

Created by T. Madas

INDICES

Exam Questions

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Question 1 (**)

- a) Evaluate the following indicial expression, giving the final answer as an exact simplified fraction.

$$4^{\frac{3}{2}} + 4^{-\frac{1}{2}}.$$

- b) Simplify fully the following expression

$$\frac{12y^{-5}}{3y^{-2}}.$$

$$\boxed{}, \boxed{\frac{17}{2}}, \boxed{4y^{-3} = \frac{4}{y^3}}$$

$$\begin{aligned} \text{a) } 4^{\frac{3}{2}} + 4^{-\frac{1}{2}} &= (\sqrt{4})^3 + \frac{1}{\sqrt{4}} = 2^3 + \frac{1}{2} = 8 + \frac{1}{2} = \frac{17}{2} \quad (\text{ex 8.1}) \\ \text{b) } \frac{12y^{-5}}{3y^{-2}} &= 4y^{-5-(-2)} = 4y^{-3} \quad \text{or } \frac{4}{y^3} \end{aligned}$$

Question 2 (**)

- a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

i. $2^{-4} + 8^{-1}$.

ii. $\left(\frac{81}{16}\right)^{\frac{3}{4}}$.

- b) Simplify fully the following expression

$$\frac{(4xy^2)^2}{(2x)^3}.$$

$$\boxed{}, \boxed{\frac{3}{16}}, \boxed{\frac{27}{8}}, \boxed{2y^4x^{-1} = \frac{2y^4}{x}}$$

Handwritten solutions for parts (a) i, (a) ii, and (b) of the question:

(a) i) $2^{-4} + 8^{-1} = \frac{1}{2^4} + \frac{1}{8} = \frac{1}{16} + \frac{1}{8} = \frac{1}{16} + \frac{2}{16} = \frac{3}{16}$

(a) ii) $\left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\sqrt[4]{\frac{81}{16}}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

(b) $\frac{(4xy^2)^2}{(2x)^3} = \frac{16x^2y^4}{8x^3} = \frac{2y^4}{x}$ (or $2x^{-1}y^4$)

Question 3 (**+)

- a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

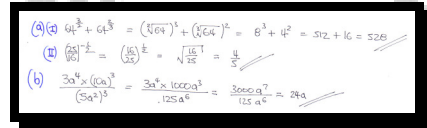
i. $64^{\frac{3}{2}} + 64^{\frac{2}{3}}$.

ii. $\left(\frac{25}{16}\right)^{-\frac{1}{2}}$.

- b) Simplify fully the following expression

$$\frac{3a^4 \times (10a)^3}{(5a^2)^3}$$

$$528, \frac{4}{5}, 24a$$



Handwritten solution for Question 3a and b:

(a) (i) $64^{\frac{3}{2}} + 64^{\frac{2}{3}} = (\sqrt[2]{64})^3 + (\sqrt[3]{64})^2 = 8^3 + 4^2 = 512 + 16 = 528$
 (ii) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} = \left(\frac{5^2}{2^4}\right)^{-\frac{1}{2}} = \sqrt[2]{\frac{2^4}{5^2}} = \frac{4}{5}$
 (b) $\frac{3a^4 \times (10a)^3}{(5a^2)^3} = \frac{3a^4 \times 1000a^3}{125a^6} = \frac{3000a^7}{125a^6} = 24a$

Question 4 (**+)

- a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

i. $16^{\frac{1}{2}} + 16^{\frac{1}{4}}$.

ii. $\left(\frac{2}{3}\right)^{-2}$.

- b) Simplify fully the following expression

$$x^{\frac{5}{2}} \times \sqrt{x}.$$

$$\boxed{6}, \boxed{\frac{9}{4}}, \boxed{x^3}$$

$$\begin{aligned} \text{(i)} \quad 16^{\frac{1}{2}} + 16^{\frac{1}{4}} &= \sqrt{16} + \sqrt[4]{16} = 4 + 2 = 6 \\ \text{(ii)} \quad \left(\frac{2}{3}\right)^{-2} &= \left(\frac{3}{2}\right)^2 = \frac{9}{4} \\ \text{(b)} \quad x^{\frac{5}{2}} \times \sqrt{x} &= x^{\frac{5}{2}} \times x^{\frac{1}{2}} = x^3 \end{aligned}$$

Question 5 (**+)

- a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

i. $2^{-5} - 8^{-2}$.

ii. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$.

- b) Solve the equation

$$y^{-\frac{1}{3}} = 8.$$

$$\boxed{}, \boxed{\frac{1}{64}}, \boxed{\frac{8}{27}}, \boxed{\frac{1}{512}}$$

Handwritten solutions for Question 5a and 5b:

a) i. $2^{-5} - 8^{-2} = \frac{1}{2^5} - \frac{1}{8^2} = \frac{1}{32} - \frac{1}{64} = \frac{2}{64} - \frac{1}{64} = \frac{1}{64}$

ii. $\left(\frac{4}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{9}}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

b) $y^{-\frac{1}{3}} = 8$
 $\Rightarrow \frac{1}{y^{\frac{1}{3}}} = 8$
 $\Rightarrow y^{\frac{1}{3}} = \frac{1}{8}$
 $\Rightarrow \sqrt[3]{y} = \frac{1}{8}$
 $\Rightarrow y = \left(\frac{1}{8}\right)^3 = \frac{1}{512}$

Question 6 (**+)

Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$\left(36^{\frac{1}{2}} + 16^{\frac{1}{4}}\right)^{-\frac{2}{3}}$$

$$\boxed{}, \boxed{\frac{1}{4}}$$

Handwritten solution for Question 6:

$$\left(36^{\frac{1}{2}} + 16^{\frac{1}{4}}\right)^{-\frac{2}{3}} = \left(\sqrt{36} + \sqrt[4]{16}\right)^{-\frac{2}{3}} = (6 + 2)^{-\frac{2}{3}} = 8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$$

Question 7 (**+)

- a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$4 \times 4^{\frac{5}{2}} + 8^{-\frac{1}{3}}.$$

The answer to this part of the question must be fully supported by a detailed method, justifying each step in the evaluation.

- b) Simplify fully the following expression

$$(2pq^2)^4 \times 5p\sqrt{q^6}.$$

$$\boxed{}, \boxed{\frac{257}{2}}, \boxed{80p^5q^{11}}$$

4) SIMULTANEOUS EQUATIONS - INDEXES

$$\begin{aligned}
 4 \times 4^{\frac{5}{2}} + 8^{-\frac{1}{3}} &= 4 \times (4^{\frac{1}{2}})^5 + \frac{1}{8^{\frac{1}{3}}} \\
 &= 4 \times (2^2)^{\frac{5}{2}} + \frac{1}{2} \\
 &= 4 \times 2^5 + \frac{1}{2} \\
 &= 128 + \frac{1}{2} \\
 &= \frac{256}{1} + \frac{1}{2} \\
 &= \frac{257}{2}
 \end{aligned}$$

4) SIMULTANEOUS EQUATIONS - INDEXES

$$\begin{aligned}
 (2pq^2)^4 \times 5p\sqrt{q^6} &= 16p^4q^8 \times 5p(q^3)^{\frac{1}{2}} \\
 &= 16p^4q^8 \times 5p q^{\frac{3}{2}} \\
 &= 80p^5q^{\frac{19}{2}}
 \end{aligned}$$

Question 8 (***)

- a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

i. $8^{\frac{1}{3}} + 8^{-\frac{1}{3}}$.

ii. $8^{-4} \times 2^{11}$.

- b) Simplify fully

$$\frac{\sqrt{9x^6y^4}}{(3x^2y^3)^2}$$

$$\boxed{}, \boxed{\frac{5}{2}}, \boxed{\frac{1}{2}}, \boxed{\frac{1}{3xy^4}}$$

$$\begin{aligned} \text{(i)} \quad 8^{\frac{1}{3}} + 8^{-\frac{1}{3}} &= \sqrt[3]{8} + \frac{1}{\sqrt[3]{8}} = 2 + \frac{1}{2} = \frac{5}{2} \\ \text{(ii)} \quad 8^{-4} \times 2^{11} &= (2^3)^{-4} \times 2^{11} = 2^{-12} \times 2^{11} = 2^{-1} = \frac{1}{2} \\ \text{(b)} \quad \frac{\sqrt{9x^6y^4}}{(3x^2y^3)^2} &= \frac{3x^3y^2}{9x^4y^6} = \frac{1}{3} \cdot \frac{x^3y^2}{x^4y^6} = \frac{1}{3} \cdot \frac{1}{xy^4} = \frac{1}{3xy^4} \end{aligned}$$

Question 9 (***)

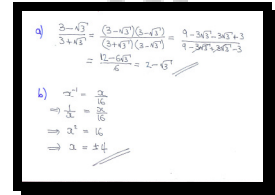
- a) Simplify the following expression, writing the final answer in the form $a + b\sqrt{3}$, where a and b are integers

$$\frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

- b) Solve the equation

$$x^{-1} = \frac{x}{16}, \quad x \neq 0.$$

$$\boxed{}, \boxed{2 - \sqrt{3}}, \boxed{x = \pm 4}$$



Handwritten solution for part a):

$$\frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})} = \frac{9 - 3\sqrt{3} + 3\sqrt{3} - 3}{9 - 3\sqrt{3} - 3\sqrt{3} + 3} = \frac{6}{6} = 1$$

