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BASIC SKILLS

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OPERATIONS WITH FRACTIONS

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Question 1

Evaluate the following giving the final answer in its simplest form without using mixed numbers.

a) $\frac{2}{5} + \frac{3}{10} =$

$\frac{2}{3} - \frac{4}{9} =$

$\frac{4}{3} \times \frac{3}{8} =$

$\frac{4}{5} \div \frac{3}{7} =$

b) $\frac{1}{2} + \frac{7}{10} =$

$\frac{2}{3} - \frac{1}{12} =$

$\frac{3}{4} \times \frac{8}{9} =$

$\frac{3}{2} \div \frac{5}{12} =$

c) $\frac{2}{3} + \frac{3}{5} =$

$\frac{5}{6} - \frac{3}{8} =$

$\frac{5}{4} \times \frac{1}{8} =$

$\frac{2}{3} \div \frac{5}{6} =$

d) $\frac{3}{4} + \frac{2}{3} =$

$\frac{7}{3} - \frac{9}{4} =$

$\frac{3}{8} \times \frac{16}{3} =$

$\frac{3}{20} \div \frac{1}{40} =$

e) $\frac{5}{3} + \frac{3}{4} =$

$\frac{5}{8} - \frac{5}{12} =$

$\frac{15}{4} \times \frac{8}{5} =$

$\frac{3}{10} \div \frac{6}{5} =$

f) $\frac{1}{2} + \frac{3}{16} =$

$\frac{9}{4} - \frac{3}{7} =$

$\frac{5}{3} \times \frac{7}{15} =$

$\frac{2}{7} \div \frac{11}{14} =$

g) $\frac{5}{3} + \frac{3}{8}$

$\frac{2}{9} - \frac{1}{10}$

$\frac{18}{5} \times \frac{7}{12}$

$\frac{12}{5} \div \frac{12}{7}$

h) $3 \times \frac{3}{4}$

$\frac{5}{4} \times 6$

$\frac{7}{6} \div 2$

$6 \div \frac{12}{5}$

Question 2

Evaluate the following giving the final answer in its simplest form using mixed numbers where appropriate.

a) $1\frac{2}{3} + \frac{7}{4}$

$1\frac{1}{4} + 3\frac{5}{6}$

$1\frac{1}{3} + 2\frac{5}{6}$

$1\frac{1}{2} + 1\frac{3}{4}$

b) $3\frac{1}{2} - 1\frac{3}{4}$

$2\frac{1}{4} - 1\frac{3}{8}$

$1\frac{3}{5} - 1\frac{1}{10}$

$2\frac{1}{3} - 1\frac{3}{4}$

c) $1\frac{5}{8} \times \frac{3}{4}$

$\frac{1}{4} \times 2\frac{1}{3}$

$2\frac{5}{8} \times \frac{3}{16}$

$3 \times 2\frac{5}{7}$

d) $1\frac{5}{6} \times 3$

$2\frac{5}{9} \div \frac{23}{3}$

$1\frac{5}{6} \times 3\frac{1}{2}$

$1\frac{2}{3} \div 3$

e) $1\frac{3}{13} \div \frac{15}{26}$

$2\frac{3}{7} \div 3\frac{1}{2}$

$\frac{4}{9} \times 2\frac{1}{3}$

$2\frac{2}{5} \div 4$

Question 3

Evaluate the following giving the final answer in its simplest form.

$$\text{a) } \frac{3}{\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{2}{6}} = \frac{3}{2} + \frac{1}{3}$$

$$\text{b) } \frac{2}{\frac{1}{3}} \times \frac{1}{4} = \frac{\frac{1}{2} - \frac{1}{4}}{\frac{3}{10} \div \frac{7}{3}} = \frac{3}{9} \times \frac{1}{5} \times \frac{10}{3}$$

$$\text{c) } \frac{2 + \frac{3}{4}}{2 - \frac{3}{4}} = \frac{\frac{1}{2} + \frac{3}{4}}{5 - \frac{1}{4}} = \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{4} + \frac{1}{6}} = \frac{\frac{1}{2} + \frac{3}{4}}{2 - \frac{2}{5}}$$

$$\text{d) } \frac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} + \frac{4}{5}} = \frac{\frac{3}{4} + \frac{1}{2}}{1 - \frac{3}{4} \times \frac{1}{2}} = \frac{1 + \frac{1}{2} \times \frac{3}{5}}{\frac{3}{4} + \frac{2}{5} + \frac{3}{20}} = \frac{\frac{1}{2} + \frac{1}{2} \times \frac{3}{5}}{\frac{3}{4} \times \frac{2}{5} - \frac{1}{6}}$$

$$\text{e) } 1\frac{3}{13} \div \frac{15}{26} = 2\frac{3}{7} \div 3\frac{1}{2} = \frac{4}{9} \times 2\frac{1}{3} = 2\frac{2}{5} \div 4$$

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SIMPLIFYING EXPRESSIONS

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Question 4

Simplify fully each of the following expressions.

a) $4(2x-1)-3(2x-7)=$

b) $2(3x-2)-2(2x-1)=$

c) $2x-4(2x-1)-(4-5x)=$

d) $6-3(2x+3)-(7-2x)=$

e) $5(2x+3)-2(2x+3)=$

Question 5

Simplify fully each of the following expressions.

a) $4(2x-3)-2(3x-8)=$

b) $5(3x-3)-3(5x-7)$

c) $9x-2(3x-4)-(7-x)=$

d) $8-2(7x+1)-(3-5x)=$

e) $7(2x-5)-5(5-2x)=$

Question 6

Simplify fully each of the following expressions.

a) $4x(x-2) - 2(x^2 + 3x - 1) =$

b) $4x(3-x) - 2x(x-2) =$

c) $2x(3x-7) - 4x - x(2-5x) =$

d) $4x(3x-2) - (10 - 4x - 5x^2) =$

e) $4(2x^2 - 3) - x(3-x) =$

Question 7

Simplify fully each of the following expressions.

a) $3x(x-3) - 2(x^2 - 3x + 1) =$

b) $7x(3-x) - 2(4 - 2x^2) =$

c) $6x(2x-5) + 9x - 7x(2+x) =$

d) $2x(2-5x) - (1 - 4x - 11x^2) =$

e) $(2x+3)(3x-2) - 2x(3+4x) - (1-2x-2x^2) =$

Question 8

Simplify fully each of the following expressions.

a) $4(x^2 - 3x) - (x+1)(x+4) =$

b) $2(3x^2 - 5) - (x+2)(x-3) =$

c) $4(2x^2 - 3) - (x-4)(x+5) =$

d) $2x(4-3x) - (2x-1)(1-3x) =$

e) $6x - x(2-x) - 2(x-1)(x+2) =$

Question 9

Expand the brackets and simplify fully each of the following expressions.

a) $(x+3)(x+1)(x+1)$

b) $(x-2)(x-5)(x+1)$

c) $(x-2)(x-3)(x+4)$

d) $(x-3)(x+2)(x+4)$

e) $(x+1)(x+2)(x-1)(x-3)$

Question 10

Expand the brackets and simplify fully each of the following expressions.

a) $(2x-1)(x-1)(x-2)$

b) $(x-1)(2x-3)(x+2)$

c) $(3x-1)(x+2)(3x+2)$

d) $(1+2x)(3-x)(1-x)$

e) $(x-3)(x-1)(x-2)(x+1)$

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QUADRATIC FACTORIZATION

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Question 11

Factorize each of the following quadratic expressions.

a) $x^2 - 8x - 9$

b) $x^2 + 7x + 10$

c) $x^2 - 7x + 12$

d) $x^2 + 4x - 12$

e) $x^2 + 8x - 20$

f) $x^2 - 6x - 16$

g) $x^2 - 11x + 24$

h) $x^2 + 10x + 24$

Question 12

Factorize each of the following quadratic expressions.

a) $x^2 + 5x - 6$

b) $x^2 + 7x + 12$

c) $x^2 + 7x - 18$

d) $x^2 - 8x + 15$

e) $x^2 - x - 20$

f) $x^2 - 11x + 18$

g) $x^2 + x - 30$

h) $x^2 - 12x + 20$

Question 13

Factorize each of the following quadratic expressions.

a) $x^2 - 12x + 36$

b) $x^2 + 4x - 12$

c) $x^2 + 9x + 18$

d) $x^2 + 2x - 15$

e) $x^2 + 8x + 15$

f) $x^2 - 7x + 12$

g) $x^2 + x - 12$

h) $x^2 - 8x + 16$

Question 14

Factorize each of the following quadratic expressions.

a) $x^2 + 3x - 28$

b) $x^2 - 2x - 24$

c) $x^2 - 10x + 21$

d) $x^2 + 14x + 24$

e) $x^2 - 12x + 32$

f) $x^2 + 13x - 30$

g) $x^2 - 11x + 28$

h) $x^2 + 12x + 32$

Question 15

Factorize each of the following quadratic expressions.

a) $x^2 - 3x - 40$

b) $x^2 - 14x + 24$

c) $x^2 + 13x + 36$

d) $x^2 - 17x + 42$

e) $x^2 + 18x + 45$

f) $x^2 + 14x + 40$

g) $x^2 - 19x + 48$

h) $x^2 - 8x - 48$

Question 16

Factorize each of the following quadratic expressions.

a) $x^2 - 14x + 45$

b) $x^2 - 15x + 50$

c) $x^2 + 6x - 40$

d) $x^2 - 30x + 56$

e) $x^2 - 9x - 52$

f) $x^2 + 4x - 45$

g) $x^2 + 15x + 56$

h) $x^2 + 37x - 360$

Question 17

Factorize each of the following quadratic expressions.

a) $2x^2 - 5x + 3$

b) $2x^2 - 11x + 15$

c) $3x^2 - 10x + 8$

d) $2x^2 - x - 10$

e) $3x^2 + 11x + 6$

f) $3x^2 - 8x + 4$

g) $5x^2 + 6x - 8$

h) $2x^2 - 9x - 18$

Question 18

Factorize each of the following quadratic expressions.

a) $3x^2 - 5x - 12$

b) $5x^2 + 19x - 4$

c) $3x^2 - 16x + 5$

d) $3x^2 - 11x + 6$

e) $3x^2 + x - 4$

f) $3x^2 + x - 2$

g) $2x^2 + 7x + 5$

h) $3x^2 + 11x - 4$

Question 19

Factorize each of the following quadratic expressions.

a) $2x^2 + 7x + 6$

b) $5x^2 - 12x - 9$

c) $2x^2 - 13x - 24$

d) $3x^2 + 2x - 8$

e) $2x^2 + 13x + 15$

f) $2x^2 + 3x - 20$

g) $2x^2 - 5x - 18$

h) $3x^2 + 22x - 16$

Question 20

Factorize each of the following quadratic expressions.

a) $3x^2 - 17x + 20$

b) $2x^2 - 3x - 5$

c) $5x^2 - 7x + 2$

d) $3x^2 + 13x + 12$

e) $2x^2 - 15x + 27$

f) $7x^2 - 9x - 10$

g) $3x^2 - 13x + 14$

h) $2x^2 + 9x - 18$

Question 21

Factorize each of the following quadratic expressions.

