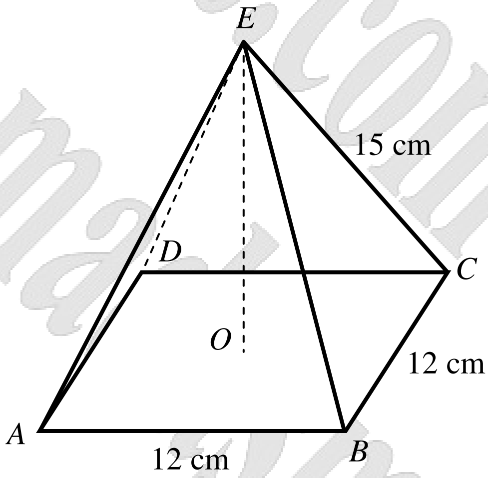
GEOMETRIC MENSURATION IN 3 DIMENSIONS

Question 1 (**)



A square pyramid $ABCDE$ stands on level horizontal ground.

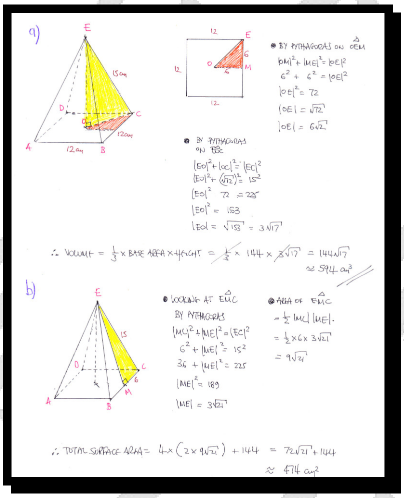
The vertex of the pyramid is at E. The points A, B, C and D are the corners of a square of side 12 cm, whose diagonals intersect at the point O.

Each of the sloping edges of the pyramid has length 15 cm.

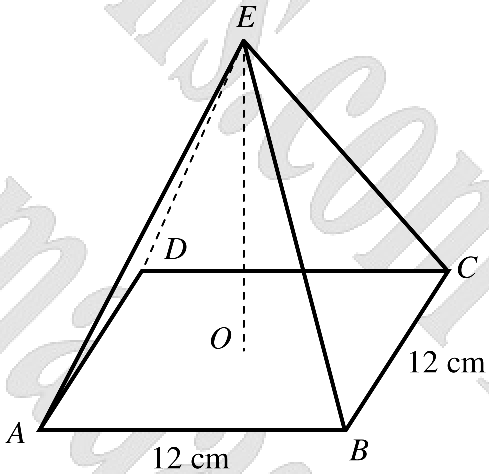
a) Calculate the volume of the pyramid.

b) Calculate the total surface area of the pyramid.

$\boxed{V = 144\sqrt{17} \approx 594 \text{ cm}^3}$, $\boxed{A = 144 + 72\sqrt{21} \approx 474 \text{ cm}^2}$



Question 2 (***)



A square pyramid $ABCDE$ stands on level horizontal ground. The points A, B, C and D are the corners of a square of side 12 cm, whose diagonals intersect at the point O. The vertex of the pyramid is at E. Each of the sloping edges of the pyramid makes an angle of $30°$ with the ground.

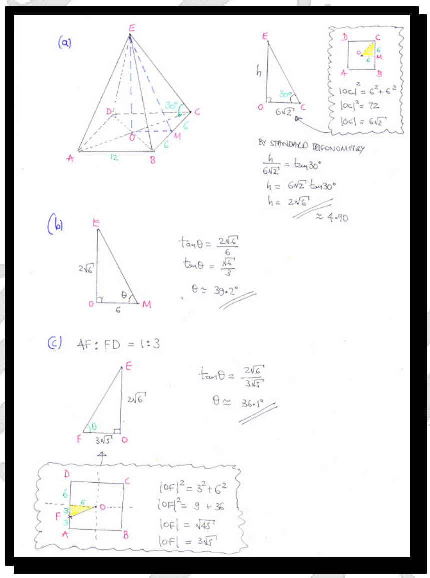
a) Determine the height of the pyramid, OE.

b) Calculate the angle the face EBC, makes with the ground.

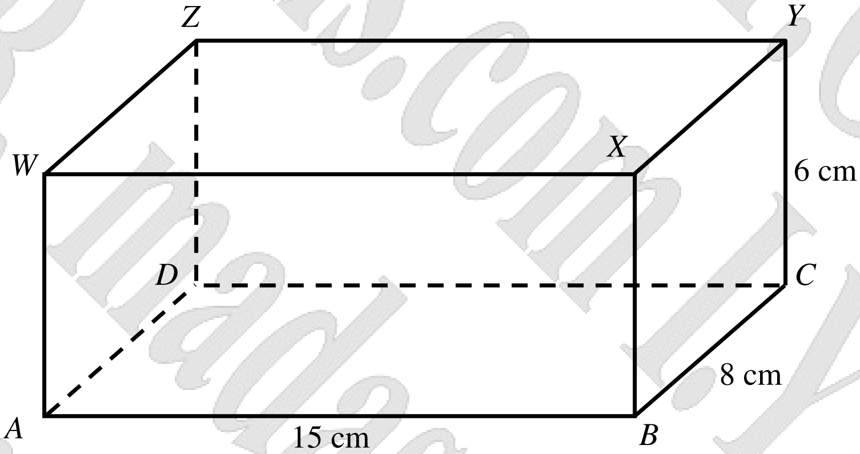
The point F lies on AD so that $AF : FD = 1:3$.

c) Calculate the angle EFO.