Question 10 (***)

- a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$\left(6\frac{1}{4}\right)^{-\frac{3}{2}}$$

- b) Simplify fully the following expression

$$\frac{12(x^3y^2z)^4}{(4x^2z^6)^2}$$

$$\boxed{\frac{8}{125}}, \quad \boxed{\frac{3}{4}x^8y^8z^{-8} = \frac{3x^8y^8}{4z^8}}$$

$$\begin{aligned} \text{a) } \left(6\frac{1}{4}\right)^{-\frac{3}{2}} &= \left(\frac{25}{4}\right)^{-\frac{3}{2}} = \left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^3 = \frac{8}{125} \\ \text{b) } \frac{12(x^3y^2z)^4}{(4x^2z^6)^2} &= \frac{12x^{12}y^8z^4}{16x^4z^{12}} = \frac{3}{4}x^8y^8z^{-8} = \frac{3x^8y^8}{4z^8} \end{aligned}$$

Question 11 (***)

- a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions, where appropriate.

i. $\frac{6}{2^{-2}}.$

ii. $\left(1\frac{7}{9}\right)^{\frac{3}{2}}.$

- b) Solve the equation

$$z^{\frac{3}{2}} = 27.$$

$$\boxed{}, \boxed{24}, \boxed{\frac{64}{27}}, \boxed{z=9}$$

(a) $\frac{6}{2^{-2}} = \frac{6}{\frac{1}{4}} = 24$
(ii) $\left(1\frac{7}{9}\right)^{\frac{3}{2}} = \left(\frac{16}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{16}{9}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$
(b) $z^{\frac{3}{2}} = 27 \Rightarrow \left(z^{\frac{1}{2}}\right)^3 = 27$
 $\Rightarrow z^{\frac{1}{2}} = \left(27\right)^{\frac{1}{3}}$
 $\Rightarrow z = 9$

Question 12 (***)

- a) Simplify the following expression, writing the final answer in the form $a + b\sqrt{3}$, where a and b are integers

$$\frac{2\sqrt{3}-1}{2-\sqrt{3}}$$

- b) Solve the equation

$$2^{x+2} = 4\sqrt{2}$$

$$\boxed{4}, \boxed{4+3\sqrt{3}}, \boxed{x = \frac{1}{2}}$$

$$a) \frac{2\sqrt{3}-1}{2-\sqrt{3}} = \frac{(2\sqrt{3}-1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{4\sqrt{3}+6-2-\sqrt{3}}{4-3}$$

$$= \frac{3\sqrt{3}+4}{1} = 4+3\sqrt{3}$$

$$b) 2^{x+2} = 4\sqrt{2}$$

$$2^{x+2} = 2^2 \times 2^{\frac{1}{2}}$$

$$2^{x+2} = 2^{\frac{5}{2}}$$

$$x+2 = \frac{5}{2}$$

$$x = \frac{1}{2}$$

Question 13 (***)

- a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$27^{-4} \times 3^{11}$$

- b) Solve the equation

$$t^{-\frac{1}{2}} = \frac{1}{4}$$

$$\boxed{\frac{1}{3}}, \boxed{t=16}$$

$$a) 27^{-4} \times 3^{11} = (3^3)^{-4} \times 3^{11} = 3^{-12} \times 3^{11} = 3^{-1} = \frac{1}{3}$$

$$b) t^{-\frac{1}{2}} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{\sqrt{t}} = \frac{1}{4}$$

$$\Rightarrow \sqrt{t} = 4$$

$$\Rightarrow t = 16$$

Question 14 (***)

$$6x^{-\frac{1}{2}} - x^{\frac{1}{2}} = 5.$$

- a) Show that the substitution $y = x^{\frac{1}{2}}$ transforms the above indicial equation into the quadratic equation

$$y^2 + 5y - 6 = 0.$$

- b) Solve the quadratic equation and hence find the root of the **indicial** equation.

$$x = 1$$

$6x^{-\frac{1}{2}} - x^{\frac{1}{2}} = 5$
 $\Rightarrow \frac{6}{x^{\frac{1}{2}}} - x^{\frac{1}{2}} = 5$
 $\text{Let } y = x^{\frac{1}{2}}$
 $\Rightarrow \frac{6}{y} - y = 5$
 $\Rightarrow 6 - y^2 = 5y$
 $\Rightarrow 0 = y^2 + 5y - 6$
 $\Rightarrow (y+6)(y-1) = 0$
 $y = -6$
 $y = 1$
 $x^{\frac{1}{2}} = 1$
 $x = 1$

Question 15 (***)

The points $(2, 14)$ and $(6, 126)$ lie on the curve with equation

$$y = ax^n, \quad x \in \mathbb{R}$$

where a and n are non zero constants.

Find the value of a and the value of n .

$$a = \frac{7}{2}, \quad n = 2$$

$y = ax^n$
 $(2, 14) \Rightarrow 14 = a \times 2^n$
 $(6, 126) \Rightarrow 126 = a \times 6^n$
 $\Rightarrow \frac{14}{126} = \frac{a \times 2^n}{a \times 6^n} = \frac{2^n}{6^n} = \left(\frac{2}{6}\right)^n = \left(\frac{1}{3}\right)^n$
 $\frac{1}{9} = \left(\frac{1}{3}\right)^n$
 $3^{-2} = 3^{-n}$
 $n = 2$
 $14 = a \times 2^2$
 $14 = 4a$
 $7 = 2a$
 $a = \frac{7}{2}$

Question 16 (***)

- a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

i. $4^{\frac{1}{2}} + 9^{-\frac{1}{2}}$.

ii. $32^5 \times 8^{-10}$.

- b) Simplify fully the following expression

$$\sqrt{\frac{3a^3bc \times 6a^2b^2c^3}{2abc^4}}$$

$$\frac{7}{3}, \frac{1}{32}, 3a^2b$$

Handwritten solutions for Question 16(a) and (b):

(a) i. $4^{\frac{1}{2}} + 9^{-\frac{1}{2}} = \sqrt{4} + \frac{1}{\sqrt{9}} = 2 + \frac{1}{3} = \frac{7}{3}$

ii. $32^5 \times 8^{-10} = (2^5)^5 \times (2^3)^{-10} = 2^{25} \times 2^{-30} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

(b) $\sqrt{\frac{3a^3bc \times 6a^2b^2c^3}{2abc^4}} = \sqrt{\frac{18a^5b^3c^4}{2abc^4}} = \sqrt{9a^4b^2} = 3a^2b$

Question 17 (***)

$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}.$$

- a) Show that the substitution $x = t^{\frac{1}{3}}$ transforms the above indicial equation into the quadratic equation

$$x^2 - 2x - 15 = 0.$$

- b) Solve the quadratic equation and hence find the two solutions of the **indicial** equation.

$$t = -27, \quad t = 125$$

Handwritten solution for Question 17:

a) $t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}$
 $\Rightarrow t^{\frac{1}{3}} = 2 + \frac{15}{t^{\frac{1}{3}}}$
 Let $x = t^{\frac{1}{3}}$
 $\Rightarrow x = 2 + \frac{15}{x}$
 $\Rightarrow x^2 = 2x + 15$
 $\Rightarrow x^2 - 2x - 15 = 0$

b) $(x-5)(x+3) = 0$
 $\Rightarrow x = 5$ or $x = -3$
 $\Rightarrow t^{\frac{1}{3}} = 5$ or $t^{\frac{1}{3}} = -3$
 $\Rightarrow t = 125$ or $t = -27$

Question 18 (***)

- a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

i. $8^{\frac{5}{3}} - 16^{\frac{3}{4}}$.

ii. $(2.25)^{-\frac{3}{2}}$.

- b) Simplify fully the following expression

$$\left(2a^{\frac{1}{2}}b^3\right)^4 \times \left(4a^6b^2\right)^{-\frac{1}{2}}.$$

$$\boxed{24}, \quad \boxed{\frac{8}{27}}, \quad \boxed{8a^{-1}b^{11} = \frac{8b^{11}}{a}}$$

Question 19 (***)

Given that the curve with equation

$$y = ax - x^{\frac{1}{3}}, \quad x \geq 0,$$

passes through the point with coordinates $\left(\frac{1}{8}, 0\right)$, find the value of the constant a .

$$\boxed{a = 4}$$

Question 20 (***)

a) Evaluate the following indicial expressions, giving the answers as integers.

i. $\left(36^{\frac{1}{2}} + 16^{\frac{1}{4}}\right)^{\frac{4}{3}}$.

ii. $\left(\frac{1}{4}\right)^{-2}$.

b) Simplify fully the following expression

$$\left(k^{\frac{3}{2}} \times 8k^{-3}\right)^{\frac{1}{3}}.$$

$$\boxed{16}, \boxed{16}, \boxed{2k^{-\frac{1}{2}} = \frac{2}{\sqrt{k}}}$$

(i) $\left(36^{\frac{1}{2}} + 16^{\frac{1}{4}}\right)^{\frac{4}{3}} = \left[\sqrt{36} + \sqrt[4]{16}\right]^{\frac{4}{3}} = [6 + 2]^{\frac{4}{3}} = 8^{\frac{4}{3}}$
 $= (\sqrt[3]{8})^4 = 2^4 = 16 //$
 (ii) $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 16 //$
 (b) $\left(k^{\frac{3}{2}} \times 8k^{-3}\right)^{\frac{1}{3}} = \left(8k^{-\frac{3}{2}}\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}(k^{-\frac{3}{2}})^{\frac{1}{3}} = 2 \times k^{-\frac{1}{2}}$
 $= \frac{2}{k^{\frac{1}{2}}} = \frac{2}{\sqrt{k}} //$

Question 21 (***)

- a) Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{3}$.

i. $\sqrt{108} + \sqrt{3}$.

ii. $\frac{\sqrt{6} + \sqrt{3}}{\sqrt{2} + 1}$.

