ARITHMETIC SERIES
Worded Questions

Question 1 (**) non calculator

A ball bearing is rolling down an inclined groove.

It rolls down by 1 cm during the first second of its motion, and in each subsequent second it rolls down by an extra 3 cm than in the previous second.

Given it takes 12 seconds for the ball bearing to roll down the groove, find in metres the length of the groove.

$\boxed{2.1 \text{ m}}$

1cm, 4cm, 7cm, 10cm, ...

THIS AN A.P WITH $a=1$, $d=3$, $n=12$

$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$S_{12} = \frac{12}{2}\left[2\times 1 + 11\times 3\right]$

$S_{12} = 6(2+33)$

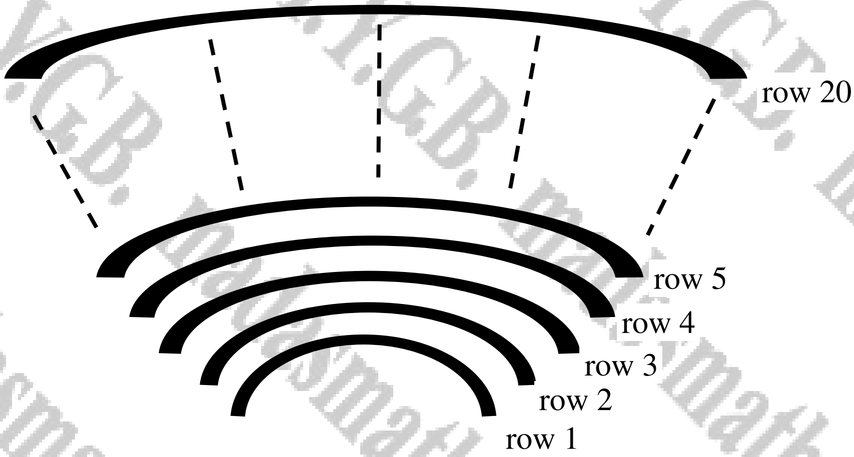
$S_{12} = 6\times 35$

$S_{12} = 210$

$\therefore$ 210 cm OR 2.1 m

Question 2 (**+)

Seats in a theatre are arranged in rows. The number of seats in each row form the terms of an arithmetic series.



The sixth row has 23 seats and the fifteenth row has 50 seats.

a) Find the number of seats in the first row.

The theatre has 20 rows of seats in total.

b) Find the number of seats in this theatre.

$\boxed{8}$, $\boxed{730}$

a) USING STANDARD SEQUENCE/SERIES FORMULA $u_n = a + (n-1)d$

- $u_6 = 23$
  $a + 5d = 23$
  $a = 23 - 5d$
- $u_{15} = 50$
  $a + 14d = 50$
  $a = 50 - 14d$

$23 - 5d = 50 - 14d$
$9d = 27$
$d = 3$ & $a = 8$

∴ THE FIRST ROW HAS 8 SEATS

b) SUMMING UP THE FIRST 20 TERMS OF AN A.P WITH $a=8, d=3$

$\Rightarrow S_n = \frac{n}{2}\left[2a + (n-1)d\right]$
$\Rightarrow S_{20} = \frac{20}{2}\left[2 \times 8 + 19 \times 3\right]$
$\Rightarrow S_{20} = 10\left[16 + 57\right]$
$\Rightarrow S_{20} = 730$

∴ A TOTAL OF 730 SEATS

Question 3 (***)

Arnold is planning to save for the next 48 months in order to raise a deposit to buy a flat. He plans to save £300 this month and each successive month thereafter, to save an extra £5 compared to the previous month.

a) Find the amount he will save on the twelfth month.

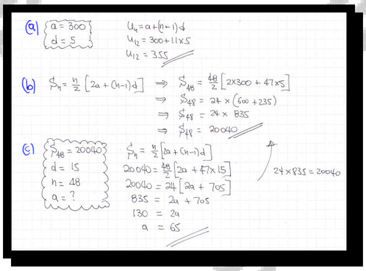
b) Find the total amount he will save at the end of the 48 months.

Franco is also planning to save for the next 48 months in order to buy a car.

He plans to save £a this month and each successive month thereafter, to save an extra £15 compared to the previous month.

c) Find the value of a, if Franco saves the same amount of money as Arnold does in the next 48 months.

$\boxed{£355}$, $\boxed{£20040}$, $\boxed{a = 65}$



(a) $a = 300$, $d = 5$

$u_n = a + (n-1)d$

$u_{12} = 300 + 11 \times 5$

$u_{12} = 355$

(b) $S_n = \frac{n}{2}\left[2a + (n-1)d\right] \Rightarrow S_{48} = \frac{48}{2}\left[2 \times 300 + 47 \times 5\right]$

$\Rightarrow S_{48} = 24 \times (600 + 235)$

$\Rightarrow S_{48} = 24 \times 835$

$\Rightarrow S_{48} = 20040$

(c) $S_{48} = 20040$, $d = 15$, $n = 48$, $a = ?$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$20040 = \frac{48}{2}\left[2a + 47 \times 15\right]$

$20040 = 24\left[2a + 705\right]$

$835 = 2a + 705$

$130 = 2a$

$a = 65$

$24 \times 835 = 20040$

Question 4 (***)

Andrew is planning to pay money into a pension scheme for the next 40 years.

He plans to pay into the pension scheme £800 in the first year and each successive year thereafter, an extra £100 compared to the previous year.

a) Calculate the amount Andrew will pay into the scheme on the tenth year.

