

INDICES

Created by T. Madas

NUMBER INDICES

(Non Calculator)

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Question 1

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $4^{-1} + 2^{-3}$

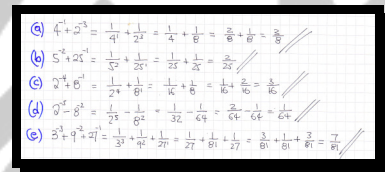
b) $5^{-2} + 25^{-1}$

c) $2^{-4} + 8^{-1}$

d) $2^{-5} - 8^{-2}$

e) $3^{-3} + 9^{-2} + 27^{-1}$

$$\frac{3}{8}, \frac{2}{25}, \frac{3}{16}, \frac{1}{64}, \frac{7}{81}$$



Handwritten solutions for Question 1:

- a) $4^{-1} + 2^{-3} = \frac{1}{4^1} + \frac{1}{2^3} = \frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$
- b) $5^{-2} + 25^{-1} = \frac{1}{5^2} + \frac{1}{25^1} = \frac{1}{25} + \frac{1}{25} = \frac{2}{25}$
- c) $2^{-4} + 8^{-1} = \frac{1}{2^4} + \frac{1}{8^1} = \frac{1}{16} + \frac{1}{8} = \frac{1}{16} + \frac{2}{16} = \frac{3}{16}$
- d) $2^{-5} - 8^{-2} = \frac{1}{2^5} - \frac{1}{8^2} = \frac{1}{32} - \frac{1}{64} = \frac{2}{64} - \frac{1}{64} = \frac{1}{64}$
- e) $3^{-3} + 9^{-2} + 27^{-1} = \frac{1}{3^3} + \frac{1}{9^2} + \frac{1}{27^1} = \frac{1}{27} + \frac{1}{81} + \frac{1}{27} = \frac{1}{27} + \frac{1}{27} + \frac{1}{81} = \frac{2}{27} + \frac{1}{81} = \frac{6}{81} + \frac{1}{81} = \frac{7}{81}$

Question 2

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $4^{\frac{1}{2}} + 9^{\frac{1}{2}}$

b) $64^{\frac{1}{2}} + 64^{\frac{1}{3}}$

c) $16^{\frac{1}{2}} + 16^{\frac{1}{4}}$

d) $9^{\frac{1}{2}} + 9^{\frac{3}{2}}$

e) $4^{\frac{1}{2}} + 4^{\frac{5}{2}}$

5, 12, 6, 30, 34

Handwritten solutions for Question 2:

- a) $4^{\frac{1}{2}} + 9^{\frac{1}{2}} = \sqrt{4} + \sqrt{9} = 2 + 3 = 5$
- b) $64^{\frac{1}{2}} + 64^{\frac{1}{3}} = \sqrt{64} + \sqrt[3]{64} = 8 + 4 = 12$
- c) $16^{\frac{1}{2}} + 16^{\frac{1}{4}} = \sqrt{16} + \sqrt[4]{16} = 4 + 2 = 6$
- d) $9^{\frac{1}{2}} + 9^{\frac{3}{2}} = \sqrt{9^2} + (\sqrt{9})^3 = 3 + 27 = 30$
- e) $4^{\frac{1}{2}} + 4^{\frac{5}{2}} = \sqrt{4} + (\sqrt{4})^5 = 2 + 2^5 = 2 + 32 = 34$

Question 3

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $9^{\frac{1}{2}} + 9^{-\frac{1}{2}}$

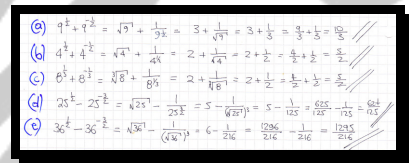
b) $4^{\frac{1}{2}} + 4^{-\frac{1}{2}}$

c) $8^{\frac{1}{3}} + 8^{-\frac{1}{3}}$

d) $25^{\frac{1}{2}} - 25^{-\frac{3}{2}}$

e) $36^{\frac{1}{2}} - 36^{-\frac{3}{2}}$

$$\frac{10}{3}, \frac{5}{2}, \frac{5}{2}, \frac{624}{125}, \frac{1295}{216}$$



Handwritten solutions for Question 3:

- a) $9^{\frac{1}{2}} + 9^{-\frac{1}{2}} = \sqrt{9} + \frac{1}{\sqrt{9}} = 3 + \frac{1}{3} = 3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$
- b) $4^{\frac{1}{2}} + 4^{-\frac{1}{2}} = \sqrt{4} + \frac{1}{\sqrt{4}} = 2 + \frac{1}{2} = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$
- c) $8^{\frac{1}{3}} + 8^{-\frac{1}{3}} = \sqrt[3]{8} + \frac{1}{\sqrt[3]{8}} = 2 + \frac{1}{2} = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$
- d) $25^{\frac{1}{2}} - 25^{-\frac{3}{2}} = \sqrt{25} - \frac{1}{\sqrt{25^3}} = 5 - \frac{1}{\sqrt{15625}} = 5 - \frac{1}{125} = \frac{625}{125} - \frac{1}{125} = \frac{624}{125}$
- e) $36^{\frac{1}{2}} - 36^{-\frac{3}{2}} = \sqrt{36} - \frac{1}{\sqrt{36^3}} = 6 - \frac{1}{\sqrt{46656}} = \frac{7296}{216} - \frac{1}{216} = \frac{7295}{216}$

Question 4

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $16^{\frac{3}{2}} + 8^{\frac{2}{3}}$

b) $27^{\frac{2}{3}} + 25^{\frac{3}{2}}$

c) $8^{\frac{4}{3}} + 16^{\frac{1}{4}}$

d) $8^{\frac{5}{3}} - 16^{\frac{3}{4}}$

e) $27^{\frac{4}{3}} - 81^{\frac{3}{4}}$

68, **134**, **18**, **24**, **54**

Handwritten solutions for Question 4:

- (a) $16^{\frac{3}{2}} + 8^{\frac{2}{3}} = (\sqrt[4]{16})^3 + (\sqrt[3]{8})^2 = 4^3 + 2^2 = 64 + 4 = 68$
- (b) $27^{\frac{2}{3}} + 25^{\frac{3}{2}} = (\sqrt[3]{27})^2 + (\sqrt{25})^3 = 3^2 + 5^3 = 9 + 125 = 134$
- (c) $8^{\frac{4}{3}} + 16^{\frac{1}{4}} = (\sqrt[3]{8})^4 + \sqrt[4]{16} = 2^4 + 2 = 16 + 2 = 18$
- (d) $8^{\frac{5}{3}} - 16^{\frac{3}{4}} = (\sqrt[3]{8})^5 - (\sqrt[4]{16})^3 = 2^5 - 2^3 = 32 - 8 = 24$
- (e) $27^{\frac{4}{3}} - 81^{\frac{3}{4}} = (\sqrt[3]{27})^4 - (\sqrt[4]{81})^3 = 3^4 - 3^3 = 81 - 27 = 54$

