SIMPLE

RECIPROCAL

FUNCTIONS
Question 1 (**+)**

The curves $C_1$ and $C_2$ have respective equations

\[ C_1: \quad y = \frac{1}{x-3}, \quad x \neq 3 \]

\[ C_2: \quad y = \frac{1}{x - 3}, \quad x \neq 0 \]

Sketch on separate diagrams the graph of $C_1$ and the graph of $C_2$.

Indicate clearly in each graph any asymptotes and the coordinates of any intersections with the coordinate axes.
Question 2 (**+)**

The figure above shows the graph of the curve with equation

\[ y = -\frac{4}{x}, \quad x \neq 0. \]

a) Sketch the graph of the curve \( C \) with equation

\[ y = 2 - \frac{4}{x}, \quad x \neq 0. \]

Indicate clearly the coordinates of any points of intersection between \( C \) and the coordinate axes.

b) State the equations of the two asymptotes of \( C \).

\[ x = 0, \quad y = 2 \]
Question 3 (***)

The curves $C_1$ and $C_2$ have respective equations

$C_1: \quad y = -\frac{1}{x}, \quad x \neq 0$

$C_2: \quad y = \frac{1}{x-2}, \quad x \neq 2$

a) Sketch on separate diagrams the graph of $C_1$ and the graph of $C_2$.

Indicate clearly in each graph any asymptotes and the coordinates of any intersections with the coordinate axes.

b) Find the coordinates of the point of intersection between $C_1$ and $C_2$.

$(1, -1)$
Question 4 (***)

The curves \( C_1 \) and \( C_2 \) have respective equations

\[ C_1: \quad y = \frac{1}{x} + 2, \quad x \neq 0 \]

\[ C_2: \quad y = \frac{1}{x+2}, \quad x \neq -2 \]

a) Sketch on separate diagrams the graph of \( C_1 \) and the graph of \( C_2 \).

Indicate clearly in each graph any asymptotes and the coordinates of any intersections with the coordinate axes.

b) Find the coordinates of the point of intersection between \( C_1 \) and \( C_2 \).

\( (1,1), (1,1) \)
The figure above shows the graph of the curve $C$ with equation

$$y = \frac{2}{x}, \ x \neq 0.$$  

a) Describe the geometric transformation which maps the graph of $C$ onto the graph with equation

$$y = \frac{2}{x-2}, \ x \neq 0.$$  

b) Sketch the graph of the curve with equation

$$y = \frac{2}{x} + 2, \ x \neq 0.$$  

Indicate clearly the coordinates of any points of intersections between the curve and the coordinate axes. State the equations of the two asymptotes of the curve.

[continues overleaf]
c) Show that the $x$ coordinates of the points of intersection between the graph of $y = \frac{2}{x-2}$ and the graph of $y = \frac{2}{x} + 2$ are the roots of the quadratic equation $x^2 - 2x - 2 = 0$.

d) Hence find, in exact surd form, the $x$ coordinates of the points of intersection between the graph of $y = \frac{2}{x-2}$ and the graph of $y = \frac{2}{x} + 2$.

\[ x = 1 \pm \sqrt{3} \]
Question 6 (***)

A curve $C$ has equation

$$y = \frac{1}{x^2}, \ x \in \mathbb{R}, \ x \neq 0.$$ 

a) Sketch the graph of $C$.

b) Sketch on separate set of axes the graph of ...

i. ... $y = \frac{1}{x^2} + 1, \ x \in \mathbb{R}, \ x \neq 0$.

ii. ... $y = \frac{1}{(x + 1)^2}, \ x \in \mathbb{R}, \ x \neq -1$.

Mark clearly in each sketch the equations of any asymptotes to these curves and the coordinates of any intersections with the coordinate axes.
Question 7 (***)

A curve $C$ has equation

$$f(x) = -\frac{1}{x^2}, \; x \in \mathbb{R}, \; x \neq 0.$$ 

a) Sketch the graph of $C$.

b) Sketch on separate set of axes the graph of ...

i. $f(x-1)$.

ii. $f'(x)$.