- b) Solve the equation

$$(5-x)^{\frac{3}{2}} = 8.$$

$$7\sqrt{3}, \sqrt{3}, x=1$$

(a) i. $\sqrt{108} + \sqrt{3} = \sqrt{36 \cdot 3} + \sqrt{3} = 6\sqrt{3} + \sqrt{3} = 7\sqrt{3}$
 ii. $\frac{\sqrt{6} + \sqrt{3}}{\sqrt{2} + 1} = \frac{(\sqrt{2} + \sqrt{3})(\sqrt{2} - 1)}{(\sqrt{2} + 1)(\sqrt{2} - 1)} = \frac{\sqrt{2} - \sqrt{2} + \sqrt{6} - \sqrt{3}}{2 - 1} = \frac{\sqrt{6} - \sqrt{3}}{1} = \sqrt{6} - \sqrt{3} = \sqrt{3}$
 (b) $(5-x)^{\frac{3}{2}} = 8$
 $\Rightarrow [(5-x)^{\frac{1}{2}}]^3 = 8^{\frac{2}{3}}$
 $\Rightarrow (5-x)^{\frac{1}{2}} = (\sqrt[3]{8})^{\frac{2}{3}}$
 $\Rightarrow 5-x = 4$
 $\Rightarrow x = 1$

Question 22 (***)

a) Evaluate the following indicial expressions, giving the answers as integers.

i. $\left(\frac{1}{3}\right)^{-3}$

ii. $\frac{8^6}{32^3}$

b) Simplify fully the following expression

$$\left(\frac{6a^7b^2 \times 9b}{2a}\right)^{\frac{1}{3}}$$

$$\boxed{108}, \boxed{8}, \boxed{3a^2b}$$

Handwritten solution for Question 22b:

$$(a) \left(\frac{1}{3}\right)^{-3} + 27^{\frac{1}{3}} = (3)^3 + (\sqrt[3]{27})^1 = 27 + 3^1 = 27 + 3^1 = 108$$

$$(ii) \frac{8^6}{32^3} = \frac{(2^3)^6}{(2^5)^3} = \frac{2^{18}}{2^{15}} = 2^3 = 8$$

$$(b) \left(\frac{6a^7b^2 \times 9b}{2a}\right)^{\frac{1}{3}} = \left(\frac{54a^6b^3}{2a}\right)^{\frac{1}{3}} = (27a^5b^3)^{\frac{1}{3}} = 3a^{\frac{5}{3}}b$$

Final answer: $\boxed{108}, \boxed{8}, \boxed{3a^2b}$

Question 23 (***)

- a)** Simplify fully each of the following expressions, writing the final answer as a single simplified surd.

i. $(2 + \sqrt{3})(2\sqrt{3} - 3)$.

ii. $\frac{\sqrt{6} + 3\sqrt{2}}{\sqrt{6} + \sqrt{2}}$.

- b) Solve the equation**

$$8w^{\frac{1}{2}} - w^{-1} = 0, \quad w \neq 0.$$

$$\square, \square{\sqrt{3}}, \square{\sqrt{3}}, \square{w = \frac{1}{4}}$$

③ (1) $(2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$ $\therefore \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$
 (2) $\frac{\sqrt{2}+3\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{2}+3\sqrt{3})(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$
 $= \frac{2-3\sqrt{6}+3\sqrt{6}-9}{2-3} = \frac{-7}{-1} = 7$
 $= \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}\sqrt{3}}{2}$
 $= \frac{2\sqrt{6}}{4} = \sqrt{6}$
 b) $8W^{\frac{1}{2}} - W^{\frac{1}{2}} = 0$
 $\Rightarrow 8W^{\frac{1}{2}} = W^{\frac{1}{2}}$
 $\Rightarrow 8W^{\frac{1}{2}} = \frac{1}{W}$
 $\Rightarrow 8W^{\frac{1}{2}} \cdot W = 1$
 $\Rightarrow W^{\frac{3}{2}} = \frac{1}{8}$
 $\Rightarrow (W^{\frac{1}{2}})^3 = (\frac{1}{8})^{\frac{1}{3}}$
 $\Rightarrow W^{\frac{1}{2}} = (\frac{1}{8})^{\frac{1}{3}}$
 $\Rightarrow W = (\frac{1}{8})^{\frac{2}{3}}$
 $\Rightarrow W = \frac{1}{4}$

Question 24 (***)

$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}, \quad t \neq 0.$$

Use the substitution $x = t^{\frac{1}{3}}$ to solve the above indicial equation.

$$t = -27, \quad t = 125$$

Handwritten solution for Question 24:

$$\begin{aligned}
 t^{\frac{1}{3}} &= 2 + 15t^{-\frac{1}{3}} \\
 \Rightarrow t^{\frac{1}{3}} &= 2 + \frac{15}{t^{\frac{1}{3}}} \\
 \text{Let } x &= t^{\frac{1}{3}} \\
 \Rightarrow x &= 2 + \frac{15}{x} \\
 \Rightarrow x^2 &= 2x + 15 \\
 \Rightarrow x^2 - 2x - 15 &= 0 \\
 \Rightarrow (x+3)(x-5) &= 0 \\
 \Rightarrow x &= -3 \text{ or } x = 5 \\
 \Rightarrow t^{\frac{1}{3}} &= -3 \text{ or } t^{\frac{1}{3}} = 5 \\
 \Rightarrow t &= -27 \text{ or } t = 125
 \end{aligned}$$

Question 25 (***)

An exponential curve has equation

$$y = ab^x, \quad x \in \mathbb{R},$$

where a and b are non zero constants.

The points $A\left(\frac{1}{2}, 1\right)$, $B(2, 8)$ and $C\left(-\frac{1}{2}, k\right)$ lie on this curve.

a) Find the values of a and b .

b) Find the value of k .

$$a = \frac{1}{2}, \quad b = 4, \quad k = \frac{1}{4}$$

Handwritten solution for Question 25:

$$\begin{aligned}
 y &= ab^x \\
 A\left(\frac{1}{2}, 1\right) &\Rightarrow 1 = a \cdot b^{\frac{1}{2}} \\
 B(2, 8) &\Rightarrow 8 = a \cdot b^2 \\
 \text{Divide side by side} \\
 \frac{1}{8} &= \frac{a \cdot b^{\frac{1}{2}}}{a \cdot b^2} = \frac{1}{b^{\frac{3}{2}}} \\
 \Rightarrow \frac{1}{8} &= \frac{1}{b^{\frac{3}{2}}} \\
 \Rightarrow b^{\frac{3}{2}} &= 8 \\
 \Rightarrow (b^{\frac{3}{2}})^{\frac{2}{3}} &= 8^{\frac{2}{3}} \\
 \Rightarrow b &= 4 \\
 \text{Now } 8 &= a \cdot 4^2 \\
 8 &= a \cdot 16 \\
 a &= \frac{1}{2} \\
 \therefore y &= \frac{1}{2} \times 4^x \\
 y &= \frac{1}{2} \times 4^{\frac{1}{2}} \\
 y &= \frac{1}{2} \times 2 \\
 y &= \frac{1}{1} \\
 y &= 1
 \end{aligned}$$

Question 26 (***)

$$\left(125^{\frac{1}{3}} \times 25^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{-\frac{1}{2}}} \right)^{-\frac{2}{3}}$$

Evaluate the above indicial expression, giving the final answer as a simplified fraction.

You may not use any calculating aid in the above question, and detailed workings must support the answer.

$$\boxed{}, \boxed{\frac{1}{16}}$$

Handwritten solution for Question 26:

$$\begin{aligned} & \left(125^{\frac{1}{3}} \times 25^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{-\frac{1}{2}}} \right)^{-\frac{2}{3}} \\ &= \left[\sqrt[3]{125} \times \sqrt{25} + (\sqrt[4]{16})^3 + \sqrt[3]{64} + 49^{\frac{1}{2}} \right]^{-\frac{2}{3}} \\ &= \left[5 \times 5 + 2 \times 4 + \sqrt{49} \right]^{-\frac{2}{3}} \\ &= (25 + 8 + 7)^{-\frac{2}{3}} \\ &= (25 + 32 + 7)^{-\frac{2}{3}} \\ &= 64^{-\frac{2}{3}} \\ &= \frac{1}{64^{\frac{2}{3}}} \\ &= \frac{1}{(\sqrt[3]{64})^2} \\ &= \frac{1}{4^2} \\ &= \frac{1}{16} \end{aligned}$$

Boxed formulas used:

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

Question 27 (***)

a) Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{2}$.

i. $\sqrt{98} + \sqrt{2}$.

ii. $(\sqrt{2} + 3)(2 - 3\sqrt{2})$.

b) Solve the equation

$$\frac{27^t}{3^{t-1}} = 3\sqrt{3}.$$

$$\boxed{}, \boxed{8\sqrt{2}}, \boxed{-7\sqrt{2}}, \boxed{t = \frac{1}{4}}$$

a) i) $\sqrt{98} + \sqrt{2} = \sqrt{49 \cdot 2} + \sqrt{2} = 7\sqrt{2} + \sqrt{2} = 8\sqrt{2}$
 ii) $(\sqrt{2} + 3)(2 - 3\sqrt{2}) = 2\sqrt{2} - 3\sqrt{2} \cdot \sqrt{2} + 3 \cdot 2 - 9\sqrt{2} = 2\sqrt{2} - 6 + 6 - 9\sqrt{2} = -7\sqrt{2}$
 b) $\frac{27^t}{3^{t-1}} = 3\sqrt{3}$
 $\Rightarrow \frac{(3^3)^t}{3^{t-1}} = 3^{3t+1/2}$
 $\Rightarrow \frac{3^{3t}}{3^{t-1}} = 3^{3t+1/2}$
 $\Rightarrow 3^{3t-(t-1)} = 3^{3t+1/2}$
 $\Rightarrow 3^{2t+1} = 3^{3t+1/2}$
 $\Rightarrow 2t+1 = 3t+1/2$
 $\Rightarrow t = \frac{1}{2}$
 (Note: The handwritten solution shows a contradiction, but the final answer is $t = \frac{1}{4}$.)

