QUADRATIC INEQUALITIES

EXAM QUESTIONS

Question 1 (**)

$$f(x) = 2x^2 - 9x + 4, \; x \in \mathbb{R}.$$

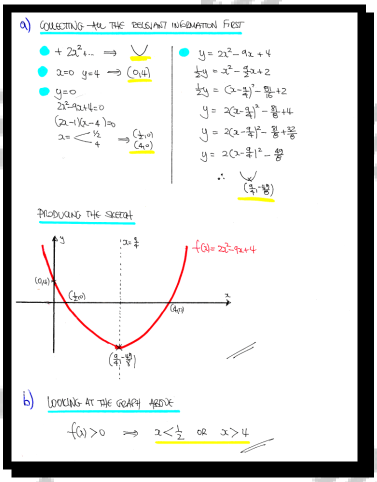
a) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes.

b) Solve the inequality

$$f(x) > 0.$$

$\boxed{}$, $\boxed{x < \frac{1}{2} \cup x > 4}$



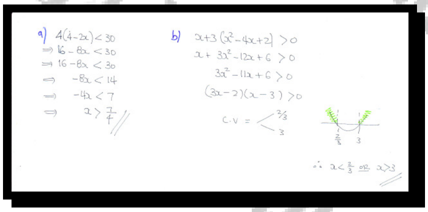
Question 2 (**+)

Solve each of the following inequalities.

a) $4(4-2x)<30$.

b) $x+3\left(x^2-4x+2\right)>0$.

$\boxed{}$, $\boxed{x>\frac{7}{4}}$, $\boxed{x<\frac{2}{3} \cup x>3}$


a) $4(4-2x)<30$
$\Rightarrow 16-8x<30$
$\Rightarrow 16-8x<30$
$\Rightarrow -8x<14$
$\Rightarrow -4x<7$
$\Rightarrow x>\frac{7}{4}$

b) $x+3(x^2-4x+2)>0$
$x+3x^2-12x+6>0$
$3x^2-11x+6>0$
$(3x-2)(x-3)>0$
C.V $= \frac{2}{3},\ 3$
$\therefore x<\frac{2}{3}$ OR $x>3$

Question 3 (**+)

a) Solve the linear inequality

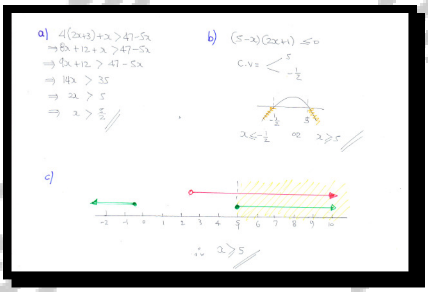
$$4(2x+3)+x>47-5x.$$

b) Solve the quadratic inequality

$$(5-x)(2x+1)\le 0.$$

c) Hence determine the range of values of x that satisfy both the inequalities of part **(a)** and part **(b)**.

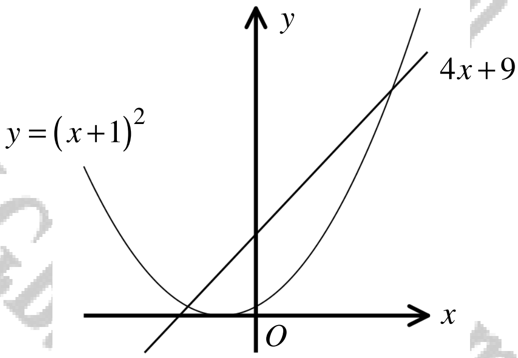
$\boxed{}$, $\boxed{x>\frac{5}{2}}$, $\boxed{x\le -\frac{1}{2} \cup x\ge 5}$, $\boxed{x\ge 5}$


a) $4(2x+3)+x>47-5x$
$\Rightarrow 8x+12+x>47-5x$
$\Rightarrow 9x+12>47-5x$
$\Rightarrow 14x>35$
$\Rightarrow 2x>5$
$\Rightarrow x>\frac{5}{2}$

b) $(5-x)(2x+1)\le 0$
C.V $= 5,\ -\frac{1}{2}$
$x\le -\frac{1}{2}$ OR $x\ge 5$

c)
$\therefore x\ge 5$

Question 4 (**+)



The figure above shows the graphs of $y=(x+1)^2$ and $y=4x+9$.

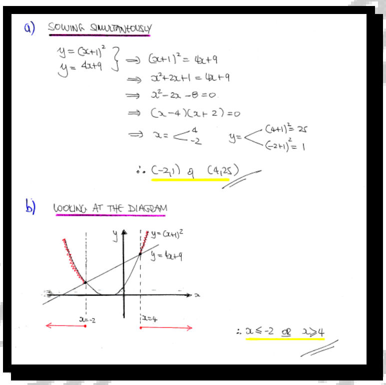
a) Find the coordinates of the points of intersection between the two graphs.

b) Hence solve the inequality

$$(x+1)^2 \geq 4x+9,$$

fully justifying the answer.

$\boxed{}$, $\boxed{(-2,1),(4,25)}$, $\boxed{x \leq -2 \;\cup\; x \geq 4}$



Question 5 **(**+)**

Solve each of the following inequalities.

a) $\frac{x+2}{3} < 3x-1$.

b) $x+6\left(x^2+x\right) > 20$.