Mark clearly in each sketch the equations of any asymptotes to these curves and the coordinates of any intersections with the coordinate axes.
Question 8 (***+)

\( f(x) = \frac{1}{x}, \ x \in \mathbb{R}, \ x \neq 0. \)

\( g(x) = \frac{1}{x-1} + 1, \ x \in \mathbb{R}, \ x \neq 1. \)

a) Describe mathematically the two transformations that map the graph of \( f(x) \) onto the graph of \( g(x) \).

b) Sketch the graph of \( g(x) \).

The sketch must include …

• ... the coordinates of any points where \( g(x) \) meet the coordinate axes.
• ... the equations of any asymptotes of \( g(x) \).

c) Solve the equation

\( g(x) = x - 1, \)

giving the answers in the form \( a + b\sqrt{5} \), where \( a \) and \( b \) are constants.

\[
\text{translation "right" by 1 unit, followed by translation "upwards" by 1 unit,} \]

\[
\begin{array}{c}
\underline{x = \frac{3}{2} \pm \frac{1}{2} \sqrt{5}}
\end{array}
\]
Question 9  (***)

A curve has equation \( y = f(x) \) given by

\[
f(x) = 2 + \frac{1}{2x - 1}, \quad x \neq \frac{1}{2}.
\]

a) Express \( f(x) \) as a single simplified fraction.

Consider the following sequence of transformations \( T_1, T_2 \) and \( T_3 \).

\[
\begin{align*}
\frac{1}{x} & \quad \xrightarrow{T_1} \quad \frac{1}{x-1} \\
& \quad \xrightarrow{T_2} \quad \frac{1}{2x-1} \\
& \quad \xrightarrow{T_3} \quad 2 + \frac{1}{2x-1}.
\end{align*}
\]

b) Describe geometrically the transformations \( T_1, T_2 \) and \( T_3 \).

c) Hence sketch the graph of \( f(x) \).

Indicate clearly any asymptotes and the coordinates of any intersections with the coordinate axes.

d) Find the coordinates of the point of intersection of \( f(x) \) and the line \( y = 3 \).

\( T_1 = \text{translation, "right", 1 unit} \), \( T_2 = \text{horizontal stretch by scale factor } \frac{1}{2} \),
\( T_3 = \text{translation, "upwards", 2 units} \), \((1,3)\)
Question 10  (***)

Consider the following sequence of transformations $T_1$, $T_2$ and $T_3$.

$$\frac{1}{x} \xrightarrow{T_1} \frac{1}{x} \xrightarrow{T_2} \frac{1}{x+1} \xrightarrow{T_3} 2 - \frac{1}{x+1}$$

a) Describe geometrically the transformations $T_1$, $T_2$ and $T_3$.

b) Hence sketch the graph of

$$y = 2 - \frac{1}{x+1}, \quad x \neq -1.$$

Indicate clearly any asymptotes and the coordinates of any intersections with the coordinate axes.

c) Solve the equation

$$2 - \frac{1}{x+1} = \frac{1}{x}.$$
Question 11  (***)

Consider a sequence of geometric transformations $T_1$, $T_2$ and $T_3$ which map the graph of the curve with equation $y_1 = \frac{1}{x}$ onto the graph of $y_2$.

$T_1$: reflection in the $x$ axis.

$T_2$: translation in the negative $x$ direction by 2 units.

$T_3$: translation in the positive $y$ direction by 2 units.

a) Show that the equation of $y_2$ is given by

$$y_2 = \frac{2x + 3}{x + 2}, \quad x \neq -2.$$

b) Sketch the graph of $y_2$.

Indicate clearly any asymptotes and coordinates of intersections with the axes.

c) Solve the equation

$$\frac{2x + 3}{x + 2} = 2 + \frac{2}{x - 1}.$$
Question 12  (***)

The curve $C_1$ has equation

$$y = -\frac{2}{x}, \ x \in \mathbb{R}, \ x \neq 0.$$ 

a) Sketch the graph of $C_1$.

The curve $C_2$ has equation

$$y = x^3 - 3x, \ x \in \mathbb{R}.$$ 

b) Sketch the graph of $C_2$.

