

INTRODUCTION TO POLYNOMIALS

SIMPLIFYING EXPRESSIONS

Question 8

Simplify fully the following expressions.

a) $4(x^2 - 3x) - (x+1)(x+4) =$

b) $2(3x^2 - 5) - (x+2)(x-3) =$

c) $4(2x^2 - 3) - (x-4)(x+5) =$

d) $2x(4 - 3x) - (2x-1)(1-3x) =$

e) $6x - x(2 - x) - 2(x-1)(x+2) =$

Question 9

Expand the brackets and simplify fully the following expressions.

a) $(x+3)(x+1)(x+1)$

b) $(x-2)(x-5)(x+1)$

c) $(x-2)(x-3)(x+4)$

d) $(x-3)(x+2)(x+4)$

e) $(x+1)(x+2)(x-1)(x-3)$

Question 10

Expand the brackets and simplify fully the following expressions.

a) $(2x-1)(x-1)(x-2)$

b) $(x-1)(2x-3)(x+2)$

c) $(3x-1)(x+2)(3x+2)$

d) $(1+2x)(3-x)(1-x)$

e) $(x-3)(x-1)(x-2)(x+1)$

FACTORIZING POLYNOMIALS

Question 1

$$y = x^3 - 6x^2 + 11x - 6.$$

Express y as a product of three linear factors.

$$y = (x-1)(x-2)(x-3)$$

Handwritten solution for Question 1:

$$y = x^3 - 6x^2 + 11x - 6$$

Factors of constant term -6 : $\pm 1, \pm 2, \pm 3, \pm 6$

Test $f(1) = 1 - 6 + 11 - 6 = 0$. $\therefore (x-1)$ is a factor.

Polynomial division:

$$\begin{array}{r} x^2 - 5x + 6 \\ x-1 \overline{) x^3 - 6x^2 + 11x - 6} \\ \underline{x^2 - x + 6} \\ -5x + 6 \\ \underline{-5x + 5} \\ 1 \\ \underline{1x - 1} \\ 0 \end{array}$$

$\therefore y = (x-1)(x^2 - 5x + 6)$

Factor the quadratic: $y = (x-1)(x-2)(x-3)$

Question 2

$$y = x^3 - 2x^2 - 11x + 12.$$

Express y as a product of three linear factors.

$$y = (x-1)(x+3)(x-4)$$

Handwritten solution for Question 2:

$$y = x^3 - 2x^2 - 11x + 12$$

Factors of constant term 12 : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Test $f(1) = 1 - 2 - 11 + 12 = 0$. $\therefore (x-1)$ is a factor.

Polynomial division:

$$\begin{array}{r} x^2 - 3x - 12 \\ x-1 \overline{) x^3 - 2x^2 - 11x + 12} \\ \underline{x^2 - x + 12} \\ -x - 12 \\ \underline{-x - 12} \\ 0 \end{array}$$

$\therefore y = (x-1)(x^2 - 3x - 12)$

Factor the quadratic: $y = (x-1)(x+3)(x-4)$

Question 3

$$y = x^3 - 6x^2 + 12x - 8.$$

Express y as a product of three linear factors.

$$y = (x-2)(x-2)(x-2)$$

Handwritten solution for Question 3:

$$y = x^3 - 6x^2 + 12x - 8$$

Factorise by inspection:

$$f(2) = 2^3 - 6(2)^2 + 12(2) - 8 = 8 - 24 + 24 - 8 = 0$$

$\therefore (x-2)$ is a factor.

