

Created by T. Madas

# IDENTITIES

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**Question 1 (\*\*)**

Use the identity

$$(a \pm b)^3 \equiv a^3 \pm 3a^2b + 3ab^2 \pm b^3,$$

to simplify  $(2 - 3x)^3$ .

$$(2 - 3x)^3 = 8 - 36x + 54x^2 - 27x^3$$

**Question 2 (\*\*)**

$$f(x) = (2x - 1)^3 - (x + 2)^2.$$

Use algebraic identities to simplify  $f(x)$ .

$$f(x) = 8x^3 - 13x^2 + 2x - 5$$

**Question 3 (\*\*)**

It is given that for some constants  $A$  and  $B$

$$6\sin x \equiv A(\cos x + \sin x) + B(\cos x - \sin x).$$

Find the value of  $A$  and the value of  $B$ .

$$\boxed{A=3}, \quad \boxed{B=-3},$$

Handwritten solution for Question 3:

$$\begin{aligned} 6\sin x &\equiv A(\cos x + \sin x) + B(\cos x - \sin x) \\ 6\sin x &\equiv A\cos x + A\sin x + B\cos x - B\sin x \\ 6\sin x &\equiv (A+B)\cos x + (A-B)\sin x \\ \therefore A+B &= 0 \quad \text{and} \quad 2A = 6 \\ A-B &= 6 \quad \text{Add} \quad 2A = 6 \\ A &= 3 \\ B &= -3 \end{aligned}$$

**Question 4 (\*\*)**

Use the identity

$$(a \pm b)^3 \equiv a^3 \pm 3a^2b + 3ab^2 \pm b^3,$$

to simplify  $(2 - \sqrt{2})^3$ .

$$\boxed{(2 - \sqrt{2})^3 = 20 - 14\sqrt{2}}$$

Question 5 (\*\*)

$$A - (Bx + C)^2 \equiv 140 + 12x - 9x^2, \quad x \in \mathbb{R}.$$

Find the values of each of the constants  $A$ ,  $B$  and  $C$  in the above identity.

$$A = 144, \quad B = \pm 3, \quad C = \pm 2$$

Handwritten solution for Question 5:

$$A - (Bx + C)^2 \equiv 140 + 12x - 9x^2$$

$$A - B^2x^2 - 2BCx - C^2 \equiv 140 + 12x - 9x^2$$

Equating coefficients:

- $B^2 = 9$
- $2BC = 12$
- $A - C^2 = 140$
- $B = \pm 3$
- $6C = 12$
- $A - C^2 = 140$
- $C = \pm 2$
- $A = 144$

Final result:

$$\therefore A = 144, \quad B = \pm 3, \quad C = \pm 2$$

Question 6 (\*\*+)

$$f(x) = \left(x^2 + \frac{2}{x}\right)^3, \quad x \in \mathbb{R}.$$

Use algebraic identities to simplify  $f(x)$ .

$$f(x) = x^6 + 6x^3 + 12 + \frac{8}{x^3}$$

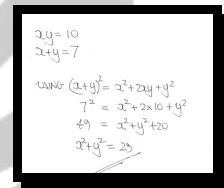
**Question 7 (\*\*+)**

It is given that

$$x + y = 7 \quad \text{and} \quad xy = 10.$$

Use a suitable algebraic identity to find the value of  $x^2 + y^2$ .

$$x^2 + y^2 = 29$$



Handwritten solution for Question 7:

$$\begin{aligned} xy &= 10 \\ x + y &= 7 \\ \text{using } (x+y)^2 &= x^2 + 2xy + y^2 \\ 7^2 &= x^2 + 2 \times 10 + y^2 \\ 49 &= x^2 + y^2 + 20 \\ x^2 + y^2 &= 29 \end{aligned}$$

**Question 8 (\*\*\*)**

Determine the value of each of the constants  $P$ ,  $Q$  and  $R$  in the identity

$$P(x+1)^2 + Qx(x+1) + Rx \equiv 1.$$

$$P=1, \quad Q=-1, \quad R=-1$$

**Question 9 (\*\*\*)**

Factorize fully

$$2x - 2x^4.$$

$$2x(1-x)(1+x+x^2)$$

**Question 10 (\*\*\*)**

Determine the value of each of the constants  $p$  and  $q$  in the identity

$$(2x+p)(6x^2-7x+2) \equiv (4x^2-1)(3x+q).$$

$$\boxed{p=1}, \boxed{q=-2}$$

**Question 11 (\*\*\*)**

$$f(x) = 9x^2 - 1, \quad x \in \mathbb{R}.$$

- a) Express  $f(x)$  as a product of two linear factors.
- b) Hence express 899 as a product of two prime factors.