, $\boxed{x<\frac{5}{8}}$, $\boxed{x<-\frac{5}{2} \;\cup\; x>\frac{4}{3}}$

a) SOLVING THE LINEAR INEQUALITY

$\Rightarrow \frac{x+2}{3} < 3x-1$

$\Rightarrow x+2 < 3(3x-1)$

$\Rightarrow x+2 < 9x-3$

$\Rightarrow -8x < -5$

$\Rightarrow x > \frac{5}{8}$ //

b) SOLVING THE QUADRATIC INEQUALITY

$\Rightarrow x+6(x^2+x) > 20$

$\Rightarrow x+6x^2+6x > 20$

$\Rightarrow 6x^2+7x-20 > 0$

FACTORIZE OR USE THE QUADRATIC FORMULA TO FIND THE CRITICAL VALUES

$\Rightarrow (3x-4)(2x+5) > 0$

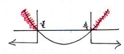
$\Rightarrow \text{C.V} = \begin{cases} \frac{4}{3} \\ -\frac{5}{2} \end{cases}$

$x = \frac{-7 \pm \sqrt{7^2-4\times6\times(-20)}}{2\times6}$

$x = \frac{-7 \pm \sqrt{529}}{12}$

$x = \frac{-7 \pm 23}{12}$

$x = \begin{cases} 4/3 \\ -5/2 \end{cases}$



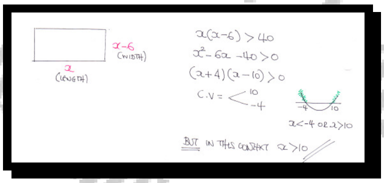
$x < -\frac{5}{2}$ OR $x > \frac{4}{3}$ //

Question 6 **(***)**

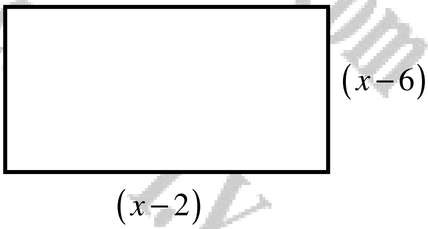
A rectangle is such so that its length is 6 cm greater than its width.

Given the area of the rectangle is at least 40 cm^2, determine the range of the possible values of the **length** of the rectangle.

$\boxed{\text{length} > 10}$



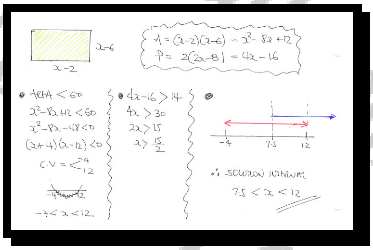
Question 7 **(***)**



The rectangle shown above measures $(x-2)$ cm by $(x-6)$ cm.

Given the area of the rectangle is at most 60 cm^2, and its perimeter at least 14 cm, determine the range of the possible values of x.

$\boxed{7.5 < x < 12}$



Question 8 (***)

a) Solve the linear inequality

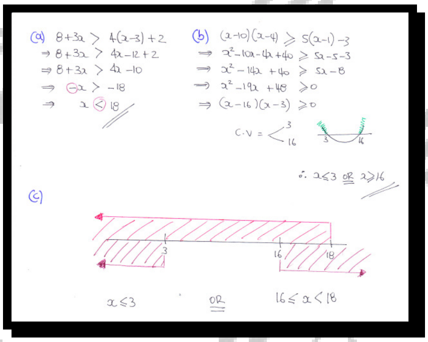
$$8+3x>4(x-3)+2.$$

b) Solve the quadratic inequality

$$(x-10)(x-4)\geq 5(x-1)-3.$$

c) Hence determine the range of values of x that satisfy both the inequalities of part **(a)** and part **(b)**.

$\boxed{}$, $\boxed{x<18}$, $\boxed{x\leq 3 \cup x\geq 16}$, $\boxed{x\leq 3 \cup 16\leq x<18}$



Question 9 (***)

a) Solve the linear inequality

$$6-2(x+2)<10.$$

b) Solve the quadratic inequality

$$(x+1)^2 \geq 4x+9.$$

c) Hence determine the range of values of x that satisfy both the inequalities of part (a) and part (b).

$\boxed{}$, $\boxed{x>-4}$, $\boxed{x \leq -2 \ \cup \ x \geq 4}$, $\boxed{-4<x\leq -2 \ \cup \ x \geq 4}$

(a) $6-2(x+2)<10$

$\Rightarrow 6-2x-4<10$

$\Rightarrow 2-2x<10$

$\Rightarrow -2x<8$

$\Rightarrow x>-4$

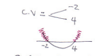
(b) $(x+1)^2 \geq 4x+9$

$\Rightarrow x^2+2x+1 \geq 4x+9$

$\Rightarrow x^2-2x-8 \geq 0$

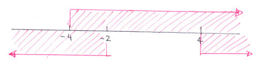
$\Rightarrow (x+2)(x-4) \geq 0$

C.V $= \begin{cases} -2 \\ 4 \end{cases}$



$x \leq -2$ OR $x \geq 4$

(c)



$-4<x\leq -2$ OR $x \geq 4$

Question 10 (***+)

The curve C has equation

$$y = (x-4)^2 + 2.$$

The line L has equation

$$y = 13 - 2x$$

a) Sketch on the same diagram the graph of C and the graph of L.
The sketch must include the coordinates of any points where these graphs meet the coordinate axes.