$\boxed{|OE| = 2\sqrt{6} \approx 4.90 \text{ cm}}$, $\boxed{\approx 39.2°}$, $\boxed{\measuredangle EFO \approx 36.1°}$



Question 3 (***)



The figure above shows a cuboid $ABCDWXYZ$, standing on level horizontal ground. The lengths of AB, BC and CY are 15 cm, 8 cm and 6 cm, respectively.

a) Find the length of AY.

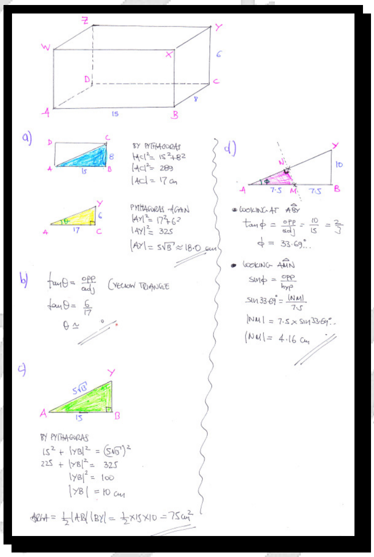
b) Calculate the angle AY makes with the ground.

c) Determine the area of the triangle ABY.

The point M is the midpoint of AB and the point N lies on AY.

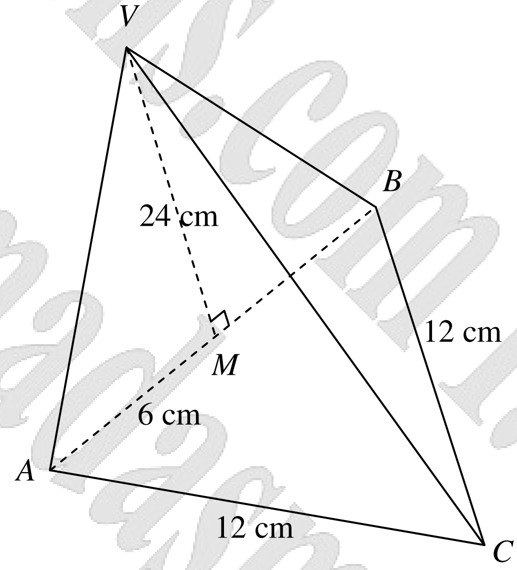
d) Calculate the length of MN, given further that MN is perpendicular to AY.

$\boxed{|AY| = 5\sqrt{13} \approx 18.0 \text{ cm}}$, $\boxed{\approx 19.4°}$, $\boxed{\text{area} = 75 \text{ cm}^2}$, $\boxed{\approx 4.16 \text{ cm}}$



Created by T. Madas

Question 4 (***)



The figure above shows a pyramid $VABC$, standing on level horizontal ground. The base of the pyramid ABC is an equilateral triangle of length 12 cm. The point M is the midpoint of AB. The vertex of the pyramid V lies vertically above M so that the length of VM is 24 cm.

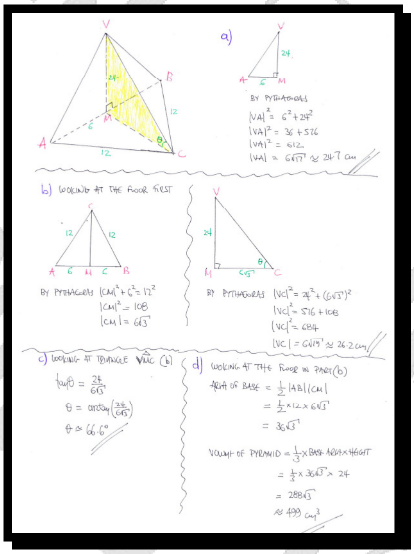
a) Find the length of VA.

b) Find the length of VC.

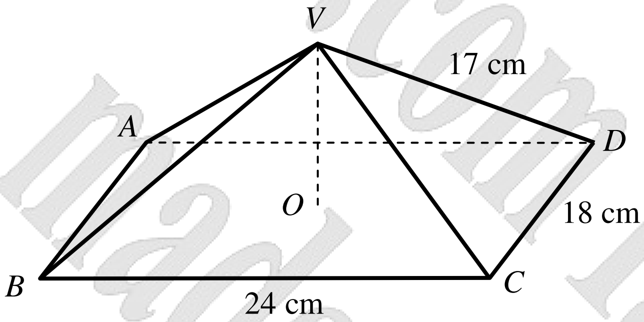
c) Calculate the angle VC makes with the ground.

d) Determine the volume of the pyramid $VABC$.