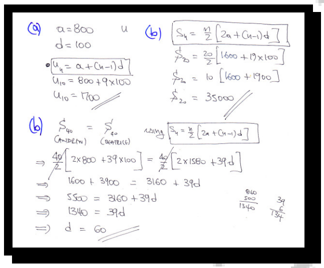
b) Find the total amount Andrew will have paid into the scheme after 20 years.

Beatrice is also planning to pay money into a pension scheme for the next 40 years.

She plans to pay £1580 in the first year and each successive year thereafter, to pay an extra £d compared to the previous year.

c) Find the value of d, if both Andrew and Beatrice paid into their pension schemes the same amount of money over the next 40 years.

$\boxed{£1700}$, $\boxed{£35000}$, $\boxed{d = 60}$



(a) $a = 800$
$d = 100$

$\boxed{u_n = a + (n-1)d}$
$u_{10} = 800 + 9 \times 100$
$u_{10} = 1700$

(b) $\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$
$S_{20} = \frac{20}{2}\left[1600 + 19 \times 100\right]$
$S_{20} = 10\left[1600 + 1900\right]$
$S_{20} = 35000$

(c) $S_{40} = S_{40}$ using $\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$
(ANDREW) (BEATRICE)

$\Rightarrow \frac{40}{2}\left[2 \times 800 + 39 \times 100\right] = \frac{40}{2}\left[2 \times 1580 + 39d\right]$
$\Rightarrow 1600 + 3900 = 3160 + 39d$
$\Rightarrow 5500 = 3160 + 39d$
$\Rightarrow 1340 = 39d$
$\Rightarrow d = 60$

Question 5 (***)

A novelist is planning to write a new book.

He plans to write 15 pages in the first week, 17 pages in the second week, 19 pages in the third week, and so on, so that he writes an extra two pages each week compared with the previous week.

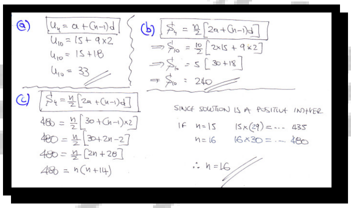
a) Find the number of pages he plans to write in the tenth week.

b) Determine how many pages he plans to write in the first ten weeks.

The novelist sticks to his plan and produces a book with 480 pages, after n weeks.

c) Use algebra to determine the value of n.

$\boxed{33}$, $\boxed{240}$, $\boxed{n=16}$


(a) $\boxed{u_n = a + (n-1)d}$
$u_{10} = 15 + 9 \times 2$
$u_{10} = 15 + 18$
$u_{10} = 33$

(b) $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$
$\Rightarrow S_{10} = \frac{10}{2}[2 \times 15 + 9 \times 2]$
$\Rightarrow S_{10} = 5[30 + 18]$
$\Rightarrow S_{10} = 240$

(c) $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$
$480 = \frac{n}{2}[30 + (n-1) \times 2]$
$480 = \frac{n}{2}[30 + 2n - 2]$
$480 = \frac{n}{2}[2n + 28]$
$480 = n(n + 14)$

SINCE SOLUTION IS A POSITIVE INTEGER
IF $n = 15$ $15 \times (29) = \dots 435$
$n = 16$ $16 \times 30 = \dots 480$

$\therefore n = 16$

Question 6 **(***+)**

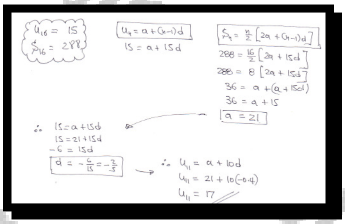
An athlete is training for a long distance race.

He is preparing by running on 16 consecutive days so that his daily running distances form an arithmetic sequence.

The athlete ran for 15 km on the 16^{th} day of his training and the total distance run over the 16 day training period was 288 km.

Find the distance the athlete ran on the 11^{th} day of his training.

$\boxed{17 \text{ km}}$



Question 7 (*+) non calculator**

On his 1^{st} birthday, Anthony was given £50 as a present by his godmother Cleo.

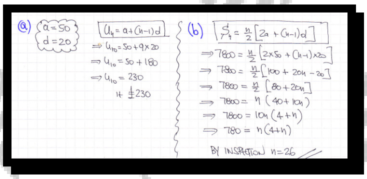
For every birthday ever since, Cleo gave Anthony £20 more than on his previous birthday. This money was saved by Anthony's mother until Anthony was n years old.

a) Find the amount of money Anthony received as a birthday present on his tenth birthday.

After Anthony's n^{th} birthday his mother gave him Cleo's presents, which was £7800 in total.

b) Determine the value of n.

$\boxed{£230}$, $\boxed{n = 26}$



(a) $a = 50$, $d = 20$

$\boxed{u_n = a + (n-1)d}$

$\Rightarrow u_{10} = 50 + 9 \times 20$

$\Rightarrow u_{10} = 50 + 180$

$\Rightarrow u_{10} = 230$

i.e. £230

(b) $\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$

$\Rightarrow 7800 = \frac{n}{2}\left[2 \times 50 + (n-1) \times 20\right]$

$\Rightarrow 7800 = \frac{n}{2}\left[100 + 20n - 20\right]$

$\Rightarrow 7800 = \frac{n}{2}\left[80 + 20n\right]$

$\Rightarrow 7800 = n(40 + 10n)$

$\Rightarrow 7800 = 10n(4 + n)$

$\Rightarrow 780 = n(4 + n)$

BY INSPECTION $n = 26$

Question 8 (***+)

A new gym opened and during its first trading month 26 people joined its membership.

A business forecast for the gym membership is drafted for the next twelve months.

