COLLISIONS

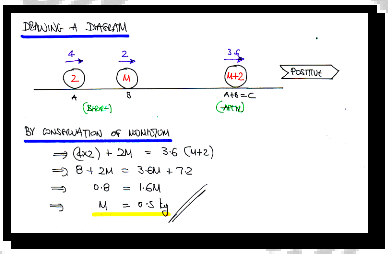
Question 1 (**)

Two particles, A and B, of respective masses 2 kg and $M\text{ kg}$ are moving on a smooth horizontal surface, in the same direction along the same straight line.

The speeds of A and B are 4 ms^{-1} and 2 ms^{-1}, respectively.

Given that when A and B collide they coalesce into a single particle C, travelling with speed 3.6 ms^{-1}, determine the value of M.

$\boxed{}$, $\boxed{M = 0.5}$



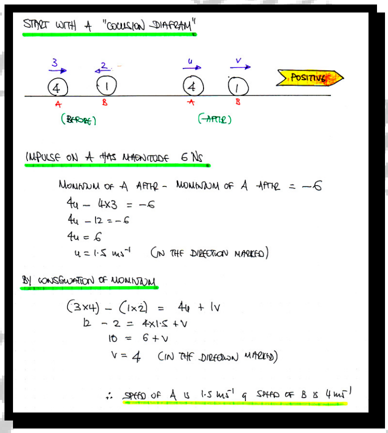
Question 2 (**)

Two particles A and B, of mass 4 kg and 1kg respectively, are moving towards each other along a straight line on a smooth horizontal plane. The particles collide directly and the magnitude of impulse exerted on A by B is 6 Ns.

Before the collision, the respective speeds of A and B are 3 ms^{-1} and 2 ms^{-1}.

Determine the speed of A and the speed of B, after the collision.

$\boxed{}$, $\boxed{v_A = 1.5\text{ ms}^{-1}}$, $\boxed{v_B = 4\text{ ms}^{-1}}$



Question 3 (**)

A rail car A of mass $3m$ is moving with constant speed $4U$ on smooth straight horizontal rails. It collides directly with another rail car B of mass $7m$ which is moving with constant speed $6U$ in the opposite direction on the same rails.

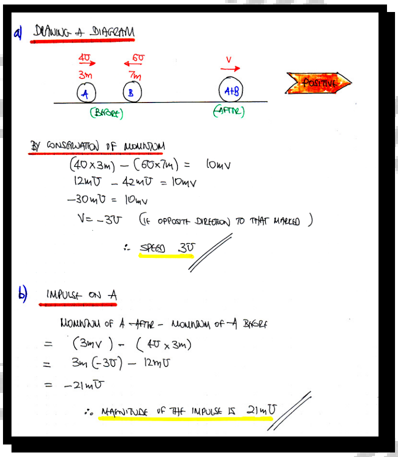
The rail cars' couples join so that immediately after the collision they move together.

The rail cars are modelled as particles.

a) Find, in terms of U, the speed of the rail cars immediately after the collision.

b) Determine, in terms of m and U, the magnitude of the impulse exerted on A by B in the collision.

$\boxed{\text{speed} = 3U}$, $\boxed{\text{impulse} = 21mU}$


a) DRAWING A DIAGRAM

BY CONSERVATION OF MOMENTUM

$(4U \times 3m) - (6U \times 7m) = 10mV$

$12mU - 42mU = 10mV$

$-30mU = 10mV$

$V = -3U$ (IE OPPOSITE DIRECTION TO THAT MARKED)

$\therefore$ SPEED $3U$

b) IMPULSE ON A

MOMENTUM OF A AFTER − MOMENTUM OF A BEFORE

$= (3mV) - (4U \times 3m)$

$= 3m(-3U) - 12mU$

$= -21mU$

$\therefore$ MAGNITUDE OF THE IMPULSE IS $21mU$

Question 4 (**)

A particle P, of mass 0.3 kg lies at the edge of a horizontal table.

It is connected by a light inextensible string of length 1.5 m to another particle Q, of mass 1.1 kg which lies the same table.

Q is at rest 0.6 m from the table edge, so that PQ is perpendicular to the table edge.

P is slightly disturbed so that it falls off the table.

The string becomes taut before P reaches the floor.

Determine the impulse received by Q when the string gets taut.

$\boxed{}$, $\boxed{I = 0.99 \text{ Ns}}$

ONCE THE PARTICLE "P" FALLS IT WILL "FREEFALL" FOR 0.9m ← 1.5 − 0.6

$u = 0 \text{ ms}^{-1}$
$a = 9.8 \text{ ms}^{-2}$
$s = 0.9 \text{ m}$
$t =$
$v = ?$

$v^2 = \cancel{u^2} + 2as$
$v^2 = 2(9.8)(0.9)$
$v^2 = 17.64$
$v = 4.2 \text{ ms}^{-1}$

BY CONSERVATION OF MOMENTUM ALONG THE TAUT STRING

$$\underbrace{\overset{P}{0.3 \times 4.2} + \overset{Q}{1.1 \times 0}}_{\text{MOMENTUM BEFORE JERK}} = \underbrace{\overset{P}{0.3V} + \overset{Q}{1.1V}}_{\text{MOMENTUM AFTER JERK}}$$

$1.26 = 1.4V$

$V = 0.9 \text{ ms}^{-1}$ ← COMMON SPEED

FINALLY IMPULSE ON Q

$I =$ MOMENTUM AFTER − MOMENTUM BEFORE
$I = mv - mu$
$I = 1.1 \times 0.9 - 1.1 \times 0$
$I = 0.99 \text{ Ns}$

Question 5 (+)**

Two particles A and B, of mass m kg and λm kg respectively, $\lambda > 0$, are moving on a smooth horizontal plane.

A and B have velocities $6\mathbf{i} - 2\mathbf{j}$ ms^{-1} and $-3\mathbf{i} + 3\mathbf{j}$ ms^{-1}, respectively.

A and B collide and coalesce to a single particle moving with velocity $k\mathbf{i} + k\mathbf{j}$ ms^{-1}.

Determine the value of λ and the value of k.

