravan are modelled as particles.

a) Given that the driving force of the car is 2000 N, determine …

i. … the acceleration of the system.

ii. … the tension in the tow bar.

Later in the journey, the car descends a hill which is declined at $10°$ to the horizontal.

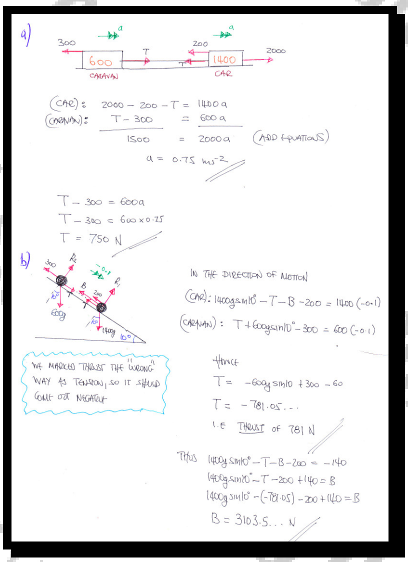
For this part of the journey the car's driving force is removed and the brakes are applied, providing a constant breaking force of B N, on the car only. The air resistance on the car and caravan are unchanged.

b) Given further that the deceleration of the system is $0.1\ \text{ms}^{-2}$, calculate …

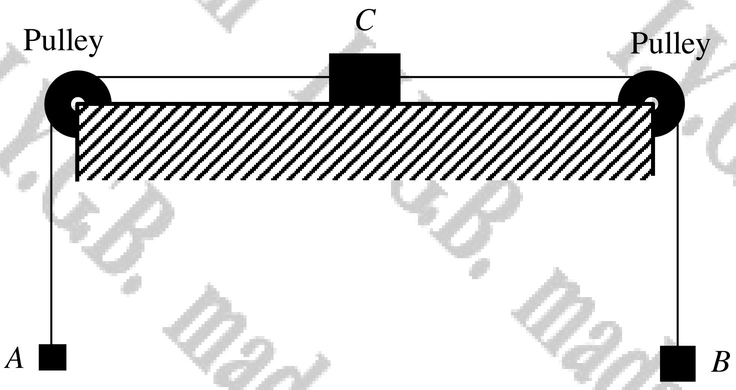
i. … the value of B.

ii. … the thrust in the towbar.

$\boxed{a = 0.75\,\text{ms}^{-2}}$, $\boxed{T = 750\ \text{N}}$, $\boxed{T \approx 781.05...\ \text{N}}$, $\boxed{B \approx 3103.5...\ \text{N}}$



Question 15 (***+)



A block C, of mass 4 kg, is placed on a rough horizontal table, where the coefficient of friction between the table and C is 0.65.

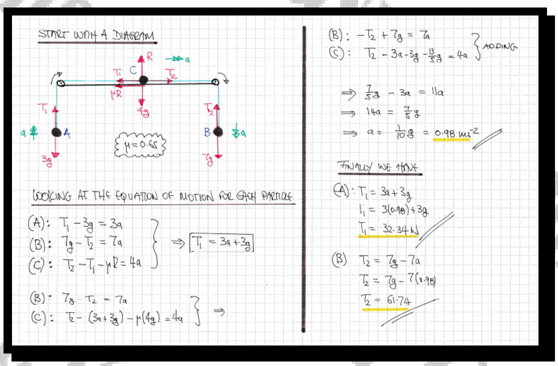
C is connected by two light inextensible strings to two more blocks, A and B, of respective masses 3 kg and 7 kg.

Each of the strings passes over two smooth pulleys, each of the pulleys located at the edge of the table, with A and B hanging freely at each of the two ends of the table, as shown in the figure above.

The system is released from rest with the strings taut.

By modelling the three blocks as particles, determine in any order the acceleration of the system and the tension in each of the two strings.

$\boxed{}$, $\boxed{a = 0.98 \text{ ms}^{-2}}$, $\boxed{T_A = 32.34 \text{ N}}$, $\boxed{T_B = 61.74 \text{ N}}$



Question 16 (***+)

Two particles A and B have masses m kg and 4 kg, respectively.

The two particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The two particles are held at rest with the string taut and the hanging parts of the string vertical.

The system is released from rest and A moves **upwards**.

a) Determine the acceleration of the system in terms of m and g.

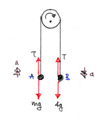
b) Show that the tension in the string, while A ascends, is $\dfrac{8mg}{m+4}$.

At the instant when A is 0.7 m above its original position, it has not yet reached the pulley and is travelling at 1.4 ms^{-1}.

c) Find the value of m.

