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DEFINITE INTEGRATION MIX

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Part 1

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1. $\int_0^2 \frac{1}{\sqrt{4x+1}} dx = 1$

2. $\int_0^{\frac{\pi}{2}} \sin 2x dx = 1$

3. $\int_0^{\frac{\pi}{6}} \sin\left(4x + \frac{\pi}{6}\right) dx = \frac{\sqrt{3}}{4}$

4. $\int_0^{\frac{\pi}{2}} \sin^2 x dx = \frac{\pi}{4}$

5. $\int_1^2 x^3 \ln x dx = 4 \ln 2 - \frac{15}{16}$

6. $\int_0^{\frac{1}{2}} \frac{x}{(2-x)^2} dx = \frac{1}{3} + \ln \frac{3}{4}$

7. $\int_1^2 \frac{x}{(2x-1)^2} dx = \frac{2 + \ln 27}{12}$

8. $\int_0^1 \frac{3x}{(x+1)(x-2)} dx = -\ln 2$

9. $\int_0^{\frac{\pi}{4}} \tan^2 x dx = \frac{1}{4}(4 - \pi)$

10. $\int_0^2 \frac{x+2}{\sqrt{4x+1}} dx = \frac{17}{6}$

11. $\int_0^1 x e^{-2x} dx = \frac{1}{4}(1 - 3e^{-2})$

$$12. \int_0^2 \frac{2x}{\sqrt{x^2+4}} dx = 4(\sqrt{2}-1)$$

$$13. \int_{\frac{1}{6}}^{\frac{1}{3}} \frac{14x+1}{(2x+1)(1-x)} dx = 3\ln\left(\frac{5}{4}\right)$$

$$14. \int_0^{36} \frac{1}{\sqrt{x}(\sqrt{x}+2)} dx = \ln 16$$

$$15. \int_0^{\frac{\pi}{4}} 12x \cos 2x dx = \frac{3}{2}(\pi-2)$$

$$16. \int_0^{\frac{\pi}{2}} (2\sin x - 3\cos x)^2 dx = \frac{1}{4}(13\pi - 24)$$

$$17. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4x \sin 2x dx = \pi - 1$$

$$18. \int_{-6}^{\frac{3}{2}} \frac{x}{\sqrt{4-2x}} dx = -\frac{9}{2}$$

$$19. \int_1^5 \frac{x+1}{(2x-1)^{\frac{3}{2}}} dx = 2$$

$$20. \int_0^{\frac{1}{12}\pi} 6\sin^2 \theta d\theta = \frac{1}{4}(\pi-3)$$

$$21. \int_0^{\frac{1}{2}} \frac{1}{(1-x)(1+x)^2} dx = \frac{1}{6} + \frac{1}{4}\ln 3$$

$$22. \int_0^{\frac{1}{4}\pi} x \sec^2 x dx = \frac{1}{4}(\pi - \ln 4)$$

$$23. \int_0^1 \frac{9}{(2x+1)^4} dx = \frac{13}{9}$$

$$24. \int_0^{\frac{1}{2}\pi} 8x \sin^2 x dx = \frac{1}{2}(\pi^2 + 4)$$

$$25. \int_0^{\frac{1}{4}\pi} 2x \cos 4x dx = -\frac{1}{4}$$

$$26. \int_0^{\frac{1}{4}\pi} (\cos x + \sec x)^2 dx = \frac{5}{8}(\pi + 2)$$

$$27. \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx = 20$$

$$28. \int_0^3 \frac{x^2}{\sqrt{x+1}} dx = \frac{76}{15}$$

$$29. \int_2^6 \frac{5x+3}{(2x-3)(x+2)} dx = \ln 54$$

$$30. \int_0^{\ln 2} 4xe^{-x} dx = 2 - \ln 4$$

$$31. \int_0^3 \frac{x}{x^2+9} dx = \frac{1}{2} \ln 2$$

$$32. \int_0^{\frac{1}{4}\pi} \sin\left(2x + \frac{\pi}{4}\right) dx = \frac{\sqrt{2}}{2}$$

$$33. \int_{\frac{2}{3}}^1 \frac{x}{2x-1} dx = \frac{1}{6} + \frac{1}{4} \ln 3$$

$$34. \int_0^4 \frac{13-2x}{(x+4)(2x+1)} dx = 4\ln 3 - 3\ln 2$$

$$35. \int_1^e \ln x dx = 1$$

$$36. \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos 3x dx = -\frac{1}{3}$$

