PRODUCT
OPERATOR
Question 1

The product operator $\prod_{i=1}^{k} u_i$ is defined as

$$\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times \cdots \times u_{k-1} \times u_k.$$ 

Find the value of

$$\prod_{r=3}^{16} \left[ 1 + \frac{4}{r-2} \right].$$
Question 2

The product operator $\prod$ is defined as

$$\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \cdots \times u_{k-1} \times u_k.$$ 

Simplify, showing a clear method

$$\prod_{r=1}^{n} \left[ \frac{r+1}{r} \right].$$
Question 3

The product operator \( \prod \), is defined as

\[
\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.
\]

Simplify, showing a clear method

\[
\prod_{r=2}^{n+1} \left[ \frac{r^2 - 1}{r^2} \right].
\]
Question 4

The product operator $\prod_{i=1}^{k}$ is defined as

$$\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.$$ 

Use a clear method to show that

$$\prod_{m=1}^{3} \prod_{n=1}^{4} (\sqrt{mn}) = k^3 \sqrt{6},$$

where $k$ is a positive integer to be found.

$k = 12$
Question 5

The product operator $\prod_{i=1}^{k} u_i$ is defined as

$$\prod_{i=1}^{k} u_i = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.$$

Given that $k \in \mathbb{N}$, use a detailed method to find the value of

$$\prod_{r=2}^{2^k-1} \left[ \log_r (r+1) \right].$$
Question 6

The product operator $\prod_{i=1}^{k}[u_i]$ is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.$$ 

Evaluate, showing a clear method

$$\prod_{n=2}^{\infty} \left[ 1 + \frac{1}{n^2 - 1} \right].$$
Question 7

The product operator $\prod_{i=1}^{k} u_i$ is defined as

$$\prod_{i=1}^{k} u_i = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.$$ 

Use a clear method to determine the value of $k$ given that

$$\prod_{m=1}^{4} \prod_{n=1}^{3} \{k m n\} = 4 \times 96^7.$$ 

\[ k = 4 \]
Question 8

The product operator \( \prod \) is defined as

\[
\prod_{i=1}^{k} u_i = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.
\]

Simplify, showing a clear method

\[
\prod_{r=1}^{n} \left[ \frac{r}{2r+1} \right].
\]
Question 9

The product operator \( \prod \) is defined as

\[
\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.
\]

Evaluate, showing a clear method

\[
\prod_{n=2}^{\infty} \left[ 1 - \frac{1}{2 - 2^n} \right].
\]
Question 10

The product operator \( \prod \) is defined as

\[
\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.
\]

Given that \( e \) is Euler's number, use a detailed method to find the exact value of

\[
\prod_{n=1}^{\infty} \left[ \frac{(3^n \sqrt{e})}{(2^n + 1)^n} \right].
\]
Question 11

The product operator $\prod_{i=1}^{k}$ is defined as

$$\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times ... \times u_{k-1} \times u_k.$$ 

Simplify, showing a clear method, the following expression.

$$\prod_{r=1}^{n} \left[ \frac{2r}{2r+1} \right].$$

Give the final answer as a single simplified fraction.

$$\frac{a^n (n!)^2}{(2n+1)!}.$$
Question 12

The product operator \( \prod \) is defined as

\[
\prod_{i=1}^{k} u_i = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.
\]

Evaluate, showing a clear method

\[
\prod_{r=2}^{\infty} \left[ 1 - \frac{2}{r(r+1)} \right].
\]
Question 13

The product operator $\prod_{i=1}^{k}$ is defined as

$$\prod_{i=1}^{k} u_i = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k .$$

Find the value of

$$\prod_{r=1}^{\infty} \left[ \frac{(-1)^{r+1}}{e^r} \right] .$$

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Question 14

The product operator $\prod_{i=1}^k u_i$ is defined as

$$\prod_{i=1}^k u_i = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.$$ 

By showing a detailed method prove that

$$\prod_{k=1}^n \left[ \frac{2k - 1}{2k + 2} \right] = \binom{2n + 1}{n} \frac{1}{4^n (2n + 1)}.$$ 

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proof
Question 15

The product operator $\prod$ is defined as

$$\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.$$ 

Evaluate, showing a clear method

$$\prod_{r=2}^{\infty} \left[ \frac{r^3 - 1}{r^3 + 1} \right].$$
Question 16

The product operator $\prod_{i=1}^{k}$ is defined as

$$\prod_{i=1}^{k}u_i = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.$$  

Solve the equation

$$\prod_{r=1}^{\infty} \left( \frac{2\sqrt{x}}{2} \right) = 2^{-(x+2)}.$$  

You may assume that the left hand side of the equation converges.