Question 28 (***)

- a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$16^{\frac{1}{2}} + 16^{-\frac{3}{4}}.$$

- b) Solve the equation

$$x^{-\frac{2}{3}} = 64.$$

- c) Simplify fully

$$\left(x^{\frac{3}{2}} + 2x^{-\frac{3}{2}}\right)^2.$$

$$\boxed{}, \boxed{\frac{33}{8}}, \boxed{x = \frac{1}{512}}, \boxed{x^3 + 4 + \frac{4}{x^3}}$$

Handwritten solutions for Question 28:

a) $16^{\frac{1}{2}} + 16^{-\frac{3}{4}} = \sqrt{16} + \frac{1}{(\sqrt[4]{16})^3} = 4 + \frac{1}{8} = \frac{33}{8}$

b) $x^{-\frac{2}{3}} = 64$
 $(x^{\frac{2}{3}})^{-1} = 64^{-1}$
 $x^{\frac{2}{3}} = \frac{1}{64}$
 $x = (\frac{1}{64})^{\frac{3}{2}} = \frac{1}{8^3} = \frac{1}{512}$

c) $(x^{\frac{3}{2}} + 2x^{-\frac{3}{2}})^2 = (x^{\frac{3}{2}} + 2x^{\frac{3}{2}})(x^{\frac{3}{2}} + 2x^{\frac{3}{2}})$
 $= x^{\frac{3}{2} \cdot \frac{3}{2}} + 2x^{\frac{3}{2} \cdot \frac{3}{2}} + 2x^{\frac{3}{2} \cdot \frac{3}{2}} + 4x^{\frac{3}{2} \cdot \frac{3}{2}}$
 $= x^2 + 2x^2 + 2x^2 + 4x^2$
 $= x^2 + 4 + \frac{4}{x^2}$

Question 29 (***)

An exponential curve has equation

$$y = ab^x, \quad x \in \mathbb{R}$$

where a and b are non zero constants.

The points $A(1,7)$ and $B(3,175)$ lie on this curve.

Given that $b > 0$, find the values of a and b .

$$a = 1.4, \quad b = 5$$

Handwritten solution for Question 29:

Given points $A(1,7)$ and $B(3,175)$ lie on the curve $y = ab^x$.

Substituting $A(1,7)$: $7 = ab^1 \Rightarrow 7 = ab$

Substituting $B(3,175)$: $175 = ab^3$

Divide the two equations to eliminate a :

$$\frac{175}{7} = \frac{ab^3}{ab} \Rightarrow 25 = b^2$$

Since $b > 0$, $b = 5$.

Substitute $b = 5$ back into $7 = ab$:

$$7 = a(5) \Rightarrow a = \frac{7}{5} = 1.4$$

Question 30 (***)

Solve the following simultaneous equations without using a calculator

$$8^y = 4^{2x+1}$$

$$27^{2y} = 9^{x-3}$$

$$\left(-\frac{5}{3}, -\frac{14}{9} \right)$$

Handwritten solution for Question 30:

Change equations into powers of 2 or 3.

Equation 1: $8^y = 4^{2x+1}$

$$2^{3y} = 2^{2(2x+1)} \Rightarrow 2^{3y} = 2^{4x+2} \Rightarrow 3y = 4x+2$$

Equation 2: $27^{2y} = 9^{x-3}$

$$3^{6y} = 3^{2(x-3)} \Rightarrow 6y = 2x-6 \Rightarrow 3y = x-3$$

Subtract Equation 2 from Equation 1:

$$3y - 3y = 4x+2 - (x-3) \Rightarrow 0 = 3x+5 \Rightarrow x = -\frac{5}{3}$$

Substitute $x = -\frac{5}{3}$ into Equation 2:

$$3y = -\frac{5}{3} - 3 \Rightarrow 3y = -\frac{14}{3} \Rightarrow y = -\frac{14}{9}$$

Question 31 (***)

a) Solve the equation

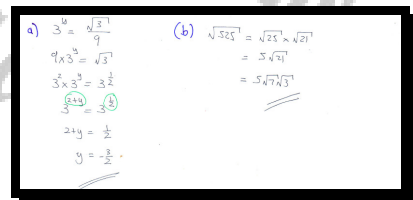
$$3^y = \frac{\sqrt{3}}{9}$$

b) Express

$$\sqrt{525},$$

in the form $a\sqrt{b}\sqrt{c}$, where a , b and c are prime numbers.

$$\boxed{}, \boxed{y = -\frac{3}{2}}, \boxed{5\sqrt{3}\sqrt{7}}$$

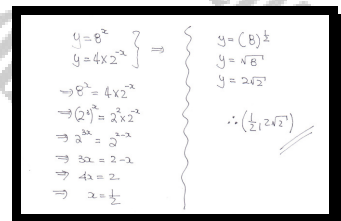


Question 32 (***)

Find the exact coordinates of the point of intersection between the curves with equations

$$y = 8^x \quad \text{and} \quad y = 4 \times 2^{-x}.$$

$$\boxed{\left(\frac{1}{2}, 2\sqrt{2}\right)}$$



Question 33 (***)

- a) If x is a real number solve the following indicial equation

$$x\left(x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}\right)^2 = 0.$$

- b) Express

$$\frac{\sqrt{98} - \sqrt{8}}{1 + \sqrt{2}},$$

in the form $a + b\sqrt{2}$, where a and b are integers.

$$\boxed{}, \boxed{x=2}, \boxed{10-5\sqrt{2}}$$

a) REWRITE THE EQUATION IN SLOPED FORM

$$\Rightarrow 2\left(x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}\right)^2 = 0$$

$$\Rightarrow 2\left(x^{\frac{1}{2}} - \frac{2}{x^{\frac{1}{2}}}\right)^2 = 0$$

SIMPLY $x > 0$, THEREFORE WE MAY WRITE

$$\Rightarrow \left(x^{\frac{1}{2}} - \frac{2}{x^{\frac{1}{2}}}\right)^2 = 0$$

$$\Rightarrow x^{\frac{1}{2}} - \frac{2}{x^{\frac{1}{2}}} = 0$$

$$\Rightarrow \frac{x - 2}{x^{\frac{1}{2}}} = 0$$

$$\Rightarrow x - 2 = 0$$

$$\Rightarrow x = 2$$

b) SIMPLIFY THE NUMERATOR BEFORE RATIONALISING

$$\Rightarrow \frac{\sqrt{98} - \sqrt{8}}{1 + \sqrt{2}} = \frac{\sqrt{49 \cdot 2} - \sqrt{4 \cdot 2}}{1 + \sqrt{2}}$$

$$= \frac{7\sqrt{2} - 2\sqrt{2}}{1 + \sqrt{2}}$$

$$= \frac{5\sqrt{2}}{1 + \sqrt{2}}$$

$$= \frac{5\sqrt{2}(1 - \sqrt{2})}{(1 + \sqrt{2})(1 - \sqrt{2})}$$

$$= \frac{5\sqrt{2} - 5\sqrt{2} \cdot \sqrt{2}}{1 - \sqrt{2} \cdot \sqrt{2} - 2}$$

$$= \frac{5\sqrt{2} - 10}{-1}$$

$$= 10 - 5\sqrt{2}$$

$a = 10$
 $b = -5$

Question 34 (***)

Find, **without** the use of any calculating aid, the solution of the equation

$$\frac{1}{2} \times 4^{2x} = 64^{64}.$$

$$x = 96.25$$

As we are not allowed calculators, to help ourselves, observe that the expression is all powers of 2.

$$\begin{aligned} \Rightarrow \frac{1}{2} \times 4^{2x} &= 64^{64} \\ \Rightarrow 2^1 \times (2^2)^{2x} &= (2^6)^{64} \\ \Rightarrow 2^1 \times 2^{4x} &= 2^{6 \times 64} \\ \Rightarrow 2^{4x+1} &= 2^{384} \\ \Rightarrow 4x+1 &= 384 \\ \Rightarrow 4x &= 383 \\ \Rightarrow x &= \frac{383}{4} = 95.75 \end{aligned}$$