It assumes that every month an extra x number of members will join, so that next month $(26+x)$ members will be added, the following month $(26+2x)$ members will be added, and so on.

Taking $x=15$, find …

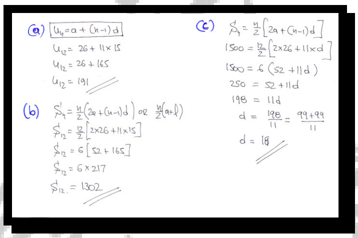
a) … the number of members that will join in the twelfth month.

b) … the total number of members that will join during the first twelve months.

The business plan recognises that in order for the business to succeed in the long term, it needs a total membership of at least 1500 during its first twelve months.

c) Using the same model, find the required value of x in order to achieve a twelve month membership target of 1500.

$\boxed{191}$, $\boxed{1302}$, $\boxed{x=18}$



(a) $\boxed{u_n = a + (n-1)d}$

$u_{12} = 26 + 11 \times 15$

$u_{12} = 26 + 165$

$u_{12} = 191$

(b) $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ or $\frac{n}{2}(a+l)$

$S_{12} = \frac{12}{2}\left[2 \times 26 + 11 \times 15\right]$

$S_{12} = 6\left[52 + 165\right]$

$S_{12} = 6 \times 217$

$S_{12} = 1302$

(c) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$1500 = \frac{12}{2}\left[2 \times 26 + 11 \times d\right]$

$1500 = 6\left(52 + 11d\right)$

$250 = 52 + 11d$

$198 = 11d$

$d = \frac{198}{11} = \frac{99+99}{11}$

$d = 18$

Question 9 (*+)**

A non regular polygon has 9 sides whose lengths, in cm, form an arithmetic sequence with common difference d.

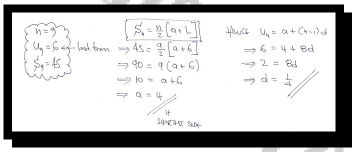
The longest side of the polygon is 6 cm and the perimeter of the polygon is 45 cm.

Find in any order …

a) … the length of the shortest side of the polygon.

b) … the value of d.

$\boxed{4\text{cm}}$, $\boxed{d = 0.25}$



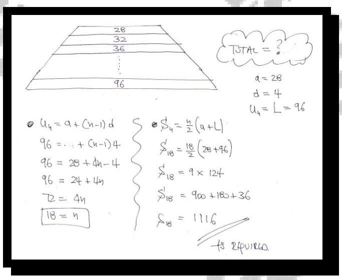
Question 10 (*+)**

The roof of a museum has a sloping shape with the roof tiles arranged neatly in horizontal rows. There are 28 roof tiles in the top row and each row below the top row has an extra 4 tiles than the row above it.

The bottom row has 96 tiles.

Show that there are 1116 tiles on the roof of the museum.

$\boxed{\text{proof}}$



Question 11 (***+)

William started receiving his annual allowance on his 13^{th} birthday. His first allowance was £750 and this amount was increased in each successive birthday by £150.

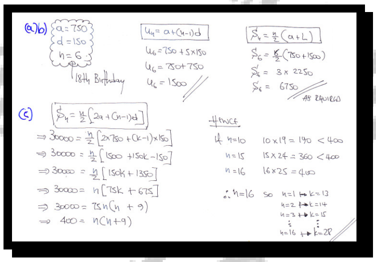
a) Use algebra to find the amount William received on his 18^{th} birthday.

b) Show that the total amount of allowances William received up and including his 18^{th} birthday was £6750.

When William turned k years old he received his last allowance. The total amount of his allowances up and including that of his k^{th} birthday was £30000.

c) Find the value of k.

$\boxed{£1500}$, $\boxed{k = 28}$



Question 12 (***+)

A non regular polygon has 10 sides whose lengths, in cm, form an arithmetic sequence with common difference d.

The longest side of the polygon is twice as long as the shortest side.

Given that the perimeter of the polygon is 405 cm, find in any order …

a) … the length of the shortest side of the polygon.

b) … the value of d.

$\boxed{27\,\text{cm}}$, $\boxed{d=3}$

$n=10$
$S_{10}=405$
$u_{10}=2a=L$

$\boxed{S_n=\frac{n}{2}(a+L)}$
$\Rightarrow 405=\frac{10}{2}(a+2a)$
$\Rightarrow 405=5(3a)$
$\Rightarrow 405=15a$
$\Rightarrow 3a=81$
$\Rightarrow a=27$

$\boxed{u_n=a+(n-1)d}$
$\Rightarrow u_{10}=a+9d$
$\Rightarrow 2a=a+9d$
$\Rightarrow a=9d$
$\Rightarrow 27=9d$
$\Rightarrow d=3$

Question 13 (***+)

The council of Broxbourne undertook a housing development scheme which started in the year 2001 and is to finish in the year 2025. Under this scheme the council will build 760 houses in 2012 and 240 houses in 2025.

The number of houses the council builds every year, forms an arithmetic sequence.

a) Determine the number of houses built in 2001.

b) Calculate the total number of houses that will be built under this scheme.

$\boxed{1200}$, $\boxed{18000}$

(a)
$u_{12}=760$
$u_{25}=240=L$
($n=25$ IN TOTAL)

$\boxed{u_n=a+(n-1)d}$
• $760=a+11d$
• $240=a+24d$

"SUBTRACT UPWARDS" $-520=13d$
$-40=d$

USING $760=a+11d$
$760=a+11(-40)$
$760=a-440$
$a=1200$

(b)
$S_n=\frac{n}{2}(a+L)$
$S_{25}=\frac{25}{2}[1200+240]=25(600+120)=25\times720=18000$

Question 14 **(***+)**

Osama starts his new job on an annual salary of £18000. His contract promises a pay rise of £1800 in each subsequent year until his salary reaches £36000. When the salary reaches £36000 Osama will receive **no more** pay rises. Osama's salary first reaches the maximum salary of £36000 in year N.

a) Determine the value of N.

b) Find Osama's total salary earnings during the first N years of his employment.