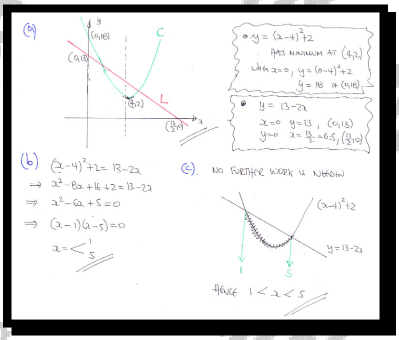
b) Solve the equation

$$(x-4)^2 + 2 = 13 - 2x.$$

c) Hence find the range of values of x for which

$$(x-4)^2 + 2 < 13 - 2x.$$

$\boxed{}$, $\boxed{x = 1,5}$, $\boxed{1 < x < 5}$



Question 11 (***+)

Find the set of values of x, that satisfy the following inequality.

$$\frac{4x+1}{x-1} > 3.$$

$\boxed{}$, $\boxed{x < -4 \cup x > 1}$

SOLVING THE RATIONAL INEQUALITY BY TRANSFORMING IT INTO A QUADRATIC

$$\Rightarrow \frac{4x+1}{x-1} > 3$$

$$\Rightarrow \frac{(4x+1)(x-1)}{(x-1)(x-1)} > 3$$

$$\Rightarrow \frac{(4x+1)(x-1)}{(x-1)^2} > 3$$

$$\Rightarrow (4x+1)(x-1) > 3(x-1)^2$$

$$\Rightarrow (4x+1)(x-1) - 3(x-1)^2 > 0$$

$$\Rightarrow (x-1)\left[(4x+1) - 3(x-1)\right] > 0$$

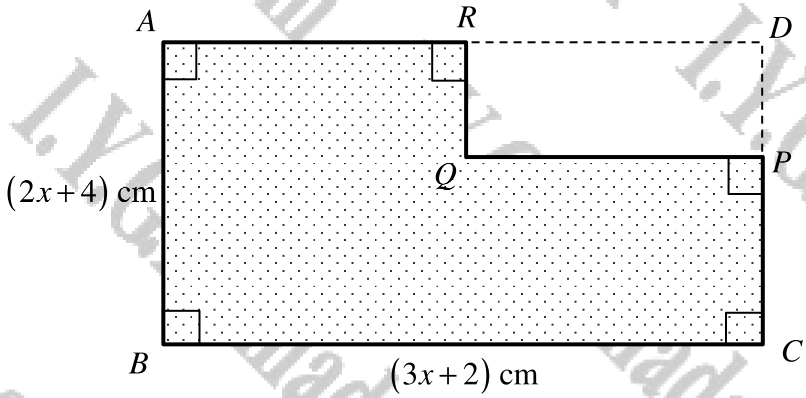
$$\Rightarrow (x-1)\left[4x+1-3x+3\right] > 0$$

$$\Rightarrow (x-1)(x+4) > 0$$

C.V. $< \begin{matrix} 1 \\ -4 \end{matrix}$

$x < -4$ OR $x > 1$ //

Question 12 (***+)



A rectangle $ABCD$ measures $(3x+2)$ cm by $(2x+4)$ cm.

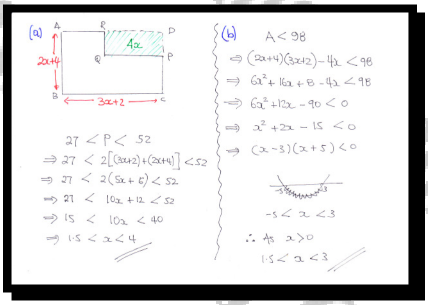
A second rectangle $PQRD$ is removed from the rectangle $ABCD$, as shown in the figure above. The perimeter of the composite shape $ABCPRQ$ is greater than 27 cm but less than 52 cm.

a) Find the range of the possible values of x.

The area of the rectangle $PQRD$ is $4x$ cm^2.

b) Given further that the area of the composite shape $ABCPRQ$ is less than 98 cm^2, determine an amended range of the possible values of x.

$\boxed{}$, $\boxed{1.5 < x < 4}$, $\boxed{1.5 < x < 3}$



(a)

$27 < P < 52$

$\Rightarrow 27 < 2[(3x+2)+(2x+4)] < 52$

$\Rightarrow 27 < 2(5x+6) < 52$

$\Rightarrow 27 < 10x+12 < 52$

$\Rightarrow 15 < 10x < 40$

$\Rightarrow 1.5 < x < 4$

(b) $A < 98$

$\Rightarrow (2x+4)(3x+2) - 4x < 98$

$\Rightarrow 6x^2 + 16x + 8 - 4x < 98$

$\Rightarrow 6x^2 + 12x - 90 < 0$

$\Rightarrow x^2 + 2x - 15 < 0$

$\Rightarrow (x-3)(x+5) < 0$

$-5 < x < 3$

$\therefore$ As $x > 0$

$1.5 < x < 3$

Question 13 (***+)

$$f(x) = x^2 - 2x - 4, \; x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $(x+a)^2 + b$, where a and b are constants.

b) Find in exact form the solutions of the equation $f(x) = 0$.

c) Hence solve the inequality

$$2(3x-4)-(x+6)(x-2) > 0.$$

$\boxed{}$, $\boxed{(x-1)^2 - 5}$, $\boxed{x = 1 \pm \sqrt{5}}$, $\boxed{1-\sqrt{5} < x < 1+\sqrt{5}}$

a) COMPLETING THE SQUARE

$f(x) = x^2 - 2x - 4 = (x-1)^2 - 1^2 - 4$
$= (x-1)^2 - 1 - 4$
$= (x-1)^2 - 5$

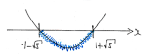
b) USING PART (a) $f(x) = 0$

$\Rightarrow (x-1)^2 - 5 = 0$
$\Rightarrow (x-1)^2 = 5$
$\Rightarrow x - 1 = \pm\sqrt{5}$
$\Rightarrow x = \begin{cases} 1+\sqrt{5} \\ 1-\sqrt{5} \end{cases}$

c) TIDYING UP THE INEQUALITY

$\Rightarrow 2(3x-4) - (x+6)(x-2) > 0$
$\Rightarrow 6x - 8 - (x^2 - 2x + 6x - 12) > 0$
$\Rightarrow \cancel{6x} - 8 - x^2 + 2x - \cancel{6x} + 12 > 0$
$\Rightarrow -x^2 + 2x + 4 > 0$
$\Rightarrow x^2 - 2x - 4 < 0$

USING PART (a) & (b)



