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# EQUATIONS BASIC TECHNIQUES

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# LINEAR EQUATIONS

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## Question 1

Solve each of the following equations.

1.  $5(4-2x) = 30$

$x = -1$

2.  $2(2x-3) + 5 = 29$

$x = \frac{15}{2}$

3.  $4(2x+3) + x = 47 - 5x$

$x = \frac{5}{2}$

4.  $3(2x-3) - 9 = 38 - x$

$x = 8$

5.  $2 + 2(4x+3) = 2(2x-1) + 2x$

$x = -5$

6.  $8 + 3(x+3) = 2(2x+1)$

$x = 15$

1. $5(4-2x) = 30$ $\Rightarrow 20 - 10x = 30$ $\Rightarrow -10 = 10x$ $\Rightarrow -1 = x$ $\Rightarrow x = -1$	4. $3(2x-3) - 9 = 38 - x$ $\Rightarrow 6x - 9 - 9 = 38 - x$ $\Rightarrow 6x - 18 = 38 - x$ $\Rightarrow 7x = 56$ $\Rightarrow x = 8$
2. $2(2x-3) + 5 = 29$ $\Rightarrow 4x - 6 + 5 = 29$ $\Rightarrow 4x - 1 = 29$ $\Rightarrow 4x = 30$ $\Rightarrow 2x = 15$ $\Rightarrow x = \frac{15}{2}$	5. $2 + 2(4x+3) = 2(2x-1) + 2x$ $\Rightarrow 2 + 8x + 6 = 4x - 2 + 2x$ $\Rightarrow 8x + 8 = 6x - 2$ $\Rightarrow 2x = -10$ $\Rightarrow x = -5$
3. $4(2x+3) + x = 47 - 5x$ $\Rightarrow 8x + 12 + x = 47 - 5x$ $\Rightarrow 9x + 12 = 47 - 5x$ $\Rightarrow 14x = 35$ $\Rightarrow 2x = 5$ $\Rightarrow x = \frac{5}{2}$	6. $8 + 3(x+3) = 2(2x+1)$ $\Rightarrow 8 + 3x + 9 = 4x + 2$ $\Rightarrow 3x + 17 = 4x + 2$ $\Rightarrow 15 = x$ $\Rightarrow x = 15$

## Question 2

Solve each of the following equations.

1.  $6 - 4(4 - x) = 4$

$x = \frac{7}{2}$

2.  $16 - 4(7 - x) = 8$

$x = 5$

3.  $6 - 2(7 - 3x) = 8 - (3x + 7)$

$x = 1$

4.  $13 - 2(1 - 4x) = 6 - (2 - x)$

$x = -1$

5.  $21 - (3 - 2x) = 33 - 4(10 - 3x)$

$x = \frac{5}{2}$

6.  $10 - (5 - 2x) = -11 - 2(3x - 2)$

$x = -\frac{3}{2}$

1. $6 - 4(4 - x) = 4$ $\Rightarrow 6 - 16 + 4x = 4$ $\Rightarrow -10 + 4x = 4$ $\Rightarrow 4x = 14$ $\Rightarrow 2x = 7$ $\Rightarrow x = \frac{7}{2}$	4. $13 - 2(1 - 4x) = 6 - (2 - x)$ $\Rightarrow 13 - 2 + 8x = 6 - 2 + x$ $\Rightarrow 11 + 8x = 4 + x$ $\Rightarrow 7x = -7$ $\Rightarrow x = -1$
2. $16 - 4(7 - x) = 8$ $\Rightarrow 16 - 28 + 4x = 8$ $\Rightarrow -12 + 4x = 8$ $\Rightarrow 4x = 20$ $\Rightarrow x = 5$	5. $21 - (3 - 2x) = 33 - 4(10 - 3x)$ $\Rightarrow 21 - 3 + 2x = 33 - 40 + 12x$ $\Rightarrow 18 + 2x = -7 + 12x$ $\Rightarrow 25 = 10x$ $\Rightarrow \frac{25}{10} = x$ $\Rightarrow x = \frac{5}{2}$
3. $6 - 2(7 - 3x) = 8 - (3x + 7)$ $\Rightarrow 6 - 14 + 6x = 8 - 3x - 7$ $\Rightarrow -8 + 6x = 1 - 3x$ $\Rightarrow 9x = 9$ $\Rightarrow x = 1$	6. $10 - (5 - 2x) = -11 - 2(3x - 2)$ $\Rightarrow 10 - 5 + 2x = -11 - 6x + 4$ $\Rightarrow 5 + 2x = -7 - 6x$ $\Rightarrow 8x = -12$ $\Rightarrow 2x = -3$ $\Rightarrow x = -\frac{3}{2}$

## Question 3

Solve each of the following equations.

1.  $3(7-4x) = 15$

$x = \frac{1}{2}$

2.  $3(2x+3) + 2 = 20$

$x = \frac{3}{2}$

3.  $4(7x+8) = -4 - 8x$

$x = -1$

4.  $5(20-3x) + x = 34 + 2(x+1)$

$x = 4$

5.  $2 + 2(4x-3) = 4(x+1) + 12x$

$x = -1$

6.  $8 - 3(x-3) = 2(3-7x)$

$x = -1$

Handwritten solutions for the six equations:

- $3(7-4x) = 15$ 

$$\begin{aligned} \Rightarrow 7-4x &= 5 \\ \Rightarrow 2 &= 4x \\ \Rightarrow 1 &= 2x \\ \Rightarrow x &= \frac{1}{2} \end{aligned}$$
- $3(2x+3) + 2 = 20$ 

$$\begin{aligned} \Rightarrow 3(2x+3) &= 18 \\ \Rightarrow 2x+3 &= 6 \\ \Rightarrow 2x &= 3 \\ \Rightarrow x &= \frac{3}{2} \end{aligned}$$
- $4(7x+8) = -4 - 8x$ 

$$\begin{aligned} \Rightarrow 7x+8 &= -1-2x \\ \Rightarrow 9x &= -9 \\ \Rightarrow x &= -1 \end{aligned}$$
- $5(20-3x) + x = 34 + 2(x+1)$ 

$$\begin{aligned} \Rightarrow 100-15x+x &= 34+2x+2 \\ \Rightarrow 100-14x &= 2x+36 \\ \Rightarrow 64 &= 16x \\ \Rightarrow x &= 4 \end{aligned}$$
- $2 + 2(4x-3) = 4(x+1) + 12x$ 

$$\begin{aligned} \Rightarrow 2 + 8x-6 &= 4x+4+12x \\ \Rightarrow 4x-4 &= 16x+4 \\ \Rightarrow -4 &= 12x \\ \Rightarrow x &= -1 \end{aligned}$$
- $8 - 3(x-3) = 2(3-7x)$ 

$$\begin{aligned} \Rightarrow 8-3x+9 &= 6-14x \\ \Rightarrow 17-3x &= 6-14x \\ \Rightarrow 11x &= -11 \\ \Rightarrow x &= -1 \end{aligned}$$

## Question 4

Solve the following equations

1.  $6 - 4(2 - x) = 2x - 1$

$x = \frac{1}{2}$

2.  $25 - 4(7 - 3x) = 9x + 6$

$x = 3$

3.  $4 - 2(1 - 3x) = 12 - (3x + 7)$

$x = \frac{1}{3}$

4.  $13 - 2(1 - 4x) = 20 - 4(2 - x)$

$x = \frac{1}{4}$

5.  $50 - (8 - 3x) = 77 - 2(1 - 7x)$

$x = -3$

6.  $20 - (9 - 2x) = -8 + 3(4x - 2)$

$x = \frac{5}{2}$

1. $6 - 4(2 - x) = 2x - 1$ $\Rightarrow 6 - 8 + 4x = 2x - 1$ $\Rightarrow -2 + 4x = 2x - 1$ $\Rightarrow 2x = 1$ $\Rightarrow x = \frac{1}{2}$	4. $13 - 2(1 - 4x) = 20 - 4(2 - x)$ $\Rightarrow 13 - 2 + 8x = 20 - 8 + 4x$ $\Rightarrow 11 + 8x = 12 + 4x$ $\Rightarrow 4x = 1$ $\Rightarrow x = \frac{1}{4}$
2. $25 - 4(7 - 3x) = 9x + 6$ $\Rightarrow 25 - 28 + 12x = 9x + 6$ $\Rightarrow -3 + 12x = 9x + 6$ $\Rightarrow 3x = 9$ $\Rightarrow x = 3$	5. $50 - (8 - 3x) = 77 - 2(1 - 7x)$ $\Rightarrow 50 - 8 + 3x = 77 - 2 + 14x$ $\Rightarrow 42 + 3x = 75 + 14x$ $\Rightarrow -33 = 11x$ $\Rightarrow x = -3$
3. $4 - 2(1 - 3x) = 12 - (3x + 7)$ $\Rightarrow 4 - 2 + 6x = 12 - 3x - 7$ $\Rightarrow 2 + 6x = 5 - 3x$ $\Rightarrow 9x = 3$ $\Rightarrow x = \frac{1}{3}$	6. $20 - (9 - 2x) = -8 + 3(4x - 2)$ $\Rightarrow 20 - 9 + 2x = -8 + 12x - 6$ $\Rightarrow 11 + 2x = 12x - 14$ $\Rightarrow 25 = 10x$ $\Rightarrow x = \frac{25}{10}$ $\Rightarrow x = \frac{5}{2}$

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# FRACTIONAL EQUATIONS

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Question 1

Solve each of the following fractional equations.

1.  $\frac{x}{2} + \frac{x}{3} = 20$   $x = 24$

2.  $\frac{x}{2} - \frac{x}{3} = 2$   $x = 12$

3.  $\frac{x}{5} + \frac{2x}{3} = 26$   $x = 30$

4.  $\frac{3}{5}x + \frac{1}{2}x = 22$   $x = 20$

5.  $\frac{x}{4} + \frac{2x}{3} = 22$   $x = 24$

6.  $\frac{1}{3}x + \frac{3}{4} = \frac{5x}{6}$   $x = \frac{3}{2}$

7.  $\frac{3}{4}x - 12 = \frac{3x}{8}$   $x = 32$

8.  $\frac{1}{3}x + 6 = 5x - \frac{17}{3}$   $x = \frac{5}{2}$

The image shows handwritten solutions for the eight fractional equations. The solutions are organized into two columns and four rows, corresponding to the numbered questions. Each solution shows the steps taken to solve the equation, including finding a common denominator, multiplying through, and simplifying to find the value of x.