$\boxed{\text{FM1-}}$, $\boxed{\lambda = \frac{4}{3}}$, $\boxed{k = \frac{6}{7}}$

- BY CONSERVATION OF MOMENTUM

$$\Rightarrow m\begin{pmatrix}6\\-2\end{pmatrix} + \lambda m\begin{pmatrix}-3\\3\end{pmatrix} = (m+\lambda m)\begin{pmatrix}k\\k\end{pmatrix}$$

- DIVIDING BY m

$$\Rightarrow \begin{pmatrix}6\\-2\end{pmatrix} + \lambda\begin{pmatrix}-3\\3\end{pmatrix} = (1+\lambda)\begin{pmatrix}k\\k\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}6-3\lambda\\-2+3\lambda\end{pmatrix} = \begin{pmatrix}k(1+\lambda)\\k(1+\lambda)\end{pmatrix}$$

- THUS WE HAVE

$$\left.\begin{array}{l}6-3\lambda = k(\lambda+1)\\3\lambda-2 = k(\lambda+1)\end{array}\right\} \Rightarrow \begin{array}{l}6-3\lambda = 3\lambda-2\\8 = 6\lambda\\\lambda = \frac{4}{3}\end{array}$$

- FINALLY WE HAVE

$$\Rightarrow 6-3\lambda = k(\lambda+1)$$
$$\Rightarrow 6-3\left(\tfrac{4}{3}\right) = k\left(\tfrac{4}{3}+1\right)$$
$$\Rightarrow 6-4 = \tfrac{7}{3}k$$
$$\Rightarrow 2 = \tfrac{7}{3}k$$
$$\Rightarrow k = \tfrac{6}{7}$$

Question 6 (**+)

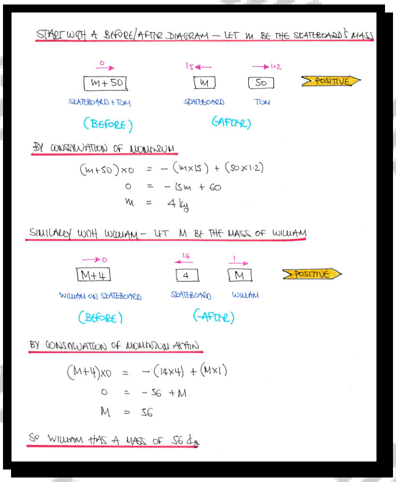
Tom, of mass 50 kg, is initially standing still on a stationary skateboard, on level horizontal ground.

He jumps off the skateboard and initially moves with a horizontal speed 1.2 ms^{-1}. The skateboard moves with a speed of 15 ms^{-1} in a direction opposite to that of Tom.

William then stands still on the same skateboard. He jumps off the skateboard and initially moves with a horizontal speed 1 ms^{-1} while the skateboard moves with a speed of 14 ms^{-1} in a direction opposite to that of William.

Find the mass of William.

$\boxed{\text{M1-Q}}$, $\boxed{m = 56 \text{ kg}}$


START WITH A BEFORE/AFTER DIAGRAM – LET m BE THE SKATEBOARD'S MASS

$\xrightarrow{0}$ $\boxed{m+50}$ SKATEBOARD + TOM (BEFORE)

$\xleftarrow{15}$ $\boxed{m}$ SKATEBOARD; $\xrightarrow{1.2}$ $\boxed{50}$ TOM (AFTER)

POSITIVE

BY CONSERVATION OF MOMENTUM

$$(m+50) \times 0 = -(m \times 15) + (50 \times 1.2)$$
$$0 = -15m + 60$$
$$m = 4 \text{ kg}$$

SIMILARLY WITH WILLIAM – LET M BE THE MASS OF WILLIAM

$\xrightarrow{0}$ $\boxed{M+4}$ WILLIAM ON SKATEBOARD (BEFORE)

$\xleftarrow{14}$ $\boxed{4}$ SKATEBOARD; $\xrightarrow{1}$ $\boxed{M}$ WILLIAM (AFTER)

POSITIVE

BY CONSERVATION OF MOMENTUM AGAIN

$$(M+4) \times 0 = -(14 \times 4) + (M \times 1)$$
$$0 = -56 + M$$
$$M = 56$$

SO WILLIAM HAS A MASS OF 56 kg

Question 7 (***)

Two particles A and B, of mass 3 kg and m kg respectively, are moving towards each other along a straight line on a smooth horizontal plane.

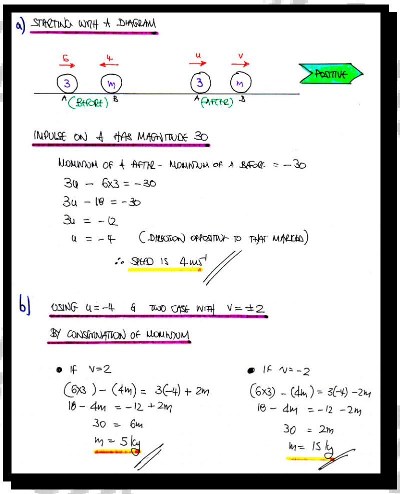
A and B collide directly.

Before the collision, the respective speeds of A and B are 6 ms^{-1} and 4 ms^{-1}.

a) If the magnitude of impulse exerted on A by B is 30 Ns, determine the speed of A after the collision.

b) Given instead that that the **speed** of B after the collision is 2 ms^{-1}, find the possible values of m.

$\boxed{|v_A| = 4 \text{ ms}^{-1}}$, $\boxed{m = 5 \text{ kg} \quad \text{or} \quad 15 \text{ kg}}$


a) STARTING WITH A DIAGRAM

IMPULSE ON A HAS MAGNITUDE 30

MOMENTUM OF A AFTER − MOMENTUM OF A BEFORE = −30

$3u - 6\times3 = -30$

$3u - 18 = -30$

$3u = -12$

$u = -4$ (DIRECTION OPPOSITE TO THAT MARKED)

∴ SPEED IS 4ms^{-1}

b) USING $u = -4$ & TWO CASE WITH $v = \pm 2$

BY CONSERVATION OF MOMENTUM

• IF $v = 2$

$(6\times3) - (4m) = 3(-4) + 2m$

$18 - 4m = -12 + 2m$

$30 = 6m$

$m = 5\text{kg}$

• IF $v = -2$

$(6\times3) - (4m) = 3(-4) - 2m$

$18 - 4m = -12 - 2m$

$30 = 2m$

$m = 15\text{kg}$

Question 8 (***)

Three smooth particles, A, B and C, of respective masses 0.5 kg, 1 kg and 2 kg, are moving in the same straight line and in the same direction. The motion takes place on a smooth horizontal surface.

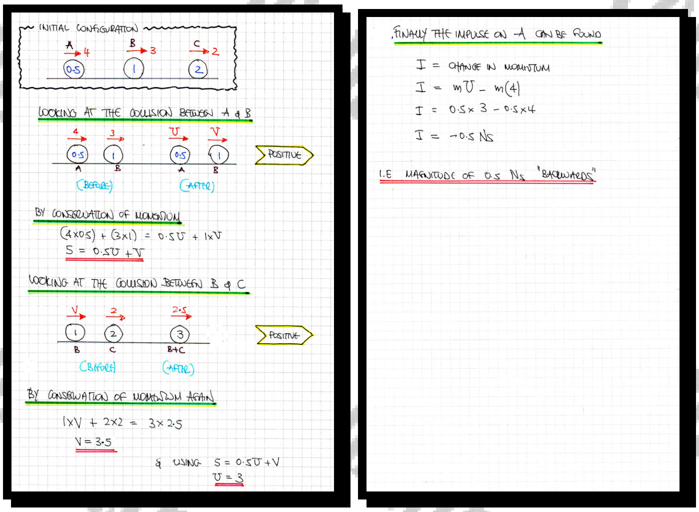
The speeds of A, B and C are 4 ms^{-1}, 3 ms^{-1} and 2 ms^{-1}, respectively.