Question 5

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $9^{\frac{3}{2}}$

b) $8^{-\frac{2}{3}}$

c) $16^{-\frac{3}{2}}$

d) $27^{\frac{4}{3}}$

e) $81^{-\frac{3}{4}}$

$$\boxed{27}, \boxed{\frac{1}{4}}, \boxed{\frac{1}{64}}, \boxed{81}, \boxed{\frac{1}{27}}$$

Handwritten solutions for Question 5:

- a) $9^{\frac{3}{2}} = (\sqrt{9})^3 = 3^3 = 27 //$
- b) $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4} //$
- c) $16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{(\sqrt{16})^3} = \frac{1}{4^3} = \frac{1}{64} //$
- d) $27^{\frac{4}{3}} = (\sqrt[3]{27})^4 = 3^4 = 81 //$
- e) $81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}} = \frac{1}{(\sqrt[4]{81})^3} = \frac{1}{3^3} = \frac{1}{27} //$

Question 6

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $\left(\frac{2}{3}\right)^{-2}$

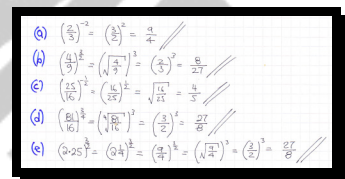
b) $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

c) $\left(\frac{25}{16}\right)^{-\frac{1}{2}}$

d) $\left(\frac{81}{16}\right)^{\frac{3}{4}}$

e) $(2.25)^{\frac{3}{2}}$

$\frac{9}{4}, \frac{8}{27}, \frac{4}{5}, \frac{27}{8}, \frac{27}{8}$



Handwritten solutions for Question 6:

- a) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$
- b) $\left(\frac{4}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{9}}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$
- c) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} = \left(\frac{16}{25}\right)^{\frac{1}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$
- d) $\left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\sqrt[4]{\frac{81}{16}}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$
- e) $(2.25)^{\frac{3}{2}} = \left(2\frac{1}{4}\right)^{\frac{3}{2}} = \left(\frac{9}{4}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{9}{4}}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

Question 7

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $\left(1\frac{7}{9}\right)^{\frac{3}{2}}$

b) $\left(5\frac{4}{9}\right)^{-\frac{1}{2}}$

c) $\left(2\frac{1}{4}\right)^{\frac{5}{2}}$

d) $\left(4\frac{17}{27}\right)^{\frac{2}{3}}$

e) $\left(6\frac{1}{4}\right)^{-\frac{3}{2}}$

$$\frac{64}{27}, \frac{3}{7}, \frac{243}{32}, \frac{25}{9}, \frac{8}{125}$$

Handwritten solutions for Question 7:

- a) $\left(1\frac{7}{9}\right)^{\frac{3}{2}} = \left(\frac{16}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{16}{9}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$
- b) $\left(5\frac{4}{9}\right)^{-\frac{1}{2}} = \left(\frac{49}{9}\right)^{-\frac{1}{2}} = \left(\frac{7}{3}\right)^{-1} = \frac{3}{7}$
- c) $\left(2\frac{1}{4}\right)^{\frac{5}{2}} = \left(\frac{9}{2}\right)^{\frac{5}{2}} = \left(\sqrt{\frac{9}{2}}\right)^5 = \left(\frac{3}{\sqrt{2}}\right)^5 = \frac{243}{32}$
- d) $\left(4\frac{17}{27}\right)^{\frac{2}{3}} = \left(\frac{125}{27}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{125}{27}}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$
- e) $\left(6\frac{1}{4}\right)^{-\frac{3}{2}} = \left(\frac{25}{4}\right)^{-\frac{3}{2}} = \left(\frac{5}{2}\right)^{-3} = \left(\sqrt{\frac{5}{2}}\right)^{-6} = \left(\frac{\sqrt{5}}{\sqrt{2}}\right)^{-6} = \frac{8}{125}$

Question 8

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $32^5 \times 8^{-9} \times 2^8$

b) $8^{-4} \times 2^{11}$

c) $\frac{8^6}{16^3}$

d) $27^{-4} \times 3^{11}$

e) $(5^6 \times 25^3 \div 125^2)^{\frac{1}{2}}$

$$\boxed{64}, \boxed{\frac{1}{2}}, \boxed{64}, \boxed{\frac{1}{3}}, \boxed{125}$$

Handwritten solutions for Question 8:

- a) $32^5 \times 8^{-9} \times 2^8 = (2^5)^5 \times (2^3)^{-9} \times 2^8 = 2^{25} \times 2^{-27} \times 2^8 = 2^{25-27+8} = 2^6 = 64$
- b) $8^{-4} \times 2^{11} = (2^3)^{-4} \times 2^{11} = 2^{-12} \times 2^{11} = 2^{-1} = \frac{1}{2}$
- c) $\frac{8^6}{16^3} = \frac{(2^3)^6}{(2^4)^3} = \frac{2^{18}}{2^{12}} = 2^6 = 64$
- d) $27^{-4} \times 3^{11} = (3^3)^{-4} \times 3^{11} = 3^{-12} \times 3^{11} = 3^{-1} = \frac{1}{3}$
- e) $(5^6 \times 25^3 \div 125^2)^{\frac{1}{2}} = [5^6 \times (5^2)^3 \div (5^3)^2]^{\frac{1}{2}} = [5^6 \times 5^6 \div 5^6]^{\frac{1}{2}} = [5^6]^{\frac{1}{2}} = 5^3 = 125$

Question 9

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $\frac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$

b) $\frac{2^6}{8^{\frac{5}{2}} \times 2^{-\frac{1}{2}}}$

c) $2^{16} \times 4^{-8} \times 8^4 \times 16^{-2}$

d) $\left(36^{\frac{1}{2}} + 16^{\frac{1}{4}}\right)^{\frac{1}{3}}$

e) $\left(125^{\frac{1}{3}} \times 25^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{-\frac{1}{2}}}\right)^{-\frac{2}{3}}$

$$\boxed{\frac{4}{27}}, \boxed{\frac{1}{2}}, \boxed{16}, \boxed{2}, \boxed{\frac{1}{16}}$$