**1.**  $\frac{x}{2} + \frac{x}{3} = 20$   
 $\frac{3x}{6} + \frac{2x}{6} = 20$   
 $\frac{5x}{6} = 20$   
 $5x = 20 \times 6$   
 $5x = 120$   
 $x = 24$

**2.**  $\frac{x}{2} - \frac{x}{3} = 2$   
 $\frac{3x}{6} - \frac{2x}{6} = 2$   
 $\frac{x}{6} = 2$   
 $x = 2 \times 6$   
 $x = 12$

**3.**  $\frac{x}{5} + \frac{2x}{3} = 26$   
 $\frac{3x}{15} + \frac{10x}{15} = 26$   
 $\frac{13x}{15} = 26$   
 $13x = 26 \times 15$   
 $13x = 390$   
 $x = \frac{390}{13}$   
 $x = 30$

**4.**  $\frac{3}{5}x + \frac{1}{2}x = 22$   
 $\frac{6x}{10} + \frac{5x}{10} = 22$   
 $\frac{11x}{10} = 22$   
 $11x = 22 \times 10$   
 $11x = 220$   
 $x = 20$

**5.**  $\frac{x}{4} + \frac{2x}{3} = 22$   
 $\frac{3x}{12} + \frac{8x}{12} = 22$   
 $\frac{11x}{12} = 22$   
 $11x = 22 \times 12$   
 $11x = 264$   
 $x = 24$

**6.**  $\frac{1}{3}x + \frac{3}{4} = \frac{5x}{6}$   
 Multiply equation through by 12 (the L.C.M. of 3, 4, 6)  
 $4x + 9 = 10x$   
 $9 = 6x$   
 $\frac{9}{6} = x$   
 $x = \frac{3}{2}$

**7.**  $\frac{3}{4}x - 12 = \frac{3x}{8}$   
 $\frac{6x}{8} - 12 = \frac{3x}{8}$   
 Multiply equation through by 8  
 $6x - 96 = 3x$   
 $3x = 96$   
 $x = 32$

**8.**  $\frac{1}{3}x + 6 = 5x - \frac{17}{3}$   
 Multiply equation through by 3  
 $\frac{1}{3}x + 3 + 6 = 5x - \frac{17}{3}$   
 $\frac{1}{3}x + 9 = 5x - \frac{17}{3}$   
 $3x + 27 = 15x - 17$   
 $3x = 15x - 44$   
 $x = \frac{44}{12}$   
 $x = \frac{11}{3}$



Question 2

Solve each of the following fractional equations.

1.  $\frac{x+1}{2} + \frac{x-3}{4} = 2$   $x = 3$

2.  $\frac{5+x}{3} + \frac{9-x}{2} = 6$   $x = 1$

3.  $\frac{2x+1}{3} + \frac{3x-1}{5} = 9$   $x = 7$

4.  $\frac{2x+1}{5} + \frac{x-6}{2} = 8$   $x = 12$

5.  $\frac{1}{2}(x-1) - \frac{1}{3}(x-1) = 2$   $x = 13$

6.  $\frac{1}{3}(4x-1) + 1 = \frac{1}{2}(3x-1)$   $x = 7$

7.  $\frac{1}{3}(2x+1) = \frac{1}{2}(x+4)$   $x = 10$

8.  $\frac{4x-1}{5} = \frac{2x+3}{2}$   $x = -\frac{17}{2}$

1.  $\frac{x+1}{2} + \frac{x-3}{4} = 2$   
 Multiply equation by 4  
 $2(x+1) + (x-3) = 8$   
 $2x+2+x-3=8$   
 $3x-1=8$   
 $3x=9$   
 $x=3$

2.  $\frac{5+x}{3} + \frac{9-x}{2} = 6$   
 Multiply equation by 6  
 $2(5+x) + 3(9-x) = 36$   
 $10+2x+27-3x=36$   
 $-x+37=36$   
 $-x=-1$   
 $x=1$

3.  $\frac{2x+1}{3} + \frac{3x-1}{5} = 9$   
 Multiply equation by 15  
 $5(2x+1) + 3(3x-1) = 135$   
 $10x+5+9x-3=135$   
 $19x+2=135$   
 $19x=133$   
 $x = \frac{133}{19}$   
 $x = 7$

4.  $\frac{2x+1}{5} + \frac{x-6}{2} = 8$   
 Multiply equation by 10  
 $2(2x+1) + 5(x-6) = 80$   
 $4x+2+5x-30=80$   
 $9x-28=80$   
 $9x=108$   
 $x=12$

5.  $\frac{1}{2}(x-1) - \frac{1}{3}(x-1) = 2$   
 Multiply equation by 6  
 $3(x-1) - 2(x-1) = 12$   
 $3x-3-2x+2=12$   
 $x-1=12$   
 $x=13$

6.  $\frac{1}{3}(4x-1) + 1 = \frac{1}{2}(3x-1)$   
 Multiply equation by 6  
 $2(4x-1) + 6 = 3(3x-1)$   
 $8x-2+6=9x-3$   
 $8x+4=9x-3$   
 $7=x$   
 $x=7$

7.  $\frac{1}{3}(2x+1) = \frac{1}{2}(x+4)$   
 Multiply equation by 6  
 $2(2x+1) = 3(x+4)$   
 $4x+2=3x+12$   
 $x=10$

8.  $\frac{4x-1}{5} = \frac{2x+3}{2}$   
 Cross multiply  
 $5(4x-1) = 2(2x+3)$   
 $20x-5=4x+6$   
 $16x=11$   
 $x = \frac{11}{16}$   
 $(x = -8.5)$

Question 3

Solve each of the following fractional equations.

1.  $\frac{x-2}{x+2} = \frac{2}{3}$   $x = 10$

2.  $\frac{3x-1}{x+1} = \frac{5}{2}$   $x = 7$

3.  $\frac{2x-4}{3x-1} = \frac{3}{4}$   $x = -13$

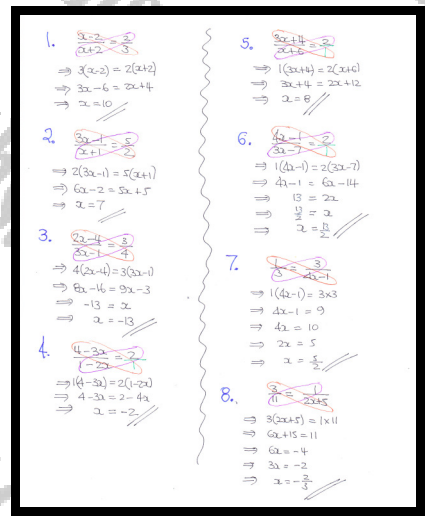
4.  $\frac{4-3x}{1-2x} = 2$   $x = -2$

5.  $\frac{3x+4}{x+6} = 2$   $x = 8$

6.  $\frac{4x-1}{3x-7} = 2$   $x = \frac{13}{2}$

7.  $\frac{1}{3} = \frac{3}{4x-1}$   $x = \frac{5}{2}$

8.  $\frac{3}{11} = \frac{1}{2x+5}$   $x = -\frac{2}{3}$



Question 4

Solve each of the following fractional equations.

1.  $\frac{2}{3} = \frac{1-3x}{4x+3}$   $x=9$

2.  $\frac{1}{2x+5} + 3 = 0$   $x = -\frac{8}{3}$

3.  $\frac{4}{4x+1} - 3 = 0$   $x = \frac{1}{12}$

4.  $\frac{3}{11} = \frac{1}{2x+5}$   $x = -\frac{2}{3}$

5.  $\frac{2x-1}{x+1} = \frac{2x+3}{x-5}$   $x = \frac{1}{8}$

6.  $\frac{x+1}{x-2} = \frac{x-3}{x+4}$   $x = \frac{1}{5}$

7.  $\frac{2x}{x-3} = \frac{2x+3}{x}$   $x = -3$

8.  $\frac{3}{x+1} + 1 = \frac{4x}{x+1}$   $x = \frac{4}{3}$

1.  $\frac{2}{3} = \frac{1-3x}{4x+3}$   
 $\Rightarrow \frac{2(4x+3)}{3} = \frac{1-3x}{1}$   
 $\Rightarrow 2(4x+3) = 3(1-3x)$   
 $\Rightarrow 8x+6 = 3-9x$   
 $\Rightarrow 9 = -2x$   
 $\Rightarrow x = -\frac{9}{2}$

2.  $\frac{1}{2x+5} + 3 = 0$   
 $\Rightarrow \frac{1}{2x+5} = -3$   
 $\Rightarrow \frac{1}{2x+5} = \frac{-3(2x+5)}{2x+5}$   
 $\Rightarrow 1 = -6x-15$   
 $\Rightarrow -16 = -6x$   
 $\Rightarrow -\frac{16}{6} = -x$   
 $\Rightarrow x = \frac{8}{3}$

3.  $\frac{4}{4x+1} - 3 = 0$   
 $\Rightarrow \frac{4}{4x+1} = 3$   
 $\Rightarrow \frac{4}{4x+1} = \frac{3(4x+1)}{4x+1}$   
 $\Rightarrow 4 = 12x+3$   
 $\Rightarrow 1 = 12x$   
 $\Rightarrow x = \frac{1}{12}$

4.  $\frac{3}{11} = \frac{1}{2x+5}$   
 $\Rightarrow \frac{3(2x+5)}{11} = \frac{1}{1}$   
 $\Rightarrow 3(2x+5) = 11$   
 $\Rightarrow 6x+15 = 11$   
 $\Rightarrow 6x = -4$   
 $\Rightarrow x = -\frac{2}{3}$

5.  $\frac{2x-1}{x+1} = \frac{2x+3}{x-5}$   
 $\Rightarrow (2x-1)(x-5) = (x+1)(2x+3)$   
 $\Rightarrow 2x^2-10x-2x+5 = 2x^2+3x+2x+3$   
 $\Rightarrow -12x+5 = 5x+3$   
 $\Rightarrow 2 = 16x$   
 $\Rightarrow \frac{2}{16} = x$   
 $\Rightarrow x = \frac{1}{8}$

6.  $\frac{x+1}{x-2} = \frac{x-3}{x+4}$   
 $\Rightarrow (x+1)(x+4) = (x-2)(x-3)$   
 $\Rightarrow x^2+4x+2x+4 = x^2-3x-2x+6$   
 $\Rightarrow 5x+4 = -5x+6$   
 $\Rightarrow 10x = 2$   
 $\Rightarrow x = \frac{2}{10} = \frac{1}{5}$

7.  $\frac{2x}{x-3} = \frac{2x+3}{x}$   
 $\Rightarrow 2x^2 = (x-3)(2x+3)$   
 $\Rightarrow 2x^2 = 2x^2+3x-6x-9$   
 $\Rightarrow 0 = -3x-9$   
 $\Rightarrow 3x = -9$   
 $\Rightarrow x = -3$

8.  $\frac{3}{x+1} + 1 = \frac{4x}{x+1}$   
 $\Rightarrow 1 = \frac{4x}{x+1} - \frac{3}{x+1}$   
 $\Rightarrow 1 = \frac{4x-3}{x+1}$   
 $\Rightarrow \frac{4x-3}{x+1} = \frac{1(x+1)}{x+1}$   
 $\Rightarrow 4x-3 = x+1$   
 $\Rightarrow 3x = 4$   
 $\Rightarrow x = \frac{4}{3}$

Question 5

Solve each of the following fractional equations.

