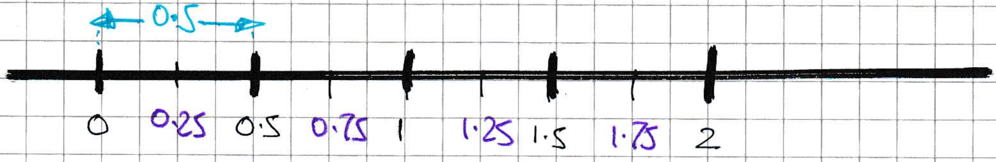
IYGB - SYNF PAPER C - QUESTION 1

DRAW A TABLE OF VALUES BASED ON MIDPOINTS



x	0.25	0.75	1.25	1.75
3^x	$3^{0.25}$	$3^{0.75}$	$3^{1.25}$	$3^{1.75}$

USING THE MID-ORDINATE RULE FORMULA

$$\int_0^2 3^x \, dx \approx (\text{THICKNESS}) \times (\text{SUM OF ALL})$$

$$\approx 0.5 \times \left[3^{0.25} + 3^{0.75} + 3^{1.25} + 3^{1.75} \right]$$

$$\approx 7.191162\ldots$$

$$\approx 7.19$$

3 sf

IYGB – SYN F PAPER C – QUESTION 2

LINEARIZING AS FOLLOWS

$$y = \frac{A}{x^2} + B$$

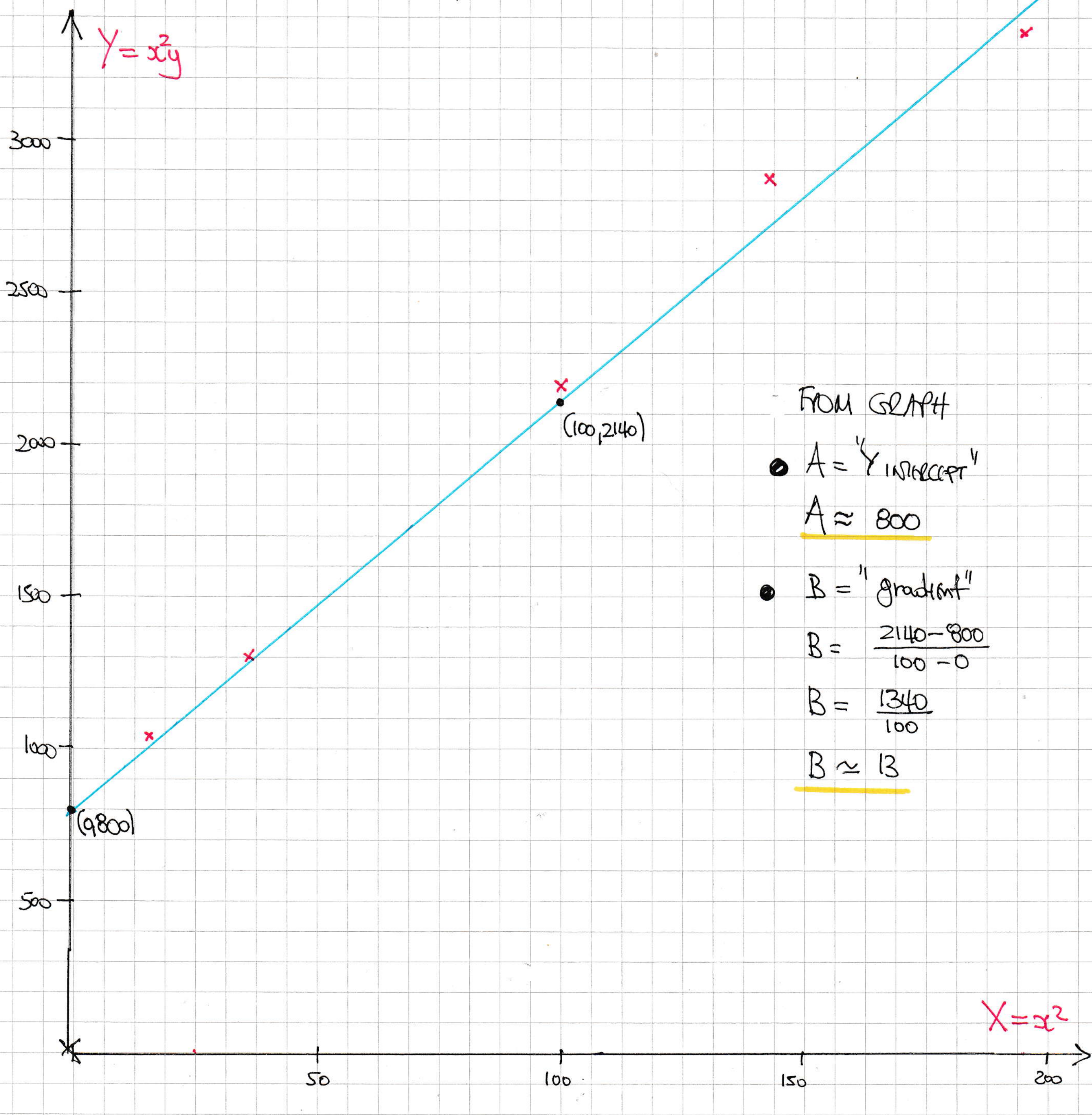
$$x^2 y = A + Bx^2$$

$$x^2 y = Bx^2 + A$$

$$Y = mX + C$$

$X = x^2$	16	36	100	144	196
$Y = x^2 y$	1056	1296	2200	2880	3332

PLOTTING ACCURATELY



FROM GRAPH

- A = "Y INTERCEPT"

  $A \approx 800$

- B = "gradient"