Obama starts his new job at the same time as Osama on an annual salary of £A. His contracts promises a pay rise of £1000 in each subsequent year until his salary reaches £36000. When the salary reaches £36000 Obama will receive **no more** pay rises. Obama's salary first reaches the maximum salary of £36000 in year 15.

c) Find the year when both Osama and Obama have the same annual salary.

d) Calculate the difference in the total salary earnings between Osama and Obama in the first 15 years of their employment.

$\boxed{N=11}$, $\boxed{S_N=297000}$, $\boxed{n=6}$, $\boxed{d=6000}$

a) $u_n = a+(n-1)d$

$36000 = 18000+(n-1)\times 1800$

$18000 = 1800(n-1)$

$10 = n-1$

$n = 11$

i.e. $N=11$

b) $S_n = \frac{n}{2}[a+L]$

$S_{11} = \frac{11}{2}[18000+36000]$

$S_{11} = \frac{11}{2}\times 54000$

$S_{11} = 11\times 27000$

$S_{11} = 270000+27000$

$S_{11} = 297000$

$\therefore$ £297000

c) FIRSTLY $u_n = a+(n-1)d$

$36000 = A+(15-1)\times 1000$

$36000 = A+14000$

$A = 22000$

THUS

$18000+(n-1)\times 1800 = 22000+(n-1)\times 1000$

$1800(n-1)-1000(n-1) = 4000$

$800(n-1) = 4000$

$n-1 = 5$

$n = 6$

d) OBAMA'S TOTAL IN 15 YEARS

$S_n = \frac{n}{2}[a+L]$

$S_{15} = \frac{15}{2}[22000+36000]$

$S_{15} = \frac{15}{2}\times 58000$

$S_{15} = 15\times 29000$

$S_{15} = 290000+145000 = 435000$

OSAMA'S TOTAL IN 15 YEARS

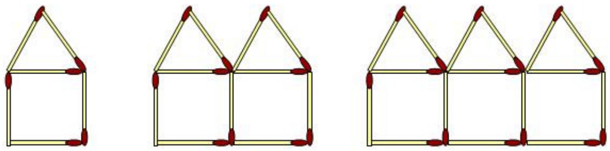
$297000+(4\times 36000)$

$297000+144000 = 441000$

$\therefore$ DIFFERENCE OF $441000-435000 =$ £6000

Question 15 (***+)

Thomas is making patterns using sticks. He uses 6 sticks for the first pattern, 11 sticks for the second pattern, 16 sticks for the third pattern and so on.



a) Find how many sticks Thomas uses to make the tenth pattern.

b) Show clearly that Thomas uses 285 sticks to make the first ten patterns.

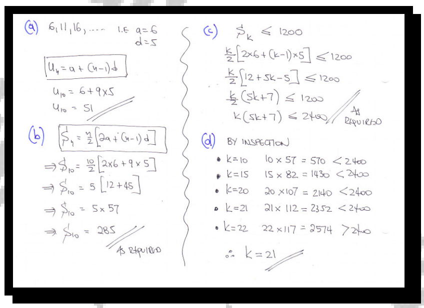
Thomas has a box with 1200 sticks. Thomas can make k **complete** patterns with the sticks in his box.

c) Show further that k satisfies the inequality

$$k(5k+7) \le 2400 .$$

d) Hence find the value of k.

$\boxed{51}$, $\boxed{k=21}$


(a) $6, 11, 16, \dots$ I.E $a=6$, $d=5$

$\boxed{u_n = a + (n-1)d}$

$u_{10} = 6 + 9 \times 5$

$u_{10} = 51$

(b) $\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$

$\Rightarrow S_{10} = \frac{10}{2}\left[2\times 6 + 9 \times 5\right]$

$\Rightarrow S_{10} = 5\left[12 + 45\right]$

$\Rightarrow S_{10} = 5 \times 57$

$\Rightarrow S_{10} = 285$

AS REQUIRED

(c) $S_k \leqslant 1200$

$\frac{k}{2}\left[2\times 6 + (k-1)\times 5\right] \leqslant 1200$

$\frac{k}{2}\left[12 + 5k - 5\right] \leqslant 1200$

$\frac{k}{2}(5k+7) \leqslant 1200$

$k(5k+7) \leqslant 2400$

AS REQUIRED

(d) BY INSPECTION

- $k=10$ $\quad 10 \times 57 = 570 < 2400$
- $k=15$ $\quad 15 \times 82 = 1430 < 2400$
- $k=20$ $\quad 20 \times 107 = 2140 < 2400$
- $k=21$ $\quad 21 \times 112 = 2352 < 2400$
- $k=22$ $\quad 22 \times 117 = 2574 > 2400$

$\therefore k = 21$

Question 16 **(***+)**

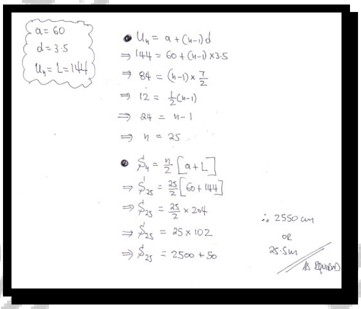
A length of rope is wrapped neatly around a circular pulley.