Divide y by $(x-2)$ using long division:

$$\begin{array}{r} x^2 - 4x + 4 \\ x-2 \overline{) x^3 - 6x^2 + 12x - 8} \\ \underline{x^2 - 2x} \\ -4x + 12x \\ \underline{-4x + 8} \\ 0 \end{array}$$

$\therefore y = (x-2)(x^2 - 4x + 4)$

Factorise the quadratic:

$$x^2 - 4x + 4 = (x-2)(x-2)$$

$\therefore y = (x-2)(x-2)(x-2)$

Question 4

$$y = x^3 - 5x^2 + 2x + 8.$$

Express y as a product of three linear factors.

$$y = (x+1)(x-4)(x-2)$$

Handwritten solution for Question 4:

$$y = x^3 - 5x^2 + 2x + 8$$

Factorise by inspection:

$$f(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 8 = -1 - 5 - 2 + 8 = 0$$

$\therefore (x+1)$ is a factor.

Divide y by $(x+1)$ using long division:

$$\begin{array}{r} x^2 - 6x + 8 \\ x+1 \overline{) x^3 - 5x^2 + 2x + 8} \\ \underline{x^3 + x^2} \\ -6x^2 + 2x \\ \underline{-6x^2 + 6x} \\ -4x + 8 \\ \underline{-4x + 4} \\ 4 \end{array}$$

Remainder 4, so $(x+1)$ is not a factor.

Try $(x-2)$:

$$f(2) = 2^3 - 5(2)^2 + 2(2) + 8 = 8 - 20 + 4 + 8 = 0$$

$\therefore (x-2)$ is a factor.

Divide y by $(x-2)$ using long division:

$$\begin{array}{r} x^2 - 3x + 4 \\ x-2 \overline{) x^3 - 5x^2 + 2x + 8} \\ \underline{x^3 - 2x^2} \\ -3x^2 + 2x \\ \underline{-3x^2 + 6x} \\ -4x + 8 \\ \underline{-4x + 8} \\ 0 \end{array}$$

$\therefore y = (x-2)(x^2 - 3x + 4)$

Factorise the quadratic:

$$x^2 - 3x + 4 = (x-4)(x+1)$$

$\therefore y = (x+1)(x-4)(x-2)$

Question 5

$$y = x^3 + 2x^2 - 5x - 6.$$

Express y as a product of three linear factors.

$$y = (x+1)(x+3)(x-2)$$

$y = x^3 + 2x^2 - 5x - 6$
 $f(x) = x^3 + 2x^2 - 5x - 6$
 $f(-1) = 1 - 2 + 5 - 6 \neq 0$
 $f(-3) = -27 + 18 + 15 - 6 = 0$
 $\therefore (x+3)$ is a factor

$$\begin{array}{r} x^3 + 2x^2 - 5x - 6 \\ -(x^3 + 3x^2 + 3x + 9) \\ \hline -x^2 - 8x - 15 \\ -(x^2 + 3x + 3) \\ \hline -5x - 18 \\ -(5x + 15) \\ \hline -3 \end{array}$$

 $f(x) = (x+3)(x^2 - x - 2)$
 $f(x) = (x+3)(x+1)(x-2)$

Question 6

$$y = -x^3 + 5x^2 - 7x + 3.$$

Express y as a product of three linear factors.

$$y = (3-x)(x-1)^2$$

$y = -x^3 + 5x^2 - 7x + 3$
 $f(x) = -x^3 + 5x^2 - 7x + 3$
 $f(1) = -1 + 5 - 7 + 3 = 0$
 $f(3) = -27 + 45 - 21 + 3 = 0$
 $\therefore (x-1)$ is a factor

$$\begin{array}{r} -x^3 + 5x^2 - 7x + 3 \\ -(x^3 - 5x^2 + 7x - 3) \\ \hline 10x^2 - 14x + 6 \\ -(2x^2 - 7x + 3) \\ \hline 8x^2 - 7x + 3 \\ -(8x^2 - 20x + 12) \\ \hline 13x - 9 \end{array}$$

 $f(x) = (x-1)(-x^2 + 6x - 3)$
 $f(x) = -(x-1)(x^2 - 6x + 3)$
 $f(x) = -(x-1)(x-1)(x-3)$
 $f(x) = -(x-1)^2(x-3)$
 $f(x) = (3-x)(x-1)^2$