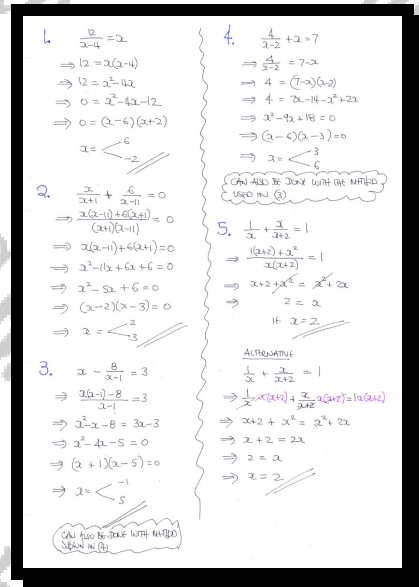
1.  $\frac{12}{x-4} = x$   $x = -2, 6$

2.  $\frac{x}{x+1} + \frac{6}{x-11} = 0$   $x = 2, 3$

3.  $x - \frac{8}{x-1} = 3$   $x = -1, 5$

4.  $\frac{4}{x-2} + x = 7$   $x = 3, 6$

5.  $\frac{1}{x} + \frac{x}{x+2} = 1$   $x = 2$



Question 6

Solve each of the following fractional equations.

1.  $x = 2 + \frac{12}{x-1}$

$x = -2, 5$

2.  $\frac{8}{x} = 1 + \frac{9}{x+1}$

$x = -4, 2$

3.  $x = 4 + \frac{10}{x-1}$

$x = -1, 6$

4.  $\frac{4}{x} + \frac{7x}{x+6} = 5$

$x = 1, 12$

5.  $\frac{4}{x+1} + \frac{x}{2x-3} = 1$

$x = 1, 9$

(1)  $x = 2 + \frac{12}{x-1}$   
 $\Rightarrow x-2 = \frac{12}{x-1}$   
 $\Rightarrow (x-2)(x-1) = 12$   
 $\Rightarrow x^2 - 3x + 2 = 12$   
 $\Rightarrow x^2 - 3x - 10 = 0$   
 $\Rightarrow (x+2)(x-5) = 0$   
 $\Rightarrow x = -2, 5$

(2)  $\frac{8}{x} = 1 + \frac{9}{x+1}$   
 $\Rightarrow \frac{8}{x} - \frac{9}{x+1} = 1$   
 $\Rightarrow \frac{8(x+1) - 9x}{x(x+1)} = 1$   
 $\Rightarrow \frac{8x+8-9x}{x^2+x} = 1$   
 $\Rightarrow \frac{-x+8}{x^2+x} = 1$   
 $\Rightarrow 0 - x = x^2 + x$   
 $\Rightarrow 0 = x^2 + 2x - 8$   
 $\Rightarrow 0 = (x-2)(x+4)$   
 $\Rightarrow x = 2, -4$

(3)  $x = 4 + \frac{10}{x-1}$   
 $\Rightarrow x-4 = \frac{10}{x-1}$   
 $\Rightarrow (x-4)(x-1) = 10$   
 $\Rightarrow x^2 - 5x + 4 = 10$   
 $\Rightarrow x^2 - 5x - 6 = 0$   
 $\Rightarrow (x-6)(x+1) = 0$   
 $\Rightarrow x = 6, -1$

(4)  $\frac{4}{x} + \frac{7x}{x+6} = 5$   
 $\Rightarrow \frac{4(x+6) + 7x^2}{x(x+6)} = 5$   
 $\Rightarrow \frac{4x+24+7x^2}{x^2+6x} = 5$   
 $\Rightarrow 7x^2 + 4x + 24 = 5(x^2 + 6x)$   
 $\Rightarrow 7x^2 + 4x + 24 = 5x^2 + 30x$   
 $\Rightarrow 2x^2 - 26x + 24 = 0$   
 $\Rightarrow 2x^2 - 13x + 12 = 0$   
 $\Rightarrow (x-1)(x-12) = 0$   
 $\Rightarrow x = 1, 12$

(5)  $\frac{4}{x+1} + \frac{x}{2x-3} = 1$   
 $\Rightarrow \frac{4(2x-3) + x(x+1)}{(x+1)(2x-3)} = 1$   
 $\Rightarrow \frac{8x-12+x^2+x}{(x+1)(2x-3)} = 1$   
 $\Rightarrow x^2 + 9x - 12 = (x+1)(2x-3)$   
 $\Rightarrow x^2 + 9x - 12 = 2x^2 - 2x - 3$   
 $\Rightarrow 0 = x^2 - 10x + 9$   
 $\Rightarrow (x-1)(x-9) = 0$   
 $\Rightarrow x = 1, 9$

Question 7

Solve each of the following fractional equations.

1.  $\frac{4}{x} + \frac{3}{x-2} = 1$

$x = 1, 8$

2.  $\frac{6}{x} + \frac{7}{x-5} = 2$

$x = \frac{3}{2}, 10$

3.  $\frac{2}{x} - \frac{3}{x+1} = 2$

$x = -2, \frac{1}{2}$

4.  $\frac{8}{x+2} - \frac{6}{x+4} = 1$

$x = 2, -6$

5.  $\frac{4}{x+3} + \frac{3}{2x-1} = 1$

$x = -1, 4$

(1)  $\frac{4}{x} + \frac{3}{x-2} = 1$   
 $\Rightarrow \frac{4(x-2) + 3x}{x(x-2)} = 1$   
 $\Rightarrow \frac{4x-8+3x}{x^2-2x} = 1$   
 $\Rightarrow 7x-8 = x^2-2x$   
 $\Rightarrow 0 = x^2-9x+8$   
 $\Rightarrow 0 = (x-8)(x-1)$   
 $\Rightarrow x = 1, 8$

ALTERNATIVE  
 $\frac{4}{x} + \frac{3}{x-2} = 1$   
 Multiply through by  $x(x-2)$   
 $\Rightarrow 4(x-2) + 3x = 1 \cdot x(x-2)$   
 $\Rightarrow 4x-8+3x = x^2-2x$   
 $\Rightarrow 7x-8 = x^2-2x$   
 $\Rightarrow 0 = x^2-9x+8$   
 $\Rightarrow 0 = (x-8)(x-1)$   
 $\Rightarrow x = 1, 8$

(2)  $\frac{6}{x} + \frac{7}{x-5} = 2$   
 $\Rightarrow \frac{6(x-5) + 7x}{x(x-5)} = 2$   
 $\Rightarrow \frac{6x-30+7x}{x^2-5x} = 2$   
 $\Rightarrow \frac{13x-30}{x^2-5x} = 2$   
 $\Rightarrow 13x-30 = 2x^2-10x$   
 $\Rightarrow 0 = 2x^2-23x+30$   
 $\Rightarrow 0 = (2x-3)(x-10)$   
 $\Rightarrow x = \frac{3}{2}, 10$

ALTERNATIVE  
 $\frac{6}{x} + \frac{7}{x-5} = 2$   
 Multiply through by  $x(x-5)$   
 $\Rightarrow 6(x-5) + 7x = 2x(x-5)$   
 $\Rightarrow 6x-30+7x = 2x^2-10x$   
 $\Rightarrow 13x-30 = 2x^2-10x$   
 $\Rightarrow 0 = 2x^2-23x+30$   
 $\Rightarrow 0 = (2x-3)(x-10)$   
 $\Rightarrow x = \frac{3}{2}, 10$

Question 8

Solve each of the following fractional equations.

1.  $\frac{1}{2x+1} + \frac{3}{2x+3} = 2$   $x = 0, -1$

2.  $\frac{3}{x+3} - \frac{4}{x-3} = \frac{5x}{x^2-9}$   $x = -\frac{7}{2}$

3.  $\frac{2}{x-3} = 1 + \frac{2}{x-2}$   $x = 1, 4$

4.  $\frac{1}{x-1} + \frac{3}{x+1} = 2$   $x = 0, 2$

5.  $\frac{6}{x(x-2)} = \frac{1}{x-2} + 1$   $x = 3, -2$

1.  $\frac{1}{2x+1} + \frac{3}{2x+3} = 2$   
 $\Rightarrow \frac{(2x+3) + 3(2x+1)}{(2x+1)(2x+3)} = 2$   
 $\Rightarrow \frac{2x+3+6x+3}{4x^2+8x+3} = 2$   
 $\Rightarrow \frac{8x+6}{4x^2+8x+3} = 2$   
 $\Rightarrow 8x+6 = 8x^2+16x+6$   
 $\Rightarrow 0 = 8x^2+8x$   
 $\Rightarrow 0 = 8x(x+1)$   
 $\Rightarrow x = 0, -1$

2.  $\frac{3}{x+3} - \frac{4}{x-3} = \frac{5x}{x^2-9}$   
 $\Rightarrow \frac{3(x-3) - 4(x+3)}{(x+3)(x-3)} = \frac{5x}{(x+3)(x-3)}$   
 $\Rightarrow 3(x-3) - 4(x+3) = 5x$   
 $\Rightarrow 3x-9-4x-12 = 5x$   
 $\Rightarrow -x-21 = 5x$   
 $\Rightarrow -21 = 6x$   
 $\Rightarrow x = -\frac{21}{6}$   
 $\Rightarrow x = -\frac{7}{2}$

3.  $\frac{2}{x-3} = 1 + \frac{2}{x-2}$   
 $\Rightarrow \frac{2}{x-3} - 1 = \frac{2}{x-2}$   
 $\Rightarrow \frac{2 - (x-3)}{x-3} = \frac{2}{x-2}$   
 $\Rightarrow \frac{5-x}{x-3} = \frac{2}{x-2}$   
 $\Rightarrow (5-x)(x-2) = 2(x-3)$   
 $\Rightarrow 5x-10-x^2+2x = 2x-6$   
 $\Rightarrow -x^2+5x-10 = 2x-6$   
 $\Rightarrow -x^2+3x-4 = 0$   
 $\Rightarrow x^2-3x+4 = 0$   
 $\Rightarrow (x-1)(x-4) = 0$   
 $\Rightarrow x = 1, 4$

4.  $\frac{1}{x-1} + \frac{3}{x+1} = 2$   
 $\Rightarrow \frac{(x+1) + 3(x-1)}{(x-1)(x+1)} = 2$   
 $\Rightarrow \frac{x+1+3x-3}{x^2-1} = 2$   
 $\Rightarrow \frac{4x-2}{x^2-1} = 2$   
 $\Rightarrow 4x-2 = 2x^2-2$   
 $\Rightarrow 0 = 2x^2-4x$   
 $\Rightarrow 0 = 2x(x-2)$   
 $\Rightarrow x = 0, 2$

5.  $\frac{6}{x(x-2)} = \frac{1}{x-2} + 1$   
 $\Rightarrow \frac{6}{x(x-2)} - \frac{1}{x-2} = 1$   
 $\Rightarrow \frac{6 - x}{x(x-2)} = 1$   
 $\Rightarrow 6-x = x(x-2)$   
 $\Rightarrow 6-x = x^2-2x$   
 $\Rightarrow 0 = x^2-2x-6$   
 $\Rightarrow 0 = (x+2)(x-3)$   
 $\Rightarrow x = -2, 3$

Question 9

Solve each of the following fractional equations.