Initially there is a direct collision between A and B, followed by another direct collision between B and C.

As a result of the second collision, B and C coalesce into a single particle moving with speed of 2.5 ms^{-1}.

Determine the magnitude of the impulse received by A during the first collision.

$\boxed{}$, $\boxed{|I| = 0.5 \text{ Ns}^{-1}}$


INITIAL CONFIGURATION

LOOKING AT THE COLLISION BETWEEN A & B

BY CONSERVATION OF MOMENTUM

$(4 \times 0.5) + (3 \times 1) = 0.5U + 1 \times V$

$5 = 0.5U + V$

LOOKING AT THE COLLISION BETWEEN B & C

BY CONSERVATION OF MOMENTUM AGAIN

$1 \times V + 2 \times 2 = 3 \times 2.5$

$V = 3.5$

& USING $5 = 0.5U + V$

$U = 3$

FINALLY THE IMPULSE ON A CAN BE FOUND

I = CHANGE IN MOMENTUM

$I = mU - m(4)$

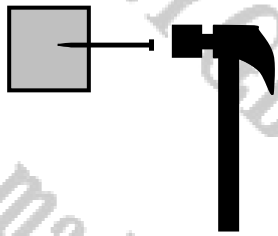
$I = 0.5 \times 3 - 0.5 \times 4$

$I = -0.5$ Ns

I.E MAGNITUDE OF 0.5 Ns "BACKWARDS"

Question 9 (***)

A large nail of mass 0.05 kg is partly driven horizontally into a bock of wood and it is to be driven further into the block.



The nail is stuck by a hammer of mass 0.4 kg. The hammer moves horizontally and impacts the nail, delivering 1.8 Ns of linear momentum. After the impact the nail and hammer move together as one object.

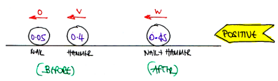
a) Calculate the speed of the nail after the impact.

The wood provides a constant resistance to the motion of the nail of 120 N. After the impact the nail moves for T s advancing a further distance D m into the block.

b) Determine the value of T and the value of D.

$\boxed{}$, $\boxed{v = 4 \text{ ms}^{-1}}$, $\boxed{T = 0.015 \text{ s}}$, $\boxed{D = 0.03 \text{ m}}$

a) BY A DIAGRAM



MOMENTUM OF NAIL BEFORE + MOMENTUM OF HAMMER BEFORE = 0.45W

$0 \times 0.05 + 1.8 = 0.45W$

$W = 4 \text{ ms}^{-1}$

b) IMPULSE = EXTERNAL FORCE × TIME ACTING

$4 \times 0.45 = 120 \times T$

$1.8 = 120T$

$T = 0.015 \text{ s}$

c) USING KINEMATICS

$u = W = 4$

$a =$

$s = ?$

$t = 0.015$

$v = 0$

$s = \frac{1}{2}(u+v)t$

$s = \frac{1}{2}(4+0) \times 0.015$

$s = 0.03 \text{ m}$

Question 10 (***)

Two particles P and Q of respective masses 2 kg and 3 kg move on a smooth horizontal surface in the same direction along a straight line.

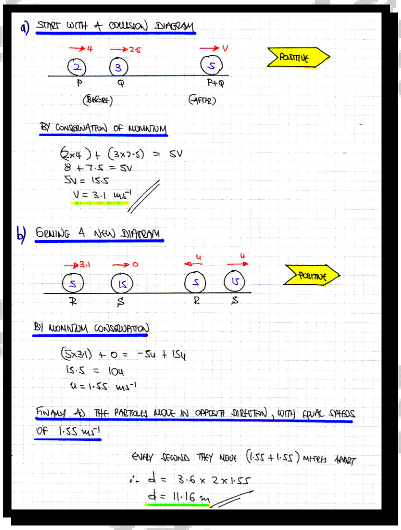
The speeds of P and Q are 4 ms^{-1} and 2.5 ms^{-1}, respectively.

a) Given that when P and Q collide they coalesce into a single particle R, determine the speed of R after the collision.

After the collision R continues in a straight line and collides directly with a third particle S of mass 15 kg which was initially at rest. After their collision R and S move in opposite directions with equal speeds.

b) Find the distance between R and S, 3.6 s after their collision.

$\boxed{v = 3.1 \text{ ms}^{-1}}$, $\boxed{d = 11.16 \text{ m}}$


a) START WITH A COLLISION DIAGRAM

BY CONSERVATION OF MOMENTUM

$(2\times4)+(3\times2.5) = 5V$
$8+7.5 = 5V$
$5V = 15.5$
$V = 3.1 \text{ ms}^{-1}$

b) GENING A NEW DIAGRAM

BY MOMENTUM CONSERVATION

$(5\times3.1)+0 = -5u+15u$
$15.5 = 10u$
$u = 1.55 \text{ ms}^{-1}$

FINALLY AS THE PARTICLES MOVE IN OPPOSITE DIRECTION, WITH EQUAL SPEEDS OF 1.55 ms^{-1}

EVERY SECOND THEY MOVE $(1.55+1.55)$ METRES APART

$\therefore d = 3.6\times2\times1.55$
$d = 11.16 \text{ m}$

Question 11 (***+)

Two smooth spheres of equal radius, A and B, of mass 3 kg and m kg respectively, are moving in the same direction, along a straight line on a smooth horizontal plane.

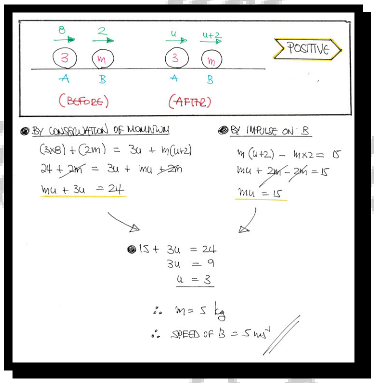
The spheres collide and the magnitude of impulse exerted on B by A is 15 Ns.

Before the collision, the respective speeds of A and B are 8 ms^{-1} and 2 ms^{-1}.

After the collision B is moving with speed 2 ms^{-1} **relative** to A.

Determine the value of m and the speed of B, after the collision.

$\boxed{}$, $\boxed{m = 5 \text{ kg}}$, $\boxed{v_B = 5 \text{ ms}^{-1}}$



Question 12 (***+)

Two smooth particles, A and B of respective masses $2m$ kg and $5m$ kg, are moving in the same straight line and in opposite directions.

The motion takes place on a smooth horizontal surface.

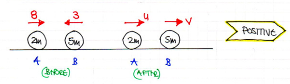
The speeds of A and B are 8 ms^{-1} and 3 ms^{-1}, respectively.

There is a direct collision between A and B.