Question 7

$$y = x^3 + 3x^2 - 4.$$

Express y as a product of three linear factors.

$$y = (x-1)(x+2)^2$$

Handwritten solution for Question 7:

$$y = x^3 + 3x^2 - 4$$

$$f(x) = x^3 + 3x^2 - 4$$

Factors of constant term: $\begin{matrix} +1 \\ +2 \\ +4 \end{matrix}$

$$f(1) = 1 + 3 - 4 = 0 \quad \therefore (x-1) \text{ is a factor}$$

$$\begin{array}{r} x^3 + 3x^2 - 4 \\ \underline{-(x^3 + x^2)} \\ 2x^2 - 4 \\ \underline{-(2x^2 + 4x)} \\ -4x - 4 \\ \underline{-(-4x - 4)} \\ 0 \end{array}$$

$$\therefore f(x) = (x-1)(x^2 + 4x + 4)$$

$$f(x) = (x-1)(x+2)^2$$

$$f(x) = (x-1)(x+2)^2$$

Question 8

$$y = -x^3 + 7x^2 - 15x + 9.$$

Express y as a product of three linear factors.

$$y = (1-x)(x-3)^2$$

Handwritten solution for Question 8:

$$y = -x^3 + 7x^2 - 15x + 9$$

$$f(x) = -x^3 + 7x^2 - 15x + 9$$

Factors of constant term: $\begin{matrix} +1 \\ +3 \\ +9 \end{matrix}$

$$f(1) = -1 + 7 - 15 + 9 = 0 \quad \therefore (x-1) \text{ is a factor}$$

$$\begin{array}{r} -x^3 + 7x^2 - 15x + 9 \\ \underline{-(x^3 - 7x^2 + 15x - 9)} \\ 14x^2 - 30x + 18 \\ \underline{-(14x^2 - 21x + 18)} \\ 9x \\ \underline{-(9x - 9)} \\ 0 \end{array}$$

$$\therefore f(x) = (x-1)(x^2 - 6x + 9)$$

$$f(x) = (x-1)(x-3)^2$$

$$f(x) = (1-x)(x-3)^2$$

Question 9

$$y = x^3 - 13x - 12.$$

Express y as a product of three linear factors.

$$y = (x+1)(x+3)(x-4)$$

Handwritten solution for Question 9:

$$y = x^3 - 13x - 12$$

Factors of 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$f(x) = x^3 - 13x - 12$

$f(1) = 1 - 13 - 12 \neq 0$

$f(-1) = -1 + 13 - 12 = 0 \quad \therefore (x+1) \text{ is a factor}$

Divide $x^3 - 13x - 12$ by $(x+1)$:

$$\begin{array}{r} x^2 - x - 12 \\ (x+1) \overline{) x^3 - 13x - 12} \\ \underline{x^2 + x} \\ -14x - 12 \\ \underline{-14x - 14} \\ 2 \end{array}$$

$\therefore y = (x+1)(x^2 - x - 12)$

$y = (x+1)(x-4)(x+3)$

Question 10

$$y = x^3 - 9x^2 + 23x - 15.$$

Express y as a product of three linear factors.

$$y = (x-1)(x-3)(x-5)$$

Handwritten solution for Question 10:

$$y = x^3 - 9x^2 + 23x - 15$$

Factors of 15: $\pm 1, \pm 3, \pm 5, \pm 15$

$f(x) = x^3 - 9x^2 + 23x - 15$

$f(1) = 1 - 9 + 23 - 15 = 0 \quad \therefore (x-1) \text{ is a factor}$

Divide $x^3 - 9x^2 + 23x - 15$ by $(x-1)$:

$$\begin{array}{r} x^2 - 8x + 15 \\ (x-1) \overline{) x^3 - 9x^2 + 23x - 15} \\ \underline{x^3 - x^2} \\ -8x^2 + 23x \\ \underline{-8x^2 + 8x} \\ 15x - 15 \\ \underline{15x - 15} \\ 0 \end{array}$$