1.  $\frac{40}{x(x+2)} = 1 + \frac{1}{x+2}$

$x = 5, -8$

2.  $\frac{1}{2x+1} + \frac{3}{2x+3} = 2$

$x = 0, -1$

3.  $\frac{4}{2x+1} - \frac{1}{3x-1} = 5$

$x = 0, \frac{1}{6}$

4.  $\frac{3}{x-1} + \frac{2}{x+1} = \frac{5}{x^2-1}$

$x = \frac{4}{5}$

5.  $\frac{6}{x+1} - \frac{1}{x} = 2$

$x = \frac{1}{2}, 1$

1.  $\frac{40}{x(x+2)} = 1 + \frac{1}{x+2}$   
 $\Rightarrow \frac{40}{x(x+2)} - 1 - \frac{1}{x+2} = 0$   
 $\Rightarrow \frac{40 - x(x+2) - (x+2)}{x(x+2)} = 0$   
 $\Rightarrow 40 - x^2 - 2x - x - 2 = 0$   
 $\Rightarrow 40 - x^2 - 3x - 2 = 0$   
 $\Rightarrow 0 = -x^2 - 3x + 38$   
 $\Rightarrow 0 = (x-5)(x+8)$   
 $x = 5, -8$

2.  $\frac{1}{2x+1} + \frac{3}{2x+3} = 2$   
 $\Rightarrow \frac{1(2x+3) + 3(2x+1)}{(2x+1)(2x+3)} = 2$   
 $\Rightarrow \frac{2x+3+6x+3}{4x^2+8x+3} = 2$   
 $\Rightarrow \frac{8x+6}{4x^2+8x+3} = 2$   
 $\Rightarrow 8x+6 = 8x^2+16x+6$   
 $\Rightarrow 0 = 8x^2+8x$   
 $\Rightarrow 0 = 8x(x+1)$   
 $x = 0, -1$

3.  $\frac{4}{2x+1} - \frac{1}{3x-1} = 5$   
 $\Rightarrow \frac{4(3x-1) - (2x+1)}{(2x+1)(3x-1)} = 5$   
 $\Rightarrow \frac{12x-4-2x-1}{6x^2+4x-1} = 5$   
 $\Rightarrow \frac{10x-5}{6x^2+4x-1} = 5$   
 $\Rightarrow 10x-5 = 30x^2+20x-5$   
 $\Rightarrow 0 = 30x^2+10x$   
 $\Rightarrow 0 = 5x(6x+2)$   
 $x = 0, -\frac{1}{3}$

4.  $\frac{3}{x-1} + \frac{2}{x+1} = \frac{5}{x^2-1}$   
 $\Rightarrow \frac{3(x+1) + 2(x-1)}{(x-1)(x+1)} = \frac{5}{(x-1)(x+1)}$   
 $\Rightarrow \frac{3x+3+2x-2}{x^2-1} = \frac{5}{x^2-1}$   
 $\Rightarrow \frac{5x+1}{x^2-1} = \frac{5}{x^2-1}$   
 $\Rightarrow 5x+1 = 5$   
 $\Rightarrow 5x = 4$   
 $\Rightarrow x = \frac{4}{5}$

5.  $\frac{6}{x+1} - \frac{1}{x} = 2$   
 $\Rightarrow \frac{6x - (x+1)}{x(x+1)} = 2$   
 $\Rightarrow \frac{6x-x-1}{x^2+x} = 2$   
 $\Rightarrow \frac{5x-1}{x^2+x} = 2$   
 $\Rightarrow 5x-1 = 2x^2+2x$   
 $\Rightarrow 0 = 2x^2-3x+1$   
 $\Rightarrow 0 = (2x-1)(x-1)$   
 $\Rightarrow x = \frac{1}{2}, 1$



Question 10

Solve each of the following fractional equations.

1.  $\frac{2x}{x-2} - \frac{4x}{x+1} = 3$

$x = -\frac{2}{5}, 3$

2.  $\frac{1}{x+2} = 3x+4$

$x = -\frac{7}{3}, -1$

3.  $\frac{3}{x+2} + \frac{1}{x-3} = \frac{3}{2}$

$x = -\frac{1}{3}, 4$

4.  $\frac{7}{x+2} + \frac{1}{x-1} = 4$

$x = -\frac{1}{2}, \frac{3}{2}$

5.  $\frac{3}{x+1} + \frac{2}{2x-3} = 3$

$x = -\frac{1}{6}, 2$

The image shows handwritten solutions for the five equations. Each solution follows a similar process: finding a common denominator, combining terms, and solving the resulting quadratic equation. The solutions are boxed in the original image.

**1)**  $\frac{2x}{x-2} - \frac{4x}{x+1} = 3$   
 $\Rightarrow \frac{2x(x+1) - 4x(x-2)}{(x-2)(x+1)} = 3$   
 $\Rightarrow \frac{2x^2 + 2x - 4x^2 + 8x}{(x-2)(x+1)} = 3$   
 $\Rightarrow \frac{-2x^2 + 10x}{(x-2)(x+1)} = 3$   
 $\Rightarrow \frac{10x - 2x^2}{x^2 - x - 2} = 3$   
 $\Rightarrow 10x - 2x^2 = 3(x^2 - x - 2)$   
 $\Rightarrow 10x - 2x^2 = 3x^2 - 3x - 6$   
 $\Rightarrow 0 = 5x^2 - 13x - 6$   
 $\Rightarrow 0 = (5x+3)(x-2)$   
 $x = -\frac{3}{5}$

**b)**  $\frac{1}{x+2} = 3x+4$   
 $\Rightarrow \frac{1}{x+2} = \frac{3x+4}{1}$   
 $\Rightarrow (x+2)(3x+4) = 1$   
 $\Rightarrow 3x^2 + 4x + 6x + 8 = 1$   
 $\Rightarrow 3x^2 + 10x + 7 = 0$   
 $\Rightarrow (3x+7)(x+1) = 0$   
 $x = -\frac{7}{3}$

**c)**  $\frac{3}{x+2} + \frac{1}{x-3} = \frac{3}{2}$   
 $\Rightarrow \frac{3(x-3) + (x+2)}{(x+2)(x-3)} = \frac{3}{2}$   
 $\Rightarrow \frac{4x-9+x+2}{(x+2)(x-3)} = \frac{3}{2}$   
 $\Rightarrow \frac{5x-7}{(x+2)(x-3)} = \frac{3}{2}$   
 $\Rightarrow \frac{5x-7}{x^2-x-6} = \frac{3}{2}$   
 $\Rightarrow 2(5x-7) = 3(x^2-x-6)$   
 $\Rightarrow 10x-14 = 3x^2-3x-18$   
 $\Rightarrow 3x^2-13x-4 = 0$   
 $\Rightarrow (3x+1)(x-4) = 0$   
 $x = -\frac{1}{3}$

**d)**  $\frac{7}{x+2} + \frac{1}{x-1} = 4$   
 $\Rightarrow \frac{7(x-1) + (x+2)}{(x+2)(x-1)} = 4$   
 $\Rightarrow \frac{7x-7+x+2}{(x+2)(x-1)} = 4$   
 $\Rightarrow \frac{8x-5}{x^2+x-2} = 4$   
 $\Rightarrow 8x-5 = 4x^2+4x-8$   
 $\Rightarrow 0 = 4x^2-4x-3$   
 $\Rightarrow 0 = (2x-3)(2x+1)$   
 $x = \frac{3}{2}$

**e)**  $\frac{3}{x+1} + \frac{2}{2x-3} = 3$   
 $\Rightarrow \frac{3(2x-3) + 2(x+1)}{(x+1)(2x-3)} = 3$   
 $\Rightarrow \frac{6x-9+2x+2}{(x+1)(2x-3)} = 3$   
 $\Rightarrow \frac{8x-7}{(x+1)(2x-3)} = 3$   
 $\Rightarrow 8x-7 = 3(2x^2-3x-3)$   
 $\Rightarrow 8x-7 = 6x^2-9x-9$   
 $\Rightarrow 0 = 6x^2-17x-2$   
 $\Rightarrow 0 = (6x+1)(x-2)$   
 $x = -\frac{1}{6}$

Question 11

Solve each of the following fractional equations.

1.  $\frac{1}{x-2} - \frac{2}{2x-3} = 1$

$x = 1, \frac{5}{2}$

2.  $\frac{1}{x+1} = \frac{6x}{11x-1}$

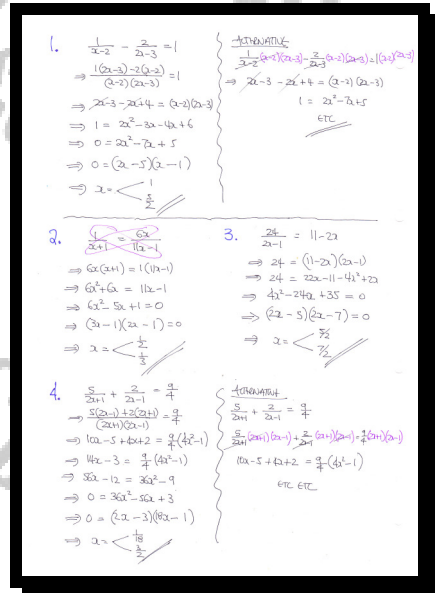
$x = \frac{1}{2}, \frac{1}{3}$

3.  $\frac{24}{2x-1} = 11 - 2x$

$x = \frac{5}{2}, \frac{7}{2}$

4.  $\frac{5}{2x+1} + \frac{2}{2x-1} = \frac{9}{4}$

$x = \frac{3}{2}, \frac{1}{18}$



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# LINEAR SIMULTANEOUS EQUATIONS

Created by T. Madas

**Question 1**

Solve each of the following sets of simultaneous equations.