$\boxed{}$, $\boxed{a = \dfrac{4-m}{4+m}g}$, $\boxed{m = 3}$

a) LOOKING AT THE DIAGRAM OPPOSITE



(A): $T - mg = ma$

(B): $4g - T = 4a$

ADDING THE EQUATIONS

$4g - mg = (m+4)a$

$(4-m)g = (m+4)a$

$\therefore a = \dfrac{4-m}{4+m}g$

b) USING RESULT FROM PART (a)

$T = 4g - 4a$

$T = 4g - \dfrac{4-m}{4+m}4g$

$T = 4g\left[1 - \dfrac{4-m}{4+m}\right]$

$T = 4g\left[\dfrac{\cancel{4} + m - \cancel{4} + m}{4+m}\right]$

$T = 4g \times \dfrac{2m}{4+m}$

$T = \dfrac{8mg}{m+4}$ AS REQUIRED

c) KINEMATICS WITH CONSTANT ACCELERATION

$u = 0$

$a = \dfrac{4-m}{4+m}$

$s = 0.7$

t

$v = 1.4$

$\Rightarrow v^2 = \cancel{u^2} + 2as$

$\Rightarrow 1.4^2 = 2 \times \dfrac{4-m}{4+m} \times 9.8 \times 0.7$

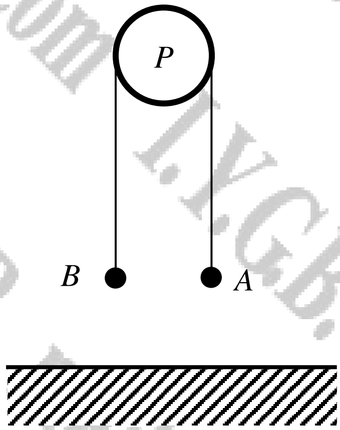
$\Rightarrow \dfrac{4-m}{4+m} = \dfrac{1}{7}$

$\Rightarrow 28 - 7m = 4 + m$

$\Rightarrow 24 = 8m$

$\Rightarrow m = 3$

Question 17 (****)



Two particles A and B of respective masses 5 kg and 2 kg are each attached to the two ends of a light inextensible string which passes over a smooth pulley P. The two particles are both held at rest, 1.54 m above a horizontal floor with the portions of the strings, not in contact with the pulley, vertical. The system is then released from rest. When in motion, each particle is subject to a constant air resistance of 7 N.

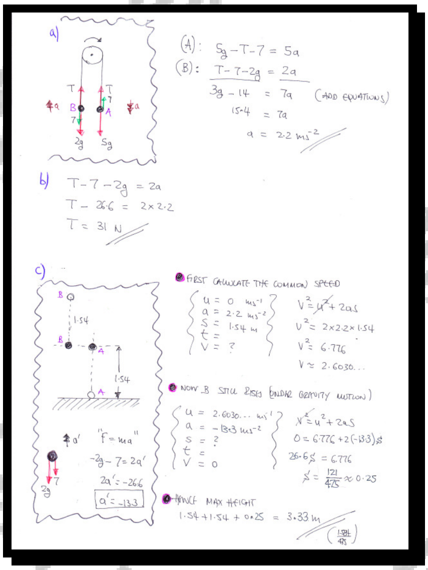
In the resulting motion A reaches the floor before B reaches P.

a) Find the acceleration of the system.

b) Calculate the tension in the string, for the period before A reaches the floor.

c) Determine the greatest height B reaches above the floor.

$\boxed{a = 2.2\,\text{ms}^{-2}}$, $\boxed{T = 31\,\text{N}}$, $\boxed{h \approx 3.33\text{ m}}$


a)

(A): $5g - T - 7 = 5a$

(B): $T - 7 - 2g = 2a$

$3g - 14 = 7a$ (ADD EQUATIONS)

$15.4 = 7a$

$a = 2.2\text{ ms}^{-2}$

b) $T - 7 - 2g = 2a$

$T - 26.6 = 2 \times 2.2$

$T = 31\text{ N}$

c)

FIRST CALCULATE THE COMMON SPEED

$u = 0\text{ ms}^{-1}$, $a = 2.2\text{ ms}^{-2}$, $s = 1.54\text{ m}$, $t =$, $v = ?$

$v^2 = u^2 + 2as$

$v^2 = 2 \times 2.2 \times 1.54$

$v^2 = 6.776$

$v \approx 2.6030\ldots$

"$F = ma$"

$-2g - 7 = 2a'$

$2a' = -26.6$

$\boxed{a' = -13.3}$

NOW B STILL RISES (UNDER GRAVITY MOTION)

$u = 2.6030\ldots\text{ ms}^{-1}$, $a = -13.3\text{ ms}^{-2}$, $s = ?$, $t =$, $v = 0$

$v^2 = u^2 + 2as$

$0 = 6.776 + 2(-13.3)s$

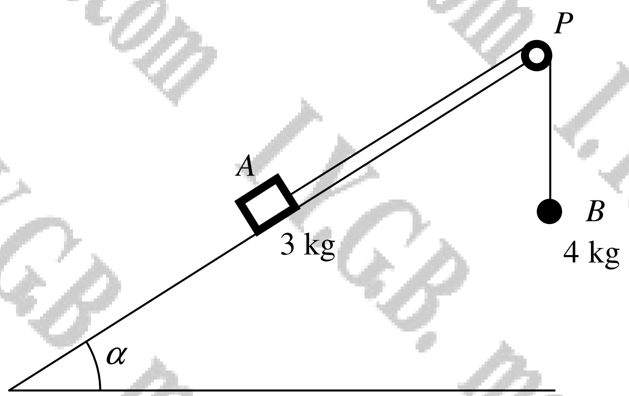
$26.6s = 6.776$

$s = \frac{121}{475} \approx 0.25$

HENCE MAX HEIGHT

$1.54 + 1.54 + 0.25 = 3.33\text{ m}$ $\left(\frac{1584}{475}\right)$

Question 18 (****)



Two particles A and B, of mass 3 kg and 4 kg respectively, are attached to each of the ends of a light inextensible string. The string passes over a smooth pulley P, at the top of a fixed rough plane, inclined at α to the horizontal, where $\tan\alpha = 0.75$.