$\boxed{|VA| = 6\sqrt{17} \approx 24.7 \text{ cm}}$, $\boxed{|VC| = 6\sqrt{19} \approx 26.2 \text{ cm}}$, $\boxed{\approx 66.6°}$, $\boxed{\text{volume} = 499 \text{ cm}^3}$



Question 5 (***)



A pyramid $VABCD$ stands on level horizontal ground. The points A, B, C and D are the corners of a rectangle, where $|AD| = |BC| = 24$ cm and $|AB| = |CD| = 18$ cm. The vertex of the pyramid is at V and the diagonals intersect at the point O.

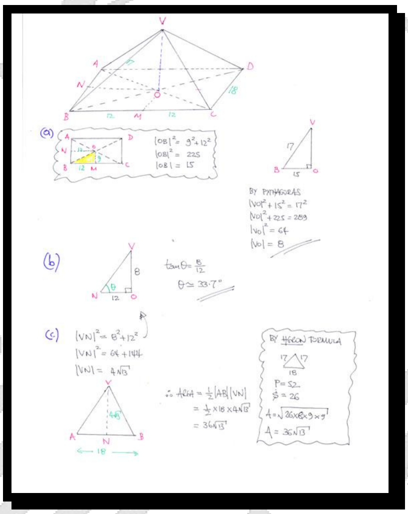
Each of the four sloping edges of the pyramid is 17 cm.

a) Determine the height of the pyramid, VO.

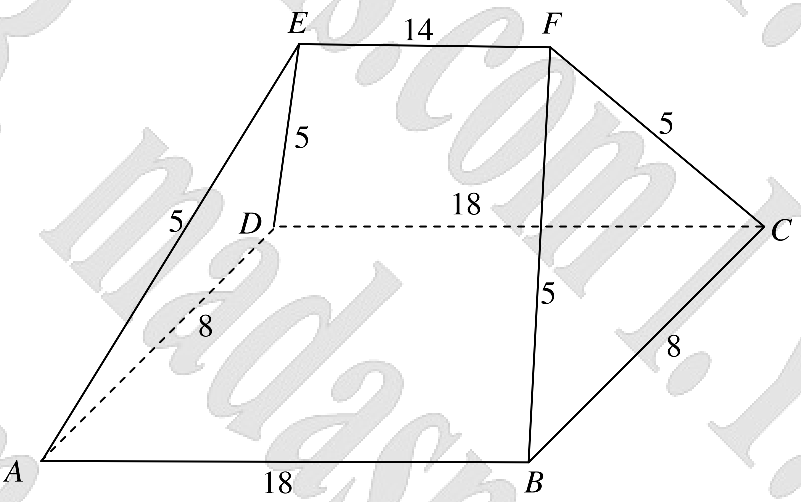
b) Calculate the angle the face VAB, makes with the base of the pyramid.

c) Calculate the exact area of the face VAB.

$\boxed{|VO| = 8 \text{ cm}}$, $\boxed{\approx 33.7°}$, $\boxed{\text{area} = 36\sqrt{13} \text{ cm}^2}$



Question 6 (***)



The figure above shows a wooden pole structure $ABCDEF$, modelling a tent, standing on level horizontal ground. The base of the tent $ABCD$ is a rectangle. The pole EF is horizontal.

The following measurements are given in metres.

$$|AB| = |CD| = 18, \quad |BC| = |DA| = 8, \quad |EF| = 14, \quad |EA| = |ED| = |FB| = |FC| = 5.$$

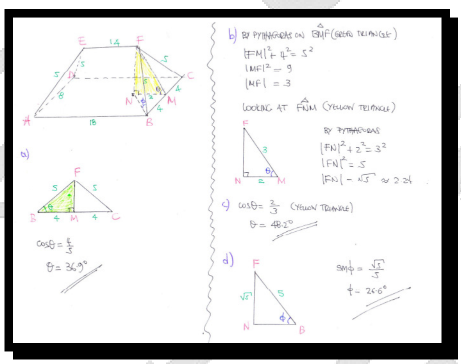
a) Calculate the angle FBC.

b) Find the height of the pole EF, from the ground.

c) Calculate the angle the face BFC makes with the ground.

d) Determine the angle the pole BF makes with the ground.

$\boxed{\approx 36.9°}$, $\boxed{|FN| = \sqrt{5} \approx 2.24 \text{ m}}$, $\boxed{\approx 48.2°}$, $\boxed{\approx 26.6°}$



Question 7 (****)

A pyramid $PQRS$ has a triangular horizontal base PQR, where $|PQ| = |PR| = 8$ m and $|RQ| = 12$ m. The vertex of the pyramid S lies directly above the level of PQR so that $|SQ| = |SR| = 10$ m and $|SP| = 8$ m.

a) Show that the shortest distance of S from the base PQR is $\sqrt{57}$ m.

b) Calculate to the nearest degree the acute angle between …

i. … the plane SQR and the plane PQR.

ii. … the edge SQ and the plane PQR.

c) Determine as an exact surd the shortest distance of P from the plane SQR.

$\boxed{71°}$, $\boxed{49°}$, $\boxed{d = \frac{1}{2}\sqrt{399}}$