1.  $2x + y = 11$

$3x - y = 14$

$x = 5, y = 1$

2.  $2x + 3y = 18$

$x - y = 4$

$x = 6, y = 2$

3.  $3k + 2h = 19$

$4k - h = 18$

$k = 5, h = 2$

4.  $3a + 2b = 25$

$a - 3b = 1$

$a = 7, b = 2$

5.  $2p + 3q = 11$

$5p - 4q = 39$

$p = 7, q = -1$

The image shows handwritten solutions for the five sets of simultaneous equations. Each set is solved using the elimination method. The solutions are as follows:

- $$\begin{array}{r} 2x + y = 11 \quad (x1) \\ 3x - y = 14 \quad (x2) \\ \hline 5x = 25 \\ x = 5 \end{array}$$

$$\begin{array}{r} 2x + y = 11 \\ 2(5) + y = 11 \\ 10 + y = 11 \\ y = 1 \end{array}$$

$\therefore x = 5, y = 1$
- $$\begin{array}{r} 2x + 3y = 18 \quad (x1) \\ 2x - y = 4 \quad (x2) \\ \hline 4y = 14 \\ y = 3.5 \end{array}$$

$$\begin{array}{r} 2x + 3y = 18 \\ 2x - y = 4 \\ \hline 4y = 14 \\ y = 3.5 \end{array}$$

$\therefore x = 6, y = 2$
- $$\begin{array}{r} 3k + 2h = 19 \quad (x1) \\ 4k - h = 18 \quad (x2) \\ \hline 15k + 2h = 19 \\ 15k - 4h = 18 \\ \hline 6h = 1 \\ h = 0.167 \end{array}$$

$$\begin{array}{r} 3k + 2h = 19 \\ 3k + 2(0.167) = 19 \\ 3k + 0.334 = 19 \\ 3k = 18.666 \\ k = 6.222 \end{array}$$

$\therefore k = 5, h = 2$
- $$\begin{array}{r} 3a + 2b = 25 \quad (x1) \\ a - 3b = 1 \quad (x2) \\ \hline 9a + 6b = 75 \\ a - 3b = 1 \\ \hline 10a = 76 \\ a = 7.6 \end{array}$$

$$\begin{array}{r} 3a + 2b = 25 \\ 3(7.6) + 2b = 25 \\ 22.8 + 2b = 25 \\ 2b = 2.2 \\ b = 1.1 \end{array}$$

$\therefore a = 7, b = 2$
- $$\begin{array}{r} 2p + 3q = 11 \quad (x1) \\ 5p - 4q = 39 \quad (x2) \\ \hline 10p + 6q = 55 \\ 10p - 12q = 78 \\ \hline 18q = -23 \\ q = -1.278 \end{array}$$

$$\begin{array}{r} 2p + 3q = 11 \\ 2p + 3(-1.278) = 11 \\ 2p - 3.834 = 11 \\ 2p = 14.834 \\ p = 7.417 \end{array}$$

$\therefore p = 7, q = -1$

**Question 2**

Solve each of the following sets of simultaneous equations.

1.  $c + 2d = 13$

$2c - 5d = -1$

$c = 7, d = 3$

2.  $4x + 3y = 14$

$2x - y = 2$

$x = 2, y = 2$

3.  $4k - h = 15$

$5k + 2h = 22$

$k = 4, h = 1$

4.  $3a + 2b = 18$

$a + 5 = 3b$

$a = 4, b = 3$

5.  $3p + 2q = 13$

$5p = 3q + 9$

$p = 3, q = 2$

The image shows handwritten solutions for the five sets of simultaneous equations. Each problem is solved using the elimination method. For example, for problem 1, the equations are  $c + 2d = 13$  (x5) and  $2c - 5d = -1$  (x2), leading to  $5c + 10d = 65$  and  $4c - 10d = -2$ . Adding these gives  $9c = 63$ , so  $c = 7$ . Substituting  $c = 7$  into the first equation gives  $7 + 2d = 13$ , so  $2d = 6$  and  $d = 3$ . Similar steps are shown for the other four problems, resulting in the solutions  $(x=2, y=2)$ ,  $(k=4, h=1)$ ,  $(a=4, b=3)$ , and  $(p=3, q=2)$ .

**Question 3**

Solve each of the following sets of simultaneous equations.

1.  $c + 2d = 9$   
 $2c = 5d + 9$   $c = 7, d = 1$

2.  $2a + b = 10$   
 $2a + 3b = 14$   $a = 4, b = 2$

3.  $3x + 2y = 14$   
 $2x - y = 7$   $x = 4, y = 1$

4.  $3a + 2b = 17$   
 $5a - 4b = 21$   $a = 5, b = 1$

5.  $m + n = 5$   
 $5m - 3n = 1$   $m = 2, n = 3$

Handwritten solutions for questions 1, 2, and 3:

(1)  $c + 2d = 9$   
 $2c = 5d + 9$   
 $2c - 5d = 9$  (x(-2))  
 $-2c - 4d = -18$   
 $2c - 5d = 9$   
 $-9d = -27$   
 $d = 3$   
 $c + 2(3) = 9$   
 $c + 6 = 9$   
 $c = 3$

(2)  $2a + b = 10$  (x(-1))  
 $2a + 3b = 14$   
 $-2a - b = -10$   
 $2a + 3b = 14$   
 $-4b = -24$   
 $b = 6$   
 $2a + 6 = 10$   
 $2a = 4$   
 $a = 2$

(3)  $3x + 2y = 14$   
 $2x - y = 7$  (x2)  
 $6x + 4y = 28$   
 $4x - 2y = 14$   
 $2x - 2y = 14$   
 $4x - 2y = 14$   
 $2x = 0$   
 $x = 0$   
 $2(0) - y = 7$   
 $-y = 7$   
 $y = -7$

Handwritten solutions for questions 4 and 5:

(4)  $3a + 2b = 17$  (x2)  
 $5a - 4b = 21$   
 $6a + 4b = 34$   
 $5a - 4b = 21$   
 $11a = 55$   
 $a = 5$   
 $3(5) + 2b = 17$   
 $15 + 2b = 17$   
 $2b = 2$   
 $b = 1$

(5)  $m + n = 5$  (x5)  
 $5m - 3n = 1$   
 $5m + 5n = 25$   
 $5m - 3n = 1$   
 $8n = 24$   
 $n = 3$   
 $m + 3 = 5$   
 $m = 2$

**Question 4**

Solve each of the following sets of simultaneous equations.

1.  $3P - 4Q = 0$

$5P - Q = 17$

$P = 4, Q = 3$

2.  $3a + 4b = 13$

$-2a + 3b = 14$

$a = -1, b = 4$

3.  $3n + 2m = 14$

$5n - 7m = 13$

$n = 4, m = 1$

4.  $w + 3v = 19$

$5v - 4w = 9$

$v = 5, w = 4$

5.  $3c + 2d = 8$

$6c + 7d = 10$

$c = 4, d = -2$

The image shows handwritten solutions for the five sets of simultaneous equations. Each problem is solved using the elimination method. For example, for problem 1, the equations are  $3P - 4Q = 0$  and  $5P - Q = 17$ . The second equation is multiplied by 4 to get  $20P - 4Q = 68$ . Subtracting the first equation from this gives  $17P = 68$ , so  $P = 4$ . Substituting  $P = 4$  into the first equation gives  $12 - 4Q = 0$ , so  $Q = 3$ . Similar steps are shown for the other four problems, leading to the solutions  $(a, b) = (-1, 4)$ ,  $(n, m) = (4, 1)$ ,  $(v, w) = (5, 4)$ , and  $(c, d) = (4, -2)$ .

Question 5

Solve each of the following sets of simultaneous equations.

1.  $7k + 2h = 22$

$3k - 4h = 7$

$k = 3, h = \frac{1}{2}$

2.  $4p - 2q = 10$

$7p - q = 10$

$p = 1, q = -3$

3.  $9t + 2r = -12$

$3r - 2t = 13$

$t = -2, r = 3$

4.  $x + 2y = 10$

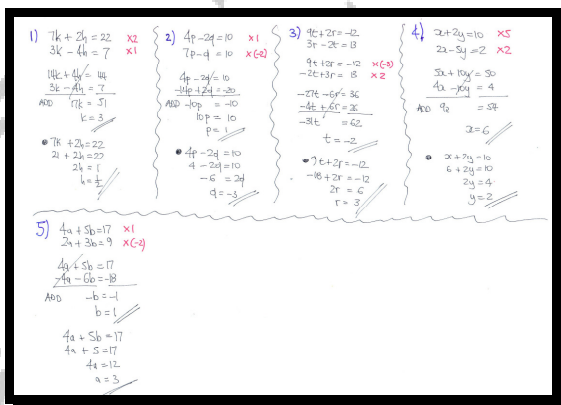
$2x - 5y = 2$

$x = 6, y = 2$

5.  $4a + 5b = 17$

$2a + 3b = 9$

$a = 3, b = 1$





**Question 6**

Solve each of the following sets of simultaneous equations.

1.  $4M + N = 13$

$$2M + 3N = 19$$

$$M = 2, N = 5$$

2.  $7t + 2v = 15$

$$3t - 4v = 4$$

$$t = 2, v = \frac{1}{2}$$

3.  $5x + 2y = 9$

$$3x - 4y = 8$$

$$x = 2, y = -\frac{1}{2}$$

4.  $5P + 2Q = 16$

$$3P - 4Q = 7$$

$$P = 3, Q = \frac{1}{2}$$

5.  $4x - y = 8$

$$6x - 4y = 17$$

$$x = \frac{3}{2}, y = -2$$

**Question 7**

Solve each of the following sets of simultaneous equations.