Particle A is placed at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P, as shown in the figure above.

The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane.

The particles are released from rest with the string taut.

Particle A begins to move up the incline plane, where the constant ground friction between A and the plane has magnitude 10.5 N.

Ignoring air resistance, calculate ...

a) ... the acceleration of the system immediately after the particles are released.

b) ... the magnitude and direction of the force exerted by the string on P.

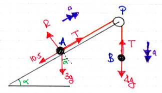
2 s after release, while both particles are moving, the string breaks.

c) Calculate the total distance A moves up the plane from the instant since the particles were released, assuming that A does not reach the pulley.

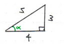
$\boxed{}$, $\boxed{a = 1.58 \text{ ms}^{-2}}$, $\boxed{F \approx 58.8 \text{ N}}$, $\boxed{26.6° \text{ to the plane}}$, $\boxed{d \approx 3.69 \text{ m}}$

[solution overleaf]

a) START WITH A DETAILED DIAGRAM



$\tan\alpha = \frac{3}{4}$



$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

LOOKING AT THE EQUATION OF MOTION FOR EACH PARTICLE

(A): $T - 10.5 - 3g\sin\alpha = 3a$ ← "F = ma" FOR EACH

(B): $4g - T = 4a$

$\Rightarrow 4g - 10.5 - 3g\sin\alpha = 7a$ ← ADDING THE EQUATIONS

$\Rightarrow 4g - 10.5 - 3g\left(\frac{3}{5}\right) = 7a$

$\Rightarrow 11.06 = 7a$

$\Rightarrow a = 1.58 \text{ ms}^{-2}$

b) FIRST WE NEED TO FIND THE TENSION IN THE STRING

$\Rightarrow 4g - T = 4a$

$\Rightarrow 4g - 4a = T$

$\Rightarrow T = 4 \times 9.8 - 4 \times 1.58$

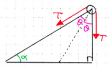
$\Rightarrow T = 32.88 \text{ N}$

LOOKING AT THE DIAGRAM BELOW

- $\theta = \frac{90 - \alpha}{2} = \frac{90 - \arctan\frac{3}{4}}{2}$

  $\theta = 26.6°$ TO THE VERTICAL

  $\left(\arctan\frac{1}{2}\right)$



- MAGNITUDE OF FORCE ON P

  $F = 2T\cos\theta = 2 \times 32.88 \times \cos(26.6\ldots) = 58.8175\ldots$

  $F \simeq 58.8 \text{ N}$

c) USE KINEMATICS FOR CONSTANT ACCELERATION

$u = 0 \text{ ms}^{-1}$
$a = 1.58 \text{ ms}^{-2}$
$s = ?$
$t = 2 \text{ s}$
$v = ?$

TWO SECONDS INTO THE MOTION

- $v = u + at$

  $v = 0 + 1.58 \times 2$

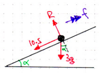
  $v = 3.16 \text{ ms}^{-1}$

- $s = \left(\frac{u+v}{2}\right)t$

  $s = \left(\frac{0 + 3.16}{2}\right) \times 2$

  $s = 3.16 \text{ m}$

NEXT RECALCULATE THE ACCELERATION (DECELERATION)



(NOTE THAT THERE IS NO TENSION AFTER 2 s)

"F = ma" IN THE DIRECTION OF MOTION

$-10.5 - 3g\sin\alpha = 3f$

$-10.5 - 17.64 = 3f$

$f = -9.38 \text{ ms}^{-2}$

FINALLY KINEMATICS UNDER THE NEW DECELERATION

$u = 3.16 \text{ ms}^{-1}$
$a = -9.38 \text{ ms}^{-2}$
$s = ?$
$t =$
$v = 0$

"$v^2 = u^2 + 2as$"

$0 = 3.16^2 + 2(-9.38)s$

$18.76s = 9.9856$

$s = 0.53228\ldots$

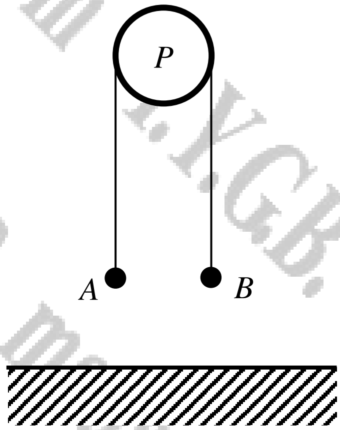
∴ TOTAL DISTANCE UP THE PLANE IS

$= 3.16 + 0.53228\ldots$

$\approx 3.69 \text{ m}$

3sf

Question 19 (****)



Two particles A and B of respective masses 0.5 kg and 0.9 kg are attached to the ends of a light inextensible string which passes over a smooth pulley P. The two particles are held at rest, at the same level, 1.4 m above a horizontal floor. The portions of the strings not in contact with the pulley are vertical. The system is then released from rest and the particles begin to move without air resistance.