• $6 \times 64 = 6 \times (2 \times 32) = 384$

• $\frac{383}{4} = \frac{380+3}{4} = \frac{380}{4} + \frac{3}{4} = 95 + \frac{3}{4} = 95.75$

Question 35 (***)

$$4^x - 2^{x+2} = 32.$$

- a) Show that the substitution $y = 2^x$ transforms the above indicial equation into the quadratic equation

$$y^2 - 4y - 32 = 0.$$

- b) Solve the quadratic equation and hence find the root of the **indicial** equation.

$$x = 3$$

a) $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

$2^{x+2} = 2^x \times 2^2 = 4 \times 2^x = 4y$

$4^x - 2^{x+2} - 32 = 0$

$y^2 - 4y - 32 = 0$ (by inspection)

b) $(y+4)(y-8) = 0$

$y = -4$ or 8

$2^x = -4$ or 8

$2^x = 8$

$x = 3$

Question 36 (****)

Solve the following simultaneous equations without using a calculator.

$$4x - 3y = 11$$

$$9^{y+3} = \frac{3\sqrt{3}}{27^x}$$

$$\left(\frac{1}{2}, -3\right)$$

$$\begin{aligned} 4x - 3y &= 117 \quad \text{R4} \\ 6x + 4y &= -9 \quad \text{R3} \end{aligned}$$

$$\frac{4x - 12y}{18x + 12y} = \frac{144}{-27}$$

$$34y = 17$$

$$y = \frac{1}{2}$$

$$\text{subst } 6x + 4y = -9$$

$$\frac{6x}{2} + \frac{4y}{2} = \frac{-9}{2}$$

$$3 + 4y = -\frac{9}{2}$$

$$\frac{4y}{4} = \frac{-12}{4}$$

$$y = -3$$

Question 37 (****)

Given that the curve with equation

$$y = kx^{\frac{1}{2}} - x^{-\frac{3}{2}}, \quad x \geq 0,$$

passes through the point with coordinates $\left(3, \frac{5}{3}\sqrt{3}\right)$, show clearly that $k = \frac{16}{9}$.

proof

$$\begin{aligned} y &= kx^{\frac{1}{3}} - x^{\frac{2}{3}} \\ (3, \frac{5}{3}\sqrt{3}) &\Rightarrow \frac{5}{3}\sqrt{3} = k \cdot 3^{\frac{1}{3}} - 3^{\frac{2}{3}} \\ &\Rightarrow \frac{5}{3}\sqrt{3} = k \cdot \sqrt{3} - \frac{1}{\sqrt{3}} \\ &\Rightarrow \frac{5}{3}\sqrt{3} = k \cdot \sqrt{3} - \frac{1}{\sqrt{3}} \quad | \cdot \sqrt{3} \\ &\Rightarrow \frac{5}{3} \cdot 3 = k \cdot \sqrt{3} \cdot \sqrt{3} - \frac{1}{\sqrt{3} \cdot \sqrt{3}} \quad | \text{ Multipl. mit } 3 \\ &\Rightarrow \frac{5}{3} \cdot 3 = k \cdot 3 - \frac{1}{1} \\ &\Rightarrow \frac{5}{3} \cdot 3 = k - \frac{1}{3} \\ &\Rightarrow \frac{5}{3} + \frac{1}{3} = k \\ &\Rightarrow \frac{5}{3} + \frac{1}{3} = k \\ &\Rightarrow k = \frac{6}{3} \end{aligned}$$

Question 38 (****)

The indicial equation

$$2^{x+1} + 2^{3-x} = 17, \quad x \in \mathbb{R},$$

is to be solved by a suitable substitution.

- a) Show clearly that the substitution $y = 2^x$ transforms the above indicial equation into the quadratic equation

$$2y^2 - 17y + 8 = 0.$$

- b) Solve the quadratic equation by factorization and hence determine the two solutions of the indicial equation.

$$\boxed{}, \quad x = -1, 3$$

Handwritten solution for Question 38:

a) $2^{x+1} + 2^{3-x} = 17$
 $\Rightarrow 2^x \cdot 2 + 2^3 \cdot 2^{-x} = 17$
 $\Rightarrow 2^x \cdot 2 + \frac{8}{2^x} = 17$
 Let $y = 2^x$
 $\therefore 2y + \frac{8}{y} = 17$
 $2y^2 + 8 = 17y$
 $2y^2 - 17y + 8 = 0$
 Solve by factorization

b) Factorize:
 $(2y - 8)(y - 1) = 0$
 $\Rightarrow y = 4$ or $y = 1$
 $2^x = 4 \Rightarrow x = 2$
 $2^x = 1 \Rightarrow x = 0$

Question 39 (****)

Solve the equation

$$(25x^2)^{-\frac{1}{2}} = 2, \quad x \neq 0.$$

$$x = \pm \frac{1}{10}$$

Handwritten solution for Question 39:

$(25x^2)^{-\frac{1}{2}} = 2$
 $\Rightarrow \frac{1}{(25x^2)^{\frac{1}{2}}} = 2$
 $\Rightarrow \frac{1}{5|x|} = 2$
 $\Rightarrow \frac{1}{|x|} = 10$
 $\Rightarrow |x| = \frac{1}{10}$
 $\Rightarrow x = \pm \frac{1}{10}$

Question 40 (****)

Given that

$$a = x^{\frac{1}{2}} + x^{-\frac{1}{2}} \quad \text{and} \quad b = x^{\frac{1}{2}} - x^{-\frac{1}{2}},$$

show clearly that

$$a^2 b^2 + 4 \equiv \left(x + \frac{1}{x} \right)^2.$$

, proof

WORKS AS FOLLOWS

$$\begin{aligned}
 a^2 b^2 + 4 &= (x^{\frac{1}{2}} + x^{-\frac{1}{2}})(x^{\frac{1}{2}} - x^{-\frac{1}{2}}) + 4 \\
 &= [x^{\frac{1}{2}} + 2(x^{\frac{1}{2}})(x^{-\frac{1}{2}}) + (x^{-\frac{1}{2}})^2][x^{\frac{1}{2}} - 2(x^{\frac{1}{2}})(x^{-\frac{1}{2}}) + (x^{-\frac{1}{2}})^2] + 4 \\
 &= [x^{\frac{1}{2}} + 2x^{\frac{1}{2}} + x^{\frac{1}{2}}][x^{\frac{1}{2}} - 2x^{\frac{1}{2}} + x^{\frac{1}{2}}] + 4 \\
 &= (x + 2 + x)(x - 2 + x) + 4 \\
 &= \frac{x^2 - 2x + 2x - 4 + 2x^2}{x^2 - 2x + x^2} + 4 \\
 &= \frac{x^2 - 2 + x^2 + 4}{x^2 - 2 + x^2} \\
 &= \frac{x^2 + 2 + x^2}{x^2 - 2 + x^2} \\
 &= \frac{x^2 + 2(x)(\frac{1}{x}) + (\frac{1}{x})^2}{(x - \frac{1}{x})^2} \\
 &= \frac{(x + \frac{1}{x})^2}{(x - \frac{1}{x})^2}
 \end{aligned}$$

As Required

Question 41 (****)

$$2^{2p-2} - 2^{p-2} - 3 = 0, \quad p \in \mathbb{R},$$

- a) Show clearly that the substitution $x = 2^p$ transforms the above indicial equation into the quadratic equation

$$x^2 - x - 12 = 0.$$

- b) Solve the quadratic equation and hence determine the value of p .

$$p = 2$$

a) $2^{2p-2} - 2^{p-2} - 3 = 0$
 let $a = 2^p$
 $\Rightarrow a^2 \times \frac{1}{4} - a \times \frac{1}{2} - 3 = 0$
 $\Rightarrow (a^2) \times \frac{1}{4} - \frac{1}{2}a - 3 = 0$
 $\Rightarrow a^2 \times \frac{1}{4} - \frac{1}{2}a - 3 = 0$
 $\Rightarrow a^2 - 2a - 12 = 0$ // as required

b) $(x+3)(x-4) = 0$
 $x = -3$
 $x = 4$
 $2^p = 4$
 $p = 2$ //

Question 42 (****)

$$100^x - 10001(10^{x-1}) + 100 = 0.$$

- a) Show that the substitution $y = 10^x$ transforms the above indicial equation into the quadratic equation

$$10y^2 - 10001y + 1000 = 0.$$

- b) Solve the quadratic equation and hence find the two solutions of the **indicial** equation.

$$\boxed{\frac{1}{10}}, \quad \boxed{x = -1}, \quad \boxed{x = 3}$$

(a) $100^x - 10001(10^{x-1}) + 100 = 0$
 $\bullet 100^x = (10^x)^2 = 10^{2x} = (10^x)^2 = y^2$
 $\bullet 10^{x-1} = 10^x \cdot 10^{-1} = \frac{1}{10} \cdot 10^x = \frac{1}{10} y$
 This
 $10y^2 - 10001 \cdot \frac{1}{10} y + 100 = 0$
 $10y^2 - 1000.1y + 100 = 0$ ~~REQUIRED~~

(b) $(10y - 1)(y - 1000) = 0$
 $y = \frac{1}{10}$
 $y = 1000$
 $x = \log_{10} \frac{1}{10} = -1$
 $x = \log_{10} 1000 = 3$

Question 43 (****)

Determine the value of k .