$\therefore y = (x-1)(x^2 - 8x + 15)$

$y = (x-1)(x-3)(x-5)$

Question 11

$$y = x^3 + 4x^2 - 4x - 16.$$

Express y as a product of three linear factors.

$$y = (x-2)(x+2)(x+4)$$

Handwritten solution for Question 11:

$$y = x^3 + 4x^2 - 4x - 16$$

$$f(x) = x^3 + 4x^2 - 4x - 16$$

$f(1) = 1 + 4 - 4 - 16 \neq 0$
 $f(-1) = -1 + 4 - 4 - 16 \neq 0$
 $f(2) = 8 + 16 - 8 - 16 = 0 \Rightarrow (x-2) \text{ is a factor.}$

$$\begin{array}{r} x-2 \overline{) \begin{array}{r} x^3 + 4x^2 - 4x - 16 \\ \underline{-(x^2 + 2x)} \\ 3x^2 - 6x - 16 \\ \underline{-(3x^2 + 6x)} \\ -12x - 16 \\ \underline{-(-12x - 24)} \\ 8 \end{array}} \end{array}$$

$y = (x-2)(x^2 + 6x + 8)$
 $y = (x-2)(x+2)(x+4)$

Question 12

$$y = x^3 - 4x^2 - 3x + 18.$$

Express y as a product of three linear factors.

$$y = (x+2)(x-3)^2$$

Handwritten solution for Question 12:

$$y = x^3 - 4x^2 - 3x + 18$$

$$f(x) = x^3 - 4x^2 - 3x + 18$$

$f(1) = 1 - 4 - 3 + 18 \neq 0$
 $f(-1) = -1 - 4 + 3 + 18 \neq 0$
 $f(3) = 27 - 36 - 9 + 18 = 0 \Rightarrow (x-3) \text{ is a factor.}$
 $f(-2) = -8 - 16 + 6 + 18 = 0 \Rightarrow (x+2) \text{ is a factor.}$

$$\begin{array}{r} x+2 \overline{) \begin{array}{r} x^3 - 4x^2 - 3x + 18 \\ \underline{-(x^2 + 2x)} \\ -5x^2 - 5x + 18 \\ \underline{-(5x^2 + 10x)} \\ 5x + 18 \\ \underline{-(5x + 10)} \\ 8 \end{array}} \end{array}$$

$y = (x+2)(x^2 - 6x + 9)$
 $y = (x+2)(x-3)^2$

Question 13

$$y = x^3 - 9x^2 + 26x - 24.$$

Express y as a product of three linear factors.

$$y = (x-2)(x-3)(x-4)$$

Handwritten solution for Question 13:

$$y = x^3 - 9x^2 + 26x - 24 \rightarrow \pm 1, \pm 2, \pm 3, \pm 4 \text{ etc.}$$

$$f(x) = x^3 - 9x^2 + 26x - 24$$

- $f(1) = 1 - 9 + 26 - 24 \neq 0$
- $f(2) = 8 - 36 + 52 - 24 = 0$

$\therefore (x-2)$ is a factor

$$\begin{array}{r} x^3 - 9x^2 + 26x - 24 \\ -(2x^3 - 18x^2 + 52x - 48) \\ \hline -x^2 + 24x - 24 \\ -(x^2 - 24x + 24) \\ \hline 0 \end{array}$$

$$\therefore y = (x-2)(x^2 - 7x + 12)$$

$$y = (x-2)(x-4)(x-3)$$

Question 14

$$y = x^3 - 13x - 12.$$

Express y as a product of three linear factors.

$$y = (x+1)(x+3)(x-4)$$

Handwritten solution for Question 14:

$$y = x^3 - 13x - 12 \rightarrow \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$f(x) = x^3 - 13x - 12$$

