INDICES

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NUMBER INDICES

(Non Calculator)

Question 1

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

- a) $4^{-1} + 2^{-3}$
- **b**) $5^{-2} + 25^{-1}$
- c) $2^{-4} + 8^{-1}$
- **d**) $2^{-5} 8^{-2}$
- **e**) $3^{-3} + 9^{-2} + 27^{-1}$

Question 2

- **a**) $4^{\frac{1}{2}} + 9^{\frac{1}{2}}$
- **b**) $64^{\frac{1}{2}} + 64^{\frac{1}{3}}$
- **c)** $16^{\frac{1}{2}} + 16^{\frac{1}{4}}$
- **d**) $9^{\frac{1}{2}} + 9^{\frac{3}{2}}$
- e) $4^{\frac{1}{2}} + 4^{\frac{5}{2}}$

Question 3

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

- **a**) $9^{\frac{1}{2}} + 9^{-\frac{1}{2}}$
- **b**) $4^{\frac{1}{2}} + 4^{-\frac{1}{2}}$
- c) $8^{\frac{1}{3}} + 8^{-\frac{1}{3}}$
- **d**) $25^{\frac{1}{2}} 25^{-\frac{3}{2}}$
- **e**) $36^{\frac{1}{2}} 36^{-\frac{3}{2}}$

Question 4

- **a**) $16^{\frac{3}{2}} + 8^{\frac{2}{3}}$
- **b**) $27^{\frac{2}{3}} + 25^{\frac{3}{2}}$
- **c**) $8^{\frac{4}{3}} + 16^{\frac{1}{4}}$
- **d**) $8^{\frac{5}{3}} 16^{\frac{3}{4}}$
- e) $27^{\frac{4}{3}} 81^{\frac{3}{4}}$

Question 5

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

- **a**) $9^{\frac{3}{2}}$
- **b**) $8^{-\frac{2}{3}}$
- **c**) $16^{-\frac{3}{2}}$
- **d**) $27^{\frac{4}{3}}$
- **e**) $81^{\frac{-3}{4}}$

Question 6

- **a**) $\left(\frac{2}{3}\right)^{-2}$
- **b**) $\left(\frac{4}{9}\right)^{\frac{2}{2}}$
- **c**) $\left(\frac{25}{16}\right)^{-\frac{1}{2}}$
- **d**) $\left(\frac{81}{16}\right)^{\frac{3}{4}}$
- e) $(2.25)^{\frac{3}{2}}$

Question 7

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

- **a**) $\left(1\frac{7}{9}\right)^{\frac{3}{2}}$
- **b**) $\left(5\frac{4}{9}\right)^{-\frac{1}{2}}$
- **c**) $\left(2\frac{1}{4}\right)^{\frac{5}{2}}$
- **d**) $\left(4\frac{17}{27}\right)^{\frac{2}{3}}$
- e) $\left(6\frac{1}{4}\right)^{-\frac{3}{2}}$

Question 8

- a) $32^5 \times 8^{-9} \times 2^8$
- **b**) $8^{-4} \times 2^{11}$
- c) $\frac{8^6}{16^3}$
- **d**) $27^{-4} \times 3^{11}$
- e) $\left(5^6 \times 25^3 \div 125^2\right)^{\frac{1}{2}}$

Question 9

a)
$$\frac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$$

b)
$$\frac{2^6}{8^{\frac{5}{2}} \times 2^{-\frac{1}{2}}}$$

c)
$$2^{16} \times 4^{-8} \times 8^4 \times 16^{-2}$$

d)
$$\left(36^{\frac{1}{2}} + 16^{\frac{1}{4}}\right)^{\frac{1}{3}}$$

e)
$$\left(125^{\frac{1}{3}} \times 25^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{-\frac{1}{2}}}\right)^{-\frac{2}{3}}$$

ALGEBRAIC INDICES

(Non Calculator)

Question 1

Simplify **fully** each of the following expressions.

- **a**) $4a^2b^3 \times 3ab^4$
- **b**) $(2a^3b^2)^4$
- c) $\frac{3a^3b^2c\times 6ab^2c^3}{2a^2bc^3}$
- **d**) $\frac{\left(4xy^2\right)^2}{\left(2x\right)^3}$
- e) $\frac{\sqrt{9x^6y^4}}{(3x^2y^3)^2}$

Question 2

Simplify **fully** each of the following expressions.

- **a**) $\frac{x^6}{x^{-2}}$
- **b**) $\frac{12y^{-5}}{3y^{-2}}$
- **c**) $(3t^3q^4)^3$
- $\mathbf{d)} \ \frac{3z^4 \times (10z)^3}{125z^5}$

Question 3

Simplify **fully** each of the following expressions.