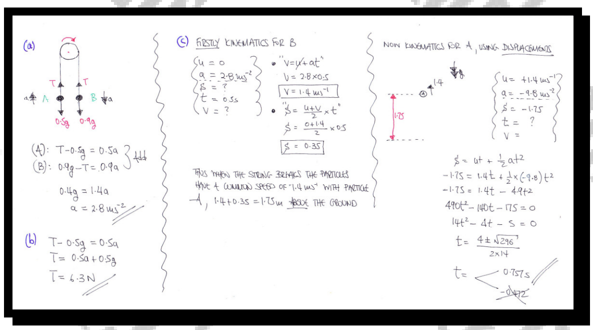
a) For the period before B reaches the floor, calculate …

- **i.** … the acceleration of the system.
- **ii.** … the tension in the string.

The string suddenly breaks 0.5 s after the particles were released.

b) Assuming A does not meet any obstacles in its consequent motion, calculate the additional time it takes A until it reaches the floor.

$\boxed{a = 2.8 \text{ ms}^{-2}}$, $\boxed{T = 6.3\,\text{N}}$, $\boxed{t = \frac{1}{2}\left(2+\sqrt{74}\right) \approx 0.757 \text{ s}}$



Question 20 (****)

Two particles A and B of respective masses $3\,\text{kg}$ and $m\,\text{kg}$ are connected by a light inextensible string which passes over a smooth pulley P.

The two particles are held at rest, at the same level above a horizontal floor, with the portions of the strings not in contact with the pulley vertical.

The system is released and B begins to decelerate at $\frac{1}{4}g\ \text{ms}^{-2}$.

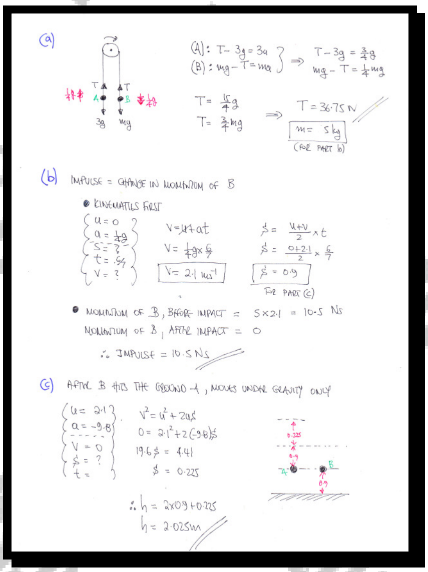
a) Find the tension in the string for the period before B reaches the ground.

The particle B hits the ground $\frac{6}{7}$ s after release and **does not** rebound.

b) Calculate the magnitude of the impulse exerted by the floor onto B.

c) Determine the greatest height of A above the ground in the subsequent motion.

$\boxed{T = 36.75\,\text{N}}$, $\boxed{I = 10.5\ \text{Ns}}$, $\boxed{h_{\text{max}} = 2.025\ \text{m}}$



Question 21 (****)

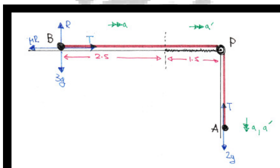
A particle A of mass 2 kg is connected to small box B of mass 3 kg by a light inextensible string. The string passes over a light smooth pulley P, which is located at the end of a horizontal house roof. The box is held on the roof with the particle hanging vertically at the end of the roof, as shown in the figure above.

The system is released from rest with the string taut, so that the distance BP is 4 m. On release, the motion of B takes place over a smooth section of the roof.

After B has moved for 2.5 m the roof becomes rough and the coefficient of friction between B and the roof is 0.75.

Calculate the speed with which B hits P.

$\boxed{}$, $\boxed{v \approx 4.26 \text{ ms}^{-1}}$



LOOKING AT THE EQUATIONS OF MOTION FOR THE BOX AND THE PARTICLE FOR THE FIRST 2.5 m OF THE MOTION

[BOX]: $T = 3a$ (NO FRICTION)
[PARTICLE]: $2g - T = 2a$
ADDING GIVES
$5a = 2g$
$a = \frac{2}{5}g$

FIND THE COMMON SPEED AT THE END OF THE FIRST 2.5 m

$u = 0$, $a = \frac{2}{5}g \text{ ms}^{-2}$, $s = 2.5 \text{ m}$, $t =$, $v = ?$

$\Rightarrow v^2 = u^2 + 2as$
$\Rightarrow v^2 = 0 + 2\left(\frac{2}{5}g\right) \times 2.5$
$\Rightarrow v^2 = 2g$
$\Rightarrow v^2 = 19.6$
$\Rightarrow v \approx 4.42718...$

NEXT OBTAIN THE EQUATION OF MOTION FOR THE BOX & PARTICLE FOR THE ROUGH SECTION OF 1.5 m

[BOX]: $T - \mu R = 3a'$
[PARTICLE]: $2g - T = 2a'$
$\Rightarrow$ ADDING GIVES
$\Rightarrow 5a' = 2g - \mu R$
$\Rightarrow 5a' = 2g - \frac{3}{4}(3g)$
$\Rightarrow 5a' = -\frac{1}{4}g$
$\Rightarrow a' = -\frac{1}{20}g$
(DECELERATION)

LOOKING AT THE KINEMATICS OF THE LAST SECTION

$u = \sqrt{2g}$, $a = -\frac{1}{20}g$, $s = 1.5$, $t =$, $v = ?$

$v^2 = u^2 + 2as$
$v^2 = 2g + 2\left(-\frac{1}{20}g\right)(1.5)$
$v^2 = 2g - \frac{3}{20}g$
$v^2 = 18.13$
$v \approx 4.26 \text{ ms}^{-1}$
(3 sf)

Question 22 (****)

A train consists of a locomotive of mass 40000 kg, pulling 20 identical carriages of mass 10000 kg each. When in motion the locomotive experiences a resistance of $4R \text{ N}$ while each carriage experiences a resistance of $R \text{ N}$.