- $f(1) = 1 - 13 - 12 \neq 0$
- $f(-1) = -1 + 13 - 12 = 0$

$\therefore (x+1)$ is a factor of $f(x)$

$$\begin{array}{r} x^3 - 13x - 12 \\ -(x^3 + x^2 + x) \\ \hline -x^2 - 14x - 12 \\ -(x^2 + 13x + 12) \\ \hline -x - 24 \\ -(x + 24) \\ \hline 0 \end{array}$$

$$\therefore y = (x+1)(x^2 - x - 12)$$

$$y = (x+1)(x+3)(x-4)$$

Question 15

$$y = x^3 + x^2 - 16x - 16.$$

Express y as a product of three linear factors.

$$y = (x+1)(x+4)(x-4)$$

$y = x^3 + x^2 - 16x - 16 \rightarrow \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$
 $f(x) = x^3 + x^2 - 16x - 16$
 $\bullet f(1) = 1 + 1 - 16 - 16 \neq 0$
 $\bullet f(-1) = -1 + 1 + 16 - 16 = 0 \quad \therefore (x+1) \text{ is a factor of } f(x)$

$$\begin{array}{r} x^3 + x^2 - 16x - 16 \\ \underline{-(x^3 + x^2)} \\ -16x - 16 \\ \underline{-(-16x - 16)} \\ 0 \end{array}$$

 $\therefore y = (x+1)(x^2 - 16)$
 $y = (x+1)(x-4)(x+4)$
 This can be checked by expansion
 $y = x^3 + x^2 - 16x - 16$
 $y = x^2(x+1) - 16(x+1)$
 $y = (x+1)(x^2 - 16)$
 $y = (x+1)(x-4)(x+4)$

SOLVING POLYNOMIAL EQUATIONS

Question 1

Factorize each of the following cubics into three linear factors.

a) $2x^3 + x^2 - 13x + 6$

b) $9x^3 - 9x^2 - x + 1$

c) $25x^3 - 50x^2 - 4x + 8$

$$(x-2)(x+3)(2x-1), (x-1)(3x+1)(3x-1), (x-2)(5x+2)(5x-2)$$

(a) Let $f(x) = 2x^3 + x^2 - 13x + 6$

- $f(1) = 2 + 1 - 13 + 6 = -4 \neq 0$
- $f(-1) = -2 + 1 + 13 + 6 = 18 \neq 0$
- $f(2) = 16 + 4 - 26 + 6 = 0 \quad \therefore (x-2) \text{ is a factor of } f(x)$

$$\begin{array}{r} 2x^3 + x^2 - 13x + 6 \\ x-2 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 - 4x^2 + 8x - 16} \\ 5x^2 - 13x + 22 \\ \underline{5x^2 - 10x + 20} \\ -3x + 2 \\ \underline{-3x + 6} \\ -4 \end{array}$$

$\therefore f(x) = (x-2)(2x^2 + 5x - 3)$

$$f(x) = (x-2)(2x-1)(x+3)$$

(b) Let $f(x) = 9x^3 - 9x^2 - x + 1$

$f(1) = 9 - 9 - 1 + 1 = 0 \quad \therefore (x-1) \text{ is a factor}$

$$\begin{array}{r} 9x^3 - 9x^2 - x + 1 \\ x-1 \overline{) 9x^3 - 9x^2 - x + 1} \\ \underline{9x^3 - 9x^2 + 9x - 9} \\ -10x + 10 \\ \underline{-10x + 10} \\ 0 \end{array}$$