If, after the collision, the speed of one particle is twice as large as the speed of the other particle determine the possible values of he speed of B, after the collision.

$\boxed{}$, $\boxed{\text{speed} \approx 0.167 \text{ ms}^{-1} \cup \ = 0.25 \text{ ms}^{-1} \cup \ = 1 \text{ ms}^{-1}}$

USING A STANDARD COLLISION DIAGRAM

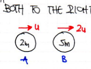


FIRST LET US NOTE THAT B HAS TO MOVE TO THE "RIGHT"

(MOMENTUM BEFORE $= 16m - 15m = +m$)

HENCE WE HAVE THE FOLLOWING CASES

- "BOTH TO THE RIGHT"



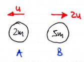
$\Rightarrow 2mu + 10mu = m$

$\Rightarrow 12mu = m$

$\Rightarrow u = \frac{1}{12}$

$\therefore$ SPEED OF $B = 2 \times \frac{1}{12} = \frac{1}{6} = 0.167 \text{ ms}^{-1}$

- "BOTH REBOUND, WITH B THE FASTEST"



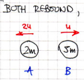
$\Rightarrow -2mu + 10mu = m$

$\Rightarrow 8mu = m$

$\Rightarrow u = \frac{1}{8}$

$\therefore$ SPEED OF $B = 2 \times \frac{1}{8} = \frac{1}{4} = 0.25 \text{ ms}^{-1}$

- "BOTH REBOUND, WITH A THE FASTEST"



$\Rightarrow -4mu + 5mu = m$

$\Rightarrow mu = m$

$\Rightarrow u = 1$

$\therefore$ SPEED OF $B = 1 \text{ ms}^{-1}$

$\therefore$ THE POSSIBLE SPEEDS OF B ARE 0.167, 0.25, 1

Question 13 (***+)

Two particles, A and B, of respective masses 2 kg and 13 kg are moving on a smooth horizontal surface in the same direction along the same straight line.

The speeds of A and B are 6 ms^{-1} and 2 ms^{-1}, respectively.

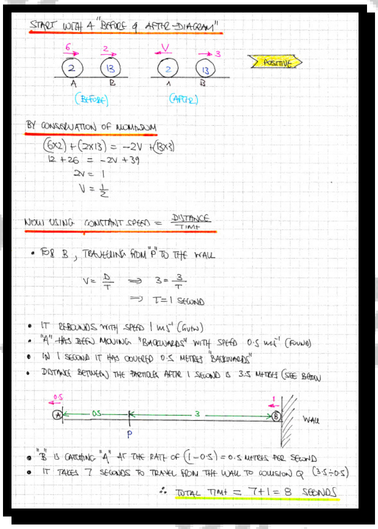
The two particles collide at the point P and **after** this collision B is moving with a speed of 3 ms^{-1}.

After the collision at P, B hits a fixed smooth vertical wall, which is perpendicular to the direction of its motion.

The wall is at a distance of 3 m from P.

If B rebounds off the wall with speed 1 ms^{-1} and collides again with A at the point Q, find the time that elapses between the collision at P and the collision at Q.

$\boxed{}$, $\boxed{t = 8 \text{ s}}$


START WITH A "BEFORE & AFTER DIAGRAM"

6 → 2 → ; ← V → 3 ; POSITIVE ; (2) (13) A B (BEFORE) ; (2) (13) A B (AFTER)

BY CONSERVATION OF MOMENTUM

$(6\times2)+(2\times13) = -2V + (13\times3)$

$12 + 26 = -2V + 39$

$2V = 1$

$V = \frac{1}{2}$

NOW USING CONSTANT SPEED = $\frac{\text{DISTANCE}}{\text{TIME}}$

- FOR B, TRAVELLING FROM "P" TO THE WALL

$V = \frac{D}{T} \Rightarrow 3 = \frac{3}{T}$

$\Rightarrow T = 1$ SECOND

- IT REBOUNDS WITH SPEED 1 ms^{-1} (GIVEN)
- "A" HAS BEEN MOVING "BACKWARDS" WITH SPEED 0.5 ms^{-1} (FOUND)
- IN 1 SECOND IT HAS COVERED 0.5 METRES "BACKWARDS"
- DISTANCE BETWEEN THE PARTICLES AFTER 1 SECOND IS 3.5 METRES (SEE BELOW

- "B" IS CATCHING "A" AT THE RATE OF $(1-0.5) = 0.5$ METRES PER SECOND
- IT TAKES 7 SECONDS TO TRAVEL FROM THE WALL TO COLLISION Q $(3.5 \div 0.5)$

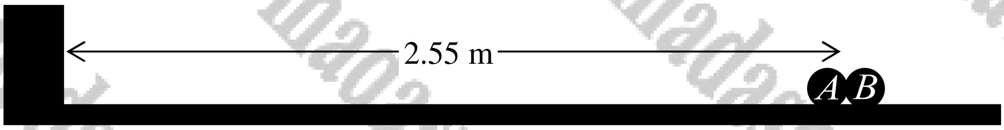
$\therefore$ TOTAL TIME $= 7 + 1 = 8$ SECONDS

Question 14 (****)

A particle is at rest on a horizontal surface when it explodes into two particle parts, A and B, of respective masses 0.4 kg and 0.6 kg.

a) Given that the speed of A immediately after the explosion is 12 ms^{-1}, determine the speed of B.

In the subsequent motion, A experiences no resistance or ground friction but B experiences **constant** ground friction.



The explosion takes place 2.55 m away from a smooth vertical wall which is perpendicular to the direction of motion of A.

A has a perfectly elastic collision with the wall, it rebounds and collides directly with B, 0.75 s after the explosion.

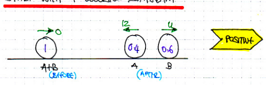
All collisions are instantaneous.

b) Show that the speed of B just before the two particles collide is 2.4 ms^{-1}.

c) Calculate the coefficient of friction between the ground and B.

$\boxed{}$, $\boxed{V_B = 8 \text{ ms}^{-1}}$, $\boxed{\mu = \frac{16}{21} \approx 0.762}$

a) START WITH A COLLISION DIAGRAM



BY CONSERVATION OF MOMENTUM

$(1\times 0) = -12\times 0.4 + u\times 0.6$

$0 = -4.8 + 0.6u$

$0.6u = 4.8$

$u = 8 \text{ ms}^{-1}$

b) PARTICLE A MOVES WITH CONSTANT SPEED OF 12 ms^{-1} FOR 0.75 s

- $d_A = 12 \times 0.75 = 9 \text{ m}$
- $9 - 2\times 2.55 = 3.9 \text{ m}$ ← "TO THE RIGHT" OF THE COLLISION)

NOW KINEMATICS FOR B – AS FRICTION IS CONSTANT DECELERATION MUST ALSO BE CONSTANT

$u = 8 \text{ ms}^{-1}$
$a = ?$
$s = 3.9 \text{ m}$
$t = 0.75 \text{ s}$
$v = ?$

$\Rightarrow$ "$s = \frac{u+v}{2}\times t$"