$$37. \int_0^{\frac{1}{2}\pi} 4\cos x (1+\sin x)^3 dx = 15$$

$$38. \int_{\frac{1}{8}}^{\frac{1}{6}\pi} \cot^2 2x dx = \frac{1}{2} - \frac{1}{6}\sqrt{3} - \frac{1}{24}\pi$$

$$39. \int_0^{\frac{1}{2}} \frac{3-5x}{(1-x)(2-3x)} dx = \frac{4}{3}\ln 2$$

$$40. \int_{\ln 2}^{\ln 4} (e^{2x}-2)^2 dx = 4(9+\ln 2)$$

$$41. \int_0^{\frac{1}{2}\pi} x \sin 2x dx = \frac{1}{4}\pi$$

$$42. \int_{-1}^1 \frac{9+4x^2}{9-4x^2} dx = -2+3\ln 5$$

$$43. \int_{-1}^7 \frac{x^2}{\sqrt{x+2}} dx = \frac{652}{15}$$

$$44. \int_0^2 \frac{6}{3x+2} dx = \ln 16$$

$$45. \int_2^5 \frac{1}{4 + \sqrt{x-1}} dx = 2 + 8 \ln\left(\frac{5}{6}\right)$$

$$46. \int_0^{\frac{1}{4}\pi} \frac{\cos 2x}{\cos^2 x} dx = \frac{1}{2}(\pi - 2)$$

$$47. \int_0^{\frac{\pi}{4}} \cos\left(3x + \frac{\pi}{4}\right) dx = -\frac{\sqrt{2}}{6}$$

$$48. \int_2^4 \frac{8}{(3x-4)^3} dx = \frac{5}{16}$$

$$49. \int_1^{\frac{5}{2}} \frac{4x}{\sqrt{2x-1}} dx = \frac{20}{3}$$

$$50. \int_0^{\frac{\pi}{3}} \cos\left(3x + \frac{\pi}{3}\right) dx = -\frac{\sqrt{3}}{3}$$

$$51. \int_0^1 \frac{18 - 4x - x^2}{(4 - 3x)(1 + x)^2} dx = \frac{7}{3} \ln 2 + \frac{3}{2}$$

$$52. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x + \cot x)^2 dx = \frac{1}{8}(26 - \pi - 4\sqrt{2})$$

$$53. \int_0^{\frac{\pi}{3}} \tan^3 x dx = \frac{3}{2} - \ln 2$$

$$54. \int_{\frac{1}{e}}^1 x \ln x dx = \frac{1}{4}\left(\frac{3}{e^2} - 1\right)$$

$$55. \int_0^1 \frac{x}{(1+x)^2} dx = \ln 2 - \frac{1}{2}$$

$$56. \int_2^3 \frac{x^2 - 4x + 9}{(4-x)(1-x)^2} dx = 1 + \ln 2$$

$$57. \int_0^{\frac{\pi}{12}} 10 \sin 8\theta \cos 2\theta d\theta = \frac{1}{12}(16 + 3\sqrt{3})$$

$$58. \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin\left(4x + \frac{\pi}{6}\right) dx = -\frac{\sqrt{3}}{8}$$

$$59. \int_1^e (x^2 + 1) \ln x dx = \frac{2}{9}(e^3 + 5)$$

$$60. \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2 dx = 4\pi + 3\sqrt{3}$$

$$61. \int_0^3 x\sqrt{x+1} dx = \frac{116}{15}$$

$$62. \int_2^3 \frac{x^2 + x + 2}{x^2 + 2x - 3} dx = 1 + \ln\left(\frac{25}{18}\right)$$

$$63. \int_0^1 \frac{9}{(2x+1)^2} dx = 3$$

$$64. \int_0^{\frac{\pi}{6}} \sin x \sin 3x dx = \frac{\sqrt{3}}{16}$$

$$65. \int_0^{\sqrt{2}} x^3 \ln(x^2 + 2) dx = \frac{1}{2} + \ln 2$$

$$66. \int_0^{\frac{1}{4}} \frac{4}{(1+2x)(1-2x)} dx = \ln 3$$

$$67. \int_0^{\frac{\pi}{2}} \cos^3 x \, dx = \frac{2}{3}$$

$$68. \int_0^3 \frac{4}{2x+3} \, dx = \ln 9$$

$$69. \int_0^2 \frac{6x^3}{\sqrt{x^2+1}} \, dx = 4(1+\sqrt{5})$$

$$70. \int_5^8 \frac{2x^2}{x^2-16} \, dx = 6 + 4 \ln 3$$

$$71. \int_1^2 \frac{\ln x}{x} \, dx = \frac{1}{2}(\ln 2)^2$$

$$72. \int_0^1 \frac{17-5x}{(3+2x)(2-x)^2} \, dx = \frac{1}{2} + \ln\left(\frac{10}{3}\right)$$