a) $2x^2 + 7x - 30$

b) $2x^2 + 11x - 30$

c) $4x^2 + 8x + 3$

d) $4x^2 - 11x - 3$

e) $4x^2 + 4x - 15$

f) $4x^2 - 7x - 2$

g) $4x^2 + 5x - 6$

h) $4x^2 - 8x - 5$

Question 22

Factorize each of the following quadratic expressions.

a) $4x^2 - 5x - 9$

b) $4x^2 + 16x + 15$

c) $4x^2 - 19x - 5$

d) $4x^2 - 33x + 8$

e) $4x^2 + 5x - 9$

f) $4x^2 - 20x + 9$

g) $4x^2 - 11x - 20$

h) $4x^2 + 5x - 21$

Question 23

Factorize each of the following quadratic expressions.

a) $4x^2 - 27x + 18$

b) $6x^2 + 7x - 3$

c) $6x^2 - 5x - 4$

d) $6x^2 + 19x + 10$

e) $6x^2 + 7x - 10$

f) $6x^2 - 13x - 15$

g) $6x^2 - 17x + 10$

h) $6x^2 - 13x + 6$

Question 24

Factorize each of the following quadratic expressions.

a) $6x^2 + x - 12$

b) $6x^2 + 11x - 10$

c) $8x^2 - 10x - 3$

d) $8x^2 + 14x + 3$

e) $8x^2 - 5x - 3$

f) $8x^2 - 22x + 5$

g) $8x^2 - 6x - 5$

h) $8x^2 + 3x - 5$

Question 25

Factorize each of the following quadratic expressions.

a) $12x^2 + 7x + 1$

b) $12x^2 + 4x - 1$

c) $12x^2 + 5x - 3$

d) $12x^2 - 17x + 5$

e) $12x^2 - 7x - 5$

f) $12x^2 + 5x - 7$

g) $12x^2 - 16x + 5$

h) $12x^2 - 13x - 4$

Question 26

Factorize each of the following quadratic expressions.

a) $12x^2 - 29x - 60$

b) $21x^2 - 17x + 2$

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SQUARING BRACKETS

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Question 27

Square the following brackets.

a) $(x+3)^2$

b) $(y-9)^2$

c) $(a+1)^2$

d) $(k-6)^2$

e) $(1+w)^2$

f) $(2-v)^2$

Question 28

Square the following brackets.

a) $(5+x)^2$

b) $(4-b)^2$

c) $(t+2)^2$

d) $(y-5)^2$

e) $(b+7)^2$

f) $(c-8)^2$

Question 29

Square the following brackets.

a) $(6+k)^2$

b) $(3-p)^2$

c) $(8+d)^2$

d) $(7-m)^2$

e) $(q+10)^2$

f) $(2n-1)^2$

Question 30

Square the following brackets.

a) $(3x+2)^2$

b) $(2z-3)^2$

c) $(4n+1)^2$

d) $(3p+4)^2$

e) $(2k-2)^2$

f) $(4-2n)^2$

Question 31

Square the following brackets.

a) $(3+4p)^2$

b) $(2y-9)^2$

c) $(5a+1)^2$

d) $(6b+2)^2$

e) $(4h-2)^2$

f) $(6-2y)^2$

Question 32

Square the following brackets.

a) $(1+4q)^2$

b) $(3-5k)^2$

c) $(7d-1)^2$

d) $(6x+4)^2$

e) $(3y-3)^2$

f) $(7-2w)^2$

Question 33

Square the following brackets.

a) $(8+2n)^2$

b) $(1-5c)^2$

c) $(4m-1)^2$

d) $(8p+1)^2$

e) $(6n-1)^2$

f) $(3m+2)^2$

Question 34

Square the following brackets.

a) $(4h-4)^2$

b) $(3-2x)^2$

c) $(1+4c)^2$

d) $(5-2p)^2$

e) $(7t+5)^2$

f) $(5d-2)^2$

Question 35

Square the following brackets.

a) $(3z - 4)^2$

b) $(1 - 7w)^2$

c) $(2 + 4y)^2$

d) $(2 - 6k)^2$

e) $(5b - 1)^2$

f) $(5m - 4)^2$

Question 36

Square the following brackets.

a) $(8v - 1)^2$

b) $(6 - 2t)^2$

c) $(9 + 2c)^2$

d) $(1 - 10n)^2$

e) $(4x - 5)^2$

f) $(12a + 1)^2$

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DIFFERENCE OF SQUARES

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Question 37

Factorize:

a) $x^2 - 16$

b) $25 - a^2$

c) $y^2 - 1$

d) $81 - b^2$

e) $t^2 - 36$

f) $64 - w^2$

g) $n^2 - 100$

h) $4 - d^2$

i) $p^2 - 9$

j) $49 - c^2$

Question 38

Factorize:

a) $36w^2 - 16$

b) $9x^2 - 25$

c) $4y^2 - 1$

d) $16k^2 - 81$

e) $25v^2 - 36$

f) $9w^2 - 64$

g) $9m^2 - 100$

h) $9z^2 - 4$

i) $16p^2 - 9$

j) $25c^2 - 49$

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COMPLETING THE SQUARE

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Question 39

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 - 8x - 2$

b) $y = x^2 + 6x + 10$

c) $y = x^2 - 4x + 1$

d) $y = x^2 + 4x + 9$

e) $y = x^2 + 8x + 20$

Question 40

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 - 6x + 10$

b) $y = x^2 - 10x + 24$

c) $y = x^2 + 10x + 30$

d) $y = x^2 - 12x - 4$

e) $y = x^2 + 14x + 29$

Question 41

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 + 2x$

b) $y = x^2 - 8x$

c) $y = x^2 - 2x + 7$

d) $y = x^2 + 6x + 10$

e) $y = x^2 - 12x + 27$

Question 42

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 + 12x + 36$

b) $y = x^2 - 4x + 6$

c) $y = x^2 - 16x + 40$

d) $y = x^2 - 8x + 18$

e) $y = x^2 + 14x + 50$

Question 43

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 - 7x + 10$

b) $y = x^2 + 3x + 10$

c) $y = x^2 + x + 1$

d) $y = 2x^2 - 8x + 10$

e) $y = 3x^2 - 18x + 24$

Question 44

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = 4x^2 - 8x + 28$

b) $y = 5x^2 + 20x + 15$

c) $y = 3x^2 + 24x + 45$

d) $y = 4x^2 - 8x + 16$

e) $y = 5x^2 + 10x$

Question 45

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = -x^2 + 4x - 3$

b) $y = -x^2 - 12x - 32$

c) $y = -x^2 - 6x - 7$

d) $y = -x^2 + 14x$

e) $y = -x^2 + 2x - 15$

Question 46

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = -2x^2 + 12x + 36$

b) $y = -4x^2 + 4x - 3$

c) $y = -x^2 - x - 2$

d) $y = -x^2 - 3x + \frac{1}{4}$

e) $y = -2x^2 + \frac{1}{2}x - \frac{9}{8}$

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REARRANGE FORMULAE

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Question 47

Rearrange the following formulae for the variable written inside the square bracket.

a) $v = u + at$ [a] (*)

b) $y = 2x + 3$ [x] (*)

c) $y = bt - 3$ [t] (*)

d) $A = \pi r^2$ [r] (*)

e) $v^2 = u^2 + 2as$ [u] (*)

f) $P = T - 7Q$ [Q] (*+)

g) $X = R - QT$ [Q] (*+)

h) $F = Mr^3$ [M] (*+)

i) $V = Bx^3$ [x] (*+)

Question 48

Rearrange the following formulae for the variable written inside the square bracket.

a) $s = \frac{v}{t}$ [t] (*+)

b) $h = \frac{V}{mg}$ [m] (*+)

c) $A = \frac{xy}{2t}$ [t] (*+)

d) $s = \frac{1}{2}at^2$ [t] (*+)

e) $E = \frac{1}{2}mv^2$ [m] (*+)

f) $V = \frac{4}{3}\pi r^3$ [r] (**)

g) $V = \frac{1}{3}\pi r^2 h$ [r] (**)

h) $E = \frac{3}{4}x^2$ [x] (**)

i) $T = \lambda \frac{x}{a}$ [x] (**+)

Question 49

Rearrange the following formulae for the variable written inside the square bracket.

a) $E = \frac{\lambda}{2a}x^2$ [x] (**+)

b) $F = G\frac{mM}{r^2}$ [m] (**+)

c) $\sqrt{y} = 2x$ [y] (**+)

d) $\sqrt{y-1} = 3x$ [y] (**+)

e) $A = 2\pi r(r+h)$ [h] (**+)

f) $s = \frac{u+v}{2}t$ [u] (**+)

g) $B = Ax^2 - T$ [x] (***)

h) $Y = \frac{1}{2}Bx^2$ [x] (***)

Question 50

Rearrange the following formulae for the variable written inside the square bracket.

a) $v^2 = \omega^2(a^2 - x^2)$ [x] (***)

b) $s = ut + \frac{1}{2}at^2$ [a] (***)

c) $2\pi rh + \pi r^2 = 600$ [h] (***)

d) $\pi rt + \frac{1}{3}\pi kr^2 = A$ [t] (***)

e) $C = \frac{a}{x^2}$ [x] (***)

f) $S = \frac{1}{a^2}$ [a] (***)

g) $\sqrt{y} - 1 = 2x$ [y] (***)

h) $kt - kh = c$ [k] (***)

Question 51

Rearrange the following formulae for the variable written inside the square bracket.

a) $2R + QR = QT$ [R] (***)

b) $2R + QR = QT$ [Q] (***)

c) $2x - 5y = y - mx$ [x] (***)

d) $(u - 1)^2 = v$ [u] (***)

e) $x = \frac{2}{\sqrt{y}}$ [y] (***)

f) $x = \frac{1}{\sqrt{y+1}}$ [y] (***)

g) $T = \frac{8}{L^3}$ [L] (***)

h) $y + 5 = x(y + 2)$ [y] (***)

Question 52

Rearrange the following formulae for the variable written inside the square bracket.

a) $y = \frac{x+a}{x+b}$ [x] (****)

b) $\frac{tk}{h} = k-1$ [k] (****)

c) $T = \frac{Q}{Q+2}$ [Q] (****)

d) $y = \frac{x+k}{2x+k}$ [x] (****)

e) $\frac{x+1}{x} = \frac{y}{y+1}$ [y] (****)

f) $\frac{x+1}{x-1} = \frac{y-2}{y+3}$ [y] (****)

g) $T = \frac{2(1+x)}{a(1-x)}$ [x] (****+)

h) $y = x\sqrt{y^2+1}$ [y] (****+)

Question 53

Rearrange the following formulae for the variable written inside the square bracket.

a) $y = \frac{x^2}{a^2 - x^2}$ [x] (****+)

b) $x = \sqrt{\frac{y}{y+1}}$ [y] (****+)

c) $x = \sqrt{\frac{y-1}{2y}}$ [y] (****+)

d) $x = \sqrt{\frac{4y}{3-y}}$ [y] (****+)

e) $x = \sqrt{\frac{y+1}{y-1}}$ [y] (****+)

f) $x = 2\sqrt{\frac{y}{y-1}}$ [y] (****+)

g) $x = \frac{4}{y}\sqrt{y^2-1}$ [y] (****+)

h) $x = \sqrt{y^2-9}$ [y] (****+)

Question 54

Rearrange the following formulae for the variable written inside the square bracket.

a) $\frac{y^2}{y^2+1} = \frac{x^2-1}{x^2}$ [y] (****+)

b) $T = 2\pi\sqrt{\frac{L}{g}}$ [L] (****+)

c) $V = \frac{1}{3}\pi r^2\sqrt{L^2-r^2}$ [L] (****+)

d) $A = \frac{h}{2}(a+b) + \frac{b}{2}(a+h)$ [a] (****+)

e) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ [u] (****+)

f) $u^2 = v - 2u$ [u] (****)

g) $x^2 + y^2 = 2xy + z^2$ [x] (****)

h) $x + \sqrt{x} = y$ [x] (****)

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OPERATIONS WITH FRACTIONS

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Question 1

Evaluate the following giving the final answer in its simplest form without using mixed numbers.