$\Rightarrow 3.9 = \frac{8+v}{2}\times 0.75$

$\Rightarrow 3.9 = \frac{3}{8}(v+8)$

$\Rightarrow v+8 = 10.4$

$\Rightarrow v = 2.4 \text{ ms}^{-1}$ AS REQUIRED

c) OBTAIN THE DECELERATION FOR B

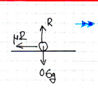
$\Rightarrow$ "$v = u + at$"

$\Rightarrow 2.4 = 8 + a\times 0.75$

$\Rightarrow 0.75a = -5.6$

$\Rightarrow a = -\frac{112}{15}$

FINALLY A DYNAMICS DIAGRAM FOR B



$\Rightarrow$ "$F = ma$"

$\Rightarrow -\mu R = 0.6\times\left(-\frac{112}{15}\right)$

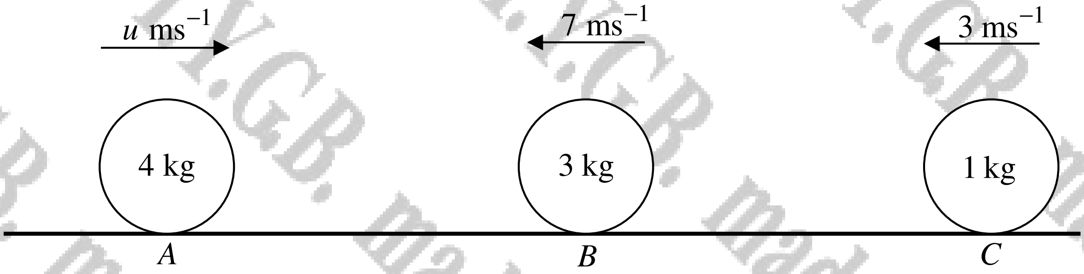
$\Rightarrow -\mu(0.6g) = -\frac{112}{15}\times 0.6$

$\Rightarrow \mu g = \frac{112}{15}$

$\Rightarrow \mu = \frac{16}{21} \approx 0.762$

Question 15 (****)

Three particles A, B and C, of respective masses 4 kg, 3 kg and 1 kg, are moving along the same straight line, on a smooth horizontal plane.



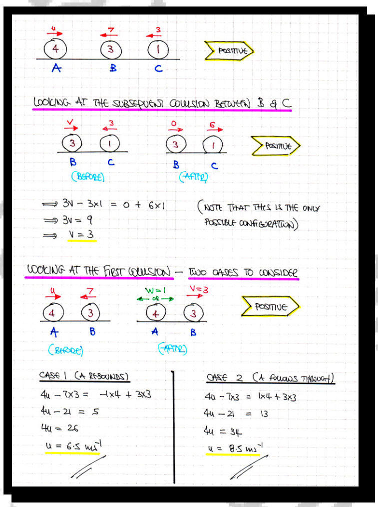
The figure above, shows the particles at a certain instant when A and B are moving towards each other with respective speeds $u\text{ ms}^{-1}$ and 7 ms^{-1}, and C is moving in the same direction as B with speed 3 ms^{-1}.

An initial collision take place between A and B, followed by a second collision between B and C. It is not known whether more collisions take place.

Immediately after the second collision, B is at rest and the speeds of A and C are 1 ms^{-1} and 6 ms^{-1}, respectively.

Determine the possible values of u.

$\boxed{}$, $\boxed{u = 6.5\text{ ms}^{-1} \cup u = 8.5\text{ ms}^{-1}}$



Question 16 (****)

Two particles, A and B, of respective mass 3 kg and 2 kg are moving in the same direction in the same straight line on a smooth surface.

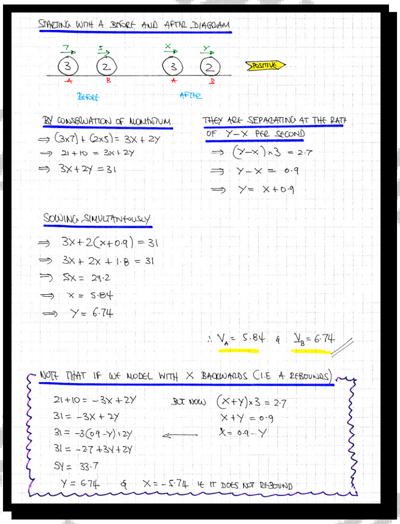
The particles collide.

Before the collision the speed of A is 7 ms^{-1} and the speed of B is 5 ms^{-1}.

The distance between the two particles 3 s after the collision is 2.7 m.

Determine the speed of A and the speed of B after the collision.

$\boxed{}$, $\boxed{V_A = 5.84 \text{ ms}^{-1}}$, $\boxed{V_B = 6.74 \text{ ms}^{-1}}$



BY CONSERVATION OF MOMENTUM

$\Rightarrow (3\times7)+(2\times5) = 3x+2y$

$\Rightarrow 21+10 = 3x+2y$

$\Rightarrow 3x+2y = 31$

THEY ARE SEPARATING AT THE RATE OF $y-x$ PER SECOND

$\Rightarrow (y-x)\times3 = 2.7$

$\Rightarrow y-x = 0.9$

$\Rightarrow y = x+0.9$

SOLVING SIMULTANEOUSLY

$\Rightarrow 3x+2(x+0.9) = 31$

$\Rightarrow 3x+2x+1.8 = 31$

$\Rightarrow 5x = 29.2$

$\Rightarrow x = 5.84$

$\Rightarrow y = 6.74$

$\therefore V_A = 5.84$ & $V_B = 6.74$

NOTE THAT IF WE MODEL WITH x BACKWARDS (I.E A REBOUNDS)

$21+10 = -3x+2y$

$31 = -3x+2y$

$31 = -3(0.9-y)+2y$

$31 = -2.7+3y+2y$

$5y = 33.7$

$y = 6.74$ & $x = -5.74$ IE IT DOES NOT REBOUND

BUT NOW $(x+y)\times3 = 2.7$

$x+y = 0.9$

$x = 0.9-y$

Question 17 (****)

Two small smooth spheres of equal radii, A and B, are moving on the same straight line and in the **same** direction.

A has mass 5 kg and speed 4 ms^{-1} and B has mass 2 kg and speed 3.5 ms^{-1}.

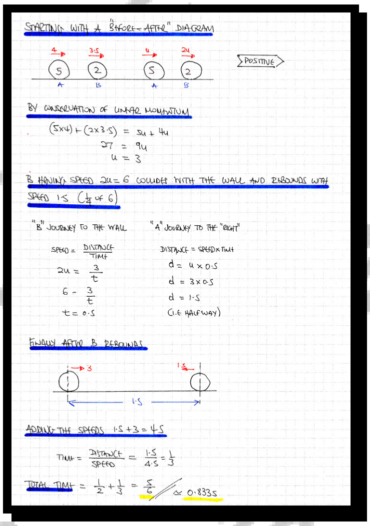
The spheres collide directly and after the impact the direction of their motion remains unchanged, with the speed of B twice as large as the speed of A.