$\therefore 1-\sqrt{5} < x < 1+\sqrt{5}$

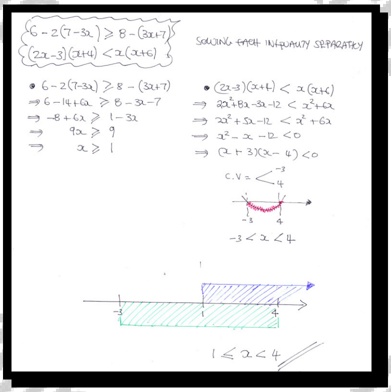
Question 14 (***+)

Determine the range of values of x that satisfy **both** the inequalities given below.

$$6-2(7-3x) \geq 8-(3x+7)$$

$$(2x-3)(x+4) < x(x+6).$$

$\boxed{}$, $\boxed{1 \leq x < 4}$

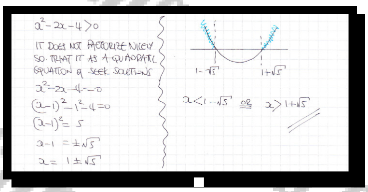


Question 15 (***+)

Solve the following quadratic inequality.

$$x^2-2x-4>0.$$

$\boxed{}$, $\boxed{x<1-\sqrt{5} \ \cup \ x>1+\sqrt{5}}$



Question 16 (***+)

$$f(x) = x^2 - 12x + 30, \quad x \in \mathbb{R}.$$

a) Find in exact surd form the solutions of the equation $f(x) = 0$.

b) Hence solve the inequality …

i. … $f(x) < 0$.

ii. … $n^2 - 12n + 30 < 0$, where n is an **integer**.

$\boxed{}$, $\boxed{x = 6 \pm \sqrt{6}}$, $\boxed{6 - \sqrt{6} < x < 6 + \sqrt{6}}$, $\boxed{n = 4,5,6,7,8}$

a) AS THE SOLUTIONS ARE SURDS, COMPLETE THE SQUARE OR USE THE QUADRATIC FORMULA

$f(x) = 0 \Rightarrow x^2 - 12x + 30 = 0$

$\Rightarrow (x-6)^2 - 6^2 + 30 = 0$

$\Rightarrow (x-6)^2 - 36 + 30 = 0$

$\Rightarrow (x-6)^2 = 6$

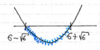
$\Rightarrow x - 6 = \pm\sqrt{6}$

$\Rightarrow x = \begin{cases} 6+\sqrt{6} \\ 6-\sqrt{6} \end{cases}$

b) I) USING PART (a)

$x^2 - 12 + 30 < 0$

$\text{C.V} = \begin{cases} 6+\sqrt{6} \\ 6-\sqrt{6} \end{cases}$



$6 - \sqrt{6} < x < 6 + \sqrt{6}$

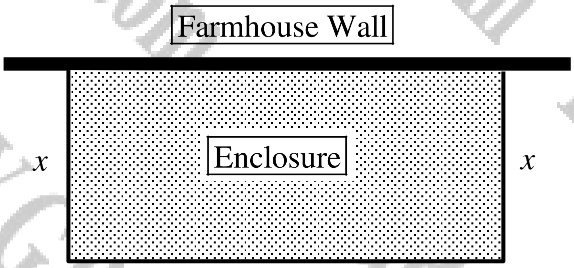
II) USING PART (bI)

$6 - \sqrt{6} < n < 6 + \sqrt{6}$

$3.5505\ldots < n < 8.4494\ldots$

$\therefore n = 4,5,6,7,8$ OR $4 \leqslant n \leqslant 8$, $n \in \mathbb{N}$

Question 17 (***+)



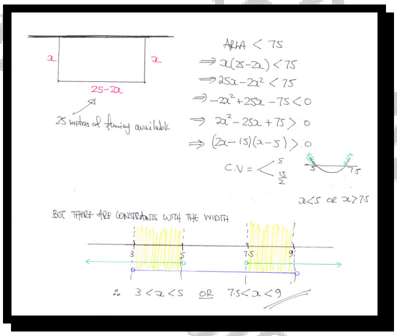
The figure above shows the plan of a rectangular enclosure to be built next to a farmhouse. One of the farmhouse's walls will form one of the sides of the enclosure and 25 metres of fencing will form the other three sides.

The width of the enclosure is x metres, as shown in the figure.

The area of the enclosure must be at most 75 m^2.

Given further that the width of the enclosure must be at least 3 metres but no more than 9 metres, determine the range of the possible values of x.

$\boxed{}$, $\boxed{3 < x < 5 \cup 7.5 < x < 9}$



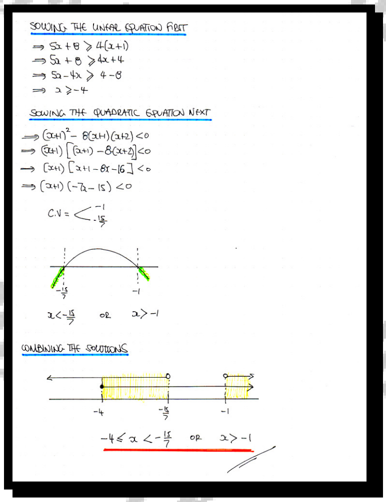
Question 18 (***+)

Determine the range of values of x that satisfy **both** the inequalities given below.

$$5x+8 \geq 4(x+1)$$

$$(x+1)^2 - 8(x+1)(x+2) < 0 .$$

$$\boxed{-4 \leq x < -\frac{15}{7} \ \cup \ x > -1}$$



SOLVING THE LINEAR EQUATION FIRST

$\Rightarrow 5x+8 \geqslant 4(x+1)$
$\Rightarrow 5x+8 \geqslant 4x+4$
$\Rightarrow 5x-4x \geqslant 4-8$
$\Rightarrow x \geqslant -4$