1. $\frac{2}{5} + \frac{3}{10} = \frac{7}{10}$ $\frac{2}{3} - \frac{4}{9} = \frac{2}{9}$ $\frac{4}{3} \times \frac{3}{8} = \frac{1}{2}$ $\frac{4}{5} \div \frac{3}{7} = \frac{28}{15}$

2. $\frac{1}{2} + \frac{7}{10} = \frac{6}{5}$ $\frac{2}{3} - \frac{1}{12} = \frac{7}{12}$ $\frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$ $\frac{3}{2} \div \frac{5}{12} = \frac{18}{5}$

3. $\frac{2}{3} + \frac{3}{5} = \frac{19}{15}$ $\frac{5}{6} - \frac{3}{8} = \frac{11}{24}$ $\frac{5}{4} \times \frac{1}{8} = \frac{5}{32}$ $\frac{2}{3} \div \frac{5}{6} = \frac{4}{5}$

4. $\frac{3}{4} + \frac{2}{3} = \frac{17}{12}$ $\frac{7}{3} - \frac{9}{4} = \frac{1}{12}$ $\frac{3}{8} \times \frac{16}{3} = 2$ $\frac{3}{20} \div \frac{1}{40} = 6$

5. $\frac{5}{3} + \frac{3}{4} = \frac{29}{12}$ $\frac{5}{8} - \frac{5}{12} = \frac{5}{24}$ $\frac{15}{4} \times \frac{8}{5} = 6$ $\frac{3}{10} \div \frac{6}{5} = \frac{1}{4}$

6. $\frac{1}{2} + \frac{3}{16} = \frac{11}{16}$ $\frac{9}{4} - \frac{3}{7} = \frac{51}{28}$ $\frac{5}{3} \times \frac{7}{15} = \frac{7}{9}$ $\frac{2}{7} \div \frac{11}{14} = \frac{4}{11}$

7. $\frac{5}{3} + \frac{3}{8} = \frac{49}{24}$ $\frac{2}{9} - \frac{1}{10} = \frac{11}{90}$ $\frac{18}{5} \times \frac{7}{12} = \frac{21}{10}$ $\frac{12}{5} \div \frac{12}{7} = \frac{7}{5}$

8. $3 \times \frac{3}{4} = \frac{9}{4}$ $\frac{5}{4} \times 6 = \frac{15}{2}$ $\frac{7}{6} \div 2 = \frac{7}{12}$ $6 \div \frac{12}{5} = \frac{5}{2}$

Question 2

Evaluate the following giving the final answer in its simplest form using mixed numbers where appropriate.

1. $1\frac{2}{3} + \frac{7}{4} = 3\frac{5}{12}$ $1\frac{1}{4} + 3\frac{5}{6} = 5\frac{1}{12}$ $1\frac{1}{3} + 2\frac{5}{6} = 4\frac{1}{6}$ $1\frac{1}{2} + 1\frac{3}{4} = 3\frac{1}{4}$

2. $3\frac{1}{2} - 1\frac{3}{4} = 1\frac{3}{4}$ $2\frac{1}{4} - 1\frac{3}{8} = \frac{7}{8}$ $1\frac{3}{5} - 1\frac{1}{10} = \frac{1}{2}$ $2\frac{1}{3} - 1\frac{3}{4} = \frac{7}{12}$

3. $1\frac{5}{8} \times \frac{3}{4} = \frac{39}{32} = 1\frac{7}{32}$ $\frac{1}{4} \times 2\frac{1}{3} = \frac{7}{12}$ $2\frac{5}{8} \times \frac{3}{16} = \frac{63}{128}$ $3 \times 2\frac{5}{7} = \frac{57}{7} = 8\frac{1}{7}$

4. $1\frac{5}{6} \times 3 = \frac{11}{2} = 5\frac{1}{2}$ $2\frac{5}{9} \div \frac{23}{3} = \frac{1}{3}$ $1\frac{5}{6} \times 3\frac{1}{2} = \frac{77}{12} = 6\frac{5}{12}$ $1\frac{2}{3} \div 3 = \frac{5}{9}$

5. $1\frac{3}{13} \div \frac{15}{26} = \frac{32}{15} = 2\frac{2}{15}$ $2\frac{3}{7} \div 3\frac{1}{2} = \frac{34}{49}$ $\frac{4}{9} \times 2\frac{1}{3} = \frac{4}{21}$ $2\frac{2}{5} \div 4 = \frac{3}{5}$

Question 3

Evaluate the following giving the final answer in its simplest form.

1. $\frac{3}{\frac{3}{4}} = 4$

$\frac{\frac{3}{4}}{4} = \frac{3}{16}$

$\frac{\frac{2}{3}}{\frac{1}{6}} = 4$

$\frac{3}{\frac{1}{2}} + \frac{1}{3} = \frac{37}{6}$

2. $\frac{2}{\frac{1}{3}} \times \frac{1}{4} = \frac{3}{4}$

$\frac{\frac{1}{2}}{\frac{1}{4}} - \frac{1}{\frac{1}{2}} = \frac{3}{2}$

$\frac{\frac{3}{5}}{\frac{7}{10}} \div \frac{7}{3} = \frac{4}{49}$

$\frac{3}{\frac{2}{9}} \times \frac{1}{5} \times \frac{10}{\frac{3}{5}} = \frac{15}{2}$

3. $\frac{2 + \frac{3}{4}}{2 - \frac{3}{4}} = \frac{11}{5}$

$\frac{\frac{1}{2} + \frac{3}{4}}{5 - \frac{1}{4}} = \frac{5}{19}$

$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{4} + \frac{1}{6}} = 2$

$\frac{\frac{1}{2} + \frac{3}{4}}{\frac{1}{2} - \frac{2}{5}} = \frac{25}{2}$

4. $\frac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} + \frac{4}{5}} = \frac{70}{93}$

$\frac{\frac{3}{4} + \frac{1}{2}}{1 - \frac{3}{4} \times \frac{1}{2}} = 2$

$\frac{1 + \frac{1}{2} \times \frac{3}{5}}{\frac{3}{4} + \frac{2}{5} + \frac{3}{20}} = 1$

$\frac{\frac{1}{2} + \frac{1}{2} \times \frac{3}{5}}{\frac{3}{4} \times \frac{2}{5} - \frac{1}{6}} = 6$

5. $1\frac{3}{13} \div \frac{15}{26}$

$2\frac{3}{7} \div 3\frac{1}{2}$

$\frac{4}{9} \times 2\frac{1}{3}$

$2\frac{2}{5} \div 4$

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SIMPLIFYING EXPRESSIONS

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Question 4

Simplify fully the following expressions.

1. $4(2x-1) - 3(2x-7) = 2x+17$

2. $2(3x-2) - 2(2x-1) = 2x-2$

3. $2x - 4(2x-1) - (4-5x) = -x$

4. $6 - 3(2x+3) - (7-2x) = -4x-10$

5. $5(2x+3) - 2(2x+3) = 6x+9$

Handwritten solutions for the five algebraic problems:

(a) $4(2x-1) - 3(2x-7)$
 $= 8x - 4 - 6x + 21$
 $= 2x + 17$

(b) $2(3x-2) - 2(2x-1)$
 $= 6x - 4 - 4x + 2$
 $= 2x - 2$

(c) $2x - 4(2x-1) - (4-5x)$
 $= 2x - 8x + 4 - 4 + 5x$
 $= -x$

(d) $6 - 3(2x+3) - (7-2x)$
 $= 6 - 6x - 9 - 7 + 2x$
 $= -4x - 10$

(e) $5(2x+3) - 2(2x+3)$
 $= 10x + 15 - 4x - 6$
 $= 6x + 9$

Question 5

Simplify fully the following expressions.

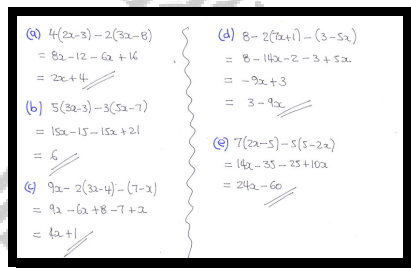
a) $4(2x-3) - 2(3x-8) = 2x + 4$

b) $5(3x-3) - 3(5x-7) = 6$

c) $9x - 2(3x-4) - (7-x) = 4x + 1$

d) $8 - 2(7x+1) - (3-5x) = 3 - 9x$

e) $7(2x-5) - 5(5-2x) = 24x - 60$



Question 6

Simplify fully the following expressions.

$$\text{a) } 4x(x-2) - 2(x^2 + 3x - 1) = 2x^2 - 14x + 2$$

$$\text{b) } 4x(3-x) - 2x(x-2) = -6x^2 + 16x$$

$$\text{c) } 2x(3x-7) - 4x - x(2-5x) = 11x^2 - 20x$$

$$\text{d) } 4x(3x-2) - (10 - 4x - 5x^2) = 17x^2 - 4x - 10$$

$$\text{e) } 4(2x^2 - 3) - x(3-x) = 9x^2 - 3x - 12$$

$\begin{aligned} \text{a) } & 4x(x-2) - 2(x^2 + 3x - 1) \\ & = 4x^2 - 8x - 2x^2 - 6x + 2 \\ & = 2x^2 - 14x + 2 \end{aligned}$	$\begin{aligned} \text{d) } & 4x(3x-2) - (10 - 4x - 5x^2) \\ & = 12x^2 - 8x - 10 + 4x + 5x^2 \\ & = 17x^2 - 4x - 10 \end{aligned}$
$\begin{aligned} \text{b) } & 4x(3-x) - 2x(x-2) \\ & = 12x - 4x^2 - 2x^2 + 4x \\ & = 16x - 6x^2 \end{aligned}$	$\begin{aligned} \text{e) } & 4(2x^2 - 3) - x(3-x) \\ & = 8x^2 - 12 - 3x + x^2 \\ & = 9x^2 - 3x - 12 \end{aligned}$
$\begin{aligned} \text{c) } & 2x(3x-7) - 4x - x(2-5x) \\ & = 6x^2 - 14x - 4x - 2x + 5x^2 \\ & = 11x^2 - 20x \end{aligned}$	