$$\boxed{f(x) = (3x+1)(3x-1)}, \boxed{899 = 29 \times 31}$$

**Question 12 (\*\*\*)**

Determine the value of each of the constants  $a$ ,  $b$  and  $c$  in the identity

$$4(x+a)^2 + b(x+3) + c \equiv 4x^2 - 21x.$$

$$\boxed{a = -2}, \boxed{b = -5}, \boxed{c = -1}$$

**Question 13 (\*\*\*)**

Determine the value of each of the constants  $P$ ,  $Q$  and  $R$  in the identity

$$P(x^2 + 1) + (x+1)(Qx+R) + x \equiv 1.$$

$$\boxed{P = 1}, \boxed{Q = -1}, \boxed{R = 0}$$

**Question 14 (\*\*\*)**

Determine the value of each of the constants  $A$ ,  $B$  and  $C$  in the identity

$$(Ax+B)(x-1)+C(x^2+1)+x+3\equiv 0.$$

$$\boxed{A=2}, \boxed{B=1}, \boxed{C=-2}$$

**Question 15 (\*\*\*)**

$$f(x) = x^4 - 27x, \quad x \in \mathbb{R}.$$

Express  $f(x)$  as a product of three factors.

$$\boxed{f(x) = x(x-3)(x^2+x+9)}$$



**Question 16 (\*\*\*)**

Determine the value of each of the constants  $A$ ,  $B$  and  $C$  in the identity

$$(A - 25)x + 8(Bx - 1) + A \equiv C + 3(2x - 1)(x + 1).$$

$$\boxed{A = 1}, \boxed{B = 3}, \boxed{C = -1}$$

**Question 17 (\*\*\*)**

Determine the value of each of the constants  $A$ ,  $B$  and  $C$  in the identity

$$x^3 \equiv A + B(x - 1)^2 + (x + 2)(x + C)^2.$$

$$\boxed{A = 1}, \boxed{B = 3}, \boxed{C = -1}$$

**Question 18 (\*\*\*)**

Determine the value of each of the constants  $A$ ,  $B$ ,  $C$  and  $D$  in the identity

$$(x+3)(Ax^2+Bx+C) \equiv 2x^3+2x^2+Dx+9.$$

$$\boxed{A=2}, \boxed{B=-4}, \boxed{C=3}, \boxed{D=-9}$$

**Question 19 (\*\*\*)**

Determine the value of each of the constants  $a$ ,  $b$ ,  $c$  and  $d$  in the identity

$$a(x-2)^3+b(x+4)^2 \equiv 3x^3+cx^2+dx+8.$$

$$\boxed{a=3}, \boxed{b=2}, \boxed{c=-16}, \boxed{d=52}$$

**Question 20** (\*\*\*)

Determine the value of each of the constants  $A$ ,  $B$ ,  $C$  and  $D$  in the identity

$$(x^2 + A)(x + B) + (x + C)(x + D) \equiv x^3 - 3x^2 + 4x.$$

$$\boxed{A=3}, \boxed{B=-2}, \boxed{C=2}, \boxed{D=-3}$$

**Question 21** (\*\*\*)

Determine the value of each of the constants  $p$  and  $q$  in the identity

$$(x^2 + p)^2 \equiv x(x^2 + q)\left(x - \frac{6}{x}\right).$$

$$\boxed{p=q=-6}$$

**Question 22** (\*\*\*)

Determine the value of each of the constants  $a$ ,  $b$ ,  $c$  and  $d$  in the identity

$$a(x+b)^3 + c \equiv 4x^3 - 24x^2 + 48x + d.$$

$$\boxed{a=4}, \boxed{b=-2}, \boxed{c=29}, \boxed{d=-3}$$

**Question 23** (\*\*\*)

Determine the value of each of the constants  $A$ ,  $B$ ,  $C$  and  $D$  in the identity

$$(x+A)(x+2)(x^2+B) \equiv Cx^4 + D.$$

$$\boxed{A=-2}, \boxed{B=4}, \boxed{C=1}, \boxed{D=-16}$$

## Question 24 (\*\*\*\*)

$$f(x) = x^3 + 3x^2 - 4, \quad x \in \mathbb{R}.$$

Use algebraic identities to express  $f(x)$  as a product of three linear factors.

$$f(x) = (x-1)(x+2)^2$$

## Question 25 (\*\*\*\*+)

Solve the following equation

$$\frac{x^3 - 1}{x^2 - 1} - x = \frac{2}{5}, \quad x \neq \pm 1.$$

$$x = \frac{3}{2}$$

Handwritten solution for Question 25:

$$\begin{aligned} \frac{x^3 - 1}{x^2 - 1} - x &= \frac{2}{5} \\ \Rightarrow \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} - x &= \frac{2}{5} \\ \Rightarrow \frac{x^2+x+1}{x+1} - x &= \frac{2}{5} \\ \Rightarrow \frac{x^2+x+1 - x(x+1)}{x+1} &= \frac{2}{5} \\ \Rightarrow \frac{x^2+x+1 - x^2 - x}{x+1} &= \frac{2}{5} \\ \Rightarrow \frac{1}{x+1} &= \frac{2}{5} \\ \Rightarrow 5 &= 2(x+1) \\ \Rightarrow 5 &= 2x + 2 \\ \Rightarrow 3 &= 2x \\ \Rightarrow x &= \frac{3}{2} \end{aligned}$$

Identities used:

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$x^3 - 1 = (x-1)(x^2 + x + 1)$$

Question 26 (\*\*\*\*+)

$$f(x) = x^3 - 3x^2 - 3x + 1, \quad x \in \mathbb{R}.$$

Express  $f(x)$  as a product of **three linear** factors

$$f(x) = (x+1)(x-2-\sqrt{3})(x-2+\sqrt{3})$$