ADVANCED MENSURATION ASSINGUIS COM I. Y. C.B. MARIASINGUIS COM I. Y. C.B. MARIASING

Question 1 (**)

. R.B.

Į.G.B.

A water tank has a square base of length 48 cm and height 35 cm.

It is filled with water to a height of 25 cm.

When a solid sphere is placed in the tank the water level rises by π cm.

11_{21/2}

Assuming the sphere is fully submerged determine the radius of the sphere.

r = 12 cm

Ĉ.p.

202.sm

madasn.

М

Α

Question 2 (**+)

The figure above shows two circles, with centres at M and N, with respective radii 7 cm and 3 cm touch each other. The points M, N and C lie in a straight line and the straight line ABC is a common tangent to the circles.

 $\left|MC\right| = \frac{35}{2}$

nadasn.

Ν

В

Determine the length of MC.

i.C.B.

Question 3 (***+)

Ĉ.Ŗ

I.C.p

A circular cylinder and a sphere both have radius r cm.

The total surface area of the cylinder is twice as large as the surface area of the sphere.

9:4

1+

212sm2

madasn,

i.C.p.

Determine the ratio of the volume of the cylinder to the volume of the sphere.

12/12

Question 4 (****)



The figure above shows a circle with centre at O, inscribed in a square ABCD. The diagonal AC meets the circle at the points P and Q.

- **a**) Show clearly that $\measuredangle PDQ = \arccos \frac{1}{3}$.
- **b)** Given that the triangle PDQ is of unit area, determine the exact area of the triangle APD.

area = √2 -

×	$ \begin{array}{l} \label{eq:constraints} \left(\mathcal{H}_{c}^{c} \ \mathcal{H}_{c}^{c} \ \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \right) = \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \\ \left(\mathcal{H}_{c}^{c} \right) = \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \\ \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \\ \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \\ \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \\ \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \\ \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{c} \\ \mathcal{H}_{c}^{c} \mathcal{H}_{c}^{$
6	$\frac{1}{2} = \frac{2}{2} \frac{1}{2} $
e	• Now let $\eta = po_{q} \longrightarrow q = 2g$ $\Rightarrow comp = comp = comp = comp = $
Baul	~//
$D_{i} = \frac{1}{2}$	
F HAE THE OLIVAL	

Question 5 (*****)

The following information is given.

- The straight line l_1 is a tangent to a circle at the point T and the point C is another point on l_1 .
- The straight line l_2 passes through C, intersecting the circle at two distinct points A and B.
- The straight line l_3 is the angle bisector of $\measuredangle TCA$.

Given further that l_3 intersects *TA* and *TB* at the points *P* and *Q* respectively, prove that the triangle *TPQ* is isosceles.



, proof



The figure above shows a Reuleaux heptagon, ABCDEFG, constructed as follows.

Firstly a regular heptagon ABCDEFG with centre at O and radius r is constructed. This is shown dotted in the figure.

A circular arc \widehat{AB} is drawn with centre at E and radius EA. A second circular arc \widehat{BC} is drawn with centre at F and radius FB.

A third circular arc CD is drawn with centre at G and radius GC and the process is repeated, forming a curved heptagon known as a Reuleaux heptagon.

Show that the area of this Reuleaux heptagon is

 $r^2 \left[2\pi \cos^2\left(\frac{\pi}{14}\right) - \sin\left(\frac{\pi}{7}\right) \right]$



Mark soul of the state in the Difference $\frac{1}{2}\nabla^{2}(\overline{\mathfrak{T}}) = \frac{1}{2}(2\pi\alpha\xi_{0}^{2})^{2}\overline{\mathfrak{T}}^{2}$ Acts = $\frac{1}{2}x^{2}\overline{\mathfrak{T}} = \frac{1}{2}x(\alpha\xi_{0}^{2})^{2}\overline{\mathfrak{T}}$ Now we have $H \subseteq [= \mathcal{R} = 2x^{\frac{R}{2}} = 2 \times 10x^{\frac{R}{2}}$ Net or state Asc $\frac{1}{2}\nabla^{2}(\overline{\mathfrak{T}}) = \frac{1}{2}(2\pi\alpha\xi_{0}^{2})^{2}\overline{\mathfrak{T}}^{2}$

$$\begin{split} &= \sum_{n}^{+} \sqrt{2} \cos \frac{\pi}{n} + v r c \sin \frac{\pi}{n} \\ &= r^2 \cos \frac{\pi}{n} s u t \frac{\pi}{n} \\ &\max \left(\frac{2}{2} \cos \frac{\pi}{n} + \frac{2}{2} \sin \frac{\pi}{n} + \frac{2}{2} \sin \frac{\pi}{n} + \frac{2}{2} \sin \frac{\pi}{n} + \frac{2}{2} \sin \frac{\pi}{n} \right) \end{split}$$

 $\frac{1}{2} \left[\frac{2}{2} \left$

Question 7 (*****)

 $\hat{\mathcal{O}}$

The figure above shows the cross sectional view of a solid sphere that **just fits** inside a right circular conical shell of radius 6 cm and height h cm.

If the sphere occupies $\frac{3}{8}$ of the volume of the conical shell determine the two possible values of h.

 $h = 8, 24\sqrt{3}$



2a

θ

С

В

Question 8 (*****)

The figure above shows a triangle ABC, where |AB| = a and |AC| = 2a.

D

The angle *BAC* is α , where $\tan \alpha = \frac{3}{4}$.

The side BC is extended to the point D so that the angle ACD is denoted by θ .