SOLVING THE QUADRATIC EQUATION NEXT

$\Rightarrow (x+1)^2 - 8(x+1)(x+2) < 0$
$\Rightarrow (x+1)\left[(x+1) - 8(x+2)\right] < 0$
$\Rightarrow (x+1)\left[x+1-8x-16\right] < 0$
$\Rightarrow (x+1)(-7x-15) < 0$

C.V. $= -1,\ -\frac{15}{7}$

$x < -\frac{15}{7}$ OR $x > -1$

COMBINING THE SOLUTIONS

$-4 \leqslant x < -\frac{15}{7}$ OR $x > -1$

Question 19 (****)

Find, as exact simplified surds, the solution interval that satisfies both the following inequalities.

$$(x+2)(x+4) > 10x+7$$

$$x\sqrt{3} < 2 + \frac{2(2x-1)}{\sqrt{3}}.$$

$\boxed{}$, $\boxed{2-2\sqrt{3} < x < 2-\sqrt{3} \ \cup \ x > 2+\sqrt{3}}$

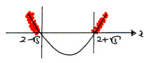
SOLVING THE FIRST INEQUALITY

$\Rightarrow (x+2)(x+4) > 10x+7$
$\Rightarrow x^2+6x+8 > 10x+7$
$\Rightarrow x^2-4x+1 > 0$

THIS DOES NOT FACTORIZE NICELY — OBTAIN CRITICAL VALUES BY COMPLETING THE SQUARE IN THE CORRESPONDING EQUATION — OR USE QUADRATIC FORMULA

$\Rightarrow$ "$x^2-4x+1=0$"
$\Rightarrow (x-2)^2-4+1=0$
$\Rightarrow (x-2)^2 = 3$
$\Rightarrow x-2 = \pm\sqrt{3}$
$\Rightarrow x = 2\pm\sqrt{3}$

LOOKING AT THE DIAGRAM

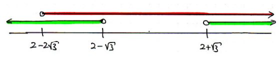


$x < 2-\sqrt{3}$ OR $x > 2+\sqrt{3}$ //

SOLVING THE SECOND INEQUALITY

$\Rightarrow x\sqrt{3} < 2 + \frac{2(2x-1)}{\sqrt{3}}$
$\Rightarrow 3x < 2\sqrt{3} + 2(2x-1)$
$\Rightarrow 3x < 2\sqrt{3} + 4x - 2$
$\Rightarrow -x < -2 + 2\sqrt{3}$
$\Rightarrow x > 2 - 2\sqrt{3}$ MULTIPLIED BY NEGATIVE

FINALLY TO OBTAIN THE COMMON SOLUTION INTERVAL



$\therefore 2-2\sqrt{3} < x < 2-\sqrt{3}$ OR $x > 2+\sqrt{3}$ //

Question 20 (****)

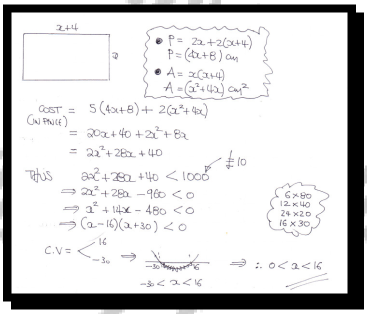
The cost for framing a picture is

- 2 pence per cm^2 of glass.
- 5 pence per cm of wooden frame.

A rectangular picture is such so that its length is 4 cm greater than its width, $x \text{ cm}$.

If a **maximum** of £10 is available for framing, determine the range of the possible values of x.

$\boxed{}$, $\boxed{0 < x < 16}$


$x+4$

x

- $P = 2x + 2(x+4)$
  $P = (4x+8)$ cm
- $A = x(x+4)$
  $A = (x^2+4x)$ cm^2

COST (IN PENCE) $= 5(4x+8) + 2(x^2+4x)$
$= 20x + 40 + 2x^2 + 8x$
$= 2x^2 + 28x + 40$

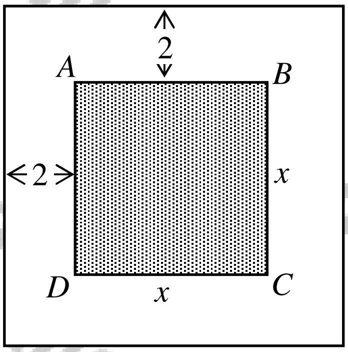
THUS $2x^2 + 28x + 40 < 1000$ £10
$\Rightarrow 2x^2 + 28x - 960 < 0$
$\Rightarrow x^2 + 14x - 480 < 0$
$\Rightarrow (x-16)(x+30) < 0$

6×80
12×40
24×20
16×30

C.V $= \begin{cases} 16 \\ -30 \end{cases} \Rightarrow$ -30 16 $\Rightarrow \therefore 0 < x < 16$

$-30 < x < 16$

Question 21 (****)



The figure above shows a square piece of lawn $ABCD$ of side length x metres.

The lawn is surrounded by a path which is 2 metres wide, as shown in the figure.

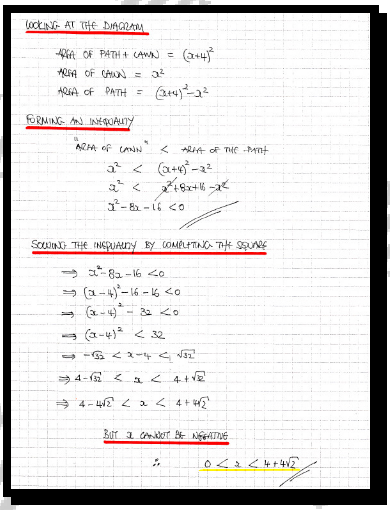
The area of the lawn must be less than the area of the path.

a) Show clearly that

$$x^2 - 8x - 16 < 0 .$$

b) Hence determine the range of the possible values of x, in terms of surds where appropriate.