The length of the rope in the first coil (the nearest to the pulley) is 60 cm, and each successive coil of rope (outwards) is 3.5 cm longer than the previous one.

The outer coil has a length of 144 cm.

Show that total length of the rope is 25.5 metres.

$\boxed{\text{proof}}$



$a = 60$
$d = 3.5$
$u_n = L = 144$

- $u_n = a + (n-1)d$
$\Rightarrow 144 = 60 + (n-1) \times 3.5$
$\Rightarrow 84 = (n-1) \times \frac{7}{2}$
$\Rightarrow 12 = \frac{1}{2}(n-1)$
$\Rightarrow 24 = n - 1$
$\Rightarrow n = 25$

- $S_n = \frac{n}{2}\left[a + L\right]$
$\Rightarrow S_{25} = \frac{25}{2}\left[60 + 144\right]$
$\Rightarrow S_{25} = \frac{25}{2} \times 204$
$\Rightarrow S_{25} = 25 \times 102$
$\Rightarrow S_{25} = 2500 + 50$

$\therefore$ 2550 cm
or
25.5 m
AS REQUIRED

Question 17 (****)

A farmer has difficulty persuading strawberry pickers to work for the entire 40 day strawberry picking season. He devises a wage plan to make the pay of the workers more attractive the more days they work.

He pays £a on the first day, £$(a+d)$ on the second day, £$(a+2d)$ on the third day, and so on, increasing the daily wages by £d every day.

A strawberry picker that worked for forty days got paid £53.40 on the last day and earned £1668 in total.

a) Show clearly that

$$10(a+53.4)=834.$$

b) Calculate the wages that this strawberry picker received on the twentieth day.

$\boxed{£41.40}$

(a) $u_{40} = 53.4$ $\quad S_{40} = 1668$

LAST TERM

$S_n = \frac{n}{2}\left[a + L\right]$

$\Rightarrow 1668 = \frac{40}{2}\left[a + 53.4\right)$

$\Rightarrow 1668 = 20(a + 53.4)$

$\Rightarrow 834 = 10(a + 53.4)$ // AS REQUIRED

(b)

$\Rightarrow 83.4 = a + 53.4$

$\Rightarrow \boxed{a = 30}$

USING $u_n = a + (n-1)d$ WITH $u_{40} = 53.4$

$53.4 = 30 + 39d$

$23.4 = 39d$

$\frac{23.4}{39} = d$ (BY LONG DIVISION OR....)

$d = \underbrace{\frac{234}{390} = \frac{117}{195}}_{\div 2} = \underbrace{\frac{39}{65}}_{\div 3} = \underbrace{\frac{3}{5}}_{\div 13} = 0.6$

$\therefore \boxed{d = 0.6}$

HENCE $u_{20} = 30 + 19 \times 0.6$

$= 30 + 11.4$

$= 41.40$ $\quad \therefore$ £41.40

Question 18 (****)

Tyler is repaying a loan over a period of n months in such a way so that his monthly repayments form an arithmetic series.

He repays £350 in the first month, £340 in the second month, £330 in the third month and so on until the full loan is repaid.

a) Assuming it takes more than 12 months to repay his loan find …

i. … the amount he pays on the twelfth month.

ii. … the total amount of his repayments in the first twelve months.

Tyler pays back his loan of £6200 after n months.

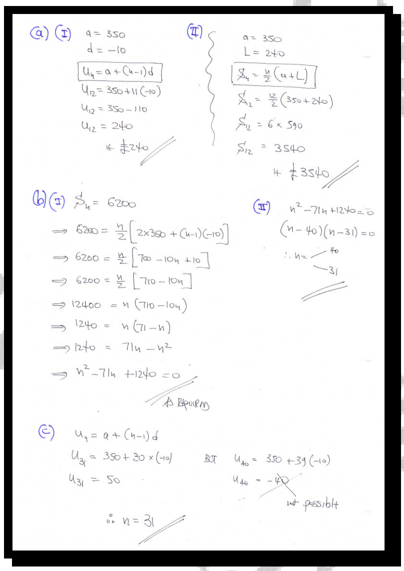
b) Show clearly that …

i. … $n^2 - 71n + 1240 = 0$

ii. … $n = 40$ is one of the solutions of this equation and find the other.

c) Determine, with a valid reason, which of the two values of n represents the actual number of months it takes Tyler to repay his loan.

$\boxed{£240}$, $\boxed{£3540}$, $\boxed{n = 31}$, $\boxed{31 \text{ months}}$



(a) (I) $a = 350$, $d = -10$

$\boxed{u_n = a + (n-1)d}$

$u_{12} = 350 + 11(-10)$

$u_{12} = 350 - 110$

$u_{12} = 240$

i.e. £240

(II) $a = 350$, $L = 240$

$\boxed{S_n = \frac{n}{2}(a + L)}$

$S_{12} = \frac{12}{2}(350 + 240)$

$S_{12} = 6 \times 590$

$S_{12} = 3540$

i.e. £3540

(b) (I) $S_n = 6200$

$\Rightarrow 6200 = \frac{n}{2}\left[2 \times 350 + (n-1)(-10)\right]$

$\Rightarrow 6200 = \frac{n}{2}\left[700 - 10n + 10\right]$

$\Rightarrow 6200 = \frac{n}{2}\left[710 - 10n\right]$

$\Rightarrow 12400 = n(710 - 10n)$

$\Rightarrow 1240 = n(71 - n)$

$\Rightarrow 1240 = 71n - n^2$

$\Rightarrow n^2 - 71n + 1240 = 0$

As required

(II) $n^2 - 71n + 1240 = 0$

$(n - 40)(n - 31) = 0$

$\therefore n = 40, 31$

(c) $u_n = a + (n-1)d$

$u_{31} = 350 + 30 \times (-10)$

$u_{31} = 50$

BUT $u_{40} = 350 + 39(-10)$

$u_{40} = \cancel{-40}$

not possible

$\therefore n = 31$

Question 19 (****)

An oil company is drilling for oil.