$\therefore f(x) = (x-1)(9x^2 - 1)$

$$f(x) = (x-1)(3x-1)(3x+1)$$

Alternative: $9x^3 - 9x^2 - x + 1 = 9x^2(x-1) - (x-1)$

$$= (x-1)(9x^2 - 1)$$

$$= (x-1)(3x-1)(3x+1)$$

Let $f(x) = 25x^3 - 50x^2 - 4x + 8$

- $f(1) = 25 - 50 - 4 + 8 = -21 \neq 0$
- $f(-1) = -25 - 50 + 4 + 8 = -63 \neq 0$
- $f(2) = 200 - 200 - 8 + 8 = 0 \quad \therefore (x-2) \text{ is a factor}$

$$\begin{array}{r} 25x^3 - 50x^2 - 4x + 8 \\ x-2 \overline{) 25x^3 - 50x^2 - 4x + 8} \\ \underline{25x^3 - 50x^2 + 50x - 100} \\ -54x + 108 \\ \underline{-54x + 108} \\ 0 \end{array}$$

$\therefore f(x) = (x-2)(25x^2 - 4)$

$$f(x) = (x-2)(5x-2)(5x+2)$$

Alternative:

$$f(x) = 25x^3 - 50x^2 - 4x + 8$$

$$f(x) = 25x^2(x-2) - 4(x-2)$$

$$f(x) = (x-2)(25x^2 - 4)$$

$$f(x) = (x-2)(5x-2)(5x+2)$$

Question 2

Solve the following cubic equations

a) $3x^3 + 8x^2 + 3x - 2 = 0$

b) $3x^3 + 5x^2 - 4x - 4 = 0$

c) $3x^3 + 2x^2 - 19x + 6 = 0$

$$x = -1, -2, \frac{1}{3}, \quad x = 1, -2, -\frac{2}{3}, \quad x = -3, 2, \frac{1}{3}$$

(a) $3x^3 + 8x^2 + 3x - 2 = 0$
 Let $f(x) = 3x^3 + 8x^2 + 3x - 2$
 $\bullet f(1) = 3 + 8 + 3 - 2 = 12 \neq 0$
 $\bullet f(-1) = -3 + 8 - 3 - 2 = 0$
 $\therefore (x+1)$ is a factor of $f(x)$

Divide $3x^3 + 8x^2 + 3x - 2$ by $(x+1)$:

$$\begin{array}{r} 3x^2 + 5x - 2 \\ (x+1) \overline{) 3x^3 + 8x^2 + 3x - 2} \\ \underline{3x^3 + 3x^2 + 3x - 2} \\ -5x^2 + 0x + 0 \\ \underline{-5x^2 - 5x + 5} \\ 5x - 5 \\ \underline{5x + 5} \\ 0 \end{array}$$

$\therefore 3x^3 + 8x^2 + 3x - 2 = (x+1)(3x^2 + 5x - 2)$
 $(x+1)(3x^2 + 5x - 2) = 0$
 $(x+1)(3x-1)(x+2) = 0$
 $\therefore x = -1, \frac{1}{3}, -2$

(b) $3x^3 + 5x^2 - 4x - 4 = 0$
 Let $f(x) = 3x^3 + 5x^2 - 4x - 4$
 $\bullet f(1) = 3 + 5 - 4 - 4 = 0$
 $\therefore (x-1)$ is a factor of $f(x)$

Divide $3x^3 + 5x^2 - 4x - 4$ by $(x-1)$:

$$\begin{array}{r} 3x^2 + 8x + 4 \\ (x-1) \overline{) 3x^3 + 5x^2 - 4x - 4} \\ \underline{3x^3 - 3x^2 + 4x - 4} \\ 8x^2 - 0x + 0 \\ \underline{8x^2 - 8x + 8} \\ 8x - 8 \\ \underline{8x - 8} \\ 0 \end{array}$$

$\therefore 3x^3 + 5x^2 - 4x - 4 = (x-1)(3x^2 + 8x + 4)$
 $(x-1)(3x^2 + 8x + 4) = 0$
 $(x-1)(3x+4)(x+1) = 0$
 $\therefore x = 1, -\frac{4}{3}, -1$