$\boxed{\text{SYN-H}}$, $\boxed{0 < x < 4 + 4\sqrt{2}}$



LOOKING AT THE DIAGRAM

AREA OF PATH + LAWN $= (x+4)^2$

AREA OF LAWN $= x^2$

AREA OF PATH $= (x+4)^2 - x^2$

FORMING AN INEQUALITY

"AREA OF LAWN" < AREA OF THE PATH

$x^2 < (x+4)^2 - x^2$

$x^2 < \cancel{x^2} + 8x + 16 - \cancel{x^2}$

$x^2 - 8x - 16 < 0$

SOLVING THE INEQUALITY BY COMPLETING THE SQUARE

$\Rightarrow x^2 - 8x - 16 < 0$

$\Rightarrow (x-4)^2 - 16 - 16 < 0$

$\Rightarrow (x-4)^2 - 32 < 0$

$\Rightarrow (x-4)^2 < 32$

$\Rightarrow -\sqrt{32} < x - 4 < \sqrt{32}$

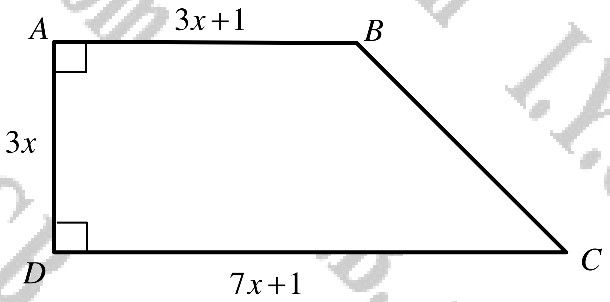
$\Rightarrow 4 - \sqrt{32} < x < 4 + \sqrt{32}$

$\Rightarrow 4 - 4\sqrt{2} < x < 4 + 4\sqrt{2}$

BUT x CANNOT BE NEGATIVE

$\therefore \quad 0 < x < 4 + 4\sqrt{2}$

Question 22 (****)



The figure above shows a right angled trapezium $ABCD$ where $|AB| = 3x+1$, $|AD| = 3x$, $|DC| = 7x+1$ and $\measuredangle DAB = \measuredangle CDA = 90°$.

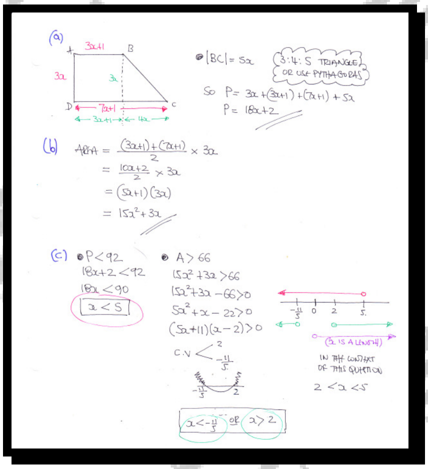
a) Express the perimeter of the trapezium in terms of x.

b) Show that the area of the trapezium is $15x^2 + 3x$.

The perimeter of the trapezium has to be less than 92 and its area greater than 66.

c) Determine the range of the possible values of x.

$\boxed{\text{C1}}$, $\boxed{P = 18x+2}$, $\boxed{2 < x < 5}$



Question 23 (****)

A rectangular piece of card has length x cm and an area of 1200 cm^2.

A square of side length 5 cm is removed from each corner and the sides of the remaining card are folded upwards to form an **open** box of height 5 cm.

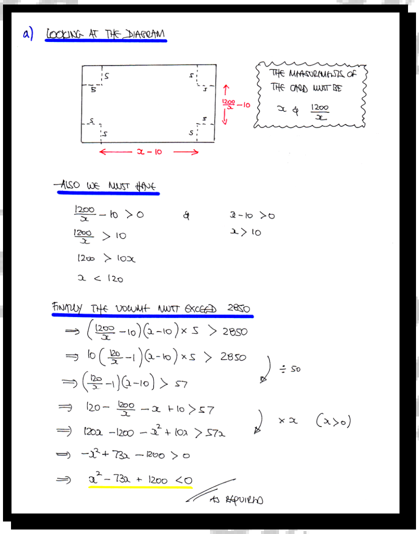
The resulting box must have a volume greater than 2850 cm^3.

a) Show clearly that

$$x^2 - 73x + 1200 < 0.$$

b) Hence determine the range of the possible values of x.

$\boxed{}$, $\boxed{25 < x < 48}$



a) LOOKING AT THE DIAGRAM

THE MEASUREMENTS OF THE CARD MUST BE x & $\frac{1200}{x}$

ALSO WE MUST HAVE

$\frac{1200}{x} - 10 > 0$ & $x - 10 > 0$

$\frac{1200}{x} > 10$ $x > 10$

$1200 > 10x$

$x < 120$

FINALLY THE VOLUME MUST EXCEED 2850

$\Rightarrow \left(\frac{1200}{x} - 10\right)(x-10) \times 5 > 2850$

$\Rightarrow 10\left(\frac{120}{x} - 1\right)(x-10) \times 5 > 2850$ $\div 50$

$\Rightarrow \left(\frac{120}{x} - 1\right)(x-10) > 57$

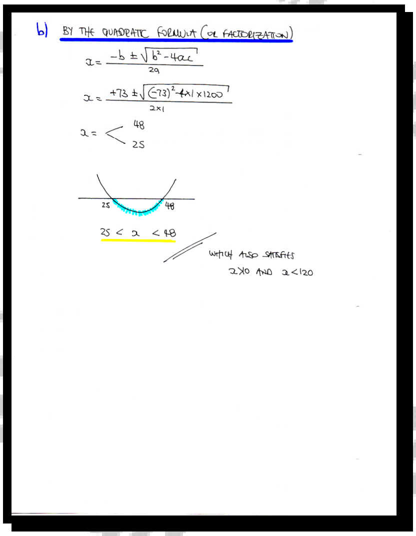
$\Rightarrow 120 - \frac{1200}{x} - x + 10 > 57$ $\times x$ $(x>0)$

