BINOMIAL EXPANSIONS PRACTICE

Question 1

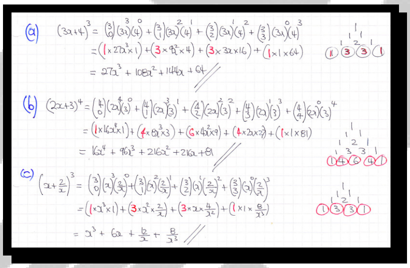
Find, without using a calculator, the binomial expansion of

a) $(3x+4)^3$

b) $(2x+3)^4$

c) $\left(x+\dfrac{2}{x}\right)^3$

$\boxed{27x^3+108x^2+144x+64}$, $\boxed{16x^4+96x^3+216x^2+216x+81}$, $\boxed{x^3+6x+\dfrac{12}{x}+\dfrac{8}{x^3}}$



Question 2

Find the binomial expansion of

a) $(2+4x)^5$

b) $(3-4x)^4$

c) $\left(2x+\dfrac{3}{x}\right)^6$

$\boxed{1024x^5+2560x^4+2560x^3+1280x^2+320x+32}$, $\boxed{256x^4-768x^3+864x^2-432x+81}$,

$\boxed{64x^6+576x^4+2160x^2+4320+\dfrac{4860}{x^2}+\dfrac{2916}{x^4}+\dfrac{729}{x^6}}$

(a) $(2+4x)^5 = \binom{5}{0}(2)^5(4x)^0+\binom{5}{1}(2)^4(4x)^1+\binom{5}{2}(2)^3(4x)^2+\binom{5}{3}(2)^2(4x)^3$
$+\binom{5}{4}(2)^1(4x)^4+\binom{5}{5}(2)^0(4x)^5$
$=(1\times 32\times 1)+(5\times 16\times 4x)+(10\times 8\times 16x^2)+(10\times 4\times 64x^3)$
$+(5\times 2\times 256x^4)+(1\times 1\times 1024x^5)$
$=32+320x+1280x^2+2560x^3+2560x^4+1024x^5$

(b) $(3-4x)^4=\binom{4}{0}(3)^4(-4x)^0+\binom{4}{1}(3)^3(-4x)^1+\binom{4}{2}(3)^2(-4x)^2+\binom{4}{3}(3)^1(-4x)^3+\binom{4}{4}(3)^0(-4x)^4$
$=(1\times 81\times 1)+[4\times 27\times(-4x)]+(6\times 9\times 16x^2)+[4\times 3\times(-64x^3)]+(1\times 1\times 256x^4)$
$=81-432x+864x^2-768x^3+256x^4$

(c) $\left(2x+\frac{3}{x}\right)^6=\binom{6}{0}(2x)^6\left(\frac{3}{x}\right)^0+\binom{6}{1}(2x)^5\left(\frac{3}{x}\right)^1+\binom{6}{2}(2x)^4\left(\frac{3}{x}\right)^2+\binom{6}{3}(2x)^3\left(\frac{3}{x}\right)^3$
$+\binom{6}{4}(2x)^2\left(\frac{3}{x}\right)^4+\binom{6}{5}(2x)^1\left(\frac{3}{x}\right)^5+\binom{6}{6}(2x)^0\left(\frac{3}{x}\right)^6$
$=(1\times 64x^6\times 1)+\left(6\times 32x^5\times\frac{3}{x}\right)+\left(15\times 16x^4\times\frac{9}{x^2}\right)+\left(20\times 8x^3\times\frac{27}{x^3}\right)$
$+\left(15\times 4x^2\times\frac{81}{x^4}\right)+\left(6\times 2x\times\frac{243}{x^5}\right)+\left(1\times 1\times\frac{729}{x^6}\right)$
$=64x^6+576x^4+2160x^2+4320+\frac{4860}{x^2}+\frac{2916}{x^4}+\frac{729}{x^6}$