- $a) \quad x^{\frac{5}{2}} \times \sqrt{x}$
- **b)** $2y^3 \times 2y^{-1}$
- **c)** $2w^{\frac{1}{2}} \times 3w^2$
- **d**) $2t^{\frac{4}{3}} \times 4\sqrt[3]{t^2}$
- **e**) $k^{\frac{3}{2}} \times 4k^{-3}$

Question 4

Simplify **fully** each of the following expressions.

- **a)** $\left(2k^{\frac{1}{2}}h^3\right)^4$
- **b**) $(9a^6b^2)^{-\frac{1}{2}}$
- $\mathbf{c)} \quad \left(2pq^2\right)^4 \times 5p\sqrt{q^6}$
- **d**) $\frac{12(x^3y^2z)^4}{(4x^2z^6)^2}$

Question 5

Simplify **fully** each of the following expressions.

- a) $(2ab^2c^3)^3$
- **b**) $\left(\frac{1}{2}x^3y^2\right)^3$
- **c**) $(9a^6b^4)^{\frac{1}{2}}$
- **d**) $\left(16p^8q^{-2}\right)^{\frac{1}{2}}$

Question 6

Simplify **fully** each of the following expressions.

a)
$$2a^3 \left(2a^{-1} + a^{\frac{1}{2}}\right)$$

b)
$$4b^{\frac{1}{2}} \left(2b + b^{\frac{1}{2}}\right)$$

c)
$$c^{\frac{3}{2}}(3c^{-1}+c)$$

d)
$$3d^{\frac{3}{2}} \left(4d^{-2} - 2d^{-\frac{1}{2}}\right)$$

Question 7

Simplify **fully** each of the following expressions.

a)
$$a\left(2a^{-1}-3a^{-\frac{1}{2}}\right)$$

b)
$$3b^2 \left(b^{-2} + 2b^{-\frac{1}{2}} \right)$$

c)
$$3c^{\frac{7}{2}} \left(2c^{-\frac{1}{2}} - c \right)$$

d)
$$2d^{\frac{7}{2}} \left(2d^{-1} + d^{\frac{1}{2}} \right)$$

Question 8

a)
$$\frac{1}{2\sqrt{x}} + \frac{4}{x^2}$$

b)
$$x\sqrt{x} - \frac{1}{x^2}$$

c)
$$\sqrt{x^3} - \frac{1}{2x^2}$$

d)
$$\sqrt[3]{x^2} - \frac{3}{2x^3}$$

$$e) \quad 4\sqrt{x} + \frac{1}{4\sqrt{x}}$$

Question 9

Write each of the following expressions as the sum of terms of the form kx^n , where k is a constant.

- **a**) $(5-x^{-2})(2x^3-x)$
- **b)** $(1-x^{\frac{1}{2}})(2-x^{\frac{1}{2}})$
- **c**) $(1+x^{\frac{1}{2}})(x^{\frac{3}{2}}+2)$

Question 10

- **a)** $\left(x^{\frac{3}{2}} + 2x^{-\frac{3}{2}}\right)^2$
- **b)** $\left(x^{\frac{1}{2}} 2x^{-\frac{1}{2}}\right)^2$
- $\mathbf{c)} \quad \left(3x^{-\frac{3}{2}} + 2x^{\frac{1}{2}}\right)^2$
- **d**) $\left(x^{\frac{5}{2}} + x^{\frac{1}{2}}\right)^2$
- $e) \quad \left(3\sqrt{x}-2\right)^2$

Question 11

Write each of the following expressions as the sum of terms of the form kx^n , where k is a constant.

- **a)** $\left(2x^{\frac{1}{2}} + 3\right)^2$
- **b)** $\left(2x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)^2$
- $\mathbf{c)} \quad \left(2x^{\frac{3}{2}} 3x^{-\frac{3}{2}}\right)^2$
- **d)** $\left(x^{\frac{1}{2}} 2x^{-\frac{3}{2}}\right)^2$
- **e**) $(x^{\frac{1}{2}}-4)(x^{-\frac{1}{2}}-1)$

Question 12

- $\mathbf{a)} \quad \frac{4+x}{2x^3}$
- **b)** $\frac{9\sqrt{x} + 6x}{3x^3}$
- c) $\frac{(x+2)(2x-3)}{4x^5}$
- $\mathbf{d)} \quad \frac{x^2 + 3x}{2\sqrt{x}}$
- $e) \quad \frac{\sqrt{x}(2x-4)}{3x^2}$

Question 13

$$\mathbf{a)} \quad \frac{(3x-2)(2x-1)}{2x^{\frac{3}{2}}}$$

b)
$$\frac{(2\sqrt{x}+3)^2}{4x}$$

$$\mathbf{c)} \quad \frac{x^2(\sqrt{x} + 4x)}{4\sqrt{x}}$$

$$\mathbf{d)} \quad \frac{\sqrt{x}\left(5x^2 - 8\right)}{4x}$$

e)
$$\frac{(x^2-3)(\sqrt{x}+4x)}{3\sqrt{x}}$$

INDICIAL EQUATIONS

(Non Calculator)

Question 1

Solve each of the following equations without using a calculator.