1.  $3k - 2h = 3$

$$5k - h = 12$$

$$k = 3, h = 3$$

2.  $4c + 5d = 16$

$$-2c + 3d = 14$$

$$c = -1, d = 4$$

3.  $5u - 8v = 6$

$$2u - 5v = 6$$

$$u = -2, v = -2$$

4.  $3K + 2L = 2$

$$2K - 5L = 14$$

$$K = 2, L = -2$$

5.  $3a - 5b = 8$

$$5a + 4b = 38$$

$$a = 6, b = 2$$

**Question 7**

Solve each of the following sets of simultaneous equations.

1.  $x + 4y = 3$

$3x + 8y = 10$

$$x = 4, y = -\frac{1}{4}$$

2.  $3x + 8y = 16$

$7x - 6y = 25$

$$x = 4, y = \frac{1}{4}$$

3.  $x + 2y = 5$

$y = 5x - 1$

$$x = \frac{7}{11}, y = \frac{24}{11}$$

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# NON LINEAR SIMULTANEOUS EQUATIONS

Created by T. Madas

**Question 1**

Solve each of the following sets of simultaneous equations.

a)  $y = 2x + 5$   
 $y = x^2 - 3$

b)  $y = 2x + 3$   
 $y = x^2 - x - 7$

c)  $y = 4x$   
 $y = x^2 + 3$

d)  $x = 2y$   
 $x^2 + 3xy = 10$

e)  $y = 3x - 7$   
 $y = x^2 - 3x + 2$

$(-2, 1), (4, 13)$ ,  $(-2, -1), (5, 13)$ ,  $(1, 4), (3, 12)$ ,  $(2, 1), (-2, -1)$ ,  $(3, 2)$

Handwritten solutions for the simultaneous equations:

(a)  $y = 2x + 5$   
 $y = x^2 - 3$   
 $x^2 - 2x - 8 = 0$   
 $(x - 4)(x + 2) = 0$   
 $x = 4$  or  $x = -2$   
 $y = 13$  or  $y = 1$   
 $\therefore (4, 13)$  &  $(-2, 1)$

(b)  $y = 2x + 3$   
 $y = x^2 - x - 7$   
 $x^2 - 3x - 10 = 0$   
 $(x - 5)(x + 2) = 0$   
 $x = 5$  or  $x = -2$   
 $y = 13$  or  $y = -1$   
 $\therefore (5, 13)$  &  $(-2, -1)$

(c)  $y = 4x$   
 $y = x^2 + 3$   
 $x^2 - 4x - 3 = 0$   
 $(x - 4.5)(x + 0.5) = 0$   
 $x = 4.5$  or  $x = -0.5$   
 $y = 18$  or  $y = -2$   
 $\therefore (4.5, 18)$  &  $(-0.5, -2)$

(d)  $x = 2y$   
 $x^2 + 3xy = 10$   
 $4y^2 + 6y^2 = 10$   
 $10y^2 = 10$   
 $y^2 = 1$   
 $y = 1$  or  $y = -1$   
 $x = 2$  or  $x = -2$   
 $\therefore (2, 1)$  &  $(-2, -1)$

(e)  $y = 3x - 7$   
 $y = x^2 - 3x + 2$   
 $x^2 - 6x + 9 = 0$   
 $(x - 3)^2 = 0$   
 $x = 3$   
 $y = 2$   
 $\therefore (3, 2)$

**Question 2**

Solve each of the following sets of simultaneous equations.

a)  $y = x^2 - 5x + 4$   
 $y = x - 1$

b)  $y = 9 - 3x$   
 $y^2 = 9(x - 1)$

c)  $x + y = 3$   
 $x^2 + y = 15$

d)  $x + y - 3 = 0$   
 $y = x^2 + 3x - 2$

e)  $2x + y = 5$   
 $2x^2 - 3x = y + 16$

$(1,0), (5,4)$ ,  $(2,3), (5,-6)$ ,  $(-3,6), (4,-1)$ ,  $(1,2), (-5,8)$ ,  $(\frac{7}{2}, -2), (-3,11)$

The image shows handwritten solutions for the five parts of Question 2. Each part is solved by substituting the linear equation into the quadratic equation, resulting in a quadratic equation in one variable. The solutions are then found using the quadratic formula or factoring, and the corresponding y-values are determined from the linear equation.

- (a)**  $y = x^2 - 5x + 4$  and  $y = x - 1$ . Substituting  $y = x - 1$  into the quadratic gives  $x^2 - 6x + 5 = 0$ , which factors to  $(x-1)(x-5) = 0$ . Solutions are  $x = 1$  or  $x = 5$ . Corresponding  $y$  values are  $y = 0$  or  $y = 4$ . Solutions:  $(1,0)$  and  $(5,4)$ .
- (b)**  $y = 9 - 3x$  and  $y^2 = 9(x - 1)$ . Substituting  $y = 9 - 3x$  into the quadratic gives  $(9 - 3x)^2 = 9(x - 1)$ , which simplifies to  $x^2 - 6x + 9 = x - 1$ , or  $x^2 - 7x + 10 = 0$ . This factors to  $(x-2)(x-5) = 0$ . Solutions are  $x = 2$  or  $x = 5$ . Corresponding  $y$  values are  $y = 3$  or  $y = -6$ . Solutions:  $(2,3)$  and  $(5,-6)$ .
- (c)**  $x + y = 3$  and  $x^2 + y = 15$ . Substituting  $y = 3 - x$  into the quadratic gives  $x^2 + 3 - x = 15$ , or  $x^2 - x - 12 = 0$ . This factors to  $(x-4)(x+3) = 0$ . Solutions are  $x = 4$  or  $x = -3$ . Corresponding  $y$  values are  $y = -1$  or  $y = 6$ . Solutions:  $(4,-1)$  and  $(-3,6)$ .
- (d)**  $x + y - 3 = 0$  and  $y = x^2 + 3x - 2$ . Substituting  $y = 3 - x$  into the quadratic gives  $(3 - x)^2 = x^2 + 3x - 2$ , which simplifies to  $9 - 6x + x^2 = x^2 + 3x - 2$ , or  $-9x + 11 = 0$ . Solution is  $x = \frac{11}{9}$ . Corresponding  $y$  value is  $y = 3 - \frac{11}{9} = \frac{16}{9}$ . Solution:  $(\frac{11}{9}, \frac{16}{9})$ .
- (e)**  $2x + y = 5$  and  $2x^2 - 3x = y + 16$ . Substituting  $y = 5 - 2x$  into the quadratic gives  $2x^2 - 3x = 5 - 2x + 16$ , or  $2x^2 - x - 21 = 0$ . This factors to  $(2x-7)(x+3) = 0$ . Solutions are  $x = \frac{7}{2}$  or  $x = -3$ . Corresponding  $y$  values are  $y = -2$  or  $y = 11$ . Solutions:  $(\frac{7}{2}, -2)$  and  $(-3, 11)$ .

**Question 3**

Solve each of the following sets of simultaneous equations.

a)  $x^2 - 3y + 11 = 0$   
 $2x - y + 1 = 0$

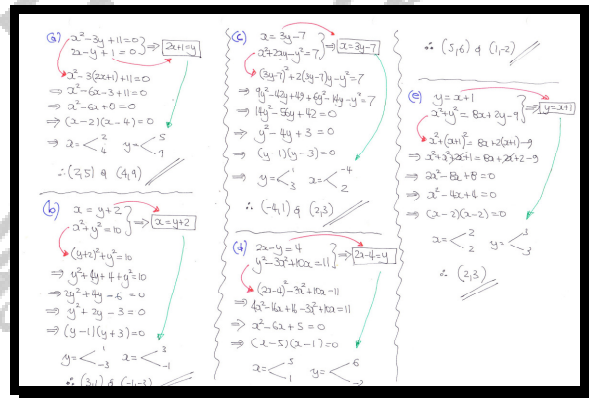
b)  $x = y + 2$   
 $x^2 + y^2 = 10$

c)  $x = 3y - 7$   
 $x^2 + 2xy - y^2 = 7$

d)  $2x - y = 4$   
 $y^2 - 3x^2 + 10x = 11$

e)  $y = x + 1$   
 $x^2 + y^2 = 8x + 2y - 9$

$(2,5), (4,9)$ ,  $(-1,-3), (3,1)$ ,  $(2,3), (-4,1)$ ,  $(1,-2), (5,6)$ ,  $(2,3)$



**Question 4**

Solve each of the following sets of simultaneous equations.

a)  $x - y = 3$   
 $x^2 + y^2 = 89$

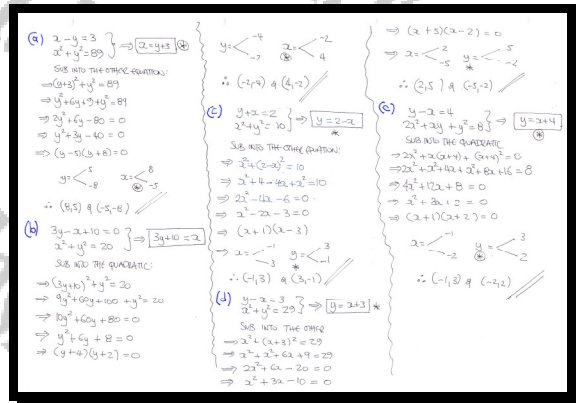
b)  $3y - x + 10 = 0$   
 $x^2 + y^2 = 20$

c)  $y + x = 2$   
 $x^2 + y^2 = 10$

d)  $y - x = 3$   
 $x^2 + y^2 = 29$

e)  $y - x = 4$   
 $2x^2 + xy + y^2 = 8$

$(8,5), (-5, -8)$ ,  $(4, -2), (-2, -4)$ ,  $(-1, 3), (3, -1)$ ,  $(-5, -2), (2, 5)$ ,  $(-1, 3), (-2, 2)$





**Question 5**

Solve each of the following sets of simultaneous equations.

a)  $x + y = 2$   
 $3x^2 + y^2 = 2(x+1)$

b)  $x = 2y + 1$   
 $y^2 + 8 = 3xy$

c)  $x + 2y = 3$   
 $4y^2 - x^2 = 33$

d)  $2y + x = 3$   
 $x^2 + 4y^2 = 2y + 18$

e)  $5x + y = 7$   
 $3x^2 + y^2 = 21$

$(1,1), \left(\frac{1}{2}, \frac{3}{2}\right), (3,1), \left(-\frac{11}{5}, -\frac{8}{5}\right), \left(-4, \frac{7}{2}\right), \left(4, -\frac{1}{2}\right), \left(-\frac{3}{2}, \frac{9}{4}\right), (2,-3), \left(\frac{1}{2}, \frac{9}{2}\right)$

The image shows handwritten solutions for the five sets of simultaneous equations. Each part (a-e) is solved by substituting one equation into the other to form a quadratic equation in one variable. The solutions are then found using the quadratic formula or factoring.