Question 7

Simplify fully the following expressions.

$$\text{a) } 3x(x-3) - 2(x^2 - 3x + 1) = x^2 - 3x - 2$$

$$\text{b) } 7x(3-x) - 2(4-2x^2) = -3x^2 + 21x - 8$$

$$\text{c) } 6x(2x-5) + 9x - 7x(2+x) = 5x^2 - 35x$$

$$\text{d) } 2x(2-5x) - (1-4x-11x^2) = x^2 + 8x - 1$$

$$\text{e) } (2x+3)(3x-2) - 2x(3+4x) - (1-2x-2x^2) = x-7$$

$$\begin{aligned} \text{a) } 3x(x-3) - 2(x^2 - 3x + 1) &= 3x^2 - 9x - 2x^2 + 6x - 2 = x^2 - 3x - 2 \\ \text{b) } 7x(3-x) - 2(4-2x^2) &= 21x - 7x^2 - 8 + 4x^2 = -3x^2 + 21x - 8 \\ \text{c) } 6x(2x-5) + 9x - 7x(2+x) &= 12x^2 - 30x + 9x - 14x - 7x^2 = 5x^2 - 35x \\ \text{d) } 2x(2-5x) - (1-4x-11x^2) &= 4x - 10x^2 - 1 + 4x + 11x^2 = x^2 + 8x - 1 \\ \text{e) } (2x+3)(3x-2) - 2x(3+4x) - (1-2x-2x^2) &= 6x^2 - 4x + 9x - 6 - 6x - 8x^2 - 1 + 2x + 2x^2 \\ &= x - 7 \end{aligned}$$

Question 8

Simplify fully the following expressions.

$$\text{a) } 4(x^2 - 3x) - (x+1)(x+4) = 3x^2 - 17x - 4$$

$$\text{b) } 2(3x^2 - 5) - (x+2)(x-3) = 5x^2 + x - 4$$

$$\text{c) } 4(2x^2 - 3) - (x-4)(x+5) = 7x^2 - x + 8$$

$$\text{d) } 2x(4-3x) - (2x-1)(1-3x) = 3x+1$$

$$\text{e) } 6x - x(2-x) - 2(x-1)(x+2) = -x^2 + 2x + 4$$

Handwritten solutions for Question 8:

a) $4(x^2 - 3x) - (x+1)(x+4)$
 $= 4x^2 - 12x - [x^2 + 4x + x + 4]$
 $= 4x^2 - 12x - [x^2 + 5x + 4]$
 $= 4x^2 - 12x - x^2 - 5x - 4$
 $= 3x^2 - 17x - 4$

b) $2(3x^2 - 5) - (x+2)(x-3)$
 $= 6x^2 - 10 - [x^2 - 3x + 2x - 6]$
 $= 6x^2 - 10 - [x^2 - x - 6]$
 $= 6x^2 - 10 - x^2 + x + 6$
 $= 5x^2 + x - 4$

c) $4(2x^2 - 3) - (x-4)(x+5)$
 $= 8x^2 - 12 - [x^2 + 5x - 4x - 20]$
 $= 8x^2 - 12 - [x^2 + x - 20]$
 $= 8x^2 - 12 - x^2 - x + 20$
 $= 7x^2 - x + 8$

d) $2x(4-3x) - (2x-1)(1-3x)$
 $= 8x - 6x^2 - [2x - 6x^2 - 1 + 3x]$
 $= 8x - 6x^2 - [2x - 6x^2 + 3x - 1]$
 $= 8x - 6x^2 - 2x + 6x^2 - 3x + 1$
 $= 3x + 1$

e) $6x - x(2-x) - 2(x-1)(x+2)$
 $= 6x - 2x + x^2 - 2(x^2 + 2x - x - 2)$
 $= 4x + x^2 - 2(x^2 + x - 2)$
 $= 4x + x^2 - 2x^2 - 2x + 4$
 $= -x^2 + 2x + 4$

Question 9

Expand the brackets and simplify fully the following expressions.

a) $(x+3)(x+1)(x+1) = x^3 + 5x^2 + 7x + 3$

b) $(x-2)(x-5)(x+1) = x^3 - 6x^2 + 3x + 10$

c) $(x-2)(x-3)(x+4) = x^3 - x^2 - 14x + 24$

d) $(x-3)(x+2)(x+4) = x^3 + 3x^2 - 10x - 24$

e) $(x+1)(x+2)(x-1)(x-3) = x^4 - x^3 - 7x^2 + x + 6$

$\begin{aligned} \text{a)} & (x+3)(x+1)(x+1) \\ &= (x+3)(x^2+2x+1) \\ &= (x+3)(x^2+2x+1) \\ &= x^3+2x^2+x+3x^2+6x+3 \\ &= x^3+5x^2+7x+3 \end{aligned}$	$\begin{aligned} \text{b)} & (x-2)(x-5)(x+1) \\ &= (x-2)(x^2-5x-5) \\ &= (x-2)(x^2-5x-5) \\ &= x^3-5x^2-5x-2x^2+10x+10 \\ &= x^3-6x^2+3x+10 \end{aligned}$	$\begin{aligned} \text{c)} & (x-2)(x-3)(x+4) \\ &= (x-2)(x^2-3x-12) \\ &= (x-2)(x^2-3x-12) \\ &= x^3-3x^2-12x-2x^2+6x+24 \\ &= x^3-x^2-14x+24 \end{aligned}$
$\begin{aligned} \text{d)} & (x-3)(x+2)(x+4) \\ &= (x-3)(x^2+6x+8) \\ &= (x-3)(x^2+6x+8) \\ &= x^3+6x^2+8x-3x^2-18x-24 \\ &= x^3+3x^2-10x-24 \end{aligned}$	$\begin{aligned} \text{e)} & (x+1)(x+2)(x-1)(x-3) \\ &= (x^2+2x+2)(x^2-3x-3) \\ &= (x^2+2x+2)(x^2-3x-3) \\ &= x^4-4x^3+5x^2+3x^2-12x^2+9x+6x-6x-6 \\ &= x^4-x^3-7x^2+x+6 \end{aligned}$	

Question 10

Expand the brackets and simplify fully the following expressions.

a) $(2x-1)(x-1)(x-2) = 2x^3 - 7x^2 + 7x - 2$

b) $(x-1)(2x-3)(x+2) = 2x^3 - x^2 - 7x + 6$

c) $(3x-1)(x+2)(3x+2) = 9x^3 + 21x^2 + 4x - 4$

d) $(1+2x)(3-x)(1-x) = 2x^3 - 7x^2 + 2x + 3$

e) $(x-3)(x-1)(x-2)(x+1) = x^4 - 5x^3 + 5x^2 + 5x - 6$

$\begin{aligned} \text{a)} \quad & (2x-1)(x-1)(x-2) \\ & = (2x-1)(x^2-2x-2) \\ & = (2x-1)(x^2-2x+2) \\ & = 2x^3 - 6x^2 + 4x \\ & \quad - 2x^2 + 2x - 2 \\ & = 2x^3 - 7x^2 + 7x - 2 \end{aligned}$	$\begin{aligned} \text{c)} \quad & (3x-1)(x+2)(3x+2) \\ & = (3x-1)(3x^2+2x+4) \\ & = (3x-1)(9x^2+6x+4) \\ & = 27x^3 + 24x^2 + 12x \\ & \quad - 9x^2 - 6x - 4 \\ & = 27x^3 + 15x^2 + 6x - 4 \end{aligned}$	$\begin{aligned} \text{e)} \quad & (x-3)(x-1)(x-2)(x+1) \\ & = (x^2-2x+3)(x^2-2x-2) \\ & = (x^2-2x+3)(x^2-2x-2) \\ & = x^4 - 2x^3 - 2x^3 + 4x^2 + 3x^2 - 6x - 2x^2 + 4x + 6 \\ & = x^4 - 4x^3 + 5x^2 + 2x + 6 \end{aligned}$
$\begin{aligned} \text{b)} \quad & (x-1)(2x-3)(x+2) \\ & = (x-1)(2x^2+2x-6) \\ & = (x-1)(2x^2+2x-6) \\ & = 2x^3 + 2x^2 - 6x \\ & \quad - 2x^2 - 2x + 6 \\ & = 2x^3 - 4x + 6 \end{aligned}$	$\begin{aligned} \text{d)} \quad & (1+2x)(3-x)(1-x) \\ & = (1+2x)(3-2x+x^2) \\ & = (1+2x)(3-2x+x^2) \\ & = 3 - 2x + 3x^2 \\ & \quad - 6x + 4x^2 + 2x^3 \\ & = 2x^3 - 4x^2 - 3x + 3 \end{aligned}$	$\begin{aligned} \text{f)} \quad & (x-3)(x-1)(x-2)(x+1) \\ & = (x^2-2x+3)(x^2-2x-2) \\ & = (x^2-2x+3)(x^2-2x-2) \\ & = x^4 - 2x^3 - 2x^3 + 4x^2 + 3x^2 - 6x - 2x^2 + 4x + 6 \\ & = x^4 - 4x^3 + 5x^2 + 2x + 6 \end{aligned}$

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QUADRATIC FACTORIZATION

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Question 11

Factorize each of the following quadratic expressions.

a) $x^2 - 8x - 9 = (x+1)(x-9)$

b) $x^2 + 7x + 10 = (x+2)(x+5)$

c) $x^2 - 7x + 12 = (x-3)(x-4)$

d) $x^2 + 4x - 12 = (x+6)(x-2)$

e) $x^2 + 8x - 20 = (x+10)(x-2)$

f) $x^2 - 6x - 16 = (x+2)(x-8)$

g) $x^2 - 11x + 24 = (x-3)(x-8)$

h) $x^2 + 10x + 24 = (x+4)(x+6)$

Question 12

Factorize each of the following quadratic expressions.

a) $x^2 + 5x - 6 = (x + 6)(x - 1)$

b) $x^2 + 7x + 12 = (x + 3)(x + 4)$

c) $x^2 + 7x - 18 = (x + 9)(x - 2)$

d) $x^2 - 8x + 15 = (x - 3)(x - 5)$

e) $x^2 - x - 20 = (x - 5)(x + 4)$

f) $x^2 - 11x + 18 = (x - 2)(x - 9)$

g) $x^2 + x - 30 = (x - 5)(x + 6)$

h) $x^2 - 12x + 20 = (x - 10)(x - 2)$

Question 13

Factorize each of the following quadratic expressions.

a) $x^2 - 12x + 36 = (x - 6)(x - 6)$

b) $x^2 + 4x - 12 = (x - 2)(x + 6)$

c) $x^2 + 9x + 18 = (x + 6)(x + 3)$

d) $x^2 + 2x - 15 = (x - 3)(x + 5)$

e) $x^2 + 8x + 15 = (x + 5)(x + 3)$

f) $x^2 - 7x + 12 = (x - 4)(x - 3)$

g) $x^2 + x - 12 = (x - 3)(x + 4)$

h) $x^2 - 8x + 16 = (x - 4)(x - 4)$

Question 14

Factorize each of the following quadratic expressions.