Question 3

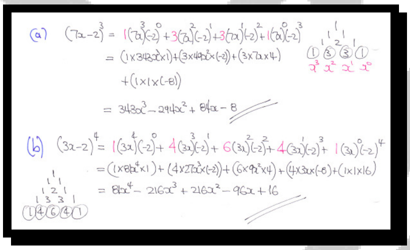
Find, without using a calculator, the binomial expansion of

a) $(7x-2)^3$

b) $(5x+2)^5$

c) $(3x-2)^4$

$\boxed{343x^3 - 294x^2 + 84x - 8}$, $\boxed{3125x^5 + 6250x^4 + 5000x^3 + 2000x^2 + 400x + 32}$, $\boxed{81x^4 - 216x^3 + 216x^2 - 96x + 16}$



(a) $(7x-2)^3 = 1(7x)^3(-2)^0 + 3(7x)^2(-2)^1 + 3(7x)^1(-2)^2 + 1(7x)^0(-2)^3$

$= (1 \times 343x^3 \times 1) + (3 \times 49x^2 \times (-2)) + (3 \times 7x \times 4) + (1 \times 1 \times (-8))$

$= 343x^3 - 294x^2 + 84x - 8$

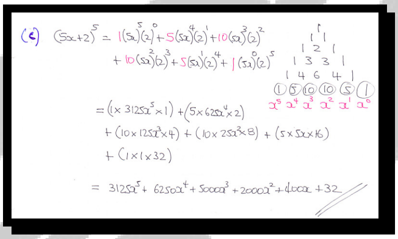
1
1 1
1 2 1
1 3 3 1
x^3 x^2 x^1 x^0

(b) $(3x-2)^4 = 1(3x)^4(-2)^0 + 4(3x)^3(-2)^1 + 6(3x)^2(-2)^2 + 4(3x)^1(-2)^3 + 1(3x)^0(-2)^4$

$= (1 \times 81x^4 \times 1) + (4 \times 27x^3 \times (-2)) + (6 \times 9x^2 \times 4) + (4 \times 3x \times (-8)) + (1 \times 1 \times 16)$

$= 81x^4 - 216x^3 + 216x^2 - 96x + 16$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1



(c) $(5x+2)^5 = 1(5x)^5(2)^0 + 5(5x)^4(2)^1 + 10(5x)^3(2)^2 + 10(5x)^2(2)^3 + 5(5x)^1(2)^4 + 1(5x)^0(2)^5$

$= (1 \times 3125x^5 \times 1) + (5 \times 625x^4 \times 2) + (10 \times 125x^3 \times 4) + (10 \times 25x^2 \times 8) + (5 \times 5x \times 16) + (1 \times 1 \times 32)$

$= 3125x^5 + 6250x^4 + 5000x^3 + 2000x^2 + 400x + 32$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
x^5 x^4 x^3 x^2 x^1 x^0

Question 4

Find the binomial expansion of

a) $(2+5x)^4$

b) $(2x-2)^5$

c) $(4+9x)^3$

$\boxed{625x^4+1000x^3+600x^2+160x+16}$, $\boxed{32x^5-160x^4+320x^3-320x^2+160x-32}$, $\boxed{729x^3+972x^2+432x+64}$

(a) $(2+5x)^4 = \binom{4}{0}(2)^4(5x)^0 + \binom{4}{1}(2)^3(5x)^1 + \binom{4}{2}(2)^2(5x)^2 + \binom{4}{3}(2)^1(5x)^3 + \binom{4}{4}(2)^0(5x)^4$

$= (1\times16\times1) + (4\times8\times5x) + (6\times4\times25x^2) + (4\times2\times125x^3) + (1\times1\times625x^4)$

$= 16 + 160x + 600x^2 + 1000x^3 + 625x^4$

(b) $(2x-2)^5 = \binom{5}{0}(2x)^5(-2)^0 + \binom{5}{1}(2x)^4(-2)^1 + \binom{5}{2}(2x)^3(-2)^2$