After the collision between A and B, B collides with a smooth vertical wall which is perpendicular to the direction AB. The wall is 3 m away from the point where the two spheres first collided.

After the impact with the wall the speed of B is $\frac{1}{4}$ of its speed before the impact.

Calculate the time that elapses between the first collision and the second collision of the two spheres.

$\boxed{}$, $\boxed{\frac{5}{6} \approx 0.833 \text{ s}}$



STARTING WITH A "BEFORE-AFTER" DIAGRAM

POSITIVE

BY CONSERVATION OF LINEAR MOMENTUM

$(5\times4)+(2\times3.5) = 5u+4u$

$27 = 9u$

$u = 3$

B HAVING SPEED $2u = 6$ COLLIDES WITH THE WALL AND REBOUNDS WITH SPEED 1.5 ($\frac{1}{4}$ of 6)

"B" JOURNEY TO THE WALL

$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$

$2u = \frac{3}{t}$

$6 = \frac{3}{t}$

$t = 0.5$

"A" JOURNEY TO THE "RIGHT"

$\text{DISTANCE} = \text{SPEED} \times \text{TIME}$

$d = u \times 0.5$

$d = 3 \times 0.5$

$d = 1.5$

(I.E HALFWAY)

FINALLY AFTER B REBOUNDS

ADDING THE SPEEDS $1.5+3 = 4.5$

$\text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}} = \frac{1.5}{4.5} = \frac{1}{3}$

$\text{TOTAL TIME} = \frac{1}{2}+\frac{1}{3} = \frac{5}{6} \approx 0.833\text{s}$

Question 18 (****+)

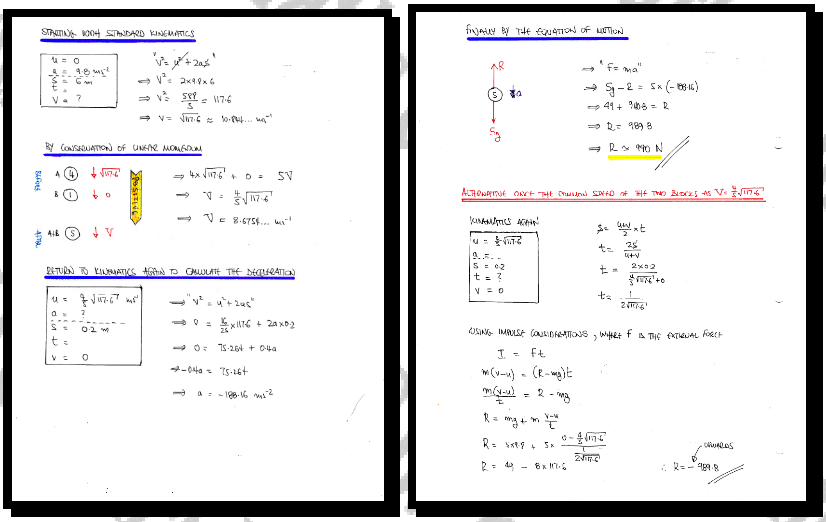
A block A of mass 4 kg is released from rest from a point P which is at a height of 6 m above soft horizontal ground.

The falling block strikes another block B of mass 1 kg which is on the ground vertically below P.

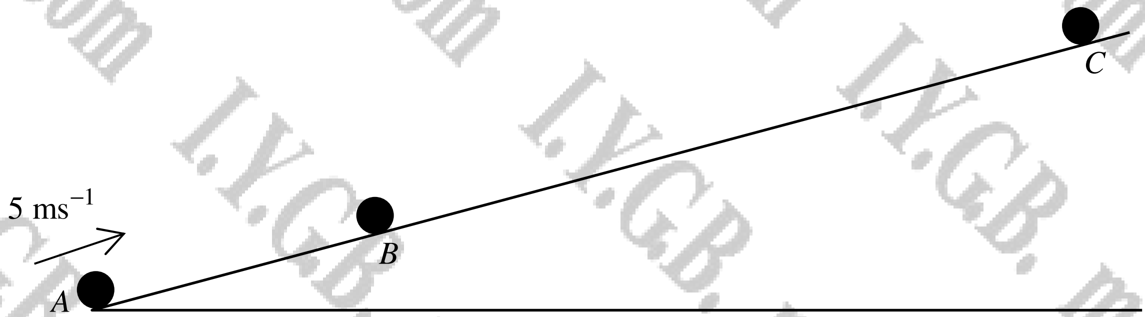
Immediately after the impact the two blocks coalesce into a single block and move downwards coming to rest after sinking a vertical distance of 20 cm into the ground.

By modelling the blocks as particles, find the magnitude of the **constant** resistance offered by the ground.

$\boxed{R = 989.8 \text{ N}}$



Question 19 (****+)



Three small spheres A, B and C, all of equal radius, have respective masses 0.9 kg, 0.6 kg an 0.1 kg. The three spheres are placed on a smooth incline plane as shown in the diagram. Sphere A is projected from the foot of the plane up the plane with speed 5 ms^{-1} while spheres B and C are released from rest at the same time as A was projected. The plane's inclination is such so that any of the spheres moving freely on it, experiences an acceleration of 2.5 ms^{-2} down the plane. Any collisions that take place are instantaneous.

a) Given that A and B collide 0.4 s after A was projected and they coalesce when into a single particle P, determine the **velocity** of P after the collision.

b) Given that P and C collide 0.6 s after A and B collided and they coalesce into a single particle Q, determine the **velocity** of Q after the collision.

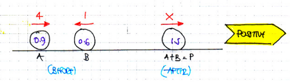
c) Find the distance from the foot of the plane that Q first comes to rest.

$\boxed{}$, $\boxed{v_P = 2 \text{ ms}^{-1}\text{, up the plane}}$, $\boxed{v_Q = 0.3125 \text{ ms}^{-1}\text{, up the plane}}$, $\boxed{d \approx 2.57 \text{ m}}$

a) DRAW A COLLISION DIAGRAM FOR A & B – IGNORE THE INCLINE AS THE ACCELERATION IS GIVEN – FIND THE PRE COLLISION SPEEDS

FOR PARTICLE A	FOR PARTICLE B
$u = 5$	$u = 0$
$a = -2.5$	$a = 2.5$
$s =$ —	$s =$ —
$t = 0.4$	$t = 0.4$
$v = ?$	$v = ?$

- $V_A = 5 - 2.5 \times 0.4$
  $V_A = 4 \text{ ms}^{-1}$
- $V_B = 0 + 2.5 \times 0.4$
  $V_B = 1$



BY MOMENTUM CONSERVATION

$\Rightarrow (4 \times 0.9) - (1 \times 0.6) = 1.5X$
$\Rightarrow 3.6 - 0.6 = 1.5X$
$\Rightarrow 1.5X = 3$
$\Rightarrow X = 2$

$\therefore$ VELOCITY 2 ms^{-1} UP THE PLANE //

b) ANALOGOUS METHOD FOR THE NEXT COLLISION

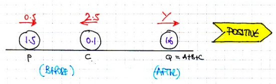
FOR A+B = P	FOR C
$u = 2$	$u = 0$
$a = -2.5$	$a = 2.5$
$s =$	$s =$
$t = 0.6$	$t = 1$
$v = ?$	$v = ?$

0.4 + 0.6 !!