$$73. \int_0^{\pi} x \cos\left(\frac{1}{4}x\right) \, dx = 2\sqrt{2}(\pi+4) - 16$$

$$74. \int_0^4 e^{\frac{1}{2}x} \, dx = 2(e^2 - 1)$$

$$75. \int_4^9 \frac{5x^2 - 8x + 1}{2x(x-1)^2} \, dx = \ln\left(\frac{32}{3}\right) - \frac{5}{24}$$

$$76. \int_0^1 \frac{3}{(\sqrt{x}-2)(\sqrt{x}+1)} \, dx = -\ln 4$$

$$77. \int_0^{\frac{\pi}{3}} \frac{1}{1-\sin x} \, dx = 1 + \sqrt{3}$$

$$78. \int_2^6 \frac{2x^2 - x + 11}{(x+2)(2x-3)} dx = 4 + 4\ln 3 - 3\ln 2$$

$$79. \int_{-1}^0 \frac{x^2}{1-x} dx = -\frac{1}{2} + \ln 2$$

$$80. \int_0^{100} \frac{1}{20 - \sqrt{x}} dx = 40\ln 2 - 20$$

$$81. \int_0^1 \frac{x^2}{x^2 - 4} dx = 1 - \ln 3$$

$$82. \int_0^{\frac{\pi}{6}} \sin^3 \theta d\theta = \frac{2}{3} - \frac{3}{8}\sqrt{3} = \frac{1}{24}(16 - 9\sqrt{3})$$

$$83. \int_0^{\ln 2} \frac{1}{1+e^x} dx = \ln\left(\frac{4}{3}\right)$$

$$84. \int_0^4 e^{\sqrt{2x+1}} dx = 2e^3$$

$$85. \int_0^1 \frac{x^3}{x+1} dx = \frac{5}{6} - \ln 2$$

$$86. \int_0^1 \frac{10}{(x+1)(x+3)(2x+1)} dx = 3\ln 3 - 3\ln 2$$

$$87. \int_0^{\frac{\pi}{2}} \left[1 + \tan\left(\frac{1}{2}x\right)\right]^2 dx = 2 + \ln 4$$

$$88. \int_0^{\frac{\pi}{3}} \frac{\sin 2x}{1 + \cos x} dx = 1 + 2\ln\left(\frac{3}{4}\right)$$

$$89. \int_0^1 3x^2 \ln(x+1) dx = -\frac{5}{6} + \ln 4$$

$$90. \int_0^{\frac{1}{4}} 2x\sqrt{1-4x} dx = \frac{1}{30}$$

$$91. \int_1^e x(1-\ln x) dx = \frac{1}{4}(e^2 - 3)$$

$$92. \int_{-\frac{1}{3}}^0 \frac{1}{3-6x-9x^2} dx = \frac{1}{12} \ln 3$$

$$93. \int_0^{\frac{\pi}{2}} x \sin^2 x dx = \frac{1}{16}(\pi^2 + 4)$$

$$94. \int_1^e x(\ln x)^2 dx = \frac{1}{4}(e^2 - 1)$$

$$95. \int_0^{\frac{\pi}{2}} \sin x \cos x (1 + \sin x)^5 dx = \frac{107}{14}$$

$$96. \int_2^5 \frac{x^2}{\sqrt{x-1}} dx = \frac{356}{15}$$

$$97. \int_0^5 \frac{1}{(x+1)(x+2)(x+3)} dx = \ln\left(\frac{8}{7}\right)$$

$$98. \int_0^{\pi} (x-1)(x+3)\sin x dx = \pi^2 + 2\pi - 10$$

$$99. \int_{-1}^0 3\ln(2x+3) dx = \frac{3}{2}(\ln 27 - 2)$$

100. $\int_0^{\frac{\pi}{6}} 12 \sec^3 x \, dx = 4 + 3 \ln 3$

101. $\int_{\sqrt{5}}^{2\sqrt{3}} \frac{\sqrt{x^2+4}}{x} \, dx = 1 + \ln\left(\frac{5}{3}\right)$

102. $\int_{e^{-1}}^e x \left[(\ln x)^2 - 1 \right] \, dx = -\frac{1}{4} (e^2 + 3e^{-2})$