$\Rightarrow 120x - 1200 - x^2 + 10x > 57x$

$\Rightarrow -x^2 + 73x - 1200 > 0$

$\Rightarrow x^2 - 73x + 1200 < 0$

AS REQUIRED



b) BY THE QUADRATIC FORMULA (OR FACTORIZATION)

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

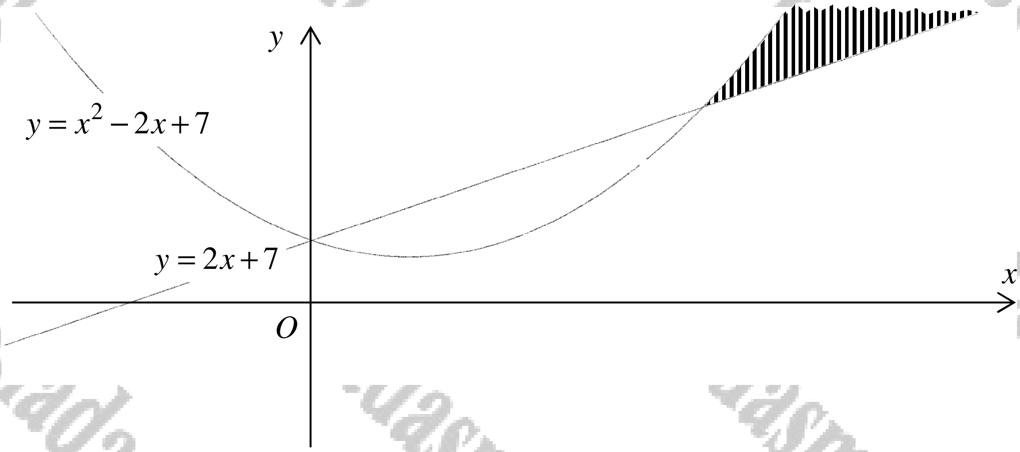
$x = \frac{+73 \pm \sqrt{(-73)^2 - 4 \times 1 \times 1200}}{2 \times 1}$

$x = \begin{cases} 48 \\ 25 \end{cases}$

$25 < x < 48$

WHICH ALSO SATISFIES $x > 10$ AND $x < 120$

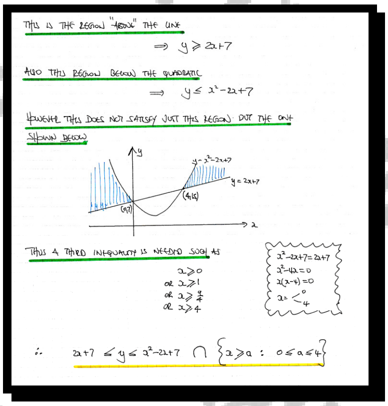
Question 24 (****)



The figure above shows the graphs of $y = x^2 - 2x + 7$ and $y = 2x + 7$.

Express the shaded region in the figure as a set of inequalities.

$\boxed{}$, $\boxed{2x + 7 \le y \le x^2 - 2x + 7 \cap \{x \ge a : 0 \le a \le 4\}}$



THIS IS THE REGION "ABOVE" THE LINE

$\Rightarrow y \ge 2x+7$

ALSO THIS REGION BELOW THE QUADRATIC

$\Rightarrow y \le x^2-2x+7$

HOWEVER THIS DOES NOT SATISFY JUST THIS REGION BUT THE ONE SHOWN BELOW

THUS A THIRD INEQUALITY IS NEEDED SUCH AS

$x \ge 0$
or $x \ge 1$
or $x \ge \frac{3}{2}$
or $x \ge 4$

$x^2-2x+7 = 2x+7$
$x^2-4x = 0$
$x(x-4) = 0$
$x = \begin{cases} 0 \\ 4 \end{cases}$

$\therefore \; 2x+7 \le y \le x^2-2x+7 \cap \{x \ge a : 0 \le a \le 4\}$

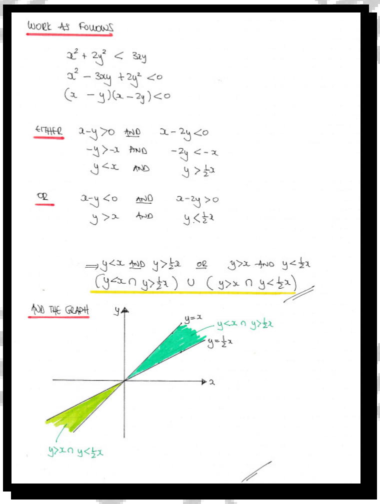
Question 25 (****)

Solve the inequality

$$x^2 + 2y^2 < 3xy \,,$$

and hence indicate the solution in a suitable sketch.

$\boxed{}$, $\boxed{\left(y < x \cap y > \frac{1}{2}x\right) \cup \left(y > x \cap y < \frac{1}{2}x\right)}$


WORK AS FOLLOWS

$x^2 + 2y^2 < 3xy$

$x^2 - 3xy + 2y^2 < 0$

$(x - y)(x - 2y) < 0$

EITHER $x - y > 0$ AND $x - 2y < 0$

$-y > -x$ AND $-2y < -x$

$y < x$ AND $y > \frac{1}{2}x$

OR $x - y < 0$ AND $x - 2y > 0$

$y > x$ AND $y < \frac{1}{2}x$

$\Longrightarrow y < x$ AND $y > \frac{1}{2}x$ OR $y > x$ AND $y < \frac{1}{2}x$

$\left(y < x \cap y > \frac{1}{2}x\right) \cup \left(y > x \cap y < \frac{1}{2}x\right)$

AND THE GRAPH

Question 26 (****+)

Consider the following inequalities

$$5x+13>4(x+2)$$

$$(x-2)^2-k(x-2)(x+3)<0,$$

where k is a non zero constant.