The sketch must include the coordinates, in exact surd form where appropriate, of all the points where the curve meets the coordinate axes.

c) Find the $x$ coordinates of the points of intersection between $C_1$ and $C_2$.

$$x = \pm 1, \ \pm \sqrt{2}$$
Question 13  (***)

\[ f(x) = \frac{1}{x}, \ x \in \mathbb{R}, \ x \neq 0. \]

\[ g(x) = 2 - \frac{1}{x}, \ x \in \mathbb{R}, \ x \neq 0. \]

a) Describe mathematically the two transformations that map the graph of \( f(x) \) onto the graph of \( g(x) \).

\[ h(x) = \frac{6}{x+2}, \ x \in \mathbb{R}, \ x \neq -2. \]

b) Sketch in the same diagram the graphs of \( g(x) \) and \( h(x) \).

The sketch must include the coordinates of …

- … all the points where the curves meet the coordinate axes.
- … the equations of any asymptotes of the curves.

c) Solve the equation \( g(x) = h(x) \).

\[ x = -\frac{1}{2}, 2 \]

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Question 14  (***)

\[ y = \frac{2}{x}, \quad x \in \mathbb{R}, \quad x \neq 0. \]

\( y = \frac{1}{x} \)

a) Describe mathematically the transformation that maps the graph of \( y = \frac{1}{x} \) onto the graph of \( y = \frac{2}{x} \).

b) Sketch the graph of \( y = \frac{2}{x} \).

Write down the equations of the asymptotes of the curve.

The straight line with equation \( y = k - 2x \), where \( k \) is a constant, is a tangent to the curve with equation \( y = \frac{2}{x} \).

c) Determine the possible values of \( k \).

\[ \text{Stretch, vertically, by scale factor of } 2, \quad k = \pm 4 \]
Question 15  (***)

\[ f(x) = \frac{1}{x}, \quad x \in \mathbb{R}, \quad x \neq 0. \]

\[ g(x) = \frac{1}{x+2}, \quad x \in \mathbb{R}, \quad x \neq -2. \]

a) Describe mathematically the two transformations that map the graph of \( f(x) \) onto the graph of \( g(x) \).

b) Sketch the graph of \( g(x) \).

The sketch must include the …

• … coordinates of all the points where the curve meet the coordinate axes.

• … equations of any asymptotes of the curve.

c) Find the coordinates of the points of intersection of \( g(x) \) and the line with equation

\[ 3y + x = 8. \]

\[ \frac{1}{3}, \quad \text{translation "left" by 2 units, followed by translation "upwards" by 2 units}, \]

\[ (1, \frac{2}{3}), (-1, 3) \]
Question 16  (***)

\[ f(x) = \frac{4x-13}{x-3}, \ x \in \mathbb{R}, \ x \neq 3. \]

a) Show that the equation of \( f(x) \) can be written as

\[ f(x) = 4 - \frac{1}{x-3}, \ x \in \mathbb{R}, \ x \neq 3. \]

b) Sketch the graph of \( f(x) \).

The sketch must include …

- ... the coordinates of the points where \( f(x) \) meets the coordinate axes.
- ... the equations of any asymptotes of the curve.

c) Solve the equation

\[ f(x) = \frac{3}{x}, \]

giving the answers in the form \( a + b\sqrt{7} \), where \( a \) and \( b \) are constants.

\[ x = 2 \pm \frac{1}{2}\sqrt{7} \]
Question 17  (***)

\[ f(x) = a - \frac{1}{b-x}, \quad x \in \mathbb{R}, \quad x \neq b, \]

where \( a \) and \( b \) are positive constants such that \( ab > 1 \).

Sketch the graph of \( f(x) \).

The sketch must include, in terms of \( a \) and \( b \), …

\begin{itemize}
  \item … the coordinates of the points where \( f(x) \) meets the coordinate axes.
  \item … the equations of any asymptotes of the curve.
\end{itemize}
The curve $C_1$ has equation

$$y = \frac{a}{x}, \ x \neq 0,$$

where $a$ is a positive constant.

a) Describe geometrically the transformation that maps the graph of $C_1$ onto the graph of $C_2$ whose equation is $y = \frac{a}{x} + 1$.

b) Sketch the graph of $C_2$.

The sketch must include the coordinates of …
• … all the points where the curves meet the coordinate axes.
• … the equations of any asymptotes of the curves.