a) $x^2 + 3x - 28 = (x - 4)(x + 7)$

b) $x^2 - 2x - 24 = (x - 6)(x + 4)$

c) $x^2 - 10x + 21 = (x - 3)(x - 7)$

d) $x^2 + 14x + 24 = (x + 2)(x + 12)$

e) $x^2 - 12x + 32 = (x - 8)(x - 4)$

f) $x^2 + 13x - 30 = (x - 2)(x + 15)$

g) $x^2 - 11x + 28 = (x - 4)(x - 7)$

h) $x^2 + 12x + 32 = (x + 8)(x + 4)$

Question 15

Factorize each of the following quadratic expressions.

a) $x^2 - 3x - 40 = (x - 8)(x + 5)$

b) $x^2 - 14x + 24 = (x - 2)(x - 12)$

c) $x^2 + 13x + 36 = (x + 9)(x + 4)$

d) $x^2 - 17x + 42 = (x - 3)(x - 14)$

e) $x^2 + 18x + 45 = (x + 15)(x + 3)$

f) $x^2 + 14x + 40 = (x + 10)(x + 4)$

g) $x^2 - 19x + 48 = (x - 16)(x - 3)$

h) $x^2 - 8x - 48 = (x - 12)(x + 4)$

Question 16

Factorize each of the following quadratic expressions.

a) $x^2 - 14x + 45 = (x - 5)(x - 9)$

b) $x^2 - 15x + 50 = (x - 5)(x - 10)$

c) $x^2 + 6x - 40 = (x - 4)(x + 10)$

d) $x^2 - 30x + 56 = (x - 2)(x - 28)$

e) $x^2 - 9x - 52 = (x - 13)(x + 4)$

f) $x^2 + 4x - 45 = (x - 5)(x + 9)$

g) $x^2 + 15x + 56 = (x + 7)(x + 8)$

h) $x^2 + 37x - 360 = (x - 8)(x + 45)$

Question 17

Factorize each of the following quadratic expressions.

a) $2x^2 - 5x + 3 = (2x + 1)(x - 3)$

b) $2x^2 - 11x + 15 = (2x - 5)(x - 3)$

c) $3x^2 - 10x + 8 = (3x - 4)(x - 2)$

d) $2x^2 - x - 10 = (2x - 5)(x + 2)$

e) $3x^2 + 11x + 6 = (3x + 2)(x + 3)$

f) $3x^2 - 8x + 4 = (3x - 2)(x - 2)$

g) $5x^2 + 6x - 8 = (5x - 4)(x + 2)$

h) $2x^2 - 9x - 18 = (2x + 3)(x - 6)$

Question 18

Factorize each of the following quadratic expressions.

a) $3x^2 - 5x - 12 = (3x + 4)(x - 3)$

b) $5x^2 + 19x - 4 = (5x - 1)(x + 4)$

c) $3x^2 - 16x + 5 = (3x - 1)(x - 5)$

d) $3x^2 - 11x + 6 = (3x - 2)(x - 3)$

e) $3x^2 + x - 4 = (3x + 4)(x - 1)$

f) $3x^2 + x - 2 = (3x - 2)(x + 1)$

g) $2x^2 + 7x + 5 = (2x + 5)(x + 1)$

h) $3x^2 + 11x - 4 = (3x - 1)(x + 4)$

Question 19

Factorize each of the following quadratic expressions.

a) $2x^2 + 7x + 6 = (2x + 3)(x + 2)$

b) $5x^2 - 12x - 9 = (5x + 3)(x - 3)$

c) $2x^2 - 13x - 24 = (2x + 3)(x - 8)$

d) $3x^2 + 2x - 8 = (3x - 4)(x + 2)$

e) $2x^2 + 13x + 15 = (2x + 3)(x + 5)$

f) $2x^2 + 3x - 20 = (2x - 5)(x + 4)$

g) $2x^2 - 5x - 18 = (2x - 9)(x + 2)$

h) $3x^2 + 22x - 16 = (3x - 2)(x + 8)$

Question 20

Factorize each of the following quadratic expressions.

a) $3x^2 - 17x + 20 = (3x - 5)(x - 4)$

b) $2x^2 - 3x - 5 = (2x - 5)(x + 1)$

c) $5x^2 - 7x + 2 = (5x - 2)(x - 1)$

d) $3x^2 + 13x + 12 = (3x + 4)(x + 3)$

e) $2x^2 - 15x + 27 = (2x - 9)(x - 3)$

f) $7x^2 - 9x - 10 = (7x + 5)(x - 2)$

g) $3x^2 - 13x + 14 = (3x - 7)(x - 2)$

h) $2x^2 + 9x - 18 = (2x - 3)(x + 6)$

Question 21

Factorize each of the following quadratic expressions.

a) $2x^2 + 7x - 30 = (2x - 5)(x + 6)$

b) $2x^2 + 11x - 30 = (2x + 15)(x - 2)$

c) $4x^2 + 8x + 3 = (2x + 1)(2x + 3)$

d) $4x^2 - 11x - 3 = (4x + 1)(x - 3)$

e) $4x^2 + 4x - 15 = (2x + 5)(2x - 3)$

f) $4x^2 - 7x - 2 = (4x + 1)(x - 2)$

g) $4x^2 + 5x - 6 = (x + 2)(4x - 3)$

h) $4x^2 - 8x - 5 = (2x - 5)(2x + 1)$

Question 22

Factorize each of the following quadratic expressions.

a) $4x^2 - 5x - 9 = (x+1)(4x-9)$

b) $4x^2 + 16x + 15 = (2x+5)(2x+3)$

c) $4x^2 - 19x - 5 = (4x+1)(x-5)$

d) $4x^2 - 33x + 8 = (x-8)(4x-1)$

e) $4x^2 + 5x - 9 = (4x+9)(x-1)$

f) $4x^2 - 20x + 9 = (2x-1)(2x-9)$

g) $4x^2 - 11x - 20 = (4x+5)(x-4)$

h) $4x^2 + 5x - 21 = (4x-7)(x+3)$

Question 23

Factorize each of the following quadratic expressions.

a) $4x^2 - 27x + 18 = (4x - 3)(x - 6)$

b) $6x^2 + 7x - 3 = (2x + 3)(3x - 1)$

c) $6x^2 - 5x - 4 = (2x + 1)(3x - 4)$

d) $6x^2 + 19x + 10 = (2x + 5)(3x + 2)$

e) $6x^2 + 7x - 10 = (x + 2)(6x - 5)$

f) $6x^2 - 13x - 15 = (6x + 5)(x - 3)$

g) $6x^2 - 17x + 10 = (x - 2)(6x - 5)$

h) $6x^2 - 13x + 6 = (2x - 3)(3x - 2)$

Question 24

Factorize each of the following quadratic expressions.

a) $6x^2 + x - 12 = (2x + 3)(3x - 4)$

b) $6x^2 + 11x - 10 = (2x + 5)(3x - 2)$

c) $8x^2 - 10x - 3 = (4x + 1)(2x - 3)$

d) $8x^2 + 14x + 3 = (4x + 1)(2x + 3)$

e) $8x^2 - 5x - 3 = (8x + 3)(x - 1)$

f) $8x^2 - 22x + 5 = (4x - 1)(2x - 5)$

g) $8x^2 - 6x - 5 = (2x + 1)(4x - 5)$

h) $8x^2 + 3x - 5 = (x + 1)(8x - 5)$

Question 25

Factorize each of the following quadratic expressions.

a) $12x^2 + 7x + 1 = (3x + 1)(4x + 1)$

b) $12x^2 + 4x - 1 = (6x - 1)(2x + 1)$

c) $12x^2 + 5x - 3 = (4x + 3)(3x - 1)$

d) $12x^2 - 17x + 5 = (12x - 5)(x - 1)$

e) $12x^2 - 7x - 5 = (12x + 5)(x - 1)$

f) $12x^2 + 5x - 7 = (x + 1)(12x - 7)$

g) $12x^2 - 16x + 5 = (2x - 1)(6x - 5)$

h) $12x^2 - 13x - 4 = (4x + 1)(3x - 4)$

Question 26

Factorize each of the following quadratic expressions.

a) $12x^2 - 29x - 60 = (4x - 15)(3x + 4)$

b) $21x^2 - 17x + 2 = (7x - 1)(3x - 2)$

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SQUARING BRACKETS

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Question 27

Square the following brackets.

a) $(x+3)^2 = x^2 + 6x + 9$

b) $(y-9)^2 = y^2 - 18y + 81$

c) $(a+1)^2 = a^2 + 2a + 1$

d) $(k-6)^2 = k^2 - 12k + 36$

e) $(1+w)^2 = 1 + 2w + w^2$

f) $(2-v)^2 = 4 - 4v + v^2$

Question 28

Square the following brackets.

a) $(5+x)^2 = 25 + 10x + x^2$

b) $(4-b)^2 = 16 - 8b + b^2$

c) $(t+2)^2 = t^2 + 4t + 4$

d) $(y-5)^2 = y^2 - 10y + 25$

e) $(b+7)^2 = b^2 + 14b + 49$

f) $(c-8)^2 = c^2 - 16c + 64$

Question 29

Square the following brackets.

a) $(6+k)^2 = 36+12k+k^2$

b) $(3-p)^2 = 9-6p+p^2$

c) $(8+d)^2 = 64+16d+d^2$

d) $(7-m)^2 = 49-14m+m^2$

e) $(q+10)^2 = q^2+20q+100$

f) $(2n-1)^2 = 4n^2-4n+1$

Question 30

Square the following brackets.

a) $(3x+2)^2 = 9x^2+12x+4$

b) $(2z-3)^2 = 4z^2-12z+9$

c) $(4n+1)^2 = 16n^2+8n+1$

d) $(3p+4)^2 = 9p^2+24p+16$

e) $(2k-2)^2 = 4k^2-8k+4$

f) $(4-2n)^2 = 16-16n+4n^2$

Question 31

Square the following brackets.

a) $(3+4p)^2 = 9+24p+16p^2$

b) $(2y-9)^2 = 4y^2-36y+81$

c) $(5a+1)^2 = 25a^2+10a+1$

d) $(6b+2)^2 = 36b^2+24b+4$

e) $(4h-2)^2 = 16h^2-16z+4$

f) $(6-2y)^2 = 36-24y+4y^2$

Question 32

Square the following brackets.

a) $(1+4q)^2 = 1+8q+16q^2$

b) $(3-5k)^2 = 9-30k+25k^2$

c) $(7d-1)^2 = 49d^2-14d+1$

d) $(6x+4)^2 = 36x^2+48x+16$

e) $(3y-3)^2 = 9y^2-18y+9$

f) $(7-2w)^2 = 49-28w+4w^2$

Question 33

Square the following brackets.

a) $(8+2n)^2 = 64+32n+4n^2$

b) $(1-5c)^2 = 1-10c+25c^2$

c) $(4m-1)^2 = 16m^2-8m+1$

d) $(8p+1)^2 = 64p^2+16p+1$

e) $(6n-1)^2 = 36n^2-12n+1$

f) $(3m+2)^2 = 9m^2+12m+4$

Question 34

Square the following brackets.