$V_P = 2 - 2.5 \times 0.6$
$V_P = 0.5 \text{ ms}^{-1}$

$V_Q = 0 + 2.5 \times 1$
$V_Q = 2.5 \text{ ms}^{-1}$

COLLISION DIAGRAM



BY CONSERVATION OF MOMENTUM

$\Rightarrow (0.5 \times 1.5) - (2.5 \times 0.1) = 1.6Y$
$\Rightarrow 0.75 - 0.25 = 1.6Y$
$\Rightarrow 1.6Y = 0.5$
$\Rightarrow Y = \frac{5}{16} = 0.3125 \text{ ms}^{-1}$

IE 0.3125 ms^{-1} UP THE PLANE

c) FINALLY THE DISTANCES – CONSIDER "A" ON ITS OWN OR AS PART

FIRST 0.4 OF A SECOND	NEXT 0.6 OF A SECOND
$u = 5$	$u = 2$
$a = -2.5$	$a = -2.5$
$s = ?$	$s =$
$t = 0.4$	$t = 0.6$
$v = 4$	$v = 0.5$

$s = \frac{1}{2}(5+4) \times 0.4$
$s = 1.8$ m

$s = \frac{1}{2}(2+0.5) \times 0.6$
$s = 0.75$

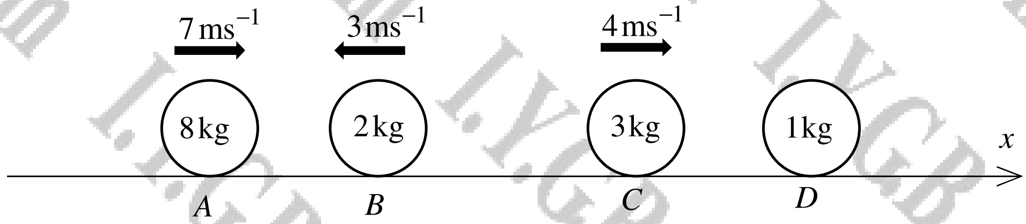
FINALLY UNTIL IT COMES TO REST

$u = 0.3125$
$a = -2.5$
$s = ?$
$t =$
$v = 0$

$v^2 = u^2 + 2as$
$0 = (0.3125)^2 + 2(-2.5)s$
$5s = \frac{25}{256}$
$s = \frac{5}{256}$

$\therefore$ TOTAL DISTANCE $= 1.8 + 0.75 + \frac{5}{256}$
$= \frac{3289}{1280}$
$\simeq 2.57$ m //

Question 20 (*****)



Four particles A, B, C and D, of respective masses 8 kg, 2 kg, 3 kg and 1 kg are constrained to move on a smooth path along the x axis.

All four particles are initially at rest, separated from each other and in the order A, B, C and D, as shown in the figure above.

At a given instant, impulses are given to A, B and C so these three particles begin to move with respective velocities 7 ms^{-1}, -3 ms^{-1} and 4 ms^{-1}.

As result of these impulses, there are **exactly three** collisions between the particles.

The first collision is between A and B.

After this collision A has velocity 4 ms^{-1}.

The second collision is between C and D.

After this collision C and D coalesce into a single particle E.

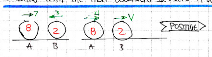
The final collision is between B and E.

Determine the range of values for the velocity of E after the third and final collision.

$\boxed{}$, $\boxed{5 < V_E < \frac{11}{2}}$

[solution overleaf]

STARTING WITH THE FIRST COLLISION BETWEEN A & B



BY CONSERVATION OF MOMENTUM

$8 \times 7 - 3 \times 2 = 4 \times 8 + 2V$

$56 - 6 = 32 + 2V$

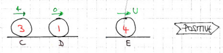
$50 = 32 + 2V$

$25 = 16 + V$

$V = 9$

$\therefore \ V_B = 9 \text{ ms}^{-1}$

NEXT THE COLLISION BETWEEN C & D



BY CONSERVATION OF MOMENTUM

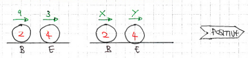
$4 \times 3 + 0 \times 1 = 4U$

$12 = 4U$

$U = 3$

$\therefore \ V_E = 3 \text{ ms}^{-1}$

NOW THE FINAL COLLISION BETWEEN B AND E



AS THERE ARE NO MORE COLLISIONS $X > V_A = 4$

BY CONSERVATION OF MOMENTUM

$\Rightarrow (9 \times 2) + (3 \times 4) = 2X + 4Y$

$\Rightarrow 18 + 12 = 2X + 4Y$

$\Rightarrow 30 = 2X + 4Y$

$\Rightarrow X + 2Y = 15$

$\Rightarrow X = 15 - 2Y$

NOW $X > 4$, IE FASTER THAN "A"

$\Rightarrow 15 - 2Y > 4$

$\Rightarrow -2Y > -11$

$\Rightarrow Y < 11/2$

BUT $Y > X$

$\Rightarrow Y > 15 - 2Y$

$\Rightarrow 3Y > 15$

$\Rightarrow Y > 5$

$\therefore \ 5 < Y < \frac{11}{2}$

$5 < V_E < \frac{11}{2}$ //

Question 21 (*****)

Two particles, A and B, of respective masses 2 kg and 13 kg are moving on a smooth horizontal surface in the same direction along the same straight line.

The speeds of A and B are 6 ms^{-1} and 2 ms^{-1}, respectively.

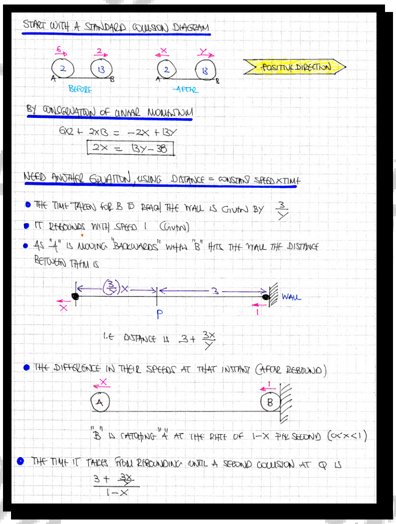
The two particles collide at the point P and after this collision A and B are moving in opposite directions.

After the collision at P, B hits a fixed smooth vertical wall, which is perpendicular to the direction of its motion. The wall is at a distance of 3 m from P.