When the driving force of the locomotive is 51000 N the trains accelerates uniformly reaching its maximum speed of 40 ms^{-1} from rest, over a distance of 16 km.

The locomotive and carriages are modelled as particles.

a) Show that $R = 1625$.

b) While the train is accelerating, calculate the tension in the couplings between

i. … the last two carriages.

ii. … the locomotive and the first carriage.

Later in the journey, the train maintains its maximum speed of 40 ms^{-1}.

The resistances to motion remain unchanged.

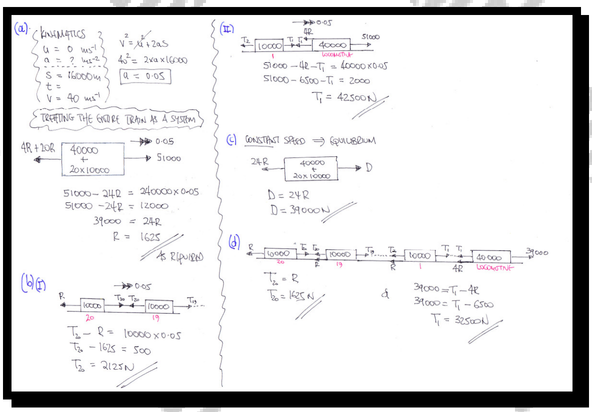
c) Find an amended figure for the driving force of the locomotive.

d) Determine an amended figure for the tension in the couplings between …

i. … the last two carriages.

ii. … the locomotive and the first carriage.

$\boxed{T_{20} = 2125 \text{ N}}$, $\boxed{T_1 = 42500 \text{ N}}$, $\boxed{D = 39000 \text{ N}}$, $\boxed{T_{20} = 1625 \text{ N}}$, $\boxed{T_1 = 32500 \text{ N}}$


(a) KINEMATICS
$u = 0 \text{ ms}^{-1}$, $a = ? \text{ ms}^{-2}$, $s = 16000 \text{ m}$, $t =$, $v = 40 \text{ ms}^{-1}$

$v^2 = u^2 + 2as$
$40^2 = 2 \times a \times 16000$
$\boxed{a = 0.05}$

TREATING THE ENTIRE TRAIN AS A SYSTEM

$51000 - 24R = 240000 \times 0.05$
$51000 - 24R = 12000$
$39000 = 24R$
$R = 1625$ AS REQUIRED

(b)(I)
$T_{20} - R = 10000 \times 0.05$
$T_{20} - 1625 = 500$
$T_{20} = 2125 \text{ N}$

(II)
$51000 - 4R - T_1 = 40000 \times 0.05$
$51000 - 6500 - T_1 = 2000$
$T_1 = 42500 \text{ N}$

(c) CONSTANT SPEED $\Rightarrow$ EQUILIBRIUM
$D = 24R$
$D = 39000 \text{ N}$

(d)
$T_{20} = R$
$T_{20} = 1625 \text{ N}$

&

$39000 = T_1 - 4R$
$39000 = T_1 - 6500$
$T_1 = 32500 \text{ N}$

Question 23 (****)

A particle A of mass 5 kg is connected to small box B of mass 7.5 kg by a light inextensible string. The string passes over a light smooth pulley P, which is located at the end of a rough horizontal house roof. The box is held on the roof with the particle hanging vertically at the end of the roof, as shown in the figure above.

The system is released from rest with the string taut.

The string, A, P and B lie in a vertical plane at right angles to the end of the roof.

a) Given that the coefficient of friction between B and the roof is 0.2, find in any order…

 i. … the acceleration of the system.

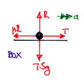
 ii. … the tension in the string.

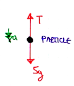
On release B is at a distance d m from P. When A has moved a distance of 2.8 m the string breaks. In the subsequent motion B comes to rest as it reaches P.

b) Calculate the value of d.

$\boxed{}$, $\boxed{a = 2.744 \text{ ms}^{-2}}$, $\boxed{T = 35.28 \text{ N}}$, $\boxed{d = 6.72 \text{ m}}$

a) LOOKING AT THE EQUATION OF MOTION FOR EACH PARTICLE SEPARATELY





"$F = ma$"
$T - \mu R = 7.5a$
$T - \mu(7.5g) = 7.5a$
$T - 0.2(7.5g) = 7.5a$
$T - 1.5g = 7.5a$

"$F = ma$"
$5g - T = 5a$

ADDING THE EQUATIONS YIELDS

$T - 1.5g = 7.5a$
$-T + 5g = 5a$
$3.5g = 12.5a$

$\Rightarrow a = 2.744 \text{ ms}^{-2}$

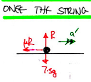
& $T - 1.5g = 7.5a$
$T - 1.5g = 7.5(2.744)$
$T = 35.28 \text{ N}$

b) FIRSTLY CALCULATE THE COMMON SPEED, WHEN THE STRING BREAKS

$u = 0 \text{ ms}^{-1}$
$a = 2.744 \text{ ms}^{-2}$
$s = 2.8 \text{ m}$
$t =$
$v = ?$