It costs £5000 to drill for the first 10 metres into the ground.

For the next 10 metres it costs an extra £1200 compared with the first 10 metres, thus it costs £6200. Each successive 10 metres drilled into the ground costs an extra £1200, compared with the cost of drilling the previous 10 metres.

a) Find the cost of drilling 200 metres into the ground.

The company has a budget of £15,000,000.

b) Determine the maximum depth, in metres, that can be reached on this budget.

$\boxed{£328,000}$, $\boxed{1540\text{ m}}$

a) FORM A MODEL

	DEPTH	COST
20 PARTS OF 10 m	10	£5000
	20	£6200
	30	£7400
	⋮	
	200	?

$\Rightarrow S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow S_{20} = \frac{20}{2}\left[2 \times 5000 + 19 \times 1200\right]$

$\Rightarrow S_{20} = 10\left[10000 + 22800\right]$

$\Rightarrow S_{20} = 328000$

ie £328,000

b) "WORKING BACKWARDS"

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow 15000000 = \frac{n}{2}\left[2 \times 5000 + (n-1) \times 1200\right]$

$\Rightarrow 15000000 = \frac{n}{2}\left[10000 + 1200n - 1200\right]$

$\Rightarrow 15000000 = \frac{n}{2}\left[8800 + 1200n\right]$

$\Rightarrow 15000000 = 4400n + 600n^2$

$\Rightarrow 600n^2 + 4400n + 15000000 = 0 \quad \div 100$

$\Rightarrow 6n^2 + 44n - 150000 = 0 \quad \div 2$

$\Rightarrow 3n^2 + 22n - 75000 = 0$

BY THE QUADRATIC FORMULA

$n = \frac{-22 \pm \sqrt{484 + 900000}}{6} = \begin{cases} \cancel{-161.823\ldots} \\ 154.489\ldots \approx 154 \end{cases}$

$\therefore$ DEPTH OF 1540 m (ACCEPT 1544 OR 1545)

Question 20 (****)

In the TV game “Extra Fifty” contestants answer a series of questions.

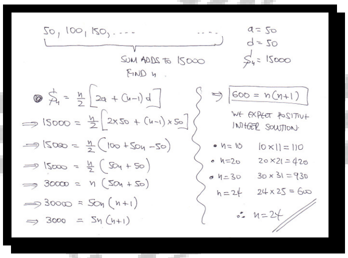
Contestants win £50 for answering the 1st question correctly, £100 for answering the 2nd question correctly, £150 for answering the 3rd question correctly, and so on.

Once an incorrect answer is given the game ends but the contestant keeps the winnings up to that point.

A contestant wins £15000.

Determine, showing all parts in the calculation, the number of the questions he or she answered correctly.

$\boxed{24}$



$\underbrace{50, 100, 150, \ldots \ldots}_{\text{SUM ADDS TO 15000, FIND } n}$

$a = 50$, $d = 50$, $S_n = 15000$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow 15000 = \frac{n}{2}\left[2 \times 50 + (n-1) \times 50\right]$

$\Rightarrow 15000 = \frac{n}{2}(100 + 50n - 50)$

$\Rightarrow 15000 = \frac{n}{2}(50n + 50)$

$\Rightarrow 30000 = n(50n + 50)$

$\Rightarrow 30000 = 50n(n+1)$

$\Rightarrow 3000 = 5n(n+1)$

$\Rightarrow \boxed{600 = n(n+1)}$

WE EXPECT POSITIVE INTEGER SOLUTION.

- $n = 10$ $10 \times 11 = 110$
- $n = 20$ $20 \times 21 = 420$
- $n = 30$ $30 \times 31 = 930$
- $n = 24$ $24 \times 25 = 600$

$\therefore n = 24$

Question 21 **(****)**

A company agrees to pay a loan back in monthly instalments, starting with £1500 .

The agreement states that the company will pay back
$£(1500-x)$ in the 2^{nd} month,
$£(1500-2x)$ in the 3^{rd} month,
$£(1500-3x)$ in the 4^{th} month,
and so on, with the repayments decreasing by $£x$ every month.

a) Given that in the first year the company repaid a total of £15360 , find the value of x .

b) Show that the total money T_n , repaid in n months, is given by

$$T_n = 20n(76-n).$$

The total value of the loan was £26000 .

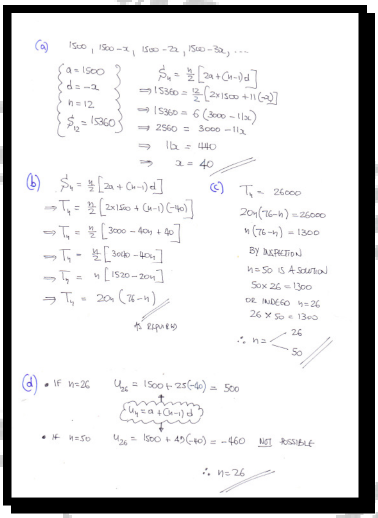
c) Show that the equation

$$T_n = 26000$$

is satisfied by two different values of n .

d) Determine, with a valid reason, which of the two values of n represents the actual number of months it takes for the company to repay the loan.