$$\frac{2^{288} + 2^{285}}{9} = 2^k.$$

You must show full workings.

$$\boxed{}, \quad \boxed{k = 285}$$

Factorize the expression as usual
 $\frac{2^{288} + 2^{285}}{9} = \frac{2^{285} \cdot 2^3 + 2^{285}}{9}$
 $= \frac{2^{285} (2^3 + 1)}{9}$
 $= \frac{2^{285} \cdot 9}{9}$
 $= 2^{285}$
 $\therefore k = 285$

Question 44 (****+)

Solve the following indicial equation

$$6^{x+2} \times 2^{1-x} = \frac{8}{3}.$$

You must show full workings.

$$\boxed{}, \boxed{x = -3}$$

Handwritten solution for Question 44:

$$\begin{aligned}
 6^{x+2} \times 2^{1-x} &= \frac{8}{3} \\
 \Rightarrow 6^2 \times 6^x \times 2^1 \times 2^{-x} &= \frac{8}{3} \\
 \Rightarrow 36 \times 6^x \times 2 \times 2^{-x} &= \frac{8}{3} \\
 \Rightarrow 72 \times \frac{6^x}{2^x} &= \frac{8}{3} \\
 \Rightarrow 72 \times \left(\frac{6}{2}\right)^x &= \frac{8}{3} \\
 \Rightarrow 72 \times 3^x &= \frac{8}{3} \\
 \Rightarrow 3 \times 72 \times 3^x &= 8 \quad (\text{Divide by 9}) \\
 \Rightarrow 3 \times 9 \times 3^x &= 1 \\
 \Rightarrow 27 \times 3^x &= 1 \\
 \Rightarrow 3^x &= \frac{1}{27} \\
 \Rightarrow x &= -3
 \end{aligned}$$

Question 45 (****+)

Show clearly that

$$\frac{1+x(x^2-1)^{-\frac{1}{2}}}{x+(x^2-1)^{\frac{1}{2}}} \equiv \frac{1}{\sqrt{x^2-1}}.$$

$$\boxed{}, \boxed{\text{proof}}$$

Handwritten solution for Question 45:

$$\begin{aligned}
 \frac{1+x(x^2-1)^{-\frac{1}{2}}}{x+(x^2-1)^{\frac{1}{2}}} &= \frac{[1+x(x^2-1)^{-\frac{1}{2}}][x-(x^2-1)^{\frac{1}{2}}]}{[x+(x^2-1)^{\frac{1}{2}}][x-(x^2-1)^{\frac{1}{2}}]} \\
 &= \frac{1 - (x^2-1)^{\frac{1}{2}} + x^2(x^2-1)^{-\frac{1}{2}} - x}{x^2 - (x^2-1)} = \frac{1 - (x^2-1)^{\frac{1}{2}} + x^2(x^2-1)^{-\frac{1}{2}} - x}{1} \\
 &= \frac{1 - (x^2-1)^{\frac{1}{2}} + x^2(x^2-1)^{-\frac{1}{2}} - x}{1} = \frac{1 - (x^2-1)^{\frac{1}{2}} + x^2(x^2-1)^{-\frac{1}{2}} - x}{1} \\
 &= \frac{1}{\sqrt{x^2-1}} \quad \text{Q.E.D.}
 \end{aligned}$$

Question 46 (****+)

Solve the following exponential equation

$$16 + 8^{x+1} - 4^{x+1} - 2^{x+5} = 0, \quad x \in \mathbb{R}.$$

$$\boxed{}, \quad \boxed{x = \pm 1}$$

AS 2, 4, 8, 16 ARE ALL POWERS OF 2 WE REWRITE THE EQUATION

$$\Rightarrow 16 + 8^{x+1} - 4^{x+1} - 2^{x+5} = 0$$

$$\Rightarrow 16 + (2^3)^{x+1} - (2^2)^{x+1} - 2^{x+5} = 0$$

$$\Rightarrow 16 + 2^{3x+3} - 2^{2x+2} - 2^{x+5} = 0$$

$$\Rightarrow 16 + 8 \times (2^x)^3 - 4 \times (2^x)^2 - 32 \times (2^x) = 0$$

$$\Rightarrow 16 + 8y^3 - 4y^2 - 32y = 0 \quad (\text{where } y = 2^x)$$

DIVIDE THE EQUATION BY 4 & REWRITE THE OBER

$$\Rightarrow 2y^3 - y^2 - 8y + 4 = 0$$

LOOK FOR FACTORS, OR NOTICE THE FACTORIZATION IN PARS

$$\Rightarrow y^2(2y-1) - 4(2y-1) = 0$$

$$\Rightarrow (2y-1)(y^2-4) = 0$$

$$\Rightarrow (2y-1)(y-2)(y+2) = 0$$

$$\Rightarrow y = \begin{cases} \frac{1}{2} \\ 2 \\ -2 \end{cases}$$

$$\Rightarrow 2^x = \begin{cases} \frac{1}{2} \\ 2 \\ -2 \end{cases}$$

$$\Rightarrow x = \begin{cases} -1 \\ 1 \\ \end{cases}$$

Question 47 (****+)Determine the value of k .

$$\frac{2^{399} - 2^{395}}{15} = 32^k.$$

You must show full workings.

$$\boxed{}, \quad \boxed{k = 79}$$

POWERS AS EXPANS

$$\Rightarrow \frac{2^{399} - 2^{395}}{15} = 32^k$$

$$\Rightarrow 2^4 \times 2^{395} - 2^{395} = (2^5)^k$$

$$\Rightarrow \frac{16 \times 2^{395} - 2^{395}}{15} = 2^{5k}$$

$$\Rightarrow \frac{15 \times 2^{395}}{15} = 2^{5k}$$

$$\Rightarrow 2^{395} = 2^{5k}$$

$$\Rightarrow 395 = 5k$$

$$\Rightarrow k = 79$$

Question 48 (***+)

Solve the following equation

$$\frac{1}{x} \times \sqrt{\frac{x^2 \sqrt{x^5}}{\left(\frac{1}{\sqrt{x}}\right)^3}} + \frac{2}{\sqrt{x}} \times \left[2\sqrt{x^3} - \left(\frac{1}{\sqrt{x}}\right)^{-3} \right] + x \left(x^3 \sqrt{x} \right)^{-\frac{2}{7}} = 4.$$

$$\boxed{x=1}$$

$$\frac{1}{2} \sqrt{\frac{2 \sqrt{2} \sqrt{2}}{(\sqrt{2})^2}} + \frac{2}{\sqrt{2}} \left[2 \sqrt{2}^3 - \left(\frac{1}{\sqrt{2}} \right)^{-2} \right] + 2 (2 \sqrt{2})^{\frac{5}{2}} = 4$$

$$\frac{1}{2} \sqrt{\frac{2 \sqrt{2} \sqrt{2}}{2 \sqrt{2}}} + 2 \sqrt{2} \left[2 \sqrt{2}^3 - (\sqrt{2})^2 \right] + \alpha (\sqrt{2} + 2 \sqrt{2})^{\frac{5}{2}} = 4$$

$$\frac{1}{2} \left[\frac{2 \sqrt{2}}{2 \sqrt{2}} \right]^{\frac{5}{2}} + 2 \sqrt{2}^2 \left[2 \sqrt{2}^3 - 2 \sqrt{2}^2 \right] + 2 \sqrt{2} (\sqrt{2})^{\frac{5}{2}} = 4$$

$$\frac{1}{2} (2 \sqrt{2})^{\frac{5}{2}} + 2 \sqrt{2}^2 \times 2 \sqrt{2} = 4$$

$$\frac{1}{2} \times 2^{\frac{5}{2}} + 2 \sqrt{2}^2 + 1 = 4$$

$$2^{\frac{5}{2}} + 2 \sqrt{2} + 1 = 4$$

$$2^{\frac{5}{2}} + 2 \sqrt{2} - 3 = 0$$

Question 49 (****+)

Solve the following indicial equation

$$\frac{2^n}{2^{\sqrt{n}} \times 2^6} = 1.$$

You must show full workings.

, $n = 9$

$$\begin{aligned} \frac{2^3}{2 \times 2^6} &= 1 \implies \frac{2^3}{2^{1+6}} = 1 \\ \implies 2^3 &= 2^{6+1} \\ \implies 8 &= 64 + 6 \end{aligned}$$

THIS IS A QUADRATIC IN \sqrt{x}

$$\begin{aligned} \implies (\sqrt{x})^2 - \sqrt{x} - 6 &= 0 \\ \implies (\sqrt{x} - 3)(\sqrt{x} + 2) &= 0 \\ \implies \sqrt{x} &= \begin{array}{c} 3 \\ \searrow \swarrow \\ \times \end{array} \end{aligned}$$

$$\implies x = 9$$

Question 50 (****+)

Show with a detailed method that

$$\frac{\sqrt[3]{16} - \sqrt[3]{2}}{\sqrt[3]{4}} = k \sqrt[3]{4}$$

where k is a constant to be found.

$$\boxed{}, \quad k = \frac{1}{2}$$

SIMPLIFYING AND INDEXES FOR EASE

$$\frac{\sqrt[3]{16} - \sqrt[3]{2}}{\sqrt[3]{4}} = \frac{16^{\frac{1}{3}} - 2^{\frac{1}{3}}}{4^{\frac{1}{3}}} = \frac{(2^4)^{\frac{1}{3}} - 2^{\frac{1}{3}}}{(2^2)^{\frac{1}{3}}} = \frac{2^{\frac{4}{3}} - 2^{\frac{1}{3}}}{2^{\frac{2}{3}}}$$

$$= \frac{2 \times 2^{\frac{1}{3}} - 2^{\frac{1}{3}}}{2^{\frac{2}{3}}} = \frac{2^{\frac{1}{3}}}{2^{\frac{2}{3}}}$$

 NOW SIMPLIFYING

$$= \frac{2^{\frac{1}{3}} \times 2^{\frac{1}{3}}}{2^{\frac{2}{3}} \times 2^{\frac{1}{3}}} = \frac{2^{\frac{2}{3}}}{2} = \frac{1}{2} \times 2^{\frac{2}{3}}$$

$$= \frac{1}{2} \times (2^{\frac{2}{3}})^{\frac{3}{2}} = \frac{1}{2} \times 4^{\frac{1}{2}} = \frac{1}{2} \sqrt{4}$$

Question 51 (*****)

$$f(x) \equiv \frac{8}{x^2} - x, \quad x \neq 0.$$

Show that $f\left(-2^{\frac{4}{3}}\right) = 3 \sqrt[3]{2}$.