Handwritten solutions for Question 9:

a) $\frac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}} = \frac{4}{(3^4)^{\frac{3}{4}}} = \frac{4}{3^3} = \frac{4}{27}$

b) $\frac{2^6}{8^{\frac{5}{2}} \times 2^{-\frac{1}{2}}} = \frac{2^6}{(2^3)^{\frac{5}{2}} \times 2^{-\frac{1}{2}}} = \frac{2^6}{2^{\frac{15}{2}} \times 2^{-\frac{1}{2}}} = \frac{2^6}{2^7} = 2^{-1} = \frac{1}{2}$

c) $2^{16} \times 4^{-8} \times 8^4 \times 16^{-2} = 2^{16} \times (2^2)^{-8} \times (2^3)^4 \times (2^4)^{-2} = 2^{16} \times 2^{-16} \times 2^{12} \times 2^{-8} = 2^4 = 16$

d) $(36^{\frac{1}{2}} + 16^{\frac{1}{4}})^{\frac{1}{3}} = (\sqrt{36} + \sqrt[4]{16})^{\frac{1}{3}} = (6 + 2)^{\frac{1}{3}} = 8^{\frac{1}{3}} = 2$

e) $\left(125^{\frac{1}{3}} \times 25^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{-\frac{1}{2}}}\right)^{-\frac{2}{3}}$
 $= \left[(5^3)^{\frac{1}{3}} \times (5^2)^{\frac{1}{2}} + (2^4)^{\frac{3}{4}} \times (2^6)^{\frac{1}{3}} + \frac{1}{7^{-1}}\right]^{-\frac{2}{3}}$
 $= \left[5 \times 5 + 2^3 \times 2 + \frac{1}{\frac{1}{7}}\right]^{-\frac{2}{3}}$
 $= [25 + 32 + 7]^{-\frac{2}{3}} = 64^{-\frac{2}{3}} = \frac{1}{(64^{\frac{2}{3}})} = \frac{1}{16}$

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ALGEBRAIC INDICES

(Non Calculator)

Created by T. Madas

Question 1

Simplify **fully** each of the following expressions.

a) $4a^2b^3 \times 3ab^4$

b) $(2a^3b^2)^4$

c) $\frac{3a^3b^2c \times 6ab^2c^3}{2a^2bc^3}$

d) $\frac{(4xy^2)^2}{(2x)^3}$

e) $\frac{\sqrt{9x^6y^4}}{(3x^2y^3)^2}$

$$\boxed{12a^3b^7}, \boxed{16a^{12}b^8}, \boxed{9a^2b^3c}, \boxed{\frac{2y^4}{x}}, \boxed{\frac{1}{3xy^4}}$$

Question 2

Simplify **fully** each of the following expressions.

a) $\frac{x^6}{x^{-2}}$

b) $\frac{12y^{-5}}{3y^{-2}}$

c) $(3t^3q^4)^3$

d) $\frac{3z^4 \times (10z)^3}{125z^5}$

x^8 , $4y^{-3}$ or $\frac{4}{y^3}$, $27t^9q^{12}$, $24z^2$

Question 3

Simplify **fully** each of the following expressions.

a) $x^{\frac{5}{2}} \times \sqrt{x}$

b) $2y^3 \times 2y^{-1}$

c) $2w^{\frac{1}{2}} \times 3w^2$

d) $2t^{\frac{4}{3}} \times 4\sqrt[3]{t^2}$

e) $k^{\frac{3}{2}} \times 4k^{-3}$

$$\boxed{x^3}, \boxed{4y^2}, \boxed{6w^{\frac{5}{2}}}, \boxed{8t^2}, \boxed{4k^{-\frac{3}{2}}}$$

Question 4Simplify **fully** each of the following expressions.

a) $\left(2k^{\frac{1}{2}}h^3\right)^4$

b) $\left(9a^6b^2\right)^{-\frac{1}{2}}$

c) $\left(2pq^2\right)^4 \times 5p\sqrt{q^6}$

d) $\frac{12\left(x^3y^2z\right)^4}{\left(4x^2z^6\right)^2}$

$$\boxed{16k^2h^{12}}, \quad \boxed{\frac{1}{3}a^{-3}b^{-1} \text{ or } \frac{1}{3a^3b}}, \quad \boxed{80p^5q^{11}}, \quad \boxed{\frac{3}{4}x^8y^8z^{-8} \text{ or } \frac{3x^8y^8}{4z^8}}$$

Question 5

Simplify **fully** each of the following expressions.

a) $(2ab^2c^3)^3$

b) $(\frac{1}{2}x^3y^2)^3$

c) $(9a^6b^4)^{\frac{1}{2}}$

d) $(16p^8q^{-2})^{\frac{1}{2}}$

$$\boxed{8a^3b^6c^9}, \boxed{\frac{1}{8}x^9y^6}, \boxed{3a^3b^2}, \boxed{4p^4q^{-1} \text{ or } \frac{4p^4}{q}}$$

Question 6

Simplify **fully** each of the following expressions.

a) $2a^3\left(2a^{-1} + a^{\frac{1}{2}}\right)$

b) $4b^{\frac{1}{2}}\left(2b + b^{\frac{1}{2}}\right)$

c) $c^{\frac{3}{2}}\left(3c^{-1} + c\right)$

d) $3d^{\frac{3}{2}}\left(4d^{-2} - 2d^{-\frac{1}{2}}\right)$

$$\boxed{4a^2 + 2a^{\frac{7}{2}}}, \boxed{8b^{\frac{3}{2}} + 4b}, \boxed{3c^{\frac{1}{2}} + c^{\frac{5}{2}}}, \boxed{12d^{-\frac{1}{2}} - 6d}$$

Question 7

Simplify **fully** each of the following expressions.

a) $a\left(2a^{-1} - 3a^{-\frac{1}{2}}\right)$

b) $3b^2\left(b^{-2} + 2b^{-\frac{1}{2}}\right)$

c) $3c^{\frac{7}{2}}\left(2c^{-\frac{1}{2}} - c\right)$

d) $2d^{\frac{7}{2}}\left(2d^{-1} + d^{\frac{1}{2}}\right)$

$$\boxed{2 - 3a^{\frac{1}{2}}}, \boxed{3 + 6b^{\frac{3}{2}}}, \boxed{6c^3 - 3c^{\frac{9}{2}}}, \boxed{4d^{\frac{5}{2}} + 2d^4}$$

Question 8

Write each of the following expressions as the sum of terms of the form kx^n , where k is a constant.