$\boxed{x=40}$, $\boxed{n=26,50}$, $\boxed{n=26}$


(a) $1500,\ 1500-x,\ 1500-2x,\ 1500-3x,\ \dots$

$a=1500$, $d=-x$, $n=12$, $S_{12}=15360$

$S_n=\frac{n}{2}[2a+(n-1)d]$
$\Rightarrow 15360=\frac{12}{2}[2\times1500+11(-x)]$
$\Rightarrow 15360=6(3000-11x)$
$\Rightarrow 2560=3000-11x$
$\Rightarrow 11x=440$
$\Rightarrow x=40$

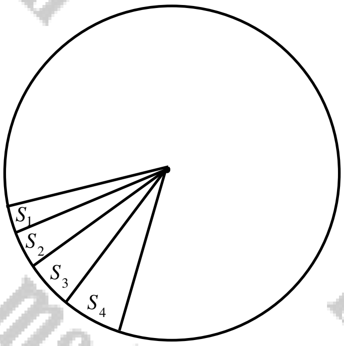
(b) $S_n=\frac{n}{2}[2a+(n-1)d]$
$\Rightarrow T_n=\frac{n}{2}[2\times1500+(n-1)(-40)]$
$\Rightarrow T_n=\frac{n}{2}[3000-40n+40]$
$\Rightarrow T_n=\frac{n}{2}[3040-40n]$
$\Rightarrow T_n=n[1520-20n]$
$\Rightarrow T_n=20n(76-n)$
AS REQUIRED

(c) $T_n=26000$
$20n(76-n)=26000$
$n(76-n)=1300$
BY INSPECTION
$n=50$ IS A SOLUTION
$50\times26=1300$
OR INDEED $n=26$
$26\times50=1300$
$\therefore n=26,\ 50$

(d) • IF $n=26$ $\quad u_{26}=1500+25(-40)=500$
$u_n=a+(n-1)d$
• IF $n=50$ $\quad u_{26}=1500+49(-40)=-460$ NOT POSSIBLE
$\therefore n=26$

Question 22 **(****)**

A machine cuts a circular sheet of plastic into **exactly** n sectors, S_1, S_2, S_3, …, S_n.



The angle that each sector subtends at the centre of the circle forms an arithmetic series.

The smallest sector and the largest sector subtend angles at the centre of $7.25°$ and $32.75°$, respectively.

Find the value of n.

$\boxed{n = 18}$

$a = 7.25$
$u_n = l = 32.75$
$S_n = 360$

$S_n = \frac{n}{2}(a + L)$
$\Rightarrow 360 = \frac{n}{2}(7.25 + 32.75)$
$\Rightarrow 360 = \frac{n}{2} \times 40$
$\Rightarrow 360 = 20n$
$\Rightarrow n = 18$

Question 23 (****)

A company offers two pay schemes for its employees.

<u>Scheme One</u>

- Annual salary in Year 1 is $£X$.
- Annual salary increases every subsequent year by $£(2Y)$, forming an arithmetic series.

<u>Scheme Two</u>

- Annual salary in Year 1 is $£(X+2000)$.
- Annual salary increases every subsequent year by $£Y$, forming an arithmetic series.

a) Show that the total salary received by an employee under Scheme One, over a nine year period is

$$9(X+8Y).$$

After nine years, the total salary received by an employee under Scheme One is £3600 larger than the total salary received by an employee under Scheme Two.

b) Show clearly that

$$Y=600.$$

c) Given further that an employee under the Scheme One earns £36000 in the eleventh year of his employment, determine the value of X.

$\boxed{X=24000}$

(a) $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$
$S_9 = \frac{9}{2}\left[2X+8(2Y)\right]$
$S_9 = 9(X+8Y)$ // AS REQUIRED

(b) $S_9 = \frac{9}{2}\left[2(X+2000)+8Y\right] \leftarrow$ SCHEME 2
$S_9 = 9\left[X+2000+4Y\right]$

HENCE
$\Rightarrow 9(X+8Y)-9(X+2000+4Y)=3600$
$\Rightarrow (X+8Y)-(X+2000+4Y)=400$
$\Rightarrow \cancel{X}+8Y-\cancel{X}-2000-4Y=400$
$\Rightarrow 4Y=2400$
$\Rightarrow Y=600$

(c) $\boxed{u_n = a+(n-1)d}$
$36000 = X+10(2\times600)$
$36000 = X+10\times1200$
$36000 = X+12000$
$X = 24000$

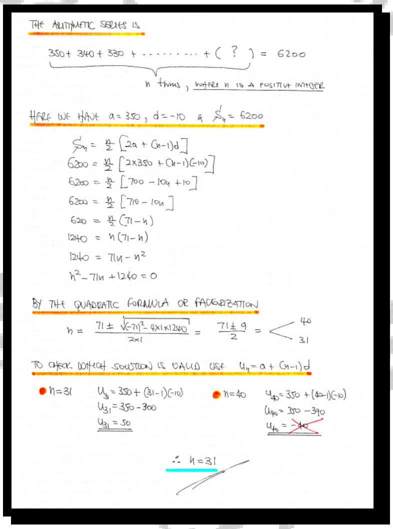
Question 24 (****+)

Ladan is repaying an interest free loan of £6200 over a period of n months, in such a way so that her monthly repayments form an arithmetic series.

She repays £350 in the first month, £340 in the second month, £330 in the third month and so on until the full loan is repaid.

Determine, showing a full algebraic method, the value of n.