You must show detailed workings.

$$\boxed{9 \text{ marks}}, \quad \boxed{\text{proof}}$$

$$f(x) \equiv \frac{8}{x^2} - x = \frac{1}{x^2} [8 - x^3]$$

$$f(-2^{\frac{4}{3}}) = \frac{1}{(-2^{\frac{4}{3}})^2} [8 - (-2^{\frac{4}{3}})^3]$$

$$= \frac{1}{2^{\frac{8}{3}}} [8 - (-2^4)]$$

$$= \frac{1}{2^{\frac{8}{3}}} [8 + 16]$$

$$= \frac{24}{2^{\frac{8}{3}}}$$

$$= \frac{3 \times 2^3}{2^{\frac{8}{3}}}$$

$$= 3 \times 2^{\frac{1}{3}}$$

$$= 3 \sqrt[3]{2}$$

$(-2^{\frac{4}{3}})^3 = (-1)^3 (2^{\frac{4}{3}})^3 = -1 \times 2^4 = -16$

Question 52 (****)

$$A = \frac{3}{2}xy + 2yz + 2xz.$$

Given that $x = \left(\frac{4}{3}\right)^{\frac{1}{3}}$, $y = \left(\frac{4}{3}\right)^{\frac{1}{3}}$ and $z = \left(\frac{3}{4}\right)^{\frac{2}{3}}$, show clearly that $A = 3\sqrt[3]{6}$

☐, ☐ proof

$$\begin{aligned}
 A &= \frac{3}{2}xy + 2yz + 2xz = \frac{3}{2}\left(\frac{4}{3}\right)^{\frac{1}{3}} + 2\left(\frac{4}{3}\right)^{\frac{1}{3}} + 2\left(\frac{3}{4}\right)^{\frac{2}{3}}\left(\frac{4}{3}\right)^{\frac{1}{3}} \\
 &= 2 \times \frac{3}{2}\left(\frac{4}{3}\right)^{\frac{1}{3}} + 2\left(\frac{4}{3}\right)^{\frac{1}{3}} + 2\left(\frac{3}{4}\right)^{\frac{2}{3}}\left(\frac{4}{3}\right)^{\frac{1}{3}} \\
 &= 2\left(\frac{4}{3}\right)^{\frac{1}{3}} + 2\left(\frac{4}{3}\right)^{\frac{1}{3}} + 2\left(\frac{4}{3}\right)^{\frac{1}{3}} \\
 &= 6\left(\frac{4}{3}\right)^{\frac{1}{3}} \\
 &= 6 \times \frac{3^{\frac{1}{3}}}{4^{\frac{1}{3}}} \times \frac{4^{\frac{1}{3}}}{4^{\frac{1}{3}}} \\
 &= 6 \times \frac{3^{\frac{1}{3}} \cdot 4^{\frac{1}{3}}}{4} \\
 &= \frac{6}{4} \times 4\sqrt[3]{6} = \frac{3}{2} \times 4\sqrt[3]{6} = 3\sqrt[3]{6}
 \end{aligned}$$

Question 53 (****)

A curve has equation

$$f(x) \equiv 3^{ax} + b, \quad x \in \mathbb{R},$$

where a and b are non zero constants.

Find the value of a and the value of b , given further that

$$f(2) = 3 - \sqrt{3} \quad \text{and} \quad f(3) = 2\sqrt{3}.$$

$$\boxed{a = \frac{1}{2}}, \quad \boxed{b = -\sqrt{3}}$$

Handwritten solution for Question 53:

Given: $f(x) = 3^{ax} + b, \quad x \in \mathbb{R}$

Using $f(2) = 3 - \sqrt{3}$ and $f(3) = 2\sqrt{3}$:

$$\begin{aligned} \Rightarrow 3^{2a} + b &= 3 - \sqrt{3} \\ \Rightarrow b &= 3 - \sqrt{3} - 3^{2a} \end{aligned} \quad \left| \quad \begin{aligned} \Rightarrow 3^{3a} + b &= 2\sqrt{3} \\ \Rightarrow b &= 2\sqrt{3} - 3^{3a} \end{aligned} \right.$$

Equating the two expressions for b :

$$\begin{aligned} 3 - \sqrt{3} - 3^{2a} &= 2\sqrt{3} - 3^{3a} \\ \Rightarrow 3^{3a} - 3^{2a} &= -3 + 2\sqrt{3} \\ \Rightarrow 3^{2a} - 3^a &= -3 + 3 \times 3^{\frac{1}{2}} \\ \Rightarrow 3^a - 3^{\frac{1}{2}} &= 3^{\frac{1}{2}} - 3^{\frac{1}{2}} \end{aligned}$$

By inspection $a = \frac{1}{2}$ or checked by raising

$$\Rightarrow 3^{\frac{1}{2}}(3^{\frac{1}{2}} - 1) = 3^{\frac{1}{2}}(3^{\frac{1}{2}} - 1)$$

$$\Rightarrow a = \frac{1}{2}$$

Finally using $b = 3 - \sqrt{3} - 3^{2a}$:

$$\begin{aligned} \Rightarrow b &= 3 - \sqrt{3} - 3^{2 \times \frac{1}{2}} \\ \Rightarrow b &= 3 - \sqrt{3} - 3 \\ \Rightarrow b &= -\sqrt{3} \end{aligned}$$

Question 54 (****)

A curve has equation

$$f(x) \equiv 2^{ax} + b, \quad x \in \mathbb{R},$$

where a and b are non zero constants.Find the value of a and the value of b , given further that

$$f(2) = \frac{5}{2} \quad \text{and} \quad f(-2) = 4.$$

$$\boxed{a = -\frac{1}{2}}, \quad \boxed{b = 2}$$

Handwritten solution for Question 54:

Given: $f(x) = 2^{ax} + b, \quad x \in \mathbb{R}$

Method 1: Using $f(2) = \frac{5}{2}$ and $f(-2) = 4$

Using $f(2) = \frac{5}{2}$:

$$\begin{aligned} 2^{2a} + b &= \frac{5}{2} \\ 2^{-2a} + b &= 4 \\ b &= \frac{5}{2} - 2^{2a} \end{aligned}$$

Using $f(-2) = 4$:

$$\begin{aligned} 2^{-2a} + b &= 4 \\ b &= 4 - 2^{-2a} \end{aligned}$$

Method 2: Solving Simultaneously

$$\begin{aligned} 4 - 2^{-2a} &= \frac{5}{2} - 2^{2a} \\ 4 - \frac{1}{2^{2a}} &= \frac{5}{2} - 2^{2a} \\ 4 - \frac{1}{A} &= \frac{5}{2} - A \quad (A = 2^{2a}) \\ 4A - 1 &= 5A - A^2 \\ 8A - 2 &= 5A - A^2 \\ 2A^2 + 3A - 2 &= 0 \\ (2A - 1)(A + 2) &= 0 \\ A &= \frac{1}{2} \end{aligned}$$

Method 3: Using $f(2) = \frac{5}{2}$ and $f(-2) = 4$ (Alternative)

$$\begin{aligned} 2^{2a} + b &= \frac{5}{2} \\ 2^{-2a} + b &= 4 \\ 2^{2a} &= \frac{1}{2^{-2a}} \\ 2^{2a} + b &= \frac{5}{2} \\ 2^{2a} + b &= 4 \\ 2^{2a} &= 4 - b \\ 2^{2a} &= 4 - 2^{-2a} \\ 2^{2a} + 2^{-2a} &= 4 \\ 2^{2a} &= 2 \\ 2a &= 1 \\ a &= \frac{1}{2} \end{aligned}$$

Final Answer: $a = -\frac{1}{2}, \quad b = 2$

Question 55 (*****)

A curve has equation

$$f(x) \equiv 4^{ax+b}, \quad x \in \mathbb{R},$$

where a and b are non zero constants.