a) $(4h-4)^2 = 16h^2-32h+16$

b) $(3-2x)^2 = 9-12x+4x^2$

c) $(1+4c)^2 = 1+8c+16c^2$

d) $(5-2p)^2 = 25-20p+4p^2$

e) $(7t+5)^2 = 49t^2+70t+25$

f) $(5d-2)^2 = 25d^2-20d+4$

Question 35

Square the following brackets.

a) $(3z-4)^2 = 9z^2 - 24z + 16$

b) $(1-7w)^2 = 1 - 14w + 49w^2$

c) $(2+4y)^2 = 4 + 16y + 16y^2$

d) $(2-6k)^2 = 4 - 24k + 36k^2$

e) $(5b-1)^2 = 25b^2 - 10b + 1$

f) $(5m-4)^2 = 25m^2 - 40m + 16$

Question 36

Square the following brackets.

a) $(8v-1)^2 = 64v^2 - 16v + 1$

b) $(6-2t)^2 = 36 - 24t + 4t^2$

c) $(9+2c)^2 = 81 + 36c + 4c^2$

d) $(1-10n)^2 = 1 - 20n + 100n^2$

e) $(4x-5)^2 = 16x^2 - 40x + 25$

f) $(12a+1)^2 = 144a^2 + 24a + 1$

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DIFFERENCE OF SQUARES

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Question 37

Factorize:

a) $x^2 - 16 = (x - 4)(x + 4)$

b) $25 - a^2 = (5 - a)(5 + a)$

c) $y^2 - 1 = (y - 1)(y + 1)$

d) $81 - b^2 = (9 - b)(9 + b)$

e) $t^2 - 36 = (t - 6)(t + 6)$

f) $64 - w^2 = (8 - w)(8 + w)$

g) $n^2 - 100 = (n - 10)(n + 10)$

h) $4 - d^2 = (2 - d)(2 + d)$

i) $p^2 - 9 = (p - 3)(p + 3)$

j) $49 - c^2 = (7 - c)(7 + c)$

Question 38

Factorize:

a) $36w^2 - 16 = (6w - 4)(6w + 4)$

b) $9x^2 - 25 = (3x - 5)(3x + 5)$

c) $4y^2 - 1 = (2y - 1)(2y + 1)$

d) $16k^2 - 81 = (4k - 9)(4k + 9)$

e) $25v^2 - 36 = (5v - 6)(5v + 6)$

f) $9w^2 - 64 = (3w - 8)(3w + 8)$

g) $9m^2 - 100 = (3m - 10)(3m + 10)$

h) $9z^2 - 4 = (3z - 2)(3z + 2)$

i) $16p^2 - 9 = (4p - 3)(4p + 3)$

j) $25c^2 - 49 = (5c - 7)(5c + 7)$

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COMPLETING THE SQUARE

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Question 39

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 - 8x - 2 = (x - 4)^2 - 18$ min(4, -18)

b) $y = x^2 + 6x + 10 = (x + 3)^2 + 1$ min(-3, 1)

c) $y = x^2 - 4x + 1 = (x - 2)^2 - 3$ min(2, -3)

d) $y = x^2 + 4x + 9 = (x + 2)^2 + 5$ min(-2, 5)

e) $y = x^2 + 8x + 20 = (x + 4)^2 + 4$ min(-4, 4)

Question 40

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 - 6x + 10 = (x - 3)^2 + 1$ min(3, 1)

b) $y = x^2 - 10x + 24 = (x - 5)^2 - 1$ min(5, -1)

c) $y = x^2 + 10x + 30 = (x + 5)^2 + 5$ min(-5, 5)

d) $y = x^2 - 12x - 4 = (x - 6)^2 - 40$ min(6, -40)

e) $y = x^2 + 14x + 29 = (x + 7)^2 - 20$ min(-7, -20)

Question 41

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 + 2x = (x+1)^2 - 1$ min(-1, -1)

b) $y = x^2 - 8x = (x-4)^2 - 16$ min(4, -16)

c) $y = x^2 - 2x + 7 = (x-1)^2 + 6$ min(1, 6)

d) $y = x^2 + 6x + 10 = (x+3)^2 + 1$ min(-3, 1)

e) $y = x^2 - 12x + 27 = (x-6)^2 - 9$ min(6, -9)

Question 42

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 + 12x + 36 = (x+6)^2$ min(-6, 0)

b) $y = x^2 - 4x + 6 = (x-2)^2 + 2$ min(2, 2)

c) $y = x^2 - 16x + 40 = (x-8)^2 - 24$ min(8, -24)

d) $y = x^2 - 8x + 18 = (x-4)^2 + 2$ min(4, 2)

e) $y = x^2 + 14x + 50 = (x+7)^2 + 1$ min(-7, 1)

Question 43

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = x^2 - 7x + 10 = \left(x - \frac{7}{2}\right)^2 - \frac{9}{4}$ $\min\left(\frac{7}{2}, -\frac{9}{4}\right)$

b) $y = x^2 + 3x + 10 = \left(x + \frac{3}{2}\right)^2 + \frac{31}{4}$ $\min\left(-\frac{3}{2}, \frac{31}{4}\right)$

c) $y = x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$ $\min\left(-\frac{1}{2}, \frac{3}{4}\right)$

d) $y = 2x^2 - 8x + 10 = 2(x - 2)^2 + 2$ $\min(2, 2)$

e) $y = 3x^2 - 18x + 24 = 3(x - 3)^2 - 3$ $\min(3, -3)$

Question 44

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

a) $y = 4x^2 - 8x + 28 = 4(x - 1)^2 + 24$ $\min(1, 24)$

b) $y = 5x^2 + 20x + 15 = 5(x + 2)^2 - 5$ $\min(-2, -5)$

c) $y = 3x^2 + 24x + 45 = 3(x + 4)^2 - 3$ $\min(-4, -3)$

d) $y = 4x^2 - 8x + 16 = 4(x - 1)^2 + 12$ $\min(1, 12)$

e) $y = 5x^2 + 10x = 5(x + 1)^2 - 5$ $\min(-1, -5)$

Question 45

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

$$\text{a) } y = -x^2 + 4x - 3 = 1 - (x - 2)^2 \quad \text{max}(2, 1)$$

$$\text{b) } y = -x^2 - 12x - 32 = 4 - (x + 6)^2 \quad \text{max}(-6, 4)$$

$$\text{c) } y = -x^2 - 6x - 7 = 2 - (x + 3)^2 \quad \text{max}(-3, 2)$$

$$\text{d) } y = -x^2 + 14x = 49 - (x - 7)^2 \quad \text{max}(7, 49)$$

$$\text{e) } y = -x^2 + 2x - 15 = -14 - (x - 1)^2 \quad \text{max}(1, -14)$$

Question 46

Complete the square in the following quadratics and then state the coordinates of their maximum or minimum point.

$$\text{a) } y = -2x^2 + 12x + 36 = 54 - 2(x - 3)^2 \quad \text{max}(3, 54)$$

$$\text{b) } y = -4x^2 + 4x - 3 = -2 - 4\left(x - \frac{1}{2}\right)^2 \quad \text{max}\left(\frac{1}{2}, -2\right)$$

$$\text{c) } y = -x^2 - x - 2 = -\frac{7}{4} - \left(x + \frac{1}{2}\right)^2 \quad \text{max}\left(-\frac{1}{2}, -\frac{7}{4}\right)$$

$$\text{d) } y = -x^2 - 3x + \frac{1}{4} = \frac{5}{2} - \left(x + \frac{3}{2}\right)^2 \quad \text{max}\left(-\frac{3}{2}, \frac{5}{2}\right)$$

$$\text{e) } y = -2x^2 + \frac{1}{2}x - \frac{9}{8} = -\frac{35}{32} - 2\left(x - \frac{1}{8}\right)^2 \quad \text{max}\left(\frac{1}{8}, -\frac{35}{32}\right)$$

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REARRANGE FORMULAE

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Question 47

Rearrange the following formulae for the variable written inside the square bracket.

a) $v = u + at$ [a] (*)

$$a = \frac{v - u}{t}$$

b) $y = 2x + 3$ [x] (*)

$$x = \frac{y - 3}{2}$$

c) $y = bt - 3$ [t] (*)

$$t = \frac{y + 3}{b}$$

d) $A = \pi r^2$ [r] (*)

$$r = \pm \sqrt{\frac{A}{\pi}}$$

e) $v^2 = u^2 + 2as$ [u] (*)

$$u = \pm \sqrt{v^2 - 2as}$$

f) $P = T - 7Q$ [Q] (*+)

$$Q = \frac{T - P}{7}$$

g) $X = R - QT$ [Q] (*+)

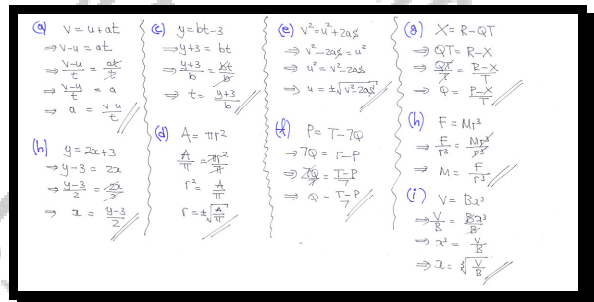
$$Q = \frac{R - X}{T}$$

h) $F = Mr^3$ [M] (*+)

$$M = \frac{F}{r^3}$$

i) $V = Bx^3$ [x] (*+)

$$x = \sqrt[3]{\frac{V}{B}}$$



Question 48

Rearrange the following formulae for the variable written inside the square bracket.

a) $s = \frac{v}{t} \quad [t]$

(*+)

$$t = \frac{v}{s}$$

b) $h = \frac{V}{mg} \quad [m]$

(*+)

$$m = \frac{v}{gh}$$

c) $A = \frac{xy}{2t} \quad [t]$

(*+)

$$t = \frac{xy}{2A}$$

d) $s = \frac{1}{2}at^2 \quad [t]$

(*+)

$$t = \pm \sqrt{\frac{2s}{a}}$$

e) $E = \frac{1}{2}mv^2 \quad [m]$

(*+)

$$m = \frac{2E}{v^2}$$

f) $V = \frac{4}{3}\pi r^3 \quad [r]$

(**)

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

g) $V = \frac{1}{3}\pi r^2 h \quad [r]$

(**)

$$r = \pm \sqrt{\frac{3V}{\pi h}}$$

h) $E = \frac{3}{4}x^2 \quad [x]$

(**)

$$x = \pm \sqrt{\frac{4E}{3}}$$

i) $T = \lambda \frac{x}{a} \quad [x]$

(**+)

$$x = \frac{Ta}{\lambda}$$

Handwritten solutions for questions a through i, showing the steps to rearrange the formulas for the variables in brackets. The solutions are as follows:

- a) $s = \frac{v}{t} \Rightarrow t = \frac{v}{s}$
- b) $h = \frac{V}{mg} \Rightarrow m = \frac{v}{gh}$
- c) $A = \frac{xy}{2t} \Rightarrow t = \frac{xy}{2A}$
- d) $s = \frac{1}{2}at^2 \Rightarrow 2s = at^2 \Rightarrow t^2 = \frac{2s}{a} \Rightarrow t = \pm \sqrt{\frac{2s}{a}}$
- e) $E = \frac{1}{2}mv^2 \Rightarrow 2E = mv^2 \Rightarrow m = \frac{2E}{v^2}$
- f) $V = \frac{4}{3}\pi r^3 \Rightarrow 3V = 4\pi r^3 \Rightarrow r^3 = \frac{3V}{4\pi} \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$
- g) $V = \frac{1}{3}\pi r^2 h \Rightarrow 3V = \pi r^2 h \Rightarrow r^2 = \frac{3V}{\pi h} \Rightarrow r = \pm \sqrt{\frac{3V}{\pi h}}$
- h) $E = \frac{3}{4}x^2 \Rightarrow 4E = 3x^2 \Rightarrow x^2 = \frac{4E}{3} \Rightarrow x = \pm \sqrt{\frac{4E}{3}}$
- i) $T = \lambda \frac{x}{a} \Rightarrow x = \frac{Ta}{\lambda}$

Handwritten solutions for questions b through i, showing the steps to rearrange the formulas for the variables in brackets. The solutions are as follows:

- b) $h = \frac{V}{mg} \Rightarrow m = \frac{v}{gh}$
- c) $A = \frac{xy}{2t} \Rightarrow t = \frac{xy}{2A}$
- d) $s = \frac{1}{2}at^2 \Rightarrow 2s = at^2 \Rightarrow t^2 = \frac{2s}{a} \Rightarrow t = \pm \sqrt{\frac{2s}{a}}$
- e) $E = \frac{1}{2}mv^2 \Rightarrow 2E = mv^2 \Rightarrow m = \frac{2E}{v^2}$
- f) $V = \frac{4}{3}\pi r^3 \Rightarrow 3V = 4\pi r^3 \Rightarrow r^3 = \frac{3V}{4\pi} \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$
- g) $V = \frac{1}{3}\pi r^2 h \Rightarrow 3V = \pi r^2 h \Rightarrow r^2 = \frac{3V}{\pi h} \Rightarrow r = \pm \sqrt{\frac{3V}{\pi h}}$
- h) $E = \frac{3}{4}x^2 \Rightarrow 4E = 3x^2 \Rightarrow x^2 = \frac{4E}{3} \Rightarrow x = \pm \sqrt{\frac{4E}{3}}$
- i) $T = \lambda \frac{x}{a} \Rightarrow x = \frac{Ta}{\lambda}$

Question 49

Rearrange the following formulae for the variable written inside the square bracket.

a) $E = \frac{\lambda}{2a} x^2$ [x]

(**+)

$$x = \pm \sqrt{\frac{2Ea}{\lambda}}$$

b) $F = G \frac{mM}{r^2}$ [m]

(**+)

$$m = \frac{Fr^2}{GM}$$

c) $\sqrt{y} = 2x$ [y]

(**+)

$$y = 4x^2$$

d) $\sqrt{y-1} = 3x$ [y]

(**+)

$$y = 9x^2 + 1$$

e) $A = 2\pi r(r+h)$ [h]

(**+)

$$h = \frac{A}{2\pi r} - r = \frac{A - 2\pi r^2}{2\pi r}$$

f) $s = \frac{u+v}{2} t$ [u]

(**+)

$$u = \frac{2s}{t} - v = \frac{2s - vt}{t}$$

g) $B = Ax^2 - T$ [x]

(***)

$$x = \pm \sqrt{\frac{B+T}{A}}$$

h) $Y = \frac{1}{2} Bx^2$ [x]

(***)

$$x = \pm \sqrt{\frac{2Y}{B}}$$

Handwritten solutions for Question 49:

- a) $E = \frac{\lambda}{2a} x^2$
 $\Rightarrow 2aE = \lambda x^2$
 $\Rightarrow \frac{2aE}{\lambda} = x^2$
 $\Rightarrow x = \pm \sqrt{\frac{2aE}{\lambda}}$
- b) $F = G \frac{mM}{r^2}$
 $\Rightarrow Fr^2 = GMm$
 $\Rightarrow m = \frac{Fr^2}{GM}$
- c) $\sqrt{y} = 2x$
 $\Rightarrow (\sqrt{y})^2 = (2x)^2$
 $\Rightarrow y = 4x^2$
- d) $\sqrt{y-1} = 3x$
 $\Rightarrow (\sqrt{y-1})^2 = (3x)^2$
 $\Rightarrow y-1 = 9x^2$
 $\Rightarrow y = 9x^2 + 1$
- e) $A = 2\pi r(r+h)$
 $\Rightarrow A = 2\pi r^2 + 2\pi rh$
 $\Rightarrow A - 2\pi r^2 = 2\pi rh$
 $\Rightarrow h = \frac{A - 2\pi r^2}{2\pi r}$
- f) $s = \frac{u+v}{2} t$
 $\Rightarrow 2s = (u+v)t$
 $\Rightarrow 2s = ut + vt$
 $\Rightarrow 2s - vt = ut$
 $\Rightarrow u = \frac{2s - vt}{t}$
- g) $B = Ax^2 - T$
 $\Rightarrow B + T = Ax^2$
 $\Rightarrow x^2 = \frac{B+T}{A}$
 $\Rightarrow x = \pm \sqrt{\frac{B+T}{A}}$
- h) $Y = \frac{1}{2} Bx^2$
 $\Rightarrow 2Y = Bx^2$
 $\Rightarrow \frac{2Y}{B} = x^2$
 $\Rightarrow x = \pm \sqrt{\frac{2Y}{B}}$

Question 50

Rearrange the following formulae for the variable written inside the square bracket.

a) $v^2 = \omega^2(a^2 - x^2)$ [x] (***)

$$x = \pm \sqrt{a^2 - \frac{v^2}{\omega^2}} = \pm \sqrt{\frac{a^2 \omega^2 - v^2}{\omega^2}}$$

b) $s = ut + \frac{1}{2}at^2$ [a] (***)

$$a = \frac{2s - 2ut}{t^2}$$

c) $2\pi rh + \pi r^2 = 600$ [h] (***)

$$h = \frac{600 - \pi r^2}{2\pi r}$$

d) $\pi r t + \frac{1}{3}\pi k r^2 = A$ [t] (***)

$$t = \frac{3A - \pi k r^2}{3\pi r}$$

e) $C = \frac{a}{x^2}$ [x] (***)

$$x = \pm \sqrt{\frac{a}{C}}$$

f) $S = \frac{1}{a^2}$ [a] (***)

$$a = \pm \frac{1}{\sqrt{S}}$$

g) $\sqrt{y} - 1 = 2x$ [y] (***)

$$y = (2x + 1)^2$$

h) $kt - kh = c$ [k] (***)

$$k = \frac{c}{t - h}$$

Handwritten solutions for Question 50:

a) $v^2 = \omega^2(a^2 - x^2)$
 $v^2 = \omega^2 a^2 - \omega^2 x^2$
 $\omega^2 x^2 = \omega^2 a^2 - v^2$
 $x^2 = \frac{\omega^2 a^2 - v^2}{\omega^2}$
 $x = \pm \sqrt{\frac{\omega^2 a^2 - v^2}{\omega^2}}$

b) $s = ut + \frac{1}{2}at^2$
 $2s = 2ut + at^2$
 $2s - 2ut = at^2$
 $\frac{2s - 2ut}{t^2} = a$
 $a = \frac{2s - 2ut}{t^2}$

c) $2\pi rh + \pi r^2 = 600$
 $2\pi r h = 600 - \pi r^2$
 $h = \frac{600 - \pi r^2}{2\pi r}$

d) $\pi r t + \frac{1}{3}\pi k r^2 = A$
 $3\pi r t + \pi k r^2 = 3A$
 $3\pi r t = 3A - \pi k r^2$
 $t = \frac{3A - \pi k r^2}{3\pi r}$

e) $C = \frac{a}{x^2}$
 $Cx^2 = a$
 $x^2 = \frac{a}{C}$
 $x = \pm \sqrt{\frac{a}{C}}$

f) $S = \frac{1}{a^2}$
 $\frac{1}{S} = a^2$
 $a^2 = \frac{1}{S}$
 $a = \pm \sqrt{\frac{1}{S}}$
 $a = \pm \frac{1}{\sqrt{S}}$

g) $\sqrt{y} - 1 = 2x$
 $\sqrt{y} = 2x + 1$
 $y = (2x + 1)^2$

h) $kt - kh = c$
 $k(t - h) = c$
 $k = \frac{c}{t - h}$

Question 51

Rearrange the following formulae for the variable written inside the square bracket.

a) $2R + QR = QT$ [R]

(***)

$$R = \frac{QT}{Q+2}$$

b) $2R + QR = QT$ [Q]

(***)

$$Q = \frac{2R}{T-R}$$

c) $2x - 5y = y - mx$ [x]

(***)

$$x = \frac{6y}{m+2}$$

d) $(u-1)^2 = v$ [u]

(***)

$$u = \pm\sqrt{v} + 1$$

e) $x = \frac{2}{\sqrt{y}}$ [y]

(***)

$$y = \frac{4}{x^2}$$

f) $x = \frac{1}{\sqrt{y+1}}$ [y]

(***)

$$y = \frac{1}{x^2} - 1$$

g) $T = \frac{8}{L^3}$ [L]

(***)

$$L = \frac{2}{\sqrt[3]{T}}$$

h) $y + 5 = x(y + 2)$ [y]

(***)

$$y = \frac{2x-5}{1-x}$$

Handwritten solutions for Question 51:

- a) $2R + QR = QT$
 $\Rightarrow R(2+Q) = QT$
 $\Rightarrow R = \frac{QT}{Q+2}$
- b) $2R + QR = QT$
 $\Rightarrow 2R = QT - QR$
 $\Rightarrow 2R = Q(T-R)$
 $\Rightarrow \frac{2R}{T-R} = \frac{Q(T-R)}{T-R}$
 $\Rightarrow Q = \frac{2R}{T-R}$
- c) $2x - 5y = y - mx$
 $\Rightarrow 2x + mx = y + y$
 $\Rightarrow x(2+m) = 2y$
 $\Rightarrow x = \frac{2y}{m+2}$
- d) $(u-1)^2 = v$
 $\Rightarrow u-1 = \pm\sqrt{v}$
 $\Rightarrow u = \pm\sqrt{v} + 1$
- e) $x = \frac{2}{\sqrt{y}}$
 $\Rightarrow x^2 = \frac{4}{y}$
 $\Rightarrow y = \frac{4}{x^2}$
- f) $x = \frac{1}{\sqrt{y+1}}$
 $\Rightarrow x^2 = \frac{1}{y+1}$
 $\Rightarrow y+1 = \frac{1}{x^2}$
 $\Rightarrow y = \frac{1}{x^2} - 1$
- g) $T = \frac{8}{L^3}$
 $\Rightarrow T L^3 = 8$
 $\Rightarrow L^3 = \frac{8}{T}$
 $\Rightarrow L = \sqrt[3]{\frac{8}{T}} = \frac{2}{\sqrt[3]{T}}$
- h) $y + 5 = x(y + 2)$
 $\Rightarrow y + 5 = xy + 2x$
 $\Rightarrow y - xy = 2x - 5$
 $\Rightarrow y(1-x) = 2x - 5$
 $\Rightarrow y = \frac{2x-5}{1-x}$