$\boxed{n = 31}$



THE ARITHMETIC SERIES IS

$$\underbrace{350 + 340 + 330 + \cdots\cdots + (\ ?\ )}_{n \text{ terms}} = 6200$$

WHERE n IS A POSITIVE INTEGER

HERE WE HAVE $a = 350$, $d = -10$ & $S_n = 6200$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$6200 = \frac{n}{2}\left[2 \times 350 + (n-1)(-10)\right]$$
$$6200 = \frac{n}{2}\left[700 - 10n + 10\right]$$
$$6200 = \frac{n}{2}\left[710 - 10n\right]$$
$$620 = \frac{n}{2}(71 - n)$$
$$1240 = n(71 - n)$$
$$1240 = 71n - n^2$$
$$n^2 - 71n + 1240 = 0$$

BY THE QUADRATIC FORMULA OR FACTORIZATION

$$n = \frac{71 \pm \sqrt{(-71)^2 - 4 \times 1 \times 1240}}{2 \times 1} = \frac{71 \pm 9}{2} = \begin{cases} 40 \\ 31 \end{cases}$$

TO CHECK WHICH SOLUTION IS VALID USE $u_n = a + (n-1)d$

- $n = 31$: $u_{31} = 350 + (31-1)(-10)$, $u_{31} = 350 - 300$, $u_{31} = 50$
- $n = 40$: $u_{40} = 350 + (40-1)(-10)$, $u_{40} = 350 - 390$, $u_{40} = \cancel{-40}$

$\therefore n = 31$

Question 25 (****+)

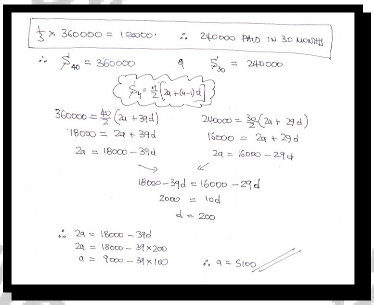
A company arranges to pay a debt of £360,000 by 40 monthly instalments.

These monthly instalments form an arithmetic series.

After 30 of these instalments were paid, the company declared themselves bankrupt leaving one third of their debt unpaid.

Find the value of the first instalment.

$\boxed{£5100}$



Question 26 (****+)

A gym has 125 members and in order to meet its outgoings it needs 600 members.

A Public Relations company is hired to re-launch the gym and increase its membership thereafter, using a variety of marketing strategies.

A preliminary model for the recruitment of new members is as follows.

It is expected that 10 new members will join in the week following the gym's re-launch, 12 new members in the second week, 14 in the third week and so on with 2 new members joining the gym in each subsequent week.

a) Find according to this preliminary model …

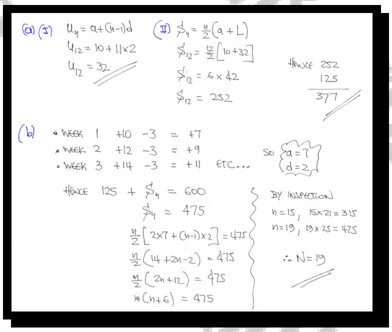
i. … the number of the new members that will join in the 12^{th} week.

ii. … the **total** number of members after 12 weeks.

The model is refined to allow for the gym losing members at the constant rate of 3 members per week. The gym **reaches** the desired target of 600 members in N weeks.

b) Determine the value of N.

$\boxed{32}$, $\boxed{377}$, $\boxed{19 \text{ weeks}}$



Question 27 (****+)

A pension broker gets paid £15 commission **per week** for every pension scheme he sells. Each week he sells a new pension scheme so that …

In the 1st week he gets paid £15 commission for the pension he just sold.

In the 2nd week he gets paid £30, £15 for the pension sold in the 1st week plus £15 for pension he sold in the 2nd week.

In the 3rd week he gets paid £45, £15 for the pension sold in the 1st week plus £15 for pension he sold in the 2nd week, plus £15 for the pension he sold in the 3rd week, and so on.

a) Find the commission he gets paid on the last week of the year.

b) Find his annual earnings after one year in this job.

His commission increases to £20 for new pension schemes sold during the 2nd year but decreases to £10 for the schemes he sold in the 1st year. The broker continues to sell at the rate of one new pension scheme every week.

c) Find his annual earnings in the 2nd year.

$\boxed{£780}$, $\boxed{£20670}$, $\boxed{£54600}$

a) LOOKING AT THE PATTERN

WEEK:	1	2	3	....	52
COMM.:	15	15 + 15 = 30	15 + 15 + 15 = 45		?

$a = 15$
$d = 15$
$n = 52$

$\Rightarrow u_n = a + (n-1)d$

$\Rightarrow u_{52} = 15 + 51 \times 15$

$\Rightarrow u_{52} = 780$

i.e. £780

b) SUMMING THE COMMISSIONS USING PART (a)

$S_n = \frac{n}{2}[a + L]$

$S_{52} = \frac{52}{2}[15 + 780] = 26 \times 795 = 20670$

i.e. £20670

c) CONTINUING THE PATTERN & LINKING WITH THE FIRST YEAR

WEEK	COMMISSION	
51	£765	FIRST YEAR
52	£780	
1	$(52 \times 10) + 20$	$a = 540$
2	$(52 \times 10) + 20 + 20$	SECOND YEAR
3	$(52 \times 10) + 20 + 20 + 20$	
⋮	⋮	
52	$(52 \times 10) + 20 \times 52$	$L = 1560$

SUMMING USING $S_n = \frac{n}{2}[a + L]$

$S_{52} = \frac{52}{2}[540 + 1560] = 54600$

i.e. £54600