a) $\frac{1}{2\sqrt{x}} + \frac{4}{x^2}$

b) $x\sqrt{x} - \frac{1}{x^2}$

c) $\sqrt{x^3} - \frac{1}{2x^2}$

d) $\sqrt[3]{x^2} - \frac{3}{2x^3}$

e) $4\sqrt{x} + \frac{1}{4\sqrt{x}}$

$$\boxed{\frac{1}{2}x^{-\frac{1}{2}} + 4x^{-2}}, \quad \boxed{x^{\frac{3}{2}} - x^{-2}}, \quad \boxed{x^{\frac{3}{2}} - \frac{1}{2}x^{-2}}, \quad \boxed{x^{\frac{2}{3}} - \frac{3}{2}x^{-3}}, \quad \boxed{4x^{\frac{1}{2}} + \frac{1}{4}x^{-\frac{1}{2}}}$$

Question 9

Write each of the following expressions as the sum of terms of the form kx^n , where k is a constant.

a) $(5 - x^{-2})(2x^3 - x)$

b) $(1 - x^{\frac{1}{2}})(2 - x^{\frac{1}{2}})$

c) $(1 + x^{\frac{1}{2}})(x^{\frac{3}{2}} + 2)$

$$\boxed{10x^3 - 7x + x^{-1}}, \quad \boxed{x - 3x^{\frac{1}{2}} + 2}, \quad \boxed{x^2 + x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 2}$$

Question 10

Write each of the following expressions as the sum of terms of the form kx^n , where k is a constant.

a) $\left(x^{\frac{3}{2}} + 2x^{-\frac{3}{2}}\right)^2$

b) $\left(x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}\right)^2$

c) $\left(3x^{-\frac{3}{2}} + 2x^{\frac{1}{2}}\right)^2$

d) $\left(x^{\frac{5}{2}} + x^{\frac{1}{2}}\right)^2$

e) $\left(3\sqrt{x} - 2\right)^2$

$$\boxed{x^3 + 4 + 4x^{-3}}, \boxed{x - 4 + 4x^{-1}}, \boxed{4x + 12x^{-1} + 9x^{-3}}, \boxed{x^5 + 2x^3 + x}, \boxed{9x - 12x^{\frac{1}{2}} + 4}$$

Question 11

Write each of the following expressions as the sum of terms of the form kx^n , where k is a constant.

a) $(2x^{\frac{1}{2}} + 3)^2$

b) $(2x^{\frac{1}{2}} - x^{-\frac{1}{2}})^2$

c) $(2x^{\frac{3}{2}} - 3x^{-\frac{3}{2}})^2$

d) $(x^{\frac{1}{2}} - 2x^{-\frac{3}{2}})^2$

e) $(x^{\frac{1}{2}} - 4)(x^{-\frac{1}{2}} - 1)$

$$\boxed{4x + 12x^{\frac{1}{2}} + 9}, \boxed{4x - 4 + x^{-1}}, \boxed{4x^3 - 12 + 9x^{-3}}, \boxed{x - 4x^{-1} + 4x^{-3}}, \boxed{-x^{\frac{1}{2}} + 5 - 4x^{-\frac{1}{2}}}$$

Question 12

Write each of the following expressions as the sum of terms of the form kx^n , where k is a constant.

a) $\frac{4+x}{2x^3}$

b) $\frac{9\sqrt{x}+6x}{3x^3}$

c) $\frac{(x+2)(2x-3)}{4x^5}$

d) $\frac{x^2+3x}{2\sqrt{x}}$

e) $\frac{\sqrt{x}(2x-4)}{3x^2}$

$$\boxed{2x^{-3} + \frac{1}{2}x^{-2}}, \quad \boxed{3x^{-\frac{5}{2}} + 2x^{-2}}, \quad \boxed{\frac{1}{2}x^{-3} + \frac{1}{4}x^{-4} - \frac{3}{2}x^{-5}}, \quad \boxed{\frac{1}{2}x^{\frac{3}{2}} + \frac{3}{2}x^{\frac{1}{2}}}, \quad \boxed{\frac{2}{3}x^{-\frac{1}{2}} - \frac{4}{3}x^{-\frac{3}{2}}}$$

Question 13

Write each of the following expressions as the sum of terms of the form kx^n , where k is a constant.

a) $\frac{(3x-2)(2x-1)}{2x^{\frac{3}{2}}}$

b) $\frac{(2\sqrt{x}+3)^2}{4x}$

c) $\frac{x^2(\sqrt{x}+4x)}{4\sqrt{x}}$

d) $\frac{\sqrt{x}(5x^2-8)}{4x}$

e) $\frac{(x^2-3)(\sqrt{x}+4x)}{3\sqrt{x}}$

$$\boxed{3x^{\frac{1}{2}} - \frac{7}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}}, \quad \boxed{1 + 3x^{-\frac{1}{2}} + \frac{9}{4}x^{-1}}, \quad \boxed{x^{\frac{5}{2}} + \frac{1}{4}x^2}, \quad \boxed{\frac{5}{4}x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}}, \quad \boxed{\frac{4}{3}x^{\frac{5}{2}} + x^2 - 4x^{\frac{1}{2}} - 3}$$

INDICIAL EQUATIONS

(Non Calculator)

Question 1

Solve each of the following equations without using a calculator.

a) $x^{\frac{1}{3}} = 2$

b) $y^{-\frac{1}{3}} = 8$

c) $z^{\frac{3}{2}} = 27$

d) $w^{\frac{2}{3}} = 64$

e) $t^{-\frac{1}{2}} = \frac{1}{4}$

$x = 8$, $y = \frac{1}{512}$, $z = 9$, $w = 512$, $t = 16$

Handwritten solutions for the five equations:

- a) $x^{\frac{1}{3}} = 2 \Rightarrow (x^{\frac{1}{3}})^3 = 2^3 \Rightarrow x^1 = 8 \Rightarrow x = 8$
- b) $y^{-\frac{1}{3}} = 8 \Rightarrow (y^{-\frac{1}{3}})^{-3} = 8^{-3} \Rightarrow y^1 = \frac{1}{512} \Rightarrow y = \frac{1}{512}$
- c) $z^{\frac{3}{2}} = 27 \Rightarrow (z^{\frac{3}{2}})^{\frac{2}{3}} = 27^{\frac{2}{3}} \Rightarrow z^1 = (27^{\frac{2}{3}})^1 \Rightarrow z = 9$
- d) $w^{\frac{2}{3}} = 64 \Rightarrow (w^{\frac{2}{3}})^{\frac{3}{2}} = 64^{\frac{3}{2}} \Rightarrow w^1 = (64^{\frac{3}{2}})^1 \Rightarrow w = 512$
- e) $t^{-\frac{1}{2}} = \frac{1}{4} \Rightarrow (t^{-\frac{1}{2}})^{-2} = (\frac{1}{4})^{-2} \Rightarrow t^1 = (\frac{1}{4})^{-2} \Rightarrow t = 16$