$v^2 = u^2 + 2as$
$v^2 = 2(2.744)(2.8)$
$v^2 = 15.3664$
$v = 3.92 \text{ ms}^{-1}$

RECALCULATE THE DECELERATION OF THE BOX ONCE THE STRING BREAKS (NO TENSION)



$\Rightarrow -\mu R = 7.5a'$
$\Rightarrow -\mu(7.5g) = 7.5a'$
$\Rightarrow a' = -\mu g$
$\Rightarrow a' = -0.2(9.8)$
$\Rightarrow a' = -1.96 \text{ ms}^{-2}$

FINALLY KINEMATICS AGAIN WITH CONSTANT DECELERATION 1.96 ms^{-2}

$u = 3.92 \text{ ms}^{-1}$
$a = -1.96 \text{ ms}^{-2}$
$s = ?$
t
$v = 0$

$v^2 = u^2 + 2as$
$0 = 3.92^2 + 2(-1.96)s$
$3.92s = 3.92^2$
$s = 3.92$

$\therefore d = 2.8 + 3.92 = 6.72 \text{ m}$

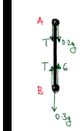
Question 24 (****)

A light rigid rod AB, where A is vertically above B, has a particle of mass 0.2 kg attached to it at A and a particle of mass 0.3 kg attached to it at B.

The loaded rod is accelerated vertically upwards by a vertical force of magnitude 6 N, applied to B.

Find the thrust in the rod.

$\boxed{}$, $\boxed{T = 2.4 \text{ N}}$


STARTING WITH A DIAGRAM – MARK THE TRUSTS AS TENSIONS

"F = ma"

(A) $0 - T - 0.2g = 0.2a$

(B) $6 + T - 0.3g = 0.3a$

ELIMINATE THE TENSION/THRUST

$6 - 0.5g = 0.5a$

$12 - g = a$

$a = 2.2 \text{ ms}^{-2}$

FINALLY WE HAVE

$-T - 0.2g = 0.2a$

$T + 0.2g = -0.2a$

$T = -0.2(g + a)$

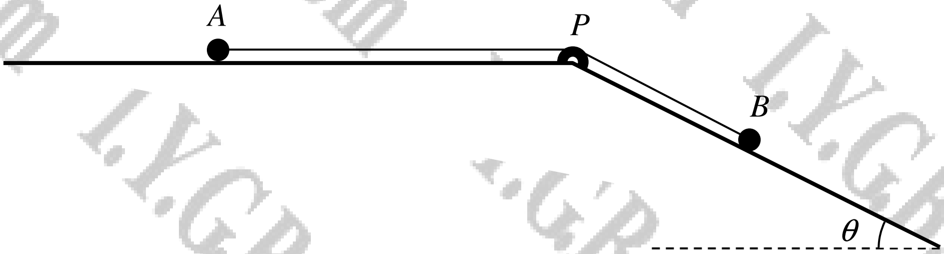
$T = -0.2\,(9.8 + 2.2)$

$T = -0.2 \times 12$

$T = -2.4$

$\therefore$ THRUST OF 2.4 N

Question 25 (****+)



Two particles A and B have masses 2 kg and 3 kg, respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a rough horizontal table. The coefficient of friction between the particle A and the table is $\frac{1}{7}$.

The string lies along the table and passes over a small smooth pulley P which is fixed to the edge of the table. Particle B is at rest on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = 0.75$.

The coefficient of friction between the particle B and the plane is also $\frac{1}{7}$.

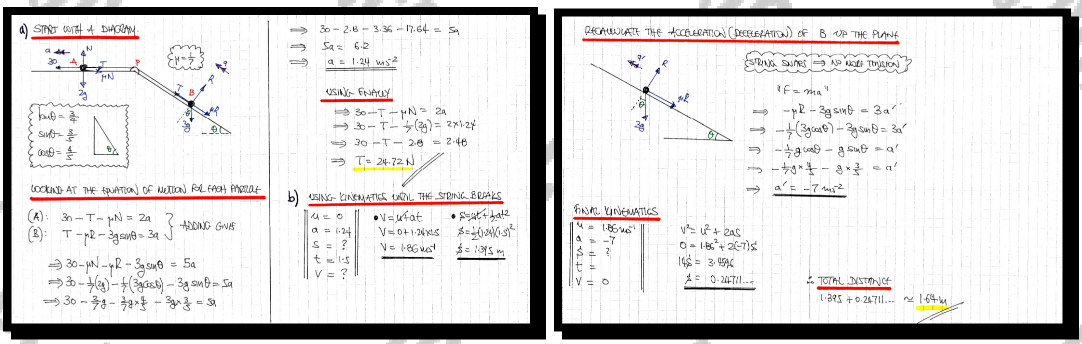
A constant force F, of magnitude 30 N, is applied to particle A, in the direction PA, while the string between the two particles is taut. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in the figure above.

a) Find the tension in the string while the system is in motion.

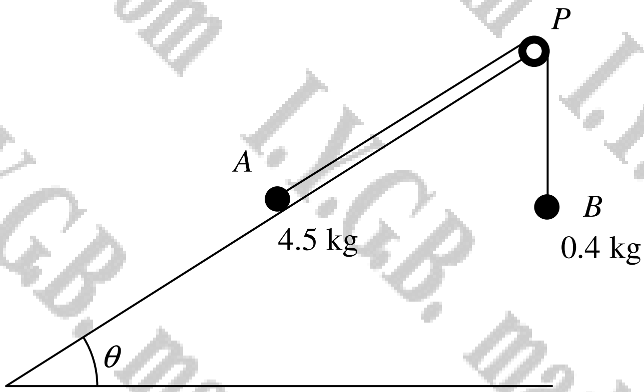
The string suddenly breaks after 1.5 s.

b) Given that B never reaches P, determine the **total** distance that B travels **up** the plane.