Question 52

Rearrange the following formulae for the variable written inside the square bracket.

a) $y = \frac{x+a}{x+b}$ [x]

(****)

$$x = \frac{a-by}{y-1}$$

b) $\frac{tk}{h} = k-1$ [k]

(****)

$$k = \frac{h}{h-t}$$

c) $T = \frac{Q}{Q+2}$ [Q]

(****)

$$Q = \frac{2T}{1-T}$$

d) $y = \frac{x+k}{2x+k}$ [x]

(****)

$$x = \frac{k(y-1)}{1-2y} = \frac{k(1-y)}{2y-1}$$

e) $\frac{x+1}{x} = \frac{y}{y+1}$ [y]

(****)

$$y = -x-1$$

f) $\frac{x+1}{x-1} = \frac{y-2}{y+3}$ [y]

(****)

$$y = \frac{-5x-1}{2}$$

g) $T = \frac{2(1+x)}{a(1-x)}$ [x]

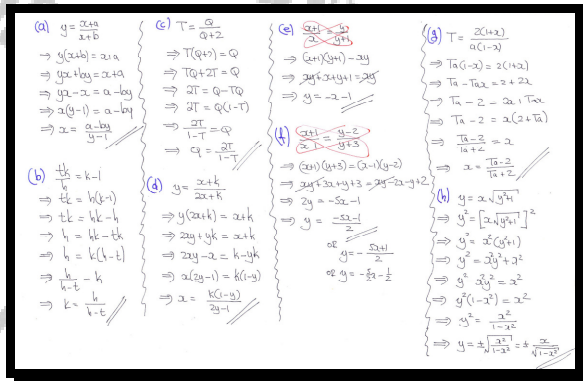
(****+)

$$x = \frac{Ta-2}{Ta+2}$$

h) $y = x\sqrt{y^2+1}$ [y]

(****+)

$$y = \pm \frac{x}{\sqrt{1-x^2}}$$



Question 53

Rearrange the following formulae for the variable written inside the square bracket.

a) $y = \frac{x^2}{a^2 - x^2}$ [x] (****+)

$$x = \pm \sqrt{\frac{ya^2}{y+1}}$$

b) $x = \sqrt{\frac{y}{y+1}}$ [y] (****+)

$$y = \frac{x^2}{1-x^2}$$

c) $x = \sqrt{\frac{y-1}{2y}}$ [y] (****+)

$$y = \frac{1}{1-2x^2}$$

d) $x = \sqrt{\frac{4y}{3-y}}$ [y] (****+)

$$y = \frac{3x^2}{x^2+4}$$

e) $x = \sqrt{\frac{y+1}{y-1}}$ [y] (****+)

$$y = \frac{x^2+1}{x^2-1}$$

f) $x = 2\sqrt{\frac{y}{y-1}}$ [y] (****+)

$$y = \frac{x^2}{x^2-4}$$

g) $x = \frac{4}{y}\sqrt{y^2-1}$ [y] (****+)

$$y = \pm \frac{4}{\sqrt{16-x^2}}$$

h) $x = \sqrt{y^2-9}$ [y] (****+)

$$y = \pm \sqrt{x^2+9}$$

Handwritten solutions for Question 53:

- a) $y = \frac{x^2}{a^2 - x^2}$ [x]
 - $\Rightarrow y(a^2 - x^2) = x^2$
 - $\Rightarrow ya^2 - yx^2 = x^2$
 - $\Rightarrow ya^2 = x^2 + yx^2$
 - $\Rightarrow ya^2 = x^2(1+y)$
 - $\Rightarrow \frac{ya^2}{1+y} = x^2$
 - $\Rightarrow x = \pm \sqrt{\frac{ya^2}{y+1}}$
- b) $x = \sqrt{\frac{y}{y+1}}$ [y]
 - $\Rightarrow x^2 = \frac{y}{y+1}$
 - $\Rightarrow x^2(y+1) = y$
 - $\Rightarrow x^2y + x^2 = y$
 - $\Rightarrow x^2y - y = -x^2$
 - $\Rightarrow y(x^2 - 1) = -x^2$
 - $\Rightarrow y = \frac{-x^2}{x^2 - 1} = \frac{x^2}{1 - x^2}$
- c) $x = \sqrt{\frac{y-1}{2y}}$ [y]
 - $\Rightarrow x^2 = \frac{y-1}{2y}$
 - $\Rightarrow 2yx^2 = y-1$
 - $\Rightarrow 2yx^2 - y = -1$
 - $\Rightarrow y(2x^2 - 1) = -1$
 - $\Rightarrow y = \frac{-1}{2x^2 - 1} = \frac{1}{1 - 2x^2}$
- d) $x = \sqrt{\frac{4y}{3-y}}$ [y]
 - $\Rightarrow x^2 = \frac{4y}{3-y}$
 - $\Rightarrow x^2(3-y) = 4y$
 - $\Rightarrow 3x^2 - x^2y = 4y$
 - $\Rightarrow 3x^2 = 4y + x^2y$
 - $\Rightarrow 3x^2 = y(4+x^2)$
 - $\Rightarrow \frac{3x^2}{4+x^2} = y$
- e) $x = \sqrt{\frac{y+1}{y-1}}$ [y]
 - $\Rightarrow x^2 = \frac{y+1}{y-1}$
 - $\Rightarrow x^2(y-1) = y+1$
 - $\Rightarrow x^2y - x^2 = y+1$
 - $\Rightarrow x^2y - y = x^2 + 1$
 - $\Rightarrow y(x^2 - 1) = x^2 + 1$
 - $\Rightarrow y = \frac{x^2 + 1}{x^2 - 1}$
- f) $x = 2\sqrt{\frac{y}{y-1}}$ [y]
 - $\Rightarrow \frac{x}{2} = \sqrt{\frac{y}{y-1}}$
 - $\Rightarrow \left(\frac{x}{2}\right)^2 = \frac{y}{y-1}$
 - $\Rightarrow \frac{x^2}{4} = \frac{y}{y-1}$
 - $\Rightarrow x^2(y-1) = 4y$
 - $\Rightarrow x^2y - x^2 = 4y$
 - $\Rightarrow x^2y - 4y = x^2$
 - $\Rightarrow y(x^2 - 4) = x^2$
 - $\Rightarrow y = \frac{x^2}{x^2 - 4}$
- g) $x = \frac{4}{y}\sqrt{y^2-1}$ [y]
 - $\Rightarrow \frac{xy}{4} = \sqrt{y^2-1}$
 - $\Rightarrow \left(\frac{xy}{4}\right)^2 = y^2-1$
 - $\Rightarrow \frac{x^2y^2}{16} = y^2-1$
 - $\Rightarrow x^2y^2 = 16y^2 - 16$
 - $\Rightarrow x^2y^2 - 16y^2 = -16$
 - $\Rightarrow y^2(x^2 - 16) = -16$
 - $\Rightarrow y^2 = \frac{-16}{x^2 - 16} = \frac{16}{16 - x^2}$
 - $\Rightarrow y = \pm \frac{4}{\sqrt{16 - x^2}}$
- h) $x = \sqrt{y^2-9}$ [y]
 - $\Rightarrow x^2 = y^2 - 9$
 - $\Rightarrow x^2 + 9 = y^2$
 - $\Rightarrow y = \pm \sqrt{x^2 + 9}$

Question 54

Rearrange the following formulae for the variable written inside the square bracket.

a) $\frac{y^2}{y^2+1} = \frac{x^2-1}{x^2}$ [y] (****+)

$$y = \pm\sqrt{x^2-1}$$

b) $T = 2\pi\sqrt{\frac{L}{g}}$ [L] (****+)

$$L = \frac{gT^2}{4\pi^2}$$

c) $V = \frac{1}{3}\pi r^2\sqrt{L^2-r^2}$ [L] (****+)

$$L = \sqrt{r^2 + \frac{9V^2}{\pi^2 r^4}}$$

d) $A = \frac{1}{2}h(a+b) + \frac{1}{2}b(a+h)$ [a] (****+)

$$a = \frac{2(A-hb)}{h+b}$$

e) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ [u] (****+)

$$u = \frac{vf}{f-v}$$

f) $u^2 = v - 2u$ [u] (****)

$$u = -1 \pm \sqrt{v+1}$$

$$\begin{aligned} u^2 &= v - 2u \\ \Rightarrow u^2 + 2u &= v \\ \Rightarrow u^2 + 2u + 1 &= v + 1 \\ \Rightarrow (u+1)^2 &= v+1 \end{aligned} \quad \left\{ \begin{array}{l} \Rightarrow u+1 = \pm\sqrt{v+1} \\ \Rightarrow u = -1 \pm\sqrt{v+1} \end{array} \right.$$

g) $x^2 + y^2 = 2xy + z^2$ [x] (****)

$$x = y \pm z$$

$$\begin{aligned} x^2 + y^2 &= 2xy + z^2 \\ \Rightarrow x^2 - 2xy + y^2 &= z^2 \\ \Rightarrow (x-y)^2 &= z^2 \end{aligned} \quad \left\{ \begin{array}{l} \Rightarrow x-y = \pm z \\ \Rightarrow x = y \pm z \end{array} \right.$$

h) $x + \sqrt{x} = y$ [x] (****)

$$x = y + \frac{1}{2} \left[1 \pm \sqrt{4y+1} \right]$$

$$\begin{aligned} x + \sqrt{x} &= y \\ \Rightarrow \sqrt{x} &= y - x \\ \Rightarrow x &= (y-x)^2 \\ \Rightarrow x &= y^2 - 2xy + x^2 \\ \Rightarrow 0 &= x^2 - 2xy + y^2 \\ \Rightarrow x^2 - (2y)x + y^2 &= 0 \\ \Rightarrow \left[x - \frac{1}{2}(2y) \right]^2 - \frac{1}{4}(2y)^2 + y^2 &= 0 \\ \Rightarrow \left[x - \frac{1}{2}(2y) \right]^2 - \frac{1}{4}(4y^2) + y^2 &= 0 \\ \Rightarrow \left[x - \frac{1}{2}(2y) \right]^2 - y^2 + y^2 &= 0 \\ \Rightarrow \left[x - \frac{1}{2}(2y) \right]^2 &= 0 + 0 \end{aligned} \quad \left\{ \begin{array}{l} \Rightarrow \left[x - \frac{1}{2}(2y) \right]^2 = \frac{4y^2 \pm 1}{4} \\ \Rightarrow x - \frac{1}{2}(2y) = \pm \frac{\sqrt{4y^2 \pm 1}}{2} \\ \Rightarrow x = \frac{1}{2}(2y) \pm \frac{\sqrt{4y^2 \pm 1}}{2} \\ \Rightarrow x = \frac{2y \pm \sqrt{4y^2 \pm 1}}{2} \\ \Rightarrow x = y \pm \frac{1}{2} \sqrt{4y^2 \pm 1} \end{array} \right.$$