Question 2

Solve each of the following equations without using a calculator.

a) $x^{-\frac{3}{4}} = 8$

b) $y^{-\frac{1}{3}} = \frac{1}{2}$

c) $(3-z)^{\frac{3}{2}} = 8$

d) $(25w^2)^{-\frac{1}{2}} = 2$

$x = \frac{1}{16}$, $y = 8$, $z = -1$, $w = (\pm)\frac{1}{10}$

Handwritten solutions for the four equations:

- a) $x^{-\frac{3}{4}} = 8$
 $\Rightarrow (x^{\frac{1}{4}})^{-3} = 8^{\frac{4}{3}}$
 $\Rightarrow x^{\frac{1}{4}} = \frac{1}{8^{\frac{4}{3}}}$
 $\Rightarrow x = \frac{1}{(8^{\frac{4}{3}})^4}$
 $\Rightarrow x = \frac{1}{2^4}$
 $\Rightarrow x = \frac{1}{16}$
- b) $y^{-\frac{1}{3}} = \frac{1}{2}$
 $\Rightarrow (y^{\frac{1}{3}})^{-1} = \frac{1}{2}$
 $\Rightarrow y^{\frac{1}{3}} = 2$
 $\Rightarrow y = 8$
- c) $(3-z)^{\frac{3}{2}} = 8$
 $\Rightarrow [(3-z)^{\frac{1}{2}}]^3 = 8^{\frac{2}{3}}$
 $\Rightarrow (3-z)^{\frac{1}{2}} = (8^{\frac{2}{3}})^{\frac{1}{3}}$
 $\Rightarrow 3-z = 2^2$
 $\Rightarrow 3-z = 4$
 $\Rightarrow -1 = -z$
 $\Rightarrow z = -1$
- d) $(25w^2)^{-\frac{1}{2}} = 2$
 $\Rightarrow [(25w^2)^{\frac{1}{2}}]^{-1} = 2^{\frac{2}{2}}$
 $\Rightarrow 25w^2 = \frac{1}{2^2}$
 $\Rightarrow 25w^2 = \frac{1}{4}$
 $\Rightarrow w^2 = \frac{1}{100}$
 $\Rightarrow w = \pm \frac{1}{10}$

Question 3

Solve each of the following equations without using a calculator.

a) $x^{-1} = \frac{x}{16}$

b) $3y^{-\frac{1}{2}} - 4 = 0$

c) $8w^{\frac{1}{2}} - w^{-1} = 0$

d) $32t^{\frac{3}{2}} - \frac{1}{t} = 0$

$x = \pm 4$, $y = \frac{9}{16}$, $w = \frac{1}{4}$, $t = \frac{1}{4}$

Handwritten solutions for the four equations:

- (a) $x^{-1} = \frac{x}{16}$
 $\Rightarrow \frac{1}{x} = \frac{x}{16}$
 $\Rightarrow x^2 = 16$
 $\Rightarrow x = \pm 4$
- (b) $3y^{-\frac{1}{2}} - 4 = 0$
 $\Rightarrow \frac{3}{\sqrt{y}} = 4$
 $\Rightarrow 4\sqrt{y} = 3$
 $\Rightarrow \sqrt{y} = \frac{3}{4}$
 $\Rightarrow y = \frac{9}{16}$
- (c) $8w^{\frac{1}{2}} - w^{-1} = 0$
 $\Rightarrow 8w^{\frac{1}{2}} = \frac{1}{w}$ ($w \neq 0$)
 $\Rightarrow 8w^{\frac{3}{2}} = 1$
 $\Rightarrow (w^{\frac{3}{2}})^{\frac{2}{3}} = (\frac{1}{8})^{\frac{2}{3}}$
 $\Rightarrow w^1 = (\frac{1}{4})^{\frac{2}{3} \cdot \frac{3}{2}}$
 $\Rightarrow w = (\frac{1}{4})^1$
 $\Rightarrow w = \frac{1}{4}$
- (d) $32t^{\frac{3}{2}} - \frac{1}{t} = 0$
 $\Rightarrow 32t^{\frac{3}{2}} = \frac{1}{t}$ ($t \neq 0$)
 $\Rightarrow t^{\frac{3}{2}} = \frac{1}{32t}$
 $\Rightarrow (t^{\frac{3}{2}})^{\frac{2}{3}} = (\frac{1}{32t})^{\frac{2}{3}}$
 $\Rightarrow t^1 = (\frac{1}{16t})^{\frac{2}{3} \cdot \frac{3}{2}}$
 $\Rightarrow t = \frac{1}{16t}$
 $\Rightarrow t = \frac{1}{4}$

Question 4

Solve each of the following equations without using a calculator.

a) $2^{3-x} = 4^x$

b) $2^{y+2} = 4\sqrt{2}$

c) $4^z = 8^{2-z}$

d) $2^w = \frac{4}{\sqrt{2}}$

e) $2^t = 8\sqrt{2}$

$x=1$, $y=\frac{1}{2}$, $z=\frac{6}{5}$, $w=\frac{3}{2}$, $t=\frac{7}{2}$

Handwritten solutions for the five equations:

- a) $2^{3-x} = 4^x$
 $\Rightarrow 2^{3-x} = (2^2)^x$
 $\Rightarrow 2^{3-x} = 2^{2x}$
 $\Rightarrow 3-x = 2x$
 $\Rightarrow 3 = 3x$
 $\Rightarrow x = 1$
- b) $2^{y+2} = 4\sqrt{2}$
 $\Rightarrow 2^{y+2} = 2^2 \times 2^{\frac{1}{2}}$
 $\Rightarrow 2^{y+2} = 2^{2.5}$
 $\Rightarrow y+2 = 2.5$
 $\Rightarrow y = 0.5$
 $\Rightarrow y = \frac{1}{2}$
- c) $4^z = 8^{2-z}$
 $\Rightarrow (2^2)^z = (2^3)^{2-z}$
 $\Rightarrow 2^{2z} = 2^{6-3z}$
 $\Rightarrow 2z = 6-3z$
 $\Rightarrow 5z = 6$
 $\Rightarrow z = \frac{6}{5}$
- d) $2^w = \frac{4}{\sqrt{2}}$
 $\Rightarrow 2^w = \frac{2^2}{2^{\frac{1}{2}}}$
 $\Rightarrow 2^w = 2^{2-\frac{1}{2}}$
 $\Rightarrow 2^w = 2^{\frac{3}{2}}$
 $\Rightarrow w = \frac{3}{2}$
- e) $2^t = 8\sqrt{2}$
 $\Rightarrow 2^t = 2^3 \times 2^{\frac{1}{2}}$
 $\Rightarrow 2^t = 2^{3.5}$
 $\Rightarrow t = 3.5$
 $\Rightarrow t = \frac{7}{2}$