$\boxed{}$, $\boxed{T = 24.72 \text{ N}}$, $\boxed{d \approx 1.64 \text{ m}}$



Question 26 (****+)

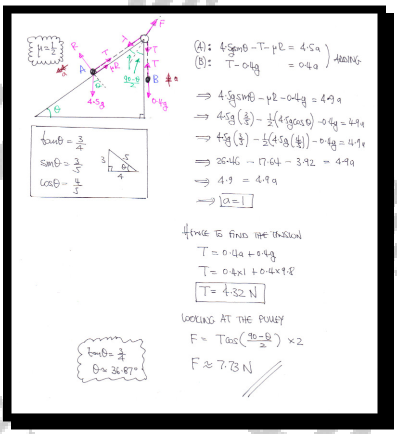


Two particles A and B, of mass 4.5 kg and 0.4 kg respectively, are attached to each of the ends of a light inextensible string. The string passes over a smooth pulley P, at the top of a fixed rough plane, inclined at θ to the horizontal, where $\tan\theta = 0.75$. Particle A is placed at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P, as shown in the figure above. The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane. The particles are released from rest with the string taut. Particle A begins to move down plane.

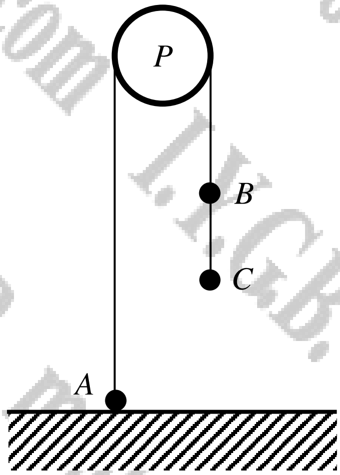
Given that the coefficient of friction between A and the plane is 0.5, determine the force exerted by the string on the pulley while the system is in motion.

Only the motion before A reaches the end of the plane and before B reaches P is to be considered.

$\boxed{F \approx 7.73 \text{ N}}$



Question 27 (****+)



Two particles A and B of respective masses m kg and 1 kg are attached to the ends of a light inextensible string which passes over a smooth pulley P. The particle B is attached to a third particle C of mass 9 kg by another light inextensible string. The three particles are held at rest, with A in contact with a horizontal floor, and the portions of the strings not in contact with the pulley vertical. The system is released from rest and C begins to accelerate towards the floor with the tension in the string BC being 50.4 N.

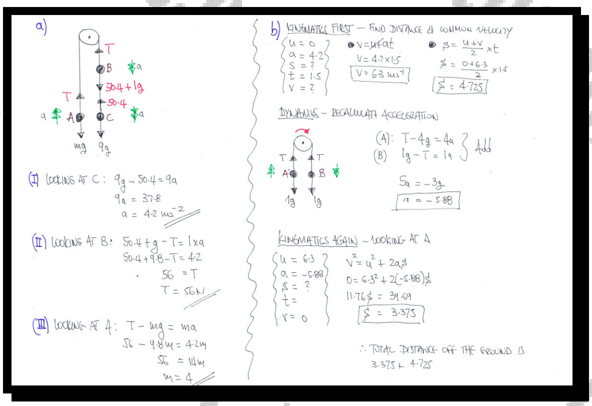
a) For the period before C reaches the floor, calculate …

- **i.** … the acceleration of the system.
- **ii.** … the tension in the string that connects A and B.
- **iii.** … the value of m.

C reaches the floor 1.5 s after release and **does not** rebound. In the ensuing motion A does not reach P and B does not reach the floor.

b) Determine the greatest height of A above the floor.

$\boxed{a = 4.2 \text{ ms}^{-2}}$, $\boxed{T = 56 \text{ N}}$, $\boxed{m = 4 \text{ kg}}$, $\boxed{h_{\max} = 8.1 \text{ m}}$



Question 28 (*****)

Two particles A and B have masses 4 kg and 1 kg, respectively.

A small, smooth light fixed pulley P, is located 1.6 m above a horizontal floor.

The two particles are connected by a light inextensible string, of length L m, which passes over P.

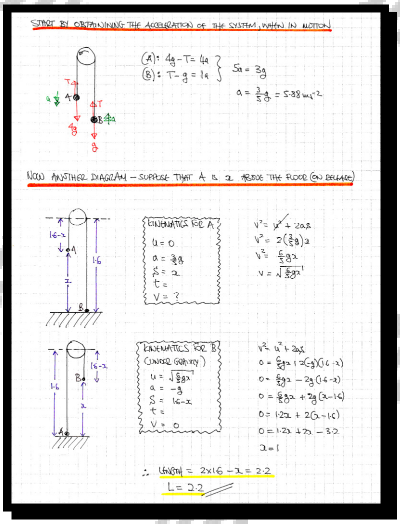
The particles are held at rest, with B level with the floor and A hanging above the floor, with the string taut and the hanging parts of the string vertical.

The system is released from rest and A hits the floor, from which it does not rebound.