The common solution interval of both these inequalities is

$$-5<x<-\frac{17}{4} \ \cup \ x>m,$$

where m is a non zero constant.

Determine, in any order, the value of k and the value of m.

$\boxed{}$, $\boxed{m=2}$, $\boxed{k=5}$

SOLVE THE LINEAR INEQUALITY FIRST

$\Rightarrow 5x+13>4(x+2)$

$\Rightarrow 5x+13>4x+8$

$\Rightarrow x>-5$

SO ONE OF THE CRITICAL VALUES NOW CAN BE ASSIGNED TO THE LINEAR INEQUALITY WHICH MEANS THAT $x=-\frac{17}{4}$ IS A CRITICAL VALUE OF THE QUADRATIC INEQUALITY

$\Rightarrow (x-2)^2-k(x-2)(x+3)<0$

SUB $x=-\frac{17}{4}$ & SOLVE AN EQUATION

$\Rightarrow \left(-\frac{17}{4}-2\right)^2-k\left(-\frac{17}{4}-2\right)\left(-\frac{17}{4}+3\right)=0$

$\Rightarrow \left(-\frac{25}{4}\right)^2-k\left(-\frac{25}{4}\right)\left(-\frac{5}{4}\right)=0$

$\Rightarrow \frac{625}{16}-\frac{125}{16}k=0$

$\Rightarrow 625-125k=0$

$\Rightarrow 125k=625$

$\Rightarrow k=5$

NEXT WE HAVE

$\Rightarrow (x-2)^2-5(x-2)(x+3)<0$

$\Rightarrow (x-2)\left[(x-2)-5(x+3)\right]<0$

$\Rightarrow (x-2)(-4x-17)<0$

$C.V = \begin{cases} -17/4 \\ 2 \end{cases}$ $\therefore m=2$

Question 27 (*****)

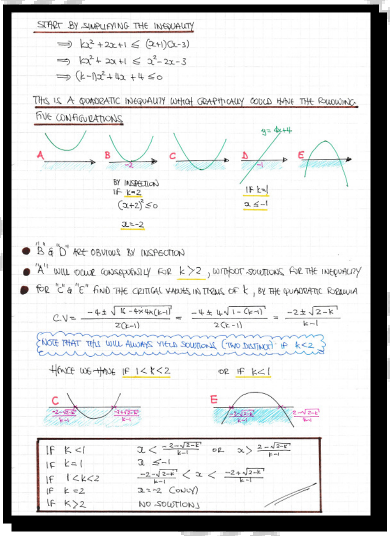
Consider the following inequality

$$kx^2 + 2x + 1 \le (x+1)(x-3),$$

where k is a real constant.

Find, in terms of k where appropriate, the solution intervals of the above inequality for all possible values of k.

$$\boxed{}, \quad \boxed{\begin{cases} k<1 & k < \dfrac{-2-\sqrt{2-k}}{k-1} \cup k > \dfrac{-2-\sqrt{2-k}}{k-1} \\ k=1 & x \le -1 \\ 1<k<2 & k < \dfrac{-2-\sqrt{2-k}}{k-1} \cup k > \dfrac{-2-\sqrt{2-k}}{k-1} \\ k=2 & x=-2 \\ k>2 & \text{no solutions} \end{cases}}$$


START BY SIMPLIFYING THE INEQUALITY

$\Rightarrow kx^2+2x+1 \le (x+1)(x-3)$

$\Rightarrow kx^2+2x+1 \le x^2-2x-3$

$\Rightarrow (k-1)x^2+4x+4 \le 0$

THIS IS A QUADRATIC INEQUALITY WHICH GRAPHICALLY COULD HAVE THE FOLLOWING FIVE CONFIGURATIONS

A B C D E

$y = 4x+4$

-2 -1

BY INSPECTION IF $k=2$

$(x+2)^2 \le 0$

$x=-2$

IF $k=1$

$x \le -1$

- "B" & "D" ARE OBVIOUS BY INSPECTION
- "A" WILL OCCUR CONSEQUENTLY FOR $k>2$, WITHOUT SOLUTIONS FOR THE INEQUALITY
- FOR "C" & "E" FIND THE CRITICAL VALUES IN TERMS OF k, BY THE QUADRATIC FORMULA

$$C.V = \frac{-4 \pm \sqrt{16-4\times4\times(k-1)}}{2(k-1)} = \frac{-4 \pm 4\sqrt{1-(k-1)}}{2(k-1)} = \frac{-2\pm\sqrt{2-k}}{k-1}$$

NOTE THAT THIS WILL ALWAYS YIELD SOLUTIONS (TWO DISTINCT) IF $k<2$

HENCE WE HAVE IF $1<k<2$ OR IF $k<1$

C $\frac{-2-\sqrt{2-k}}{k-1}$ $\frac{-2+\sqrt{2-k}}{k-1}$

E $\frac{-2-\sqrt{2-k}}{k-1}$ $\frac{2-\sqrt{2-k}}{k-1}$

IF $k<1$	$x < \frac{-2-\sqrt{2-k}}{k-1}$ OR $x > \frac{2-\sqrt{2-k}}{k-1}$
IF $k=1$	$x \le -1$
IF $1<k<2$	$\frac{-2-\sqrt{2-k}}{k-1} < x < \frac{-2+\sqrt{2-k}}{k-1}$
IF $k=2$	$x=-2$ (ONLY)
IF $k>2$	NO SOLUTIONS