$+\binom{5}{3}(2x)^2(-2)^3 + \binom{5}{4}(2x)^1(-2)^4 + \binom{5}{5}(2x)^0(-2)^5$

$= (1\times32x^5\times1) + (5\times16x^4\times(-2)) + (10\times8x^3\times4)$

$+ (10\times4x^2\times(-8)) + (5\times2x\times16) + (1\times1\times(-32))$

$= 32x^5 - 160x^4 + 320x^3 - 320x^2 + 160x - 32$

(c) $(4+9x)^3 = \binom{3}{0}(4)^3(9x)^0 + \binom{3}{1}(4)^2(9x)^1 + \binom{3}{2}(4)^1(9x)^2 + \binom{3}{3}(4)^0(9x)^3$

$= (1\times64\times1) + (3\times16\times9x) + (3\times4\times81x^2) + (1\times1\times729x^3)$

$= 64 + 432x + 972x^2 + 729x^3$

Question 5

Find the first **five** terms, in ascending order of x, of the binomial expansion of

a) $(1+2x)^{12}$

b) $(1-3x)^{10}$

c) $\left(1+\frac{1}{2}x\right)^{8}$

$\boxed{1+24x+264x^2+1760x^3+7920x^4+\dots}$, $\boxed{1-30x+405x^2-3240x^3+17010x^4-\dots}$, $\boxed{1+4x+7x^2+7x^3+\frac{35}{8}x^4+\dots}$

(a) $(1+2x)^{12} = 1 + \frac{12}{1}(2x)^1 + \frac{12\times11}{1\times2}(2x)^2 + \frac{12\times11\times10}{1\times2\times3}(2x)^3 + \frac{12\times11\times10\times9}{1\times2\times3\times4}(2x)^4 + \dots$

$= 1 + 24x + 264x^2 + 1760x^3 + 7920x^4 + \dots$

(b) $(1-3x)^{10} = 1 + \frac{10}{1}(-3x)^1 + \frac{10\times9}{1\times2}(-3x)^2 + \frac{10\times9\times8}{1\times2\times3}(-3x)^3 + \frac{10\times9\times8\times7}{1\times2\times3\times4}(-3x)^4 + \dots$

$= 1 - 30x + 405x^2 - 3240x^3 + 17010x^4 + \dots$

(c) $\left(1+\frac{1}{2}x\right)^8 = 1 + \frac{8}{1}\left(\frac{1}{2}x\right)^1 + \frac{8\times7}{1\times2}\left(\frac{1}{2}x\right)^2 + \frac{8\times7\times6}{1\times2\times3}\left(\frac{1}{2}x\right)^3 + \frac{8\times7\times6\times5}{1\times2\times3\times4}\left(\frac{1}{2}x\right)^4 + \dots$

$= 1 + 4x + 7x^2 + 7x^3 + \frac{35}{8}x^4 + \dots$

Question 6

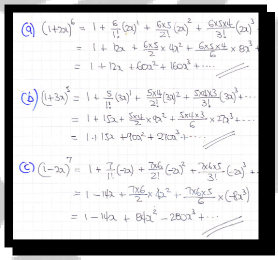
Find, without using a calculator, the first four terms in ascending order of x in the binomial expansion of

a) $(1+2x)^6$

b) $(1+3x)^5$

c) $(1-2x)^7$

$\boxed{1+12x+60x^2+160x^3+\ldots}$, $\boxed{1+15x+90x^2+270x^3+\ldots}$, $\boxed{1-14x+84x^2-280x^3+\ldots}$



(a) $(1+2x)^6 = 1 + \frac{6}{1!}(2x)^1 + \frac{6\times5}{2!}(2x)^2 + \frac{6\times5\times4}{3!}(2x)^3 + \ldots$

$= 1 + 12x + \frac{6\times5}{2}\times4x^2 + \frac{6\times5\times4}{6}\times8x^3 + \ldots$