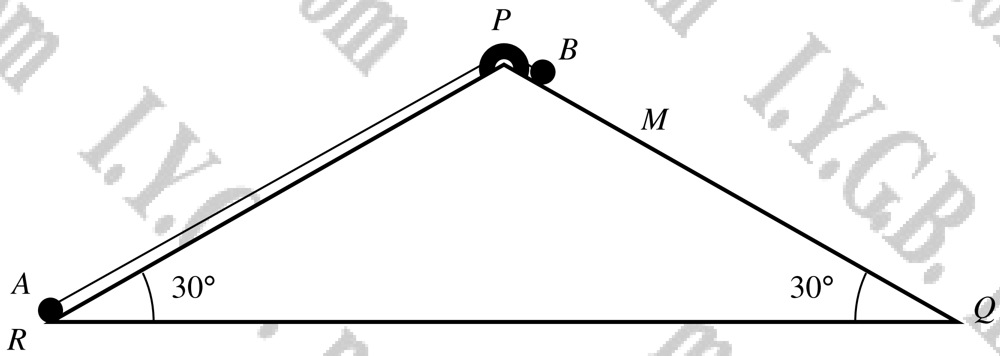
B continues moving upwards and comes to instantaneous rest as it reaches P.

Determine the value of L.

$\boxed{}$, $\boxed{L = 2.2}$



Question 29 (*****)



Two particles A and B have masses 2 kg and 5 kg, respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley P which is fixed at the top of the cross section of a triangular prism RPQ, where $\measuredangle PRQ = \measuredangle PQR = 30°$. The string lies in the vertical plane which contains the pulley and lines of greatest slope of the inclined planes, PR and PQ, as shown in the figure above. When A is held at R with the string taut, B is at P, on the line of greatest slope PQ.

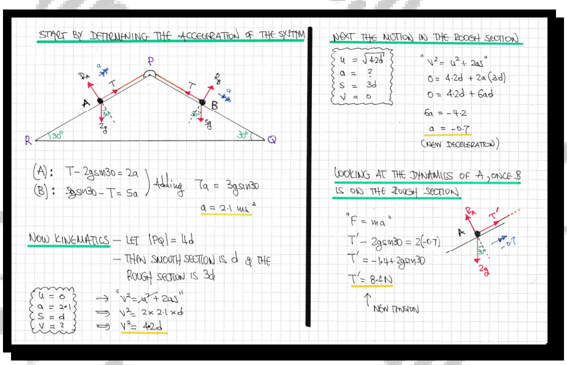
The point M, lies on PQ so that $PM : MQ = 1:3$.

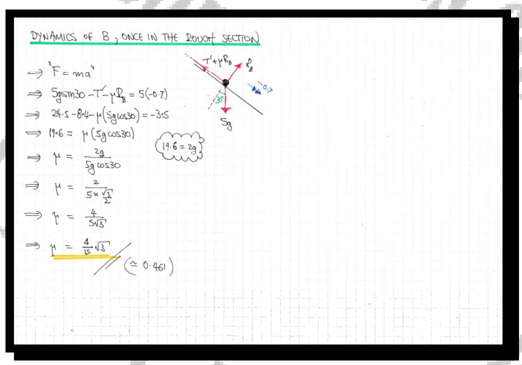
The lines of greatest slope of the inclined planes, PR and PM, are smooth but the line of greatest slope MQ is rough.

The system is released from rest with the string taut, when A is at R and B is at P, on the line of greatest slope PQ. The system initially accelerates but due to the rough section MQ, B comes to rest as it reaches Q.

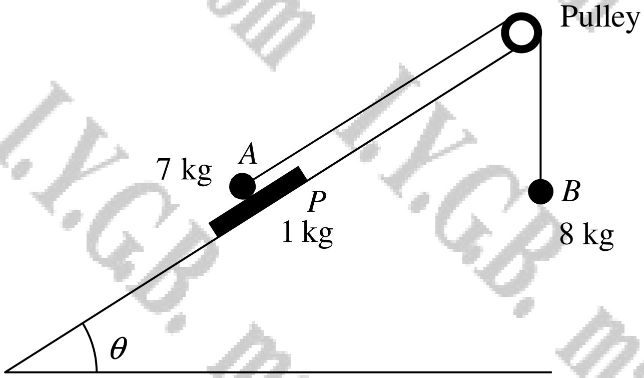
Assuming that the string remains taut throughout the motion, show that the coefficient of friction between the B and MQ is $k\sqrt{3}$, where k is a constant to be found.

$\boxed{}$, $\boxed{k = \frac{4}{15}}$





Question 30 (*****)



A rough plate P, of mass 1 kg, is placed on a fixed rough plane, inclined at an angle α to the horizontal, where $\tan\theta = 0.75$.

A particle A, of mass 7 kg, is placed on the top surface of P and is connected to another particle B, of mass 8 kg, by a light inextensible string, which passes over a smooth pulley that is located at the top the plane.

B is hanging freely at the end of the incline plane vertically below the pulley, as shown in the figure above. The two particles, the plate, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane.

When the system is released from rest with the string taut, B begins accelerate downwards at 2 ms^{-2}.

Given that P is in equilibrium, while A is accelerating on its top surface, determine the range of possible values of the coefficient of friction between P and the plane.

$\boxed{}$, $\boxed{d \approx 3.69 \text{ m}}$