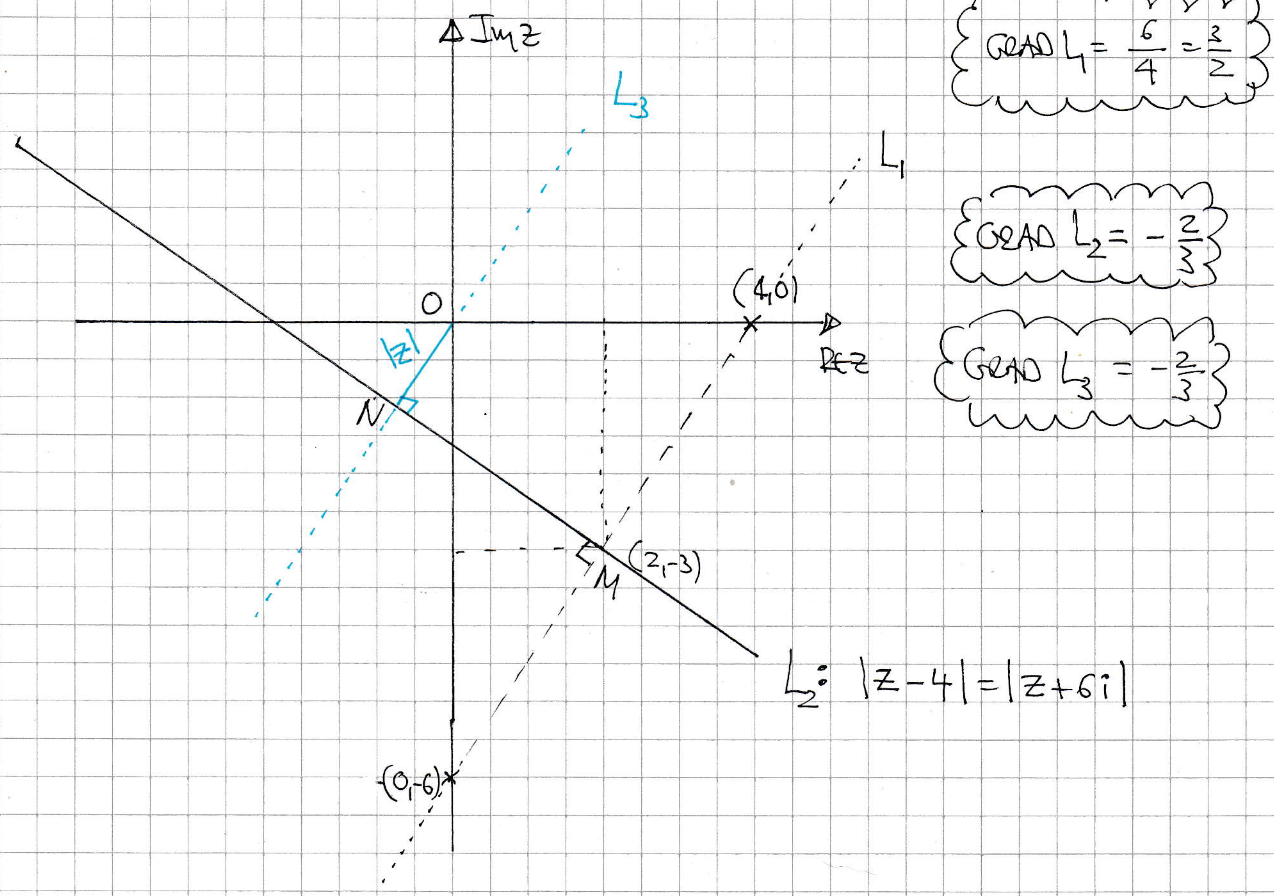
  $B = \frac{2140 - 800}{100 - 0}$

  $B = \frac{1340}{100}$

  $B \simeq 13$

IYGB - SYNF PAPER C - QUESTION 3

SKETCH THE STANDARD LOCI



FIND SOME EQUATIONS NEXT

$$\left.\begin{aligned} L_2: &\quad y+3 = -\tfrac{2}{3}(x-2) \\ L_3: &\quad y = \tfrac{3}{2}x \end{aligned}\right\} \Rightarrow \quad \frac{3}{2}x + 3 = -\frac{2}{3}x + \frac{4}{3}$$

$$9x + 18 = -4x + 8$$

$$13x = -10$$

$$x = -\frac{10}{13} \qquad y = \frac{15}{13}$$

$$\text{ie } N\left(-\frac{10}{13}, \frac{15}{13}\right)$$

$$-\frac{10}{13} + \frac{15}{13}i$$

FINALLY $|z|_{MIN} = |ON|$

$$= \left|-\frac{10}{13} + \frac{15}{13}i\right| = \frac{5}{13}\left|-2+3i\right| = \frac{5}{13}\sqrt{4+9} = \frac{5}{13}\sqrt{13}$$

$$= \frac{5}{\sqrt{13}}$$

IYGB - SYNF PAPER C - QUESTION 4

a) USING STANDARD FORMULAE FOR SUMS

$$\sum_{r=1}^{n}(r+1)(r+5) = \sum_{r=1}^{n}(r^2+6r+5) = \sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r + 5\sum_{r=1}^{n} 1$$

$$= \frac{1}{6}n(n+1)(2n+1) + 6\times\frac{1}{2}n(n+1) + 5n$$

$$= \frac{1}{6}n(n+1)(2n+1) + 3n(n+1) + 5n$$

$$= \frac{1}{6}n\left[(n+1)(2n+1) + 18(n+1) + 30\right]$$

$$= \frac{1}{6}n\left[2n^2+3n+1+18n+18+30\right]$$

$$= \frac{1}{6}n\left[2n^2+21n+49\right]$$

$$= \frac{1}{6}n(2n+7)(n+7)$$

// AS REQUIRED

b) USING THE RESULT OF PART (a)