Show clearly that $\theta = \arctan 2$.

• START WITH THE DUNGRAW, LET BEC - C	@ Finally use office-
8 BY THE OUT ON ARC 38H 201 HTT VS •	$\Rightarrow b = w + l$
=> 18c2= 1AB1 + 1Ac2- 21AB 1Ac1 were 2	$\Rightarrow + a_m \theta = b_{m_1}(\alpha + b)$
\Rightarrow $ Bc ^2 = 4a^2 + a^2 - 4a^2\cos\alpha$	= tay 0 = taw + taul
$\implies [\Re c]^2 = 5a^2 - 4a^2 \times \frac{4}{S}$ $\Rightarrow a = 67$ C	1 - ture turb
$\Longrightarrow \left(\Re c \right)^{2} = Sa^{2} - \frac{Va^{2}}{S}$	$ = t_{\text{avb}} = \frac{\pi \cdot 2}{(-\frac{3}{2} \times \frac{1}{2})} $
$\Rightarrow B_{C} ^{2} = \frac{q}{5}q^{2}$	- taul = = = = =
$\Rightarrow [Bc] = \frac{3}{\sqrt{2}}$	8
NOT BY THE SUM PITCO ART. { tong = }	= Tamb = 8-3
Since $\frac{3}{4}$	$\Rightarrow \phi_{an} \theta = \frac{10}{5}$
$ = \frac{3}{3} = \frac{3}{a} $	- fame = 2
$\rightarrow d_{SWN} = \frac{3}{2} d_{SW} l$	
	=) 0 = antw2
= = = = = = = = = = = = = = = = = = =	
\Rightarrow sing = $\frac{\sqrt{2}}{2}$	
NEXT GET THE EXACT TOLG. DATION OF R	
s $SM\theta = \frac{3T}{5}$	
$\int \int $	
$\sqrt[4]{2b}$, $0acco_{0} - \frac{a_{0}}{\sqrt{3b}} = \frac{a_{0}}{2\sqrt{3}} = \frac{1}{2}$	

proof

Question 9 (*****)

$$2x \tan x = 1, \ x \neq \frac{1}{2}n\pi, \ n \in \mathbb{N}$$

- a) Show that the above equation has a solution in the interval (0.6, 0.7).
- **b**) Use the Newton Raphson method to find the solution of this equation, correct to 5 decimal places.

The figure below shows a circle, centre at O. The points A, B and C lie on the circumference of this circle. A circular sector ABC, subtending an angle of 2θ at C, is inscribed in this circle.



c) Determine the greatest proportion of the area of the circle, which can be covered by this sector.

 $|x \approx 0.65327|, |\approx 52.45|$

You may give the answer as a percentage, correct to two decimal places

ING BE ZONO WEITH IN FULTION NOTATION = [0m205- 020] 020 + $o = \theta m 2 \theta s = \theta 2 \omega 3$ => -f(2)=22. pm2-1 $209m\theta = cos\theta$ -0.17403 ... <0 = Rust <0. + (07)= + 0.179203 ... >C <u>(220)</u> - <u>QM205</u> C r= 286050 0.6,80.7) THERE MULT ARFA OF THE CIELE II TTR2 201000 = 1 APLA OF THE SHOT A This quatton that sourceal Q= 0.65327 PREMANNE TO THE THE NEWTON - RAPHISON METLOD WITH 34 = 0.65 $\frac{1}{2}\Gamma^{2}(2\theta) = \Gamma^{2}\theta = (2R\omega\omega)^{2}\theta = 4R^{2}\theta\omega\omega^{2}\theta$ => f(x) = 2 for + 22502x THE PROPORTION COUNCED BY THE SECTOR IS $\Theta^2_{200} \Theta \frac{\mu}{11} = (\Theta) V$ $\frac{420\omega^2\theta}{\pi D^2} = \frac{4}{\pi} \theta \omega^2 \theta$ $\mathcal{X}_{n_{1}} = \mathcal{X}_{n} - \frac{f(\mathbf{x}_{n})}{f(\mathbf{x}_{n})} = \mathcal{X}_{n}$ $= \Im_{ij} - \frac{2\epsilon \log_{2i} - 1}{2 \log_{2i} + 2\chi} \frac{1}{2 \log_{2i} + 2\chi}$ $= \Im_{ij} - \frac{2 \log \chi_{ij} \log 2\eta - \log^2 \chi_{ij}}{2 \sin \chi_{ij} \log 2\eta} + 22\eta$ $V(0.65327) = \frac{1}{27}(0.15227) \cos^2(0.65327) = 0.52451$ USING-CAUDUS . MAX PRODUDARE IN $V(\Theta) = \frac{U}{\pi} \Theta \cos^2 \Theta$ $D_{4} = \frac{2.510234 - 60^{2}34}{540234 + 234}$ $\left[\left(\Theta_{M2}-\right)\Theta_{2OUS}\times\Theta+\Theta_{2O2}\times1\right]\frac{\Psi}{m}\approx\left(\Theta\right)^{1}V$ • $\Im_1 = 0.65$ • $\Im_2 = 0.6532853557$. • $\Im_3 = 0.6532711874$. $V'(\theta) = \underbrace{\mathbb{I}}_{\mathcal{H}} \left[\log^2 \theta - 2\theta \log \theta \sin \theta \right]$ $V^{I}(\theta) = \frac{\mu}{\pi} = \cos\left(\frac{1}{2} - \frac{1}{2} -$ 0.6532711871 : a= 0.65327 (5.1.p)