- **a**) $x^{\frac{1}{3}} = 2$
- **b**) $y^{-\frac{1}{3}} = 8$
- **c)** $z^{\frac{3}{2}} = 27$
- **d**) $w^{\frac{2}{3}} = 64$
- **e**) $t^{-\frac{1}{2}} = \frac{1}{4}$

Question 2

- **a**) $x^{-\frac{3}{4}} = 8$
- **b**) $y^{-\frac{1}{3}} = \frac{1}{2}$
- **c**) $(3-z)^{\frac{3}{2}} = 8$
- **d**) $\left(25w^2\right)^{-\frac{1}{2}} = 2$

Question 3

Solve each of the following equations without using a calculator.

- **a**) $x^{-1} = \frac{x}{16}$
- **b)** $3y^{-\frac{1}{2}} 4 = 0$
- $\mathbf{c)} \quad 8w^{\frac{1}{2}} w^{-1} = 0$
- **d**) $32t^{\frac{3}{2}} \frac{1}{t} = 0$

Question 4

- **a**) $2^{3-x} = 4^x$
- **b)** $2^{y+2} = 4\sqrt{2}$
- **c**) $4^z = 8^{2-z}$
- **d**) $2^w = \frac{4}{\sqrt{2}}$
- **e**) $2^t = 8\sqrt{2}$

Question 5

Solve each of the following equations without using a calculator.

- **a**) $3^{x+2} = 9^x$
- **b**) $2^{y+1} = 8^{2y-1}$
- **c**) $27^{3z+1} = 9$
- **d**) $9^{2w-3} = 27^{w+2}$
- $e) \quad 8 \times 2^{2t} = \frac{2^{5t+1}}{4^{-t}}$

Question 6

- **a**) $2^{x+2} = 4^x$
- **b**) $9^y = 27^{1-y}$
- **c**) $4^z = 8^{3-z}$
- **d**) $\frac{4^w \times 2^{5w}}{16^w} = 2^w$
- e) $\frac{27^t}{3^{t-1}} = 3\sqrt{3}$

Question 7

Solve each of the following equations without using a calculator.

$$\mathbf{a)} \quad \frac{81^{3-x}}{27^{2x+1}} = 3$$

b)
$$\frac{5^y}{25^{y-1}} = \sqrt{5}$$

$$\mathbf{c}) \quad \frac{16^z}{\sqrt{2}} = 2^{z-1}$$

d)
$$\frac{25^{t-1}}{5} = \sqrt{5}$$

Question 8

a)
$$2^{3x+4} = 4\sqrt{2}$$

b)
$$3^x = \frac{\sqrt{3}}{9}$$

c)
$$2^z = \frac{\sqrt{2}}{2^{z+1}}$$

d)
$$9^w = \frac{3^{w-1}}{27}$$

e)
$$3^{t+1} = \frac{27^t}{9}$$

Question 9

Solve each of the following equations without using a calculator.

- **a**) $3x^{\frac{1}{3}} = x^{-\frac{2}{3}}$
- **b**) $2x^{-\frac{1}{2}} \frac{3}{2}x^{\frac{1}{2}} = 0$
- $\mathbf{c)} \quad w^{\frac{3}{2}} 8x^{-\frac{1}{2}} = 0$
- **d)** $z\left(z^{\frac{1}{2}}-2z^{-\frac{1}{2}}\right)^2=0$
- **e**) $27t^{-\frac{1}{2}} = 125t$

Question 10

- $\mathbf{a)} \quad 4^x 2^{x+2} 32 = 0$
- **b**) $2^{y+2} + 2^{3-y} = 33$
- $\mathbf{c)} \quad 3^{2-z} 3^{z+1} = 26$
- **d**) $2^{2w+2} + 3 \times 2^w 1 = 0$

Question 11

Solve each of the following equations without using a calculator.

a)
$$3^{2x} - 3^{x+1} = 54$$

b)
$$100^t - 10001(10)^{t-1} + 100 = 0$$

c)
$$3(3^{2k}) - 28(3^k) + 9 = 0$$

d)
$$2^{2p-2} - 2^{p-2} - 3 = 0$$

Question 12

a)
$$2x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 12 = 0$$

b)
$$y^{\frac{1}{4}} - y^{-\frac{1}{4}} = 2$$

$$\mathbf{c)} \quad 6z^{-\frac{1}{3}} - z^{\frac{1}{3}} = 5$$

d)
$$3w + w^{\frac{1}{2}} - 2 = 0$$

e)
$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}$$