Question 5

Solve each of the following equations without using a calculator.

a) $3^{x+2} = 9^x$

b) $2^{y+1} = 8^{2y-1}$

c) $27^{3z+1} = 9$

d) $9^{2w-3} = 27^{w+2}$

e) $8 \times 2^{2t} = \frac{2^{5t+1}}{4^{-t}}$

$$\boxed{x=2}, \quad \boxed{y=\frac{4}{5}}, \quad \boxed{z=-\frac{1}{9}}, \quad \boxed{w=12}, \quad \boxed{t=\frac{2}{5}}$$

Handwritten solutions for Question 5:

- a) $3^{x+2} = 9^x$
 $\Rightarrow 3^{x+2} = (3^2)^x$
 $\Rightarrow 3^{x+2} = 3^{2x}$
 $\Rightarrow x+2 = 2x$
 $\Rightarrow 2 = x$
 $\Rightarrow x = 2$
- b) $2^{y+1} = 8^{2y-1}$
 $\Rightarrow 2^{y+1} = (2^3)^{2y-1}$
 $\Rightarrow 2^{y+1} = 2^{6y-3}$
 $\Rightarrow y+1 = 6y-3$
 $\Rightarrow 4 = 5y$
 $\Rightarrow y = \frac{4}{5}$
- c) $27^{3z+1} = 9$
 $\Rightarrow (3^3)^{3z+1} = 3^2$
 $\Rightarrow 3^{9z+3} = 3^2$
 $\Rightarrow 9z+3 = 2$
 $\Rightarrow 9z = -1$
 $\Rightarrow z = -\frac{1}{9}$
- d) $9^{2w-3} = 27^{w+2}$
 $\Rightarrow (3^2)^{2w-3} = (3^3)^{w+2}$
 $\Rightarrow 3^{4w-6} = 3^{3w+6}$
 $\Rightarrow 4w-6 = 3w+6$
 $\Rightarrow w = 12$
- e) $8 \times 2^{2t} = \frac{2^{5t+1}}{4^{-t}}$
 $\Rightarrow 8 \times 2^{2t} = 2^{5t+1} \times 2^t$
 $\Rightarrow 2^3 \times 2^{2t} = 2^{5t+1+t}$
 $\Rightarrow 2^{3+2t} = 2^{6t+1}$
 $\Rightarrow 3+2t = 6t+1$
 $\Rightarrow 2 = 4t$
 $\Rightarrow t = \frac{2}{5}$

Question 6

Solve each of the following equations without using a calculator.

a) $2^{x+2} = 4^x$

b) $9^y = 27^{1-y}$

c) $4^z = 8^{3-z}$

d) $\frac{4^w \times 2^{5w}}{16^w} = 2^w$

e) $\frac{27^t}{3^{t-1}} = 3\sqrt{3}$

$$x=2, \quad y=\frac{3}{5}, \quad z=\frac{9}{5}, \quad w=4, \quad t=\frac{1}{4}$$

Handwritten solutions for Question 6:

a) $2^{x+2} = 4^x$
 $\Rightarrow 2^{x+2} = (2^2)^x$
 $\Rightarrow 2^{x+2} = 2^{2x}$
 $\Rightarrow x+2 = 2x$
 $\Rightarrow 2 = x$
 $\Rightarrow x = 2$

b) $9^y = 27^{1-y}$
 $\Rightarrow (3^2)^y = (3^3)^{1-y}$
 $\Rightarrow 3^{2y} = 3^{3-3y}$
 $\Rightarrow 2y = 3-3y$
 $\Rightarrow 5y = 3$
 $\Rightarrow y = \frac{3}{5}$

c) $4^z = 8^{3-z}$
 $\Rightarrow (2^2)^z = (2^3)^{3-z}$
 $\Rightarrow 2^{2z} = 2^{9-3z}$
 $\Rightarrow 2z = 9-3z$
 $\Rightarrow 5z = 9$
 $\Rightarrow z = \frac{9}{5}$

d) $\frac{4^w \times 2^{5w}}{16^w} = 2^w$
 $\Rightarrow \frac{2^{2w} \times 2^{5w}}{2^{4w}} = 2^w$
 $\Rightarrow 2^{2w+5w-4w} = 2^w$
 $\Rightarrow 2^{3w} = 2^w$
 $\Rightarrow 3w = w$
 $\Rightarrow 2w = 0$
 $\Rightarrow w = 0$

e) $\frac{27^t}{3^{t-1}} = 3\sqrt{3}$
 $\Rightarrow \frac{3^{3t}}{3^{t-1}} = 3^{1+\frac{1}{2}}$
 $\Rightarrow 3^{3t-(t-1)} = 3^{\frac{3}{2}}$
 $\Rightarrow 3^{2t+1} = 3^{\frac{3}{2}}$
 $\Rightarrow 2t+1 = \frac{3}{2}$
 $\Rightarrow 2t = \frac{1}{2}$
 $\Rightarrow t = \frac{1}{4}$

Question 7

Solve each of the following equations without using a calculator.

a) $\frac{81^{3-x}}{27^{2x+1}} = 3$

b) $\frac{5^y}{25^{y-1}} = \sqrt{5}$

c) $\frac{16^z}{\sqrt{2}} = 2^{z-1}$

d) $\frac{25^{t-1}}{5} = \sqrt{5}$

$$\boxed{x = \frac{4}{5}}, \quad \boxed{y = \frac{3}{2}}, \quad \boxed{z = -\frac{1}{6}}, \quad \boxed{t = \frac{7}{4}}$$

$$\begin{array}{ll}
 \text{d)} \quad \frac{81^{-1-x}}{87^{11x}} = 3 & \text{b)} \quad \frac{5^x}{25 \cdot 4} = \sqrt{5} \\
 \Rightarrow 81^{-1-x} = 3 \cdot 3^{2x+4} & \Rightarrow \frac{5^x}{5^2 \cdot 2^2} = 5^{\frac{1}{2}} \\
 \Rightarrow (3^4)^{-1-x} = 3 \cdot (3^2)^{2x+4} & \Rightarrow \frac{5^x}{5^2 \cdot 2^2} = 5^{\frac{1}{2}} \\
 \Rightarrow 3^{4-4x} = 3 \cdot 3^{4x+8} & \Rightarrow \frac{5^x}{5^2 \cdot 2^2} = 5^{\frac{1}{2}} \\
 \Rightarrow 3^{4-4x} = 3^{4x+9} & \Rightarrow 5^x = 5^{\frac{1}{2}} \cdot 2^{x+2} \\
 \Rightarrow 12-4x = 4x+9 & \Rightarrow \text{d)} \quad \frac{25^{x+1}}{5} = \sqrt{5} \\
 8 = 10x & \Rightarrow \frac{5^{2x+2}}{5} = 5^{\frac{1}{2}} \\
 x = \frac{4}{5} & \Rightarrow 5^{2x+2} = 5^{\frac{1}{2}} \\
 & \Rightarrow 2x+2 = \frac{1}{2} \\
 & \Rightarrow 2x = -\frac{3}{2} \\
 & \Rightarrow x = -\frac{3}{4} \\
 & \Rightarrow t = \frac{7}{4}
 \end{array}$$