\[ x = -1 \]
Question 17

The product operator $\prod_{i=1}^{k}$ is defined as

$$\prod_{i=1}^{k}[u_{i}] = u_{1} \times u_{2} \times u_{3} \times u_{4} \times \ldots \times u_{k-1} \times u_{k}.$$ 

By showing a detailed method prove that if $n$ is even

$$\prod_{r=1}^{n}\left[\cos\frac{2\pi}{n} + \cos\frac{2\pi}{n} \cot\frac{(2r-1)\pi}{n}\right] = 1.$$
Question 18

The product operator \( \prod \) is defined as

\[
\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.
\]

A sequence of numbers, \( P(1), P(2), P(3) \ldots P(n) \) is defined by the equation

\[
P(n) = \frac{9}{10} \prod_{r=1}^{n} \left[ 1 + \sum_{k=1}^{r} 10^k \right]^{-1}.
\]

Express \( P(n) \) in a simplified form not involving a sigma or product operators.

\[
\boxed{P(n) = 1 - 0.1^{n+1}}
\]
Question 19

The product operator $\prod_{i=1}^{k}$, is defined as

$$\prod_{i=1}^{k} u_{i} = u_{1} \times u_{2} \times u_{3} \times u_{4} \times \ldots \times u_{k-1} \times u_{k}.$$ 

Show by a detailed method that

$$\prod_{r=1}^{\infty} \left[ 1 + \left( \frac{1}{4} \right)^{2^r} \right] = \frac{16}{15}.$$
Question 20

The product operator $\prod_{r=1}^{k}$ is defined as

$$\prod_{r=1}^{k} [u_r] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.$$ 

The integer $Z$ is a square number and defined as

$$Z = \prod_{r=1}^{20} \left( \frac{r!}{n!} \right), \quad \{n \in \mathbb{N} : 1 \leq n \leq 20\}.$$ 

By considering the terms inside the product operator in pairs, or otherwise, determine a possible value of $n$.

You must show a detailed method in this question.

\[ n = 10 \]
Question 21

\[ I = \int_0^1 \left[ \prod_{r=1}^{10} (x+r) \right] \left[ \sum_{r=1}^{10} \left( \frac{1}{x+r} \right) \right] \, dx. \]

Show by a detailed method that

\[ I = a \times b!, \]

where \( a \) and \( b \) are positive integers to be found.

The product operator \( \prod \) is defined as

\[ \prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k. \]
Question 22
The product operator $\prod_{i=1}^{k}$ is defined as

$$\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.$$ 

Show in detail that

$$\prod_{r=1}^{n} \left( \frac{1}{n - r + \frac{1}{2}} \right) = \frac{2^{2n+1} \times n!}{(2n+1)!}.$$
Question 23

The product operator $\prod_{i=1}^{k} u_i$ is defined as

$$\prod_{i=1}^{k} u_i = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.$$ 

Find the sum to infinity of the following expression

$$\sum_{k=1}^{\infty} \prod_{r=1}^{k} \left(\frac{8r - 7}{40r}\right).$$ 

$$\therefore \frac{5}{4} - 1$$
Question 24

The product operator \( \prod \) is defined as

\[
\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.
\]

a) By considering the sine double angle identity show that

\[
\frac{\sin x}{x} = \prod_{k=1}^{\infty} \cos\left(\frac{x}{2^k}\right).
\]

b) Deduce that

\[
\frac{2}{\cos\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{8}\right) \cos\left(\frac{\pi}{16}\right) \cos\left(\frac{\pi}{32}\right) \ldots} = \pi.
\]

\[\Box\] proof
Question 25

The product operator \( \prod \) is defined as

\[
\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k.
\]

By writing \( \sin x \) as an infinite factorized polynomial, where each of the factors represents a zero of \( \sin x \), derive Wallis’s formula

\[
\frac{\pi}{2} = \prod_{r=1}^{\infty} \frac{4r^2}{4r^2 - 1}.
\]

You may assume without proof that

\[
\lim_{x \to 0} \left[ \frac{\sin x}{x} \right] = 1.
\]
Question 26

The product operator $\prod_{i=1}^{k}[u_i]$ is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times \ldots \times u_{k-1} \times u_k.$$ 

The function $f(n,k)$ is defined as

$$f(n,k) = \prod_{m=1}^{n} \left[ \frac{2n-2m+1}{n!} \right] \prod_{r=1}^{k} \left[ \frac{n-r+1}{2n-2r+1} \right] \prod_{l=1}^{k} \left[ \frac{1}{(2l)!} \right], \quad n \geq 2k.$$ 

Show by a detailed method that

$$f(n,k) = \frac{(2n-2k)!}{2^n k!(n-k)!(n-2k)!}.$$ 

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**proof**