$= 1 + 12x + 60x^2 + 160x^3 + \ldots$

(b) $(1+3x)^5 = 1 + \frac{5}{1!}(3x)^1 + \frac{5\times4}{2!}(3x)^2 + \frac{5\times4\times3}{3!}(3x)^3 + \ldots$

$= 1 + 15x + \frac{5\times4}{2}\times9x^2 + \frac{5\times4\times3}{6}\times27x^3 + \ldots$

$= 1 + 15x + 90x^2 + 270x^3 + \ldots$

(c) $(1-2x)^7 = 1 + \frac{7}{1!}(-2x) + \frac{7\times6}{2!}(-2x)^2 + \frac{7\times6\times5}{3!}(-2x)^3 + \ldots$

$= 1 - 14x + \frac{7\times6}{2}\times4x^2 + \frac{7\times6\times5}{6}\times(-8x^3)$

$= 1 - 14x + 84x^2 - 280x^3 + \ldots$

Question 8

Find the first **four** terms, in ascending order of x, of the binomial expansion of

a) $(1+2x)^{11}$

b) $(1-3x)^{7}$

c) $(1-4x)^{8}$

$\boxed{1+22x+220x^2+1320x^3+\ldots}$, $\boxed{1-21x+189x^2-945x^3+\ldots}$, $\boxed{1-32x+448x^2-3584x^3+\ldots}$

(a) $(1+2x)^{11} = 1 + \frac{11}{1}(2x)^1 + \frac{11\times10}{1\times2}(2x)^2 + \frac{11\times10\times9}{1\times2\times3}(2x)^3 + \ldots$

$= 1 + 22x + (55\times4x^2) + (165\times8x^3) + \ldots$

$= 1 + 22x + 220x^2 + 1320x^3 + \ldots$

(b) $(1-3x)^7 = 1 + \frac{7}{1}(-3x)^1 + \frac{7\times6}{1\times2}(-3x)^2 + \frac{7\times6\times5}{1\times2\times3}(-3x)^3 + \ldots$

$= 1 - 21x + (21\times9x^2) + (35\times(-27x^3)) + \ldots$

$= 1 - 21x + 189x^2 - 945x^3 + \ldots$

(c) $(1-4x)^8 = 1 + \frac{8}{1}(-4x) + \frac{8\times7}{1\times2}(-4x)^2 + \frac{8\times7\times6}{1\times2\times3}(-4x)^3 + \ldots$

$= 1 - 32x + (28\times16x^2) + (56\times(-64x^3)) + \ldots$

$= 1 - 32x + 448x^2 - 3584x^3 + \ldots$

Question 9

Find the value of the constant n in each of the following binomial expansions

a) $(1+3x)^n$, if the coefficient of x^2 is 54.

b) $(1+x)^n$, if the coefficient of x^2 is 55.

$\boxed{n=4,\ n\neq -3}$, $\boxed{n=11,\ n\neq -10}$

(a) $(1+3x)^n = 1 + \frac{n}{1}(3x)^1 + \frac{n(n-1)}{1\times 2}(3x)^2 + \dots$

$\dots + \frac{n(n-1)}{2}(9x^2) + \dots$

$\dots + \underbrace{\boxed{\frac{9}{2}n(n-1)}}_{54}x^2 + \dots$

$\frac{9}{2}n(n-1) = 54$

$n(n-1) = 12$

$n^2 - n - 12 = 0$

$(n-4)(n+3) = 0$

$n = 4,\ \cancel{-3}$

(b) $(1+x)^n = 1 + \frac{n}{1}(x)^1 + \frac{n(n-1)}{1\times 2}(x)^2 + \dots$

$\dots + \underbrace{\boxed{\frac{n(n-1)}{2}}}_{55}x^2 + \dots$

$\frac{1}{2}n(n-1) = 55$

$n(n-1) = 110$

$n^2 - n - 110 = 0$

$(n+10)(n-11) = 0$

$n = \cancel{-10},\ 11$