The two particles collide again at the point Q.

If B rebounds off the wall with a speed of 1 ms^{-1} and the time that elapses between the collision at P and the collision at Q is 8 s, determine the speed of A and the speed of B after their collision at P.

$\boxed{}$, $\boxed{V_A = 0.5 \text{ ms}^{-1}}$, $\boxed{V_B = 3 \text{ ms}^{-1}}$



START WITH A STANDARD COLLISION DIAGRAM

BY CONSERVATION OF LINEAR MOMENTUM

$6 \times 2 + 2 \times 13 = -2X + 13Y$

$\boxed{2X = 13Y - 38}$

NEED ANOTHER EQUATION, USING DISTANCE = CONSTANT SPEED × TIME

- THE TIME TAKEN FOR B TO REACH THE WALL IS GIVEN BY $\frac{3}{Y}$
- IT REBOUNDS WITH SPEED 1 (GIVEN)
- AS "A" IS MOVING "BACKWARDS" WHEN "B" HITS THE WALL THE DISTANCE BETWEEN THEM IS

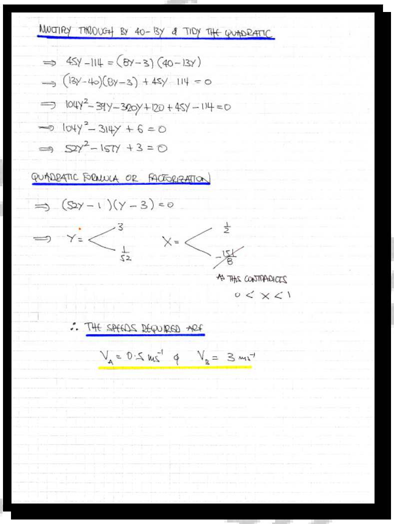
I.E DISTANCE IS $3 + \frac{3X}{Y}$

- THE DIFFERENCE IN THEIR SPEEDS AT THAT INSTANT (AFTER REBOUND)

"B" IS CATCHING "A" AT THE RATE OF $1 - X$ PER SECOND $(0 < X < 1)$

- THE TIME IT TAKES FROM REBOUNDING UNTIL A SECOND COLLISION AT Q IS

$\frac{3 + \frac{3X}{Y}}{1 - X}$



MULTIPLY THROUGH BY $40 - 13Y$ & TIDY THE QUADRATIC

$\Rightarrow 45Y - 114 = (8Y - 3)(40 - 13Y)$

$\Rightarrow (13Y - 40)(8Y - 3) + 45Y - 114 = 0$

$\Rightarrow 104Y^2 - 39Y - 320Y + 120 + 45Y - 114 = 0$

$\Rightarrow 104Y^2 - 314Y + 6 = 0$

$\Rightarrow 52Y^2 - 157Y + 3 = 0$

QUADRATIC FORMULA OR FACTORIZATION

$\Rightarrow (52Y - 1)(Y - 3) = 0$

$\Rightarrow Y = \begin{cases} 3 \\ \frac{1}{52} \end{cases} \qquad X = \begin{cases} \frac{1}{2} \\ \cancel{-\frac{151}{8}} \end{cases}$

AS THIS CONTRADICTS $0 < X < 1$

∴ THE SPEEDS REQUIRED ARE

$V_A = 0.5 \text{ ms}^{-1}$ & $V_B = 3 \text{ ms}^{-1}$

Question 22 (*****)

A bullet of mass m is fired onto a rectangular piece of foam board of mass M.

The foam board has constant thickness and a bullet is fired at right angles to one of its two rectangular faces.

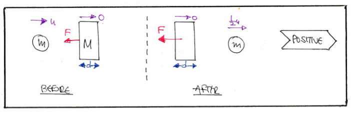
On the first occasion the foam board is fixed.
The bullet hits the board with speed u and emerges from behind with speed $\frac{1}{2}u$.

On the second occasion the foam board is free to move.
The bullet hits the board with speed u and emerges from behind with speed $\frac{1}{4}u$, relative to the board.

Show that $M = 4m$.

proof

FIRST CASE – THE BLOCK IS FIXED



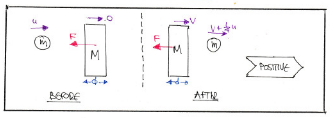
- USING THE STANDARD KINEMATICS EQUATIONS FOR CONSTANT ACCELERATION AS THE RESISTANCE F IS CONSTANT, WHILE THE BULLETS TRAVELS THROUGH

$$\Rightarrow \text{"}s = \frac{u+v}{2} \times t\text{"}$$
$$\Rightarrow d = \frac{u+\frac{1}{2}u}{2} \times T$$
$$\Rightarrow d = \frac{3}{4}uT$$
$$\Rightarrow \boxed{T = \frac{4d}{3u}}$$
← TIME FOR THE BULLET TO GET THROUGH THE FIXED BLOCK OF THICKNESS d

- NOW THE IMPULSE ON THE BULLET GIVES

$$\Rightarrow I = m\left(\tfrac{1}{2}u\right) - mu$$
$$\Rightarrow -FT = -\tfrac{1}{2}mu$$
$$\Rightarrow -F\left(\frac{4d}{3u}\right) = -\tfrac{1}{2}mu$$
$$\Rightarrow \boxed{F = \frac{3mu^2}{8d}}$$
← CONSTANT RESISTIVE FORCE F

SECOND CASE – THE BLOCK CAN MOVE AND THE BULLET NOW LEAVES WITH SPEED $\frac{1}{4}u$, RELATIVE TO THE BLOCK



- KINEMATICS AGAIN WHILE THE BULLET IS TRAVELLING THROUGH THE BLOCK

$$\Rightarrow \text{"}s = \frac{u+v}{2} \times t\text{"}$$
$$\Rightarrow d = \frac{u+\frac{1}{4}u}{2} \times T'$$
$$\Rightarrow d = \frac{5uT'}{8}$$
$$\Rightarrow \boxed{T' = \frac{8d}{5u}}$$
← TIME TO TRAVERSE THE MOVING BLOCK

- IMPULSE ON THE BULLET

$$\Rightarrow -FT' = m\left(v+\tfrac{1}{4}u\right) - mu$$
$$\Rightarrow -\left(\frac{3mu^2}{\cancel{8d}}\right)\left(\frac{\cancel{8d}}{5u}\right) = mv + \tfrac{1}{4}mu - mu$$
$$\Rightarrow -\frac{3}{5}\cancel{m}u = \cancel{m}v - \frac{3}{4}\cancel{m}u$$
$$\Rightarrow v = \frac{3}{20}u$$

- FINALLY THE IMPULSE ON THE BLOCK IS THE NEGATIVE OF THAT ON THE BULLET

$$\Rightarrow +\frac{3}{5}mu = Mv$$
$$\Rightarrow \frac{3}{5}m\cancel{u} = \frac{3}{20}M\cancel{u}$$
$$\Rightarrow M = 4m$$