Question 8

Solve each of the following equations without using a calculator.

a) $2^{3x+4} = 4\sqrt{2}$

b) $3^x = \frac{\sqrt{3}}{9}$

c) $2^z = \frac{\sqrt{2}}{2^{z+1}}$

d) $9^w = \frac{3^{w-1}}{27}$

e) $3^{t+1} = \frac{27^t}{9}$

$x = -\frac{1}{2}, y = -\frac{3}{2}, z = -\frac{1}{4}, w = -4, t = \frac{3}{2}$

Handwritten solutions for Question 8:

- a) $2^{3x+4} = 4\sqrt{2}$
 $\Rightarrow 2^{3x+4} = 2^2 \times 2^{\frac{1}{2}}$
 $\Rightarrow 2^{3x+4} = 2^{\frac{5}{2}}$
 $\Rightarrow 3x+4 = \frac{5}{2}$
 $\Rightarrow 3x = -\frac{3}{2}$
 $\Rightarrow x = -\frac{1}{2}$
- b) $3^x = \frac{\sqrt{3}}{9}$
 $\Rightarrow 3^x = \frac{3^{\frac{1}{2}}}{3^2}$
 $\Rightarrow 3^x = 3^{-\frac{3}{2}}$
 $\Rightarrow x = -\frac{3}{2}$
- c) $2^z = \frac{\sqrt{2}}{2^{z+1}}$
 $\Rightarrow 2^z = \frac{2^{\frac{1}{2}}}{2^{z+1}}$
 $\Rightarrow 2^z \times 2^{z+1} = 2^{\frac{1}{2}}$
 $\Rightarrow 2^{2z+1} = 2^{\frac{1}{2}}$
 $\Rightarrow 2z+1 = \frac{1}{2}$
 $\Rightarrow 2z = -\frac{1}{2}$
 $\Rightarrow z = -\frac{1}{4}$
- d) $9^w = \frac{3^{w-1}}{27}$
 $\Rightarrow 3^{2w} = \frac{3^{w-1}}{3^3}$
 $\Rightarrow 3^{2w} = 3^{w-4}$
 $\Rightarrow 2w = w-4$
 $\Rightarrow w = -4$
- e) $3^{t+1} = \frac{27^t}{9}$
 $\Rightarrow 3^{t+1} = \frac{3^{3t}}{3^2}$
 $\Rightarrow 3^{t+1} = 3^{3t-2}$
 $\Rightarrow t+1 = 3t-2$
 $\Rightarrow 3 = 2t$
 $\Rightarrow t = \frac{3}{2}$

Question 9

Solve each of the following equations without using a calculator.

a) $3x^{\frac{1}{3}} = x^{-\frac{2}{3}}$

b) $2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

c) $w^{\frac{3}{2}} - 8w^{-\frac{1}{2}} = 0$

d) $z\left(z^{\frac{1}{2}} - 2z^{-\frac{1}{2}}\right)^2 = 0$

e) $27t^{-\frac{1}{2}} = 125t$

$$x = \frac{1}{3}, \quad y = \frac{4}{3}, \quad w = \pm 2\sqrt{2}, \quad z = 2, \quad t = \frac{9}{25}$$

Handwritten solutions for Question 9:

a) $3x^{\frac{1}{3}} = x^{-\frac{2}{3}}$
 $\Rightarrow 3x^{\frac{1}{3}} = \frac{1}{x^{\frac{2}{3}}}$
 $\Rightarrow 3x^{\frac{1}{3}} \cdot x^{\frac{2}{3}} = 1$
 $\Rightarrow 3x = 1$
 $\Rightarrow x = \frac{1}{3}$

b) $2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$
 $\Rightarrow \frac{2}{x^{\frac{1}{2}}} = \frac{3}{2}x^{\frac{1}{2}}$
 $\Rightarrow 4 = 3x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$
 $\Rightarrow \frac{4}{3} = x$
 $\Rightarrow x = \frac{4}{3}$

c) $w^{\frac{3}{2}} - 8w^{-\frac{1}{2}} = 0$
 $\Rightarrow w^{\frac{3}{2}} = \frac{8}{w^{\frac{1}{2}}}$
 $\Rightarrow w^{\frac{3}{2}} \cdot w^{\frac{1}{2}} = 8$
 $\Rightarrow w^2 = 8$
 $\Rightarrow w = \pm \sqrt{8}$
 $\Rightarrow w = \pm 2\sqrt{2}$

d) $z\left(z^{\frac{1}{2}} - 2z^{-\frac{1}{2}}\right)^2 = 0$
 $\Rightarrow z^{\frac{1}{2}} - 2z^{-\frac{1}{2}} = 0$
 $\Rightarrow z^{\frac{1}{2}} = 2z^{-\frac{1}{2}}$
 $\Rightarrow z^{\frac{1}{2}} \cdot z^{\frac{1}{2}} = 2$
 $\Rightarrow z = 2$

e) $27t^{-\frac{1}{2}} = 125t$
 $\Rightarrow \frac{27}{t^{\frac{1}{2}}} = 125t$
 $\Rightarrow 27 = 125t^{\frac{3}{2}}$
 $\Rightarrow \frac{27}{125} = t^{\frac{3}{2}}$
 $\Rightarrow \left(\frac{27}{125}\right)^{\frac{2}{3}} = t$
 $\Rightarrow t = \left(\frac{3}{5}\right)^2$
 $\Rightarrow t = \frac{9}{25}$

Question 10

Solve each of the following equations without using a calculator.