103. $\int_0^1 \frac{x^2}{x^2+1} \, dx = 1 - \frac{\pi}{4}$

104. $\int_0^{\frac{\pi}{2}} e^{\cos x} \sin x \cos x \, dx = 1$

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Part 2

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$$1. \int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx = \frac{\pi}{2} - 1, \text{ use } x = 2 \sin \theta$$

$$2. \int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx = \frac{1}{4}(\sqrt{3}-1), \text{ use } x = 2 \cos \theta$$

$$3. \int_0^1 \frac{1}{(1+x^2)^2} dx = \frac{1}{8}(\pi+2), \text{ use } x = \tan \theta$$

$$4. \int_{\sqrt{2}}^2 \frac{1}{x^2 \sqrt{x^2-1}} dx = \frac{1}{2}(\sqrt{3}-\sqrt{2}), \text{ use } x = \sec \theta$$

$$5. \int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}} dx = \frac{\pi}{6}, \text{ use } x = \frac{\sqrt{3}}{2} \sin \theta$$

$$6. \int_0^1 \frac{1}{(1+3x^2)^{\frac{3}{2}}} dx = \frac{1}{2}, \text{ use } x = \frac{1}{\sqrt{3}} \tan \theta$$

$$7. \int_0^1 \frac{1}{\sqrt{2-x^2}} dx = \frac{\pi}{4}, \text{ use } x = \sqrt{2} \sin \theta$$

$$8. \int_0^{\frac{1}{2}} \frac{1}{4x^2+3} dx = \frac{\pi\sqrt{3}}{36}, \text{ use } x = \frac{\sqrt{3}}{2} \tan \theta$$

$$9. \int_0^1 \frac{1}{(4-x^2)^{\frac{3}{2}}} dx = \frac{\sqrt{3}}{12}, \text{ use } x = 2 \sin \theta$$

$$10. \int_{\sqrt{2}}^2 \frac{\sqrt{x^2-1}}{x} dx = \sqrt{3}-1-\frac{\pi}{12}, \text{ use } x = \operatorname{cosec} \theta$$

$$11. \int_0^1 \frac{1}{\sqrt{4-3x^2}} dx = \frac{\pi\sqrt{3}}{9}, \text{ use } x = \frac{2}{\sqrt{3}} \sin \theta$$

$$12. \int_1^{\sqrt{3}} \frac{x^2}{x^2+1} dx = \sqrt{3} - 1 - \frac{\pi}{12}, \text{ use } x = \tan \theta$$

$$13. \int_0^2 \sqrt{16-x^2} dx = \frac{1}{3}(4\pi + 6\sqrt{3}), \text{ use } x = 4 \sin \theta$$

$$14. \int_0^2 \frac{1}{(3x^2+4)^{\frac{3}{2}}} dx = \frac{1}{8}, \text{ use } x = \frac{2}{\sqrt{3}} \tan \theta$$

$$15. \int_0^2 \sqrt{16-3x^2} dx = \frac{8\pi\sqrt{3}}{9} + 2, \text{ use } x = \frac{4}{\sqrt{3}} \sin \theta$$

$$16. \int_0^3 \frac{27}{(9+x^2)^2} dx = \frac{\pi}{8} + \frac{1}{4}, \text{ use } x = 3 \tan \theta$$

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Part 3

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$$1. \int_4^8 \sqrt{x^2 - 16} \, dx = 16\sqrt{3} - 8\ln(2 + \sqrt{3})$$

$$2. \int_0^1 \frac{1}{\sqrt{x}(x+1)} \, dx = \frac{\pi}{2}$$

$$3. \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec x \, dx = \ln \left| \frac{2}{3}\sqrt{3} + 1 \right|$$

$$4. \int_0^1 x^3 \sqrt{x^2 + 1} \, dx = \frac{2}{15}(\sqrt{2} + 1)$$

$$5. \int_{\ln 2}^{\ln 3} \frac{\cosh x + 1}{\sinh x (\cosh x - 1)} \, dx = \frac{5}{2}$$

$$6. \int_1^{\sqrt{3}} x \arctan x \, dx = \frac{5\pi}{12} + \frac{1}{2}(1 - \sqrt{3})$$

$$7. \int_{\frac{4}{3}}^{\frac{5}{3}} \frac{x+1}{\sqrt{9x^2 - 16}} \, dx = \frac{1}{3}(1 + \ln 2)$$

$$8. \int_{\frac{3}{2}}^{\frac{5}{2}} \sqrt{4x^2 - 9} \, dx = 5 - \frac{9}{4}\ln 3$$