- Part (a):** Substitutes  $y = 2 - x$  into  $3x^2 + y^2 = 2(x+1)$  to get  $4x^2 - 4x + 2 = 0$ , leading to solutions  $x = \frac{1}{2}$  and  $x = 1$ .
- Part (b):** Substitutes  $x = 2y + 1$  into  $y^2 + 8 = 3xy$  to get  $4y^2 - 3y - 8 = 0$ , leading to solutions  $y = 2$  and  $y = -\frac{4}{3}$ .
- Part (c):** Substitutes  $x = 3 - 2y$  into  $4y^2 - x^2 = 33$  to get  $12y^2 - 12y - 33 = 0$ , leading to solutions  $y = \frac{1}{2}$  and  $y = -\frac{3}{2}$ .
- Part (d):** Substitutes  $x = 3 - 2y$  into  $x^2 + 4y^2 = 2y + 18$  to get  $5y^2 - 12y + 9 = 0$ , leading to solutions  $y = \frac{3}{2}$  and  $y = \frac{3}{5}$ .
- Part (e):** Substitutes  $y = 7 - 5x$  into  $3x^2 + y^2 = 21$  to get  $28x^2 - 70x + 28 = 0$ , leading to solutions  $x = \frac{1}{2}$  and  $x = \frac{3}{2}$ .

**Question 6**

Solve each of the following sets of simultaneous equations.

a)  $2y - x = 6$   
 $x^2 + y^2 = 20$

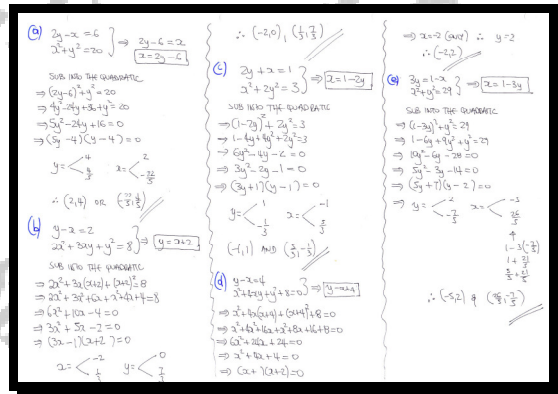
b)  $y - x = 2$   
 $2x^2 + 3xy + y^2 = 8$

c)  $2y + x = 1$   
 $x^2 + 2y^2 = 3$

d)  $y - x = 4$   
 $x^2 + 4xy + y^2 + 8 = 0$

e)  $3y = 1 - x$   
 $x^2 + y^2 = 29$

$(2, 4), (-\frac{22}{5}, \frac{4}{5}), (-2, 0), (\frac{1}{3}, \frac{7}{3}), (-1, 1), (\frac{5}{3}, -\frac{1}{3}), (-2, 2), (-5, 2), (\frac{26}{5}, -\frac{7}{5})$



**Question 7**

Solve each of the following sets of simultaneous equations.

a)  $x^2 - 3xy + y^2 = 11$   
 $3y - x = 1$

b)  $xy = 3$   
 $3x + y = 10$

c)  $2x - y = 1$   
 $4x^2 + y^2 + 4y = 9$

d)  $2y + x = 4$   
 $y = 2x^2 - 6x + 5$

e)  $3x - y = 11$   
 $2x^2 - y^2 = 31$

$(14, 5), (-7, -2), (3, 1), (\frac{1}{3}, 9), (1, 1), (-\frac{3}{2}, -4), (2, 1), (\frac{3}{4}, \frac{13}{8}), (4, 1), (\frac{38}{7}, \frac{37}{7})$

The image shows handwritten solutions for the five sets of simultaneous equations. Each solution is written in a separate box and includes the following steps:

- a)**  $x^2 - 3xy + y^2 = 11$  and  $3y - x = 1$ . The second equation is rearranged to  $x = 3y - 1$ . This is substituted into the first equation to form a quadratic in  $y$ :  $(3y-1)^2 - 3y(3y-1) + y^2 = 11$ . Simplifying gives  $4y^2 - 6y - 10 = 0$ , which is further simplified to  $2y^2 - 3y - 5 = 0$ . Factoring gives  $(y+2)(y-5) = 0$ , so  $y = -2$  or  $y = 5$ . Substituting back into  $x = 3y - 1$  gives  $x = -7$  or  $x = 14$ . The solutions are  $(-7, -2)$  and  $(14, 5)$ .
- b)**  $xy = 3$  and  $3x + y = 10$ . The second equation is rearranged to  $y = 10 - 3x$ . This is substituted into the first equation to form a quadratic in  $x$ :  $x(10 - 3x) = 3$ . Simplifying gives  $3x^2 - 10x + 3 = 0$ . Factoring gives  $(3x-1)(x-3) = 0$ , so  $x = \frac{1}{3}$  or  $x = 3$ . Substituting back into  $y = 10 - 3x$  gives  $y = 9$  or  $y = 1$ . The solutions are  $(\frac{1}{3}, 9)$  and  $(3, 1)$ .
- c)**  $2x - y = 1$  and  $4x^2 + y^2 + 4y = 9$ . The first equation is rearranged to  $y = 2x - 1$ . This is substituted into the second equation to form a quadratic in  $x$ :  $4x^2 + (2x-1)^2 + 4(2x-1) = 9$ . Simplifying gives  $8x^2 + 4x - 12 = 0$ , which is further simplified to  $2x^2 + x - 3 = 0$ . Factoring gives  $(2x+3)(x-1) = 0$ , so  $x = -\frac{3}{2}$  or  $x = 1$ . Substituting back into  $y = 2x - 1$  gives  $y = -4$  or  $y = 1$ . The solutions are  $(-\frac{3}{2}, -4)$  and  $(1, 1)$ .
- d)**  $2y + x = 4$  and  $y = 2x^2 - 6x + 5$ . The first equation is rearranged to  $x = 4 - 2y$ . This is substituted into the second equation to form a cubic in  $y$ :  $y = 2(4-2y)^2 - 6(4-2y) + 5$ . Simplifying gives  $4y^2 + 2y - 4 = 0$ , which is further simplified to  $2y^2 + y - 2 = 0$ . Factoring gives  $(y+2)(y-1) = 0$ , so  $y = -2$  or  $y = 1$ . Substituting back into  $x = 4 - 2y$  gives  $x = 8$  or  $x = 2$ . The solutions are  $(8, -2)$  and  $(2, 1)$ .
- e)**  $3x - y = 11$  and  $2x^2 - y^2 = 31$ . The first equation is rearranged to  $y = 3x - 11$ . This is substituted into the second equation to form a quadratic in  $x$ :  $2x^2 - (3x-11)^2 = 31$ . Simplifying gives  $-7x^2 + 66x - 122 = 31$ , which is further simplified to  $7x^2 - 66x + 153 = 0$ . Factoring gives  $(7x-38)(x-4) = 0$ , so  $x = \frac{38}{7}$  or  $x = 4$ . Substituting back into  $y = 3x - 11$  gives  $y = \frac{37}{7}$  or  $y = 1$ . The solutions are  $(\frac{38}{7}, \frac{37}{7})$  and  $(4, 1)$ .

**Question 8**

Solve each of the following sets of simultaneous equations.

a)  $x = y + 3$   
 $x^2 + 3y^2 = 37$

b)  $2x + y = 3$   
 $x^2 + y^2 = 5$

c)  $2y + x = 6$   
 $2x^2 + y^2 = 57$

d)  $3x = 14 - y$   
 $2x^2 = y$

e)  $3x - y = 2$   
 $4x + y^2 = 5$

$(5, 2), (-\frac{1}{2}, -\frac{7}{2}), (2, -1), (\frac{2}{5}, \frac{11}{5}), (-4, 5), (\frac{1}{3}, \frac{16}{3}), (2, 8), (-\frac{7}{2}, \frac{49}{2}), (1, 1), (-\frac{1}{9}, -\frac{7}{3})$

The image shows handwritten solutions for the five parts of Question 8. Each part (a-e) is solved by substituting the linear equation into the quadratic equation, resulting in a quadratic equation in one variable. The solutions are found using the quadratic formula or factoring, and the corresponding values for the other variable are determined.

- a)**  $x = y + 3$  and  $x^2 + 3y^2 = 37$ . Substituting  $x = y + 3$  into the quadratic equation yields  $4y^2 + 6y - 28 = 0$ , which simplifies to  $2y^2 + 3y - 14 = 0$ . Factoring gives  $(2y - 4)(y + 3.5) = 0$ , leading to  $y = 2$  and  $y = -3.5$ . The solutions are  $(5, 2)$  and  $(-1, -3.5)$ .
- b)**  $2x + y = 3$  and  $x^2 + y^2 = 5$ . Substituting  $y = 3 - 2x$  into the quadratic equation yields  $5x^2 - 12x + 4 = 0$ . Factoring gives  $(5x - 2)(x - 2) = 0$ , leading to  $x = \frac{2}{5}$  and  $x = 2$ . The solutions are  $(\frac{2}{5}, \frac{11}{5})$  and  $(2, -1)$ .
- c)**  $2y + x = 6$  and  $2x^2 + y^2 = 57$ . Substituting  $y = 6 - x$  into the quadratic equation yields  $2x^2 - 12x + 36 = 57$ , which simplifies to  $2x^2 - 12x - 21 = 0$ . Using the quadratic formula,  $x = \frac{12 \pm \sqrt{144 + 168}}{4} = \frac{12 \pm \sqrt{312}}{4}$ . The solutions are  $(\frac{1}{3}, \frac{16}{3})$  and  $(2, 8)$ .
- d)**  $3x = 14 - y$  and  $2x^2 = y$ . Substituting  $y = 14 - 3x$  into the quadratic equation yields  $2x^2 = 14 - 3x$ , which simplifies to  $2x^2 + 3x - 14 = 0$ . Factoring gives  $(2x + 7)(x - 2) = 0$ , leading to  $x = -3.5$  and  $x = 2$ . The solutions are  $(-1, -3.5)$  and  $(2, -4)$ .
- e)**  $3x - y = 2$  and  $4x + y^2 = 5$ . Substituting  $y = 3x - 2$  into the quadratic equation yields  $4x + (3x - 2)^2 = 5$ , which simplifies to  $10x^2 - 12x - 3 = 0$ . Using the quadratic formula,  $x = \frac{12 \pm \sqrt{144 + 120}}{20} = \frac{12 \pm \sqrt{264}}{20}$ . The solutions are  $(1, 1)$  and  $(-\frac{1}{9}, -\frac{7}{3})$ .