a) $4^x - 2^{x+2} - 32 = 0$

b) $2^{y+2} + 2^{3-y} = 33$

c) $3^{2-z} - 3^{z+1} = 26$

d) $2^{2w+2} + 3 \times 2^w - 1 = 0$

$x = 3$, $y = -2, 3$, $z = -1$, $w = -2$

Handwritten solutions for Question 10:

a) $4^x - 2^{x+2} - 32 = 0$
 $\Rightarrow (2^x)^2 - 2^x \times 2^2 - 32 = 0$
 $\Rightarrow (2^x)^2 - 4(2^x) - 32 = 0$
 Let $a = 2^x$
 $\Rightarrow a^2 - 4a - 32 = 0$
 $\Rightarrow (a+4)(a-8) = 0$
 $\Rightarrow a = -4$ or $a = 8$
 $\Rightarrow 2^x = 8$
 $\Rightarrow x = 3$

b) $2^{y+2} + 2^{3-y} = 33$
 $\Rightarrow 2^y \times 2^2 + 2^3 \times 2^{-y} = 33$
 $\Rightarrow 4(2^y) + 8(2^{-y}) = 33$
 Let $a = 2^y$
 $\Rightarrow 4a + \frac{8}{a} = 33$
 $\Rightarrow 4a^2 + 8 = 33a$
 $\Rightarrow 4a^2 - 33a + 8 = 0$
 $\Rightarrow (4a-1)(a-8) = 0$
 $\Rightarrow a = \frac{1}{4}$ or $a = 8$
 $\Rightarrow 2^y = \frac{1}{4}$ or $2^y = 8$
 $\Rightarrow y = -2$ or $y = 3$

c) $3^{2-z} - 3^{z+1} = 26$
 $\Rightarrow 3^2 \times 3^{-z} - 3^z \times 3^1 = 26$
 $\Rightarrow 9 \times \frac{1}{3^z} - 3^z \times 3 = 26$
 Let $a = 3^z$
 $\Rightarrow \frac{9}{a} - 3a = 26$
 $\Rightarrow 9 - 3a^2 = 26a$
 $\Rightarrow 0 = 3a^2 + 26a - 9$
 $\Rightarrow 0 = (3a-1)(a+9)$
 $\Rightarrow a = \frac{1}{3}$ or $a = -9$
 $\Rightarrow 3^z = \frac{1}{3}$
 $\Rightarrow z = -1$

d) $2^{2w+2} + 3 \times 2^w - 1 = 0$
 $\Rightarrow 2^w \times 2^2 \times 2^w + 3 \times 2^w - 1 = 0$
 $\Rightarrow (2^w)^2 \times 4 + 3(2^w) - 1 = 0$
 Let $a = 2^w$
 $\Rightarrow 4a^2 + 3a - 1 = 0$
 $\Rightarrow (4a-1)(a+1) = 0$
 $\Rightarrow a = \frac{1}{4}$ or $a = -1$
 $\Rightarrow 2^w = \frac{1}{4}$
 $\Rightarrow w = -2$

Question 11

Solve each of the following equations without using a calculator.

a) $3^{2x} - 3^{x+1} = 54$

b) $100^t - 10001(10)^{t-1} + 100 = 0$

c) $3(3^{2k}) - 28(3^k) + 9 = 0$

d) $2^{2p-2} - 2^{p-2} - 3 = 0$

$x=2$, $t=3$, -1 , $k=-1$, 2 , $p=2$

The image shows handwritten solutions for the four equations. For equation (a), it shows $3^{2x} - 3^{x+1} = 54$ leading to $3^x = 9$ and $x=2$. For equation (b), it shows $100^t - 10001(10)^{t-1} + 100 = 0$ leading to $10^t = 1000$ and $t=3$. For equation (c), it shows $3(3^{2k}) - 28(3^k) + 9 = 0$ leading to $3^k = 9$ and $k=2$. For equation (d), it shows $2^{2p-2} - 2^{p-2} - 3 = 0$ leading to $2^p = 4$ and $p=2$.

Question 12

Solve each of the following equations without using a calculator.

a) $2x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 12 = 0$

b) $y^{\frac{1}{4}} - y^{-\frac{1}{4}} = 2$

c) $6z^{-\frac{1}{3}} - z^{\frac{1}{3}} = 5$

d) $3w + w^{\frac{1}{2}} - 2 = 0$

e) $t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}$

$x = -64, \frac{27}{8}, y = 3, z = 1, -216, w = \frac{4}{9}, t = -27, 125$

Handwritten solutions for the five equations:

- a)** $2x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 12 = 0$
 $\Rightarrow 2(a^{\frac{2}{3}} + 5a^{\frac{1}{3}}) - 12 = 0$
 $\Rightarrow 2a^{\frac{2}{3}} + 5a^{\frac{1}{3}} - 12 = 0$
 $\Rightarrow (2a - 3)(a + 4) = 0$
 $a = \frac{3}{2}$
 $x = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$
- b)** $y^{\frac{1}{4}} - y^{-\frac{1}{4}} = 2$
 $\Rightarrow y^{\frac{1}{4}} - \frac{1}{y^{\frac{1}{4}}} = 2$
 $\Rightarrow a - \frac{1}{a} = 2$
 $\Rightarrow a^2 - 1 = 2a$
 $\Rightarrow a^2 - 2a - 1 = 0$
 $\Rightarrow (a-1)^2 = 0$
 $a = 1$
 $y^{\frac{1}{4}} = 1$
 $y = 1$
- c)** $6z^{-\frac{1}{3}} - z^{\frac{1}{3}} = 5$
 $\Rightarrow \frac{6}{z^{\frac{1}{3}}} - z^{\frac{1}{3}} = 5$
 $\Rightarrow \frac{6 - z^{\frac{2}{3}}}{z^{\frac{1}{3}}} = 5$
 $\Rightarrow 6 - z^{\frac{2}{3}} = 5z^{\frac{1}{3}}$
 $\Rightarrow z^{\frac{2}{3}} - 5z^{\frac{1}{3}} + 6 = 0$
 $\Rightarrow (z^{\frac{1}{3}} - 2)(z^{\frac{1}{3}} - 3) = 0$
 $z^{\frac{1}{3}} = 2$
 $z = 8$
- d)** $3w + w^{\frac{1}{2}} - 2 = 0$
 $\Rightarrow 3w + \sqrt{w} - 2 = 0$
 $\Rightarrow 3a^2 + a - 2 = 0$
 $\Rightarrow (3a - 2)(a + 1) = 0$
 $a = \frac{2}{3}$
 $w = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$
- e)** $t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}$
 $\Rightarrow t^{\frac{1}{3}} - 2 = 15t^{-\frac{1}{3}}$
 $\Rightarrow t^{\frac{1}{3}} - 2 = 15 \cdot \frac{1}{t^{\frac{1}{3}}}$
 $\Rightarrow t^{\frac{2}{3}} - 2t^{\frac{1}{3}} = 15$
 $\Rightarrow (t^{\frac{1}{3}} - 5)(t^{\frac{1}{3}} + 3) = 0$
 $t^{\frac{1}{3}} = 5$
 $t = 125$