$$9. \int_0^4 \operatorname{arsinh} \sqrt{x} \, dx = \frac{9}{2}\ln(2 + \sqrt{5}) - \sqrt{5}$$

$$10. \int_0^1 \frac{x^2}{\sqrt{x^2 + 1}} \, dx = \frac{1}{2}(\sqrt{2} - \ln(1 + \sqrt{2}))$$

$$11. \int_5^7 \frac{1}{x^2 - 10x + 29} \, dx = \frac{\pi}{8}$$

$$12. \int_5^7 \frac{1}{\sqrt{x^2 - 10x + 29}} \, dx = \ln(1 + \sqrt{2})$$

$$13. \int_{2.5}^{7.5} \frac{180}{4x^2 + 75} dx = \sqrt{3}\pi$$

$$14. \int_2^3 \frac{1}{\sqrt{3+2x-x^2}} dx = \frac{\pi}{3}$$

$$15. \int_5^7 \frac{x+1}{x^2+9} dx = \frac{1}{2} \ln 2 + \frac{\pi}{12}$$

$$16. \int_0^{\frac{\pi}{3}} \frac{1}{9\cos^2 x + \sin^2 x} dx = \frac{\pi}{18}$$

$$17. \int_0^{\frac{1}{2}\ln 3} \operatorname{sech} x dx = \frac{\pi}{6}$$

$$18. \int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1-x^4} dx = \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{3}$$

$$19. \int_0^{\frac{3}{2}} \frac{8}{4x^2+9} dx = \frac{\pi}{3}$$

$$20. \int_0^1 \frac{10}{(x+1)(x^2+4)} dx = \ln\left(\frac{16}{5}\right) + \arctan\left(\frac{1}{2}\right)$$

$$21. \int_0^3 \frac{8x}{(x+2)(x^2+4)} dx = \ln\left(\frac{26}{25}\right) + 2\left(\arctan\left(\frac{1}{2}\right) - \arctan 1\right)$$

$$22. \int_0^{\frac{9}{4}} \frac{1}{\sqrt{x(9-x)}} dx = \frac{\pi}{3}$$

$$23. \int_{-1}^0 \frac{(x+1)(x+2)}{(x-1)(x^2+1)} dx = \frac{\pi}{4} - 2\ln 2$$

$$24. \int_0^{\frac{\pi}{2}} \frac{1}{1+3\cos 3x} dx = \frac{\sqrt{2}}{6} \ln(\sqrt{2}-1)$$

$$25. \int_{-1}^1 \frac{1}{\sqrt{x^2 + 2x + 5}} dx = \ln(1 + \sqrt{2})$$

$$26. \int_0^{\sqrt{3}} \frac{x}{x^4 + 9} dx = \frac{\pi}{24}$$

$$27. \int_{-\frac{5}{3}}^{\frac{5}{6}} \frac{1}{\sqrt{25 - 9x^2}} dx = \frac{2\pi}{9}$$

$$28. \int_1^2 \sqrt{x^2 - 2x + 2} dx = \frac{1}{2} \ln(1 + \sqrt{2}) - \frac{1}{2} \sqrt{2}$$

$$29. \int_1^2 \frac{\sqrt{x}}{\sqrt{9 - x^3}} dx = \frac{2}{3} \arccos \frac{1}{3}$$

$$30. \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\sqrt{3 - \sec^2 x}} dx = \frac{\pi}{4}$$

$$31. \int_0^2 \frac{1}{(x^2 + 4)^{\frac{3}{2}}} dx = \frac{1}{8} \sqrt{2}$$

$$32. \int_1^2 \frac{x^2}{\sqrt{x^2 + 1}} dx = \frac{1}{2} [\sqrt{2} - \ln(\sqrt{2} + 1)]$$

$$33. \int_0^2 \frac{36 - 3x}{4 + 3x^2} dx = 2\pi\sqrt{3} - \ln 2$$

$$34. \int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}} dx = \frac{1}{4} [\pi - 2]$$

$$35. \int_1^4 \frac{1}{(x+9)\sqrt{x}} dx = \frac{2}{3} \arctan\left(\frac{3}{11}\right)$$

$$36. \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin x} dx = \frac{\sqrt{3}}{9} \pi$$

$$37. \int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5 \cosh x - 4 \sinh x} dx = \frac{\pi}{18}$$

$$38. \int_0^{\sqrt{12}} \operatorname{arsinh}\left(\frac{1}{2}x\right) dx = 2\sqrt{3} \ln(2 + \sqrt{3}) - 2$$