(c) $3x^3 + 2x^2 - 19x + 6 = 0$
 Let $f(x) = 3x^3 + 2x^2 - 19x + 6$
 $\bullet f(1) = 3 + 2 - 19 + 6 = -8 \neq 0$
 $\bullet f(-1) = -3 + 2 + 19 + 6 = 24 \neq 0$
 $\bullet f(2) = 24 + 8 - 38 + 6 = 0$
 $\therefore (x-2)$ is a factor of $f(x)$

Divide $3x^3 + 2x^2 - 19x + 6$ by $(x-2)$:

$$\begin{array}{r} 3x^2 + 8x - 3 \\ (x-2) \overline{) 3x^3 + 2x^2 - 19x + 6} \\ \underline{3x^3 - 6x^2 + 12x - 6} \\ 8x^2 - 11x + 12 \\ \underline{8x^2 - 16x + 16} \\ 5x - 4 \\ \underline{5x - 10} \\ 6 \end{array}$$

$\therefore 3x^3 + 2x^2 - 19x + 6 = (x-2)(3x^2 + 8x - 3)$
 $(x-2)(3x^2 + 8x - 3) = 0$
 $(x-2)(3x-1)(x+3) = 0$
 $\therefore x = 2, \frac{1}{3}, -3$

Question 3

Solve the following cubic equations

a) $6x^3 - 7x^2 - 14x + 8 = 0$

b) $8x^3 - 14x^2 - 7x + 6 = 0$

$$x = \frac{1}{2}, 2, -\frac{4}{3}, \quad x = -\frac{3}{4}, \frac{1}{2}, 2$$

(a) $6x^3 - 7x^2 - 14x + 8 = 0$
 $f(x) = 6x^3 - 7x^2 - 14x + 8$
 $f(1) = 6 - 7 - 14 + 8 = -7 \neq 0$
 $f(-1) = -6 - 7 + 14 + 8 = 9 \neq 0$
 $f(2) = 48 - 28 - 28 + 8 = 0 \quad \therefore (x-2) \text{ is a factor}$

Divide $6x^3 - 7x^2 - 14x + 8$ by $(x-2)$

$$\begin{array}{r} 6x^2 + 5x - 4 \\ (x-2) \overline{6x^3 - 7x^2 - 14x + 8} \\ \underline{6x^3 - 12x^2} \\ -13x^2 - 14x + 8 \\ \underline{-13x^2 + 26x} \\ -40x + 8 \\ \underline{-40x + 80} \\ 0 \end{array}$$

Factorise $6x^2 + 5x - 4 = 0$
 $(3x-2)(2x+4) = 0$
 $(x-2)(2x-1)(3x+4) = 0$
 $\therefore x = \frac{2}{3}, \frac{1}{2}, -\frac{4}{3}$

(b) $8x^3 - 14x^2 - 7x + 6 = 0$
 $f(x) = 8x^3 - 14x^2 - 7x + 6$
 $f(1) = 8 - 14 - 7 + 6 = -7 \neq 0$
 $f(-1) = -8 - 14 + 7 + 6 = -9 \neq 0$
 $f(2) = 64 - 56 - 14 + 6 = 0 \quad \therefore (x-2) \text{ is a factor}$

Divide $8x^3 - 14x^2 - 7x + 6$ by $(x-2)$

$$\begin{array}{r} 8x^2 + 2x - 3 \\ (x-2) \overline{8x^3 - 14x^2 - 7x + 6} \\ \underline{8x^3 - 16x^2} \\ 2x^2 - 7x + 6 \\ \underline{2x^2 - 4x} \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array}$$

Factorise $8x^2 + 2x - 3 = 0$
 $(4x+3)(2x-1) = 0$
 $(x-2)(4x+3)(2x-1) = 0$
 $\therefore x = -\frac{3}{4}, \frac{1}{2}, 2$