**Question 9**

Solve each of the following sets of simultaneous equations.

a)  $y + 2x - 1 = 0$   
 $x^2 + y^2 = 10$

b)  $2x + y = 6$   
 $x + xy = 3$

c)  $2x - y + 9 = 0$   
 $x^2 + 2xy + y^2 = 9$

d)  $y = x^2 + x + 1$   
 $x + 2y = 4$

e)  $x + y = 1$   
 $(x+1)^2 + 2y = 12$

$(-1, 3), \left(\frac{9}{5}, -\frac{13}{5}\right), (3, 0), \left(\frac{1}{2}, 5\right), (-2, 5), (-4, 1), (-2, 3), \left(\frac{1}{2}, \frac{7}{4}\right), (3, -2), (-3, 4)$

(a)  $y + 2x - 1 = 0$   
 $x^2 + y^2 = 10$   
 $\Rightarrow y = 1 - 2x$   
 SUBSTITUTE INTO THE QUADRATIC (CIRCLE)  
 $\Rightarrow x^2 + (1 - 2x)^2 = 10$   
 $\Rightarrow x^2 + 1 - 4x + 4x^2 = 10$   
 $\Rightarrow 5x^2 - 4x - 9 = 0$   
 $\Rightarrow (5x - 9)(x + 1) = 0$   
 $\Rightarrow x = \frac{-1}{5}$  or  $x = -1$   
 $y = 1 - 2\left(\frac{-1}{5}\right) = \frac{7}{5}$  or  $y = 1 - 2(-1) = 3$   
 $\therefore \left(\frac{-1}{5}, \frac{7}{5}\right)$  and  $(-1, 3)$

(b)  $2x + y = 6$   
 $x + xy = 3$   
 $\Rightarrow y = 6 - 2x$  SUBSTITUTE INTO THE OTHER  
 $\Rightarrow x + x(6 - 2x) = 3$   
 $\Rightarrow x + 6x - 2x^2 = 3$   
 $\Rightarrow -2x^2 + 7x - 3 = 0$   
 $\Rightarrow 2x^2 - 7x + 3 = 0$   
 $\Rightarrow (2x - 1)(x - 3) = 0$   
 $\Rightarrow x = \frac{1}{2}$  or  $x = 3$   
 $y = 6 - 2\left(\frac{1}{2}\right) = 5$  or  $y = 6 - 2(3) = 0$   
 $\therefore \left(\frac{1}{2}, 5\right)$  and  $(3, 0)$

(c)  $2x - y + 9 = 0$   
 $x^2 + 2xy + y^2 = 9$   
 $\Rightarrow 2x + 9 = y$  SUB INTO THE QUADRATIC  
 $\Rightarrow x^2 + 2x(2x + 9) + (2x + 9)^2 = 9$   
 $\Rightarrow x^2 + 4x^2 + 18x + 4x^2 + 36x + 81 = 9$   
 $\Rightarrow 9x^2 + 54x + 72 = 0$   
 $\Rightarrow x^2 + 6x + 8 = 0$   
 $\Rightarrow (x + 2)(x + 4) = 0$   
 $\Rightarrow x = -2$  or  $x = -4$   
 $y = 2(-2) + 9 = 5$  or  $y = 2(-4) + 9 = 1$   
 $\therefore (-2, 5)$  and  $(-4, 1)$

(d)  $y = x^2 + x + 1$   
 $x + 2y = 4$   
 $\Rightarrow x + 2(x^2 + x + 1) = 4$   
 $\Rightarrow x + 2x^2 + 2x + 2 = 4$   
 $\Rightarrow 2x^2 + 3x - 2 = 0$   
 $\Rightarrow (2x - 1)(x + 2) = 0$   
 $\Rightarrow x = \frac{1}{2}$  or  $x = -2$   
 $y = \left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1 = \frac{7}{4}$  or  $y = (-2)^2 + (-2) + 1 = 3$   
 $\therefore \left(\frac{1}{2}, \frac{7}{4}\right)$  and  $(-2, 3)$

(e)  $x + y = 1$   
 $(x+1)^2 + 2y = 12$   
 $\Rightarrow y = 1 - x$  SUB INTO THE QUADRATIC  
 $\Rightarrow (x+1)^2 + 2(1-x) = 12$   
 $\Rightarrow x^2 + 2x + 1 + 2 - 2x = 12$   
 $\Rightarrow x^2 + 3 = 12$   
 $\Rightarrow x^2 = 9$   
 $\Rightarrow x = 3$  or  $x = -3$   
 $y = 1 - 3 = -2$  or  $y = 1 - (-3) = 4$   
 $\therefore (3, -2)$  and  $(-3, 4)$

**Question 10**

Solve each of the following sets of simultaneous equations.

a)  $2x = y + 1$   
 $x^2 + y^2 = 2$

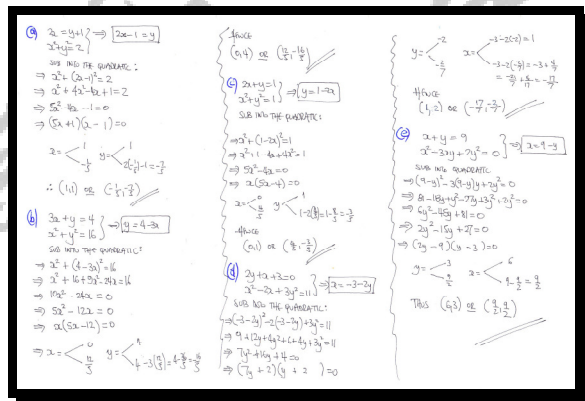
b)  $3x + y = 4$   
 $x^2 + y^2 = 16$

c)  $2x + y = 1$   
 $x^2 + y^2 = 1$

d)  $2y + x + 3 = 0$   
 $x^2 - 2x + 3y^2 = 11$

e)  $x + y = 9$   
 $x^2 - 3xy + 2y^2 = 0$

$(1,1), \left(-\frac{1}{5}, -\frac{7}{5}\right), (0,4), \left(\frac{12}{5}, -\frac{16}{5}\right), (0,1), \left(\frac{4}{5}, -\frac{3}{5}\right), (1,-2), \left(-\frac{17}{7}, -\frac{2}{7}\right), (6,3), \left(\frac{9}{2}, \frac{9}{2}\right)$



**Question 11**

Solve each of the following sets of simultaneous equations.

a)  $y + 3 = 2x$   
 $16 + y^2 = 8x$

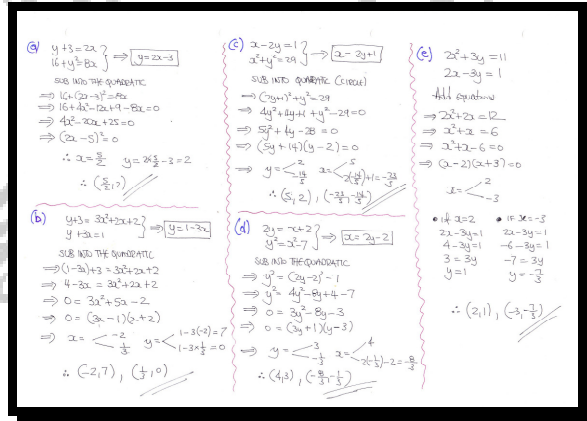
b)  $y + 3 = 3x^2 + 2x + 2$   
 $y + 3x = 1$

c)  $x - 2y = 1$   
 $x^2 + y^2 = 29$

d)  $2y = x + 2$   
 $y^2 = x^2 - 7$

e)  $2x^2 + 3y = 11$   
 $2x - 3y = 1$

$\left(\frac{5}{2}, 2\right), (-2, 7), \left(\frac{1}{3}, 0\right), (5, 2), \left(-\frac{23}{5}, -\frac{14}{5}\right), (4, 3), \left(-\frac{8}{3}, -\frac{1}{3}\right), (2, 1), \left(-3, -\frac{7}{3}\right)$



**Question 12**

Solve each of the following sets of simultaneous equations.

- a)  $3x + y = 8$   
 $xy + y^2 = 8$
- b)  $3x = y - 4$   
 $x^2 + y^2 = 34$
- c)  $2x - y = 1$   
 $x^2 + 2x + y^2 = 1$
- d)  $y - x - 3 = 0$   
 $xy(y - x) = 30$
- e)  $x - y = 6$   
 $(x + y)^2 = 4$

$(2, 2), \left(\frac{14}{3}, -6\right), (-3, -5), \left(\frac{3}{5}, \frac{29}{5}\right), (0, -1), \left(\frac{2}{5}, -\frac{1}{5}\right), (2, 5), (-2, -5), (2, -4), (4, -2)$

The image shows handwritten solutions for the five parts of Question 12. Each part (a-e) is solved using algebraic methods such as substitution and elimination. The solutions are as follows:

- a)**  $3x + y = 8$  and  $xy + y^2 = 8$ . Substituting  $y = 8 - 3x$  into the second equation leads to a quadratic equation in  $x$ , which is solved to find  $x = 2$  and  $x = \frac{14}{3}$ . Corresponding  $y$  values are  $y = 2$  and  $y = -6$ .
- b)**  $3x = y - 4$  and  $x^2 + y^2 = 34$ . Substituting  $y = 3x + 4$  into the second equation leads to a quadratic equation in  $x$ , which is solved to find  $x = -3$  and  $x = \frac{3}{5}$ . Corresponding  $y$  values are  $y = -5$  and  $y = \frac{29}{5}$ .
- c)**  $2x - y = 1$  and  $x^2 + 2x + y^2 = 1$ . Substituting  $y = 2x - 1$  into the second equation leads to a quadratic equation in  $x$ , which is solved to find  $x = 0$  and  $x = \frac{2}{5}$ . Corresponding  $y$  values are  $y = -1$  and  $y = -\frac{1}{5}$ .
- d)**  $y - x - 3 = 0$  and  $xy(y - x) = 30$ . Substituting  $y = x + 3$  into the second equation leads to a cubic equation in  $x$ , which is solved to find  $x = 2$  and  $x = -2$ . Corresponding  $y$  values are  $y = 5$  and  $y = -5$ .
- e)**  $x - y = 6$  and  $(x + y)^2 = 4$ . Substituting  $x = y + 6$  into the second equation leads to a quadratic equation in  $y$ , which is solved to find  $y = -4$  and  $y = -2$ . Corresponding  $x$  values are  $x = 2$  and  $x = 4$ .