Find the value of a and the value of b , given further that

$$f\left(\frac{2}{3}\right) = \frac{1}{4}\sqrt[3]{4} \quad \text{and} \quad f\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{2}.$$

$$\boxed{}, \quad \boxed{a = \frac{1}{2}}, \quad \boxed{b = -1}$$

Handwritten solution for Question 55:

Given: $f(x) \equiv 4^{ax+b}, \quad x \in \mathbb{R}$

Using $f\left(\frac{2}{3}\right) = \frac{1}{4}\sqrt[3]{4}$:

$$\begin{aligned} \Rightarrow 4^{a \cdot \frac{2}{3} + b} &= \frac{1}{4}\sqrt[3]{4} \\ \Rightarrow 4^{\frac{2a}{3} + b} &= 4^{-1} \times 4^{\frac{1}{3}} \\ \Rightarrow 4^{\frac{2a}{3} + b} &= 4^{-\frac{2}{3}} \\ \Rightarrow \frac{2a}{3} + b &= -\frac{2}{3} \\ \Rightarrow b &= -\frac{2a}{3} - \frac{2}{3} \end{aligned}$$

Using $f\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{2}$:

$$\begin{aligned} \Rightarrow 4^{a \cdot \frac{3}{2} + b} &= \frac{1}{2}\sqrt{2} \\ \Rightarrow 4^{\frac{3a}{2} + b} &= 2^{-1} \times 2^{\frac{1}{2}} \\ \Rightarrow 4^{\frac{3a}{2} + b} &= 2^{-\frac{1}{2}} \\ \Rightarrow \left(2^2\right)^{\frac{3a}{2} + b} &= 2^{-\frac{1}{2}} \\ \Rightarrow 2^{3a + 2b} &= 2^{-\frac{1}{2}} \\ \Rightarrow 3a + 2b &= -\frac{1}{2} \end{aligned}$$

Solving the equations simultaneously gives:

$$\begin{aligned} \Rightarrow 3a + 2\left(-\frac{2a}{3} - \frac{2}{3}\right) &= -\frac{1}{2} \\ \Rightarrow 3a - \frac{4a}{3} - \frac{4}{3} &= -\frac{1}{2} \\ \Rightarrow \frac{5a}{3} &= \frac{5}{6} \\ \Rightarrow \frac{1}{3}a &= \frac{1}{6} \\ \Rightarrow a &= \frac{1}{2} \\ \therefore b &= -\frac{2}{3}\left(\frac{1}{2}\right) - \frac{2}{3} = -\frac{1}{3} - \frac{2}{3} = -1 \\ \Rightarrow b &= -1 \end{aligned}$$

Question 56 (*****)

Find the solutions for the following equation.

$$(2x^2 - 7x + 4)^{x^2 + 2x - 8} = 1.$$

$$\boxed{V}, \boxed{}, \boxed{x = -4, \frac{1}{2}, 2, 3}$$

Let's set the expression as the LHS of the equation

$$a^0 = 1 \Rightarrow 2x^2 - 7x + 4 = 0$$

$$\Rightarrow (x-2)(x+4) = 0$$

$$\Rightarrow x = 2 \text{ or } -4$$

There is another answer possibility

$$1^a = 1 \Rightarrow 2x^2 - 7x + 4 = 1$$

$$\Rightarrow 2x^2 - 7x + 3 = 0$$

$$\Rightarrow (2x-1)(x-3) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } 3$$

Hence there are four solutions

$$x = -4, \frac{1}{2}, 2, 3 //$$

Question 57 (*****)

Find a solution for the following equation.

$$(2x^2 - 5x - 10)^{4x+5} = \frac{1}{2}.$$

$$\sqrt{\quad}, \quad \boxed{\quad}, \quad x = -\frac{3}{2}$$

THE QUESTION DOES NOT ASK TO SOLVE, BUT TO FIND A SOLUTION?

if $f(x) = \frac{1}{2} \iff f(x)=2 \ \& \ g(x)=-1$

LET US SEE IF THESE CAN BE SATISFIED TOGETHER

$f(x)=2$	$g(x)=-1$
$2x^2-5x-10=2$	$4x+5=-1$
$2x^2-5x-12=0$	$4x=-6$
$(2x+3)(x-4)=0$	$x=-\frac{3}{2}$
$a = <-\frac{3}{2}$	

\therefore A SOLUTION IS $x = -\frac{3}{2}$

Question 58 (*****)

Determine the value of x and the value of y in the following equation

$$15^{3x-2} \times 6^{1-2y} = 6.25, \quad (3x-2) \in \mathbb{N}.$$

$$\boxed{}, \quad x = \frac{4}{3}, \quad y = \frac{3}{2}$$

$15^{3x-2} \times 6^{1-2y} = 6.25, \quad 3x-2 \in \mathbb{N}$

- BREAK ALL BASES INTO PRIME FACTORS
 $\Rightarrow (5 \times 3)^{3x-2} \times (2 \times 3)^{1-2y} = \frac{25}{4}$
 $\Rightarrow 5^{3x-2} \times 3^{3x-2} \times 2^{1-2y} \times 3^{1-2y} = 5^2 \times 2^2$
- LOOK AT THE POWERS OF 5
 $\Rightarrow 5^{3x-2} = 5^2$
 $\Rightarrow 3x-2 = 2$
 $\Rightarrow 3x = 4$
 $\Rightarrow x = \frac{4}{3}$
- LOOK AT THE POWERS OF 2
 $\Rightarrow 2^{1-2y} = 2^2$
 $\Rightarrow 1-2y = 2$
 $\Rightarrow -2y = 1$
 $\Rightarrow y = -\frac{1}{2}$
- CHECK FOR CONSISTENCY OF THE SOLUTION BY CHECKING THE POWERS OF 3
 $\Rightarrow 3^{3x-2} \times 3^{1-2y} = 3^0$
 $\Rightarrow (3x-2) + (1-2y) = 0$
 $\Rightarrow (3 \times \frac{4}{3}) - 2 + 1 - (2 \times \frac{3}{2}) = 4 - 2 + 1 - 3 = 0$
 \therefore CONSISTENT $x = \frac{4}{3}, y = \frac{3}{2}$

Question 59 (*****)

$$2^{m+1} + 2^m = 3^{n+2} - 3^n.$$

Given that m and n are positive integers, find the value of m and the value of n .

$$\boxed{2}, \quad m = 3, \quad n = 1$$

$2^{m+1} + 2^m = 3^{n+2} - 3^n$

$\Rightarrow 2 \times 2^m + 2^m = 9 \times 3^n - 3^n$
 $\Rightarrow 2(2^m) + (2^m) = 9(3^n) - 3^n$
 $\Rightarrow 3(2^m) = 8(3^n)$
 $\Rightarrow 3 \times 2^m = 8 \times 3^n$
 $\Rightarrow 3 \times 2^m = 2^3 \times 3^n$

$\therefore \frac{3}{2^3} = \frac{3^n}{2^m} \Rightarrow m=1$
 $\frac{2^m}{2^3} = \frac{3^n}{3} \Rightarrow m=3$

Question 60 (*****)

Find the term independent of x in the expansion of

$$\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right)^{10}.$$

SP10, 210

$$\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right)^{10}$$

$$A^2 + B^2 \equiv (A+B)(A-B + B^2)$$

$$x+1 = (x^{\frac{2}{3}})^2 + 1^2 = (x^{\frac{2}{3}} + 1)(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1)$$

$$= \left[\frac{(x^{\frac{2}{3}} + 1)(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1)}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{(x-1)(x^{\frac{1}{2}} + 1)}{x^{\frac{1}{2}}(x^{\frac{1}{2}} - 1)} \right]^{10}$$

$$= \left[x^{\frac{2}{3}} + 1 - \frac{x^{\frac{1}{2}} + 1}{x^{\frac{1}{2}}} \right]^{10} = \left[x^{\frac{2}{3}} + 1 - (1 + x^{-\frac{1}{2}}) \right]^{10}$$

$$= \left[x^{\frac{2}{3}} - x^{-\frac{1}{2}} \right]^{10}$$

By inspection the term independent of x will be:

$$\binom{10}{4} (x^{\frac{2}{3}})^4 (x^{-\frac{1}{2}})^{-4} = \binom{10}{4} x^{\frac{2}{3} \cdot 4 - \frac{1}{2} \cdot (-4)}$$

$$= \binom{10}{4}$$

$$= \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4}$$

$$= \frac{10 \times 3}{1}$$

$$= 30$$

Question 61 (*****)

Simplify the following expression.

$$\frac{x^6 - yz^4}{x^4 + xz^2\sqrt{y}}$$

Give the answer in the form $x^2 - x^r y^s z^t$, where r , s and t are constants.

$$\boxed{}, \boxed{x^2 - x^{-1} y^{\frac{1}{2}} z^2}$$

MANIPULATE AS FRACTIONS

$$\frac{x^6 - yz^4}{x^4 + xz^2\sqrt{y}} = \frac{x^6 - yz^4}{x(x^3 + z^2\sqrt{y})}$$

DIFFERENCE OF SQUARES IN THE NUMERATOR

$$\begin{aligned} \therefore &= \frac{(x^3)^2 - (\sqrt{y}z^2)^2}{x(x^3 + z^2\sqrt{y})} = \frac{(x^3 - z^2\sqrt{y})(x^3 + z^2\sqrt{y})}{x(x^3 + z^2\sqrt{y})} \\ &= \frac{x^3 - z^2\sqrt{y}}{x} \\ &= x^2 - \frac{z^2\sqrt{y}}{x} \\ &= x^2 - x^{-1}y^{\frac{1}{2}}z^2 \end{aligned}$$