$$\sum_{r=11}^{40}(r+1)(r+5) = \sum_{r=1}^{40}(r+1)(r+5) - \sum_{r=1}^{10}(r+1)(r+5)$$

$$= \frac{1}{6}\times 40\times 87\times 47 - \frac{1}{6}\times 10\times 27\times 17$$

$$= 27260 - 765$$

$$= 26495$$

// AS REQUIRED

IYGB – SYNF PAPER C – QUESTION 5

DETERMINE THE VALUE OF Z IN CARTESIAN FORM

$$z = i(1+i)(1-2i)^2 = (i+i^2)(1-4i+4i^2)$$
$$= (-1+i)(-3-4i)$$
$$= 3+4i-3i+4$$
$$= 7+i$$

SUBSTITUTE INTO THE GIVEN RELATIONSHIP

$$\Rightarrow \overline{z-3i} + P(z-3i) = Q\bar{z}$$
$$\Rightarrow \overline{7+i-3i} + P(7+i-3i) = Q(\overline{7+i})$$
$$\Rightarrow \overline{7-2i} + P(7-2i) = Q(7-i)$$
$$\Rightarrow 7+2i + 7P - 2Pi = 7Q - Qi$$

EQUATE REAL AND IMAGINARY PARTS

REAL: $7+7P = 7Q$ IMAGINARY: $2-2P = -Q$

$1 + P = Q$

SOLVING BY SUBSTITUTION

$$2-2P = -1-P$$
$$3 = P$$
$$P = 3 \quad \& \quad Q = 4$$

IYGB – SYNF PAPER C – QUESTION 6

NOTING THAT $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$\Rightarrow \sin 2x = 1 + \cos 2x$$

$$\Rightarrow \sin 2x - \cos 2x = 1$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin 2x - \frac{1}{\sqrt{2}}\cos 2x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\frac{\pi}{4}\sin 2x - \sin\frac{\pi}{4}\cos 2x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(2x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$\sin(A-B) \equiv \sin A\cos B - \cos A\sin B$

SETTING UP THE SOLUTION:

$$2x - \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}$$

$\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

$$2x = \frac{\pi}{4} + n\pi + (-1)^n\frac{\pi}{4}$$

$$2x = \frac{\pi}{4}\left[1 + 4n + (-1)^n\right]$$

$$x = \frac{\pi}{8}\left[4n + 1 + (-1)^n\right]$$

IE $f(n) = 4n + 1 + (-1)^n$

IYGB - FURTHER SYNOPTIC PAPER C - QUESTION 7

a) SOLVE SIMULTANEOUS EQUATIONS OR USE INVERSES

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix}$$

$$\Rightarrow \underline{B}\ \underline{A} = \underline{C}$$

$$\Rightarrow \underline{B}\ \underline{A}\ \underline{A}^{-1} = \underline{C}\underline{A}^{-1}$$

$$\Rightarrow \underline{B}\ \underline{I} = \underline{C}\underline{A}^{-1}$$

$$\Rightarrow \underline{B} = \begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix}\frac{1}{-4-2}\begin{pmatrix} 4 & -1 \\ -2 & -1 \end{pmatrix}$$

$$\Rightarrow \underline{B} = \frac{1}{6}\begin{pmatrix} 0 & 6 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 2 & 1 \end{pmatrix}$$

$$\Rightarrow \underline{B} = \frac{1}{6}\begin{pmatrix} 12 & 6 \\ -6 & 0 \end{pmatrix}$$

$$\Rightarrow \underline{B} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

b) FIRSTLY LOOK FOR LINE OF INVARIANT POINTS

$$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} 2x+y \\ -x \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$\therefore\ y = -x$

IS LINE OF INVARIANT POINTS

NOW LOOK FOR INVARIANT LINES

$$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} t \\ mt+c \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{pmatrix} 2t+mt+c \\ -t \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix}$$

IYGB - SYNF PAPER C - QUESTION 7

$$\Rightarrow \begin{matrix} 2t + mt + c = X \\ -t = Y \end{matrix}$$

$$\Rightarrow \begin{matrix} 2t + mt = X - c \\ t = -Y \end{matrix}$$

$$\Rightarrow \frac{2+m}{1} = \frac{X-c}{-Y}$$

$$\Rightarrow Y = -\frac{1}{2+m}X - \frac{1}{2+m}c$$

COMPARE WITH $Y = mX + c$

$$m = \frac{-1}{2+m}$$

$$2m + m^2 = -1$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m = -1$$

$$\therefore Y = -\frac{1}{2-1}X + \frac{1}{2-1}c$$

$$Y = -X + c$$

$\therefore$ ALSO INVARIANT LINES PARALLEL TO $y = -x$

c) INVESTIGATE $\underline{B}$

$\det \underline{B} = (2\times 0) - (-1\times 1) = 1$ (AREA INVARIANT)

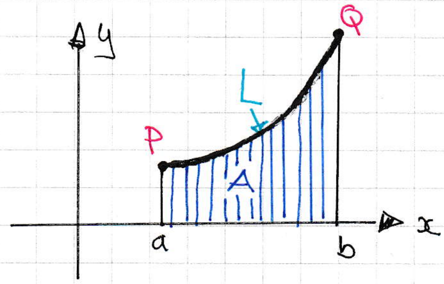
POSITIVE DETERMINANT $\Rightarrow$ ROTATION OR SHEAR (NO REFLECTION)

INVARIANT LINE OR INVARIANT LINE OF POINTS $\Rightarrow$ SHEAR (NO ROTATION)

$\therefore$ $\underline{B}$ REPRESENTS A SHEAR, WHERE $y = -x$ IS INVARIANT LINE OF POINTS

IYGB - SYNF PAPER C - QUESTION 8

LOOKING AT THE DIAGRAM



$$A = \int_a^b y(x)\,dx$$

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

NOW WE HAVE A = L (NUMERICALLY)

$$\Rightarrow \int_a^b y(x)\,dx = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\Rightarrow \frac{d}{dx}\left[\int_a^b y(x)\,dx\right] = \frac{d}{dx}\left[\int_a^b \sqrt{1+\frac{dy}{dx}}\,dx\right]$$

$$\Rightarrow y = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow y^2 = 1+\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = y^2-1$$

$$\Rightarrow \frac{dy}{dx} = \pm\sqrt{y^2-1}$$

SOLVING THE SEPARABLE O.D.E

$$\Rightarrow \frac{1}{\sqrt{y^2-1}}\,dy = \pm 1\,dx$$

$$\Rightarrow \int \frac{1}{\sqrt{y^2-1}}\,dy = \int \pm 1\,dx$$

IYGB – SYNF PAPER C – QUESTION 8