The line with equation $y = x$ intersects $C_2$ at the point $A(-2, -2)$ and $B$.

c) Determine …
 i. … the value of $a$.
 ii. … the coordinates of $B$.

\[ a = 6, \quad B(3,3) \]
Question 19  (****)
The curve \( C \) has equation
\[
y = \frac{2x + 3}{x - 2}, \; x \in \mathbb{R}, \; x \neq 2.
\]

a) Show clearly that
\[
\frac{2x + 3}{x - 2} \equiv 2 + \frac{7}{x - 2}.
\]

b) Find the coordinates of the points where \( C \) meets the coordinate axes.

c) Sketch the graph of \( C \) showing clearly the equations of any asymptotes.

d) Determine the coordinates of the points of intersection of \( C \) and the straight line with equation
\[
y = 7x - 12.
\]

\[
(0, -\frac{3}{2}), (-\frac{3}{2}, 0), (1, -5), (3, 9)
\]
Question 20

\[ f(x) = \frac{1}{x-2}, \quad x \in \mathbb{R}, \ x \neq 2. \]

\[ g(x) = 1 + \frac{1}{x}, \quad x \in \mathbb{R}, \ x \neq 0. \]

a) Describe mathematically the transformation that maps the graph of \( y = \frac{1}{x} \) onto the graph of …

i. \( f(x) \).

ii. \( g(x) \).

b) Sketch in the same diagram the graphs of \( f(x) \) and \( g(x) \).

The sketch must include …

• … the coordinates of any the points where the curves meet the coordinate axes.

• … the equations of any asymptotes of the curves.

c) Find as exact surds the coordinates of the points of intersection of the graphs of \( f(x) \) and \( g(x) \).

\[ \left( 1 \pm \sqrt{3}, \frac{1 \pm \sqrt{3}}{2} \right), \] translation "right" by 2 units, translation "upwards" by 1 unit,
Question 21  (****)

\[ f(x) = \frac{x-2}{x-3}, \ x \in \mathbb{R}, \ x \neq 3. \]

a) Express \( f(x) \) in the form

\[ f(x) = a + \frac{1}{x+b}, \]

where \( a \) and \( b \) are integers.

b) By considering a series of transformations which map the graph of \( \frac{1}{x} \) onto the graph of \( f(x) \), sketch the graph of \( f(x) \).

The sketch must include …

• … the coordinates of all the points where the curve meets the coordinate axes.
• … the equations of the two asymptotes of the curve.

\[ a = 1, \quad b = -3 \]
Question 22  (***)

The curves \( C_1 \) and \( C_2 \) have respective equations

\[
C_1: \quad y = \frac{1}{x-1}, \quad x \neq 1
\]

\[
C_2: \quad y = 1 - \frac{3}{x+3}, \quad x \neq -3
\]

a) Sketch on the same diagram the graphs of \( C_1 \) and \( C_2 \).

Indicate clearly any asymptotes and coordinates of any intersections with the coordinate axes.

b) By finding the intersections between \( C_1 \) and \( C_2 \), and considering the graphs sketched in part (a), solve the inequality

\[
\frac{1}{x-1} + \frac{3}{x+3} \geq 1.
\]

\[
-3 \leq x \leq -1 \cup 1 < x \leq 3
\]
Question 23  (***)

\[ f(x) = \frac{3x+3}{x-2}, \quad x \in \mathbb{R}, \quad x \neq 2. \]

a) Sketch the graph of \( f(x) \).

The sketch must include the coordinates of …

- … all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

b) Solve the equation \( f(x) = 2 \).

c) Hence solve the inequality \( f(x) \geq 2 \).

\[ x = -7, \quad x \leq -7 \cup x > 2 \]
A curve has equation \( y = f(x) \) given by

\[
f(x) = \frac{3x-1}{x+2}, \quad x \in \mathbb{R}, \quad x \neq 2.
\]

a) Sketch the graph of \( f(x) \).

The sketch must include the coordinates of …

• … all the points where the curve meets the coordinate axes.

• … the equations of the two asymptotes of the curve.

A different curve has equation \( y = g(x) \) given by

\[
g(x) = \frac{1}{x} + k, \quad x \in \mathbb{R}, \quad x \neq 0, \text{ where } k \text{ is a constant.}
\]

The graph of \( f(x) \) meets the graph of \( g(x) \) at the points \( A \) and \( B \).

b) Given that \( A \) lies on the \( x \) axis determine …

i. … the value of \( k \).

ii. … the coordinates of \( B \).

\[k = -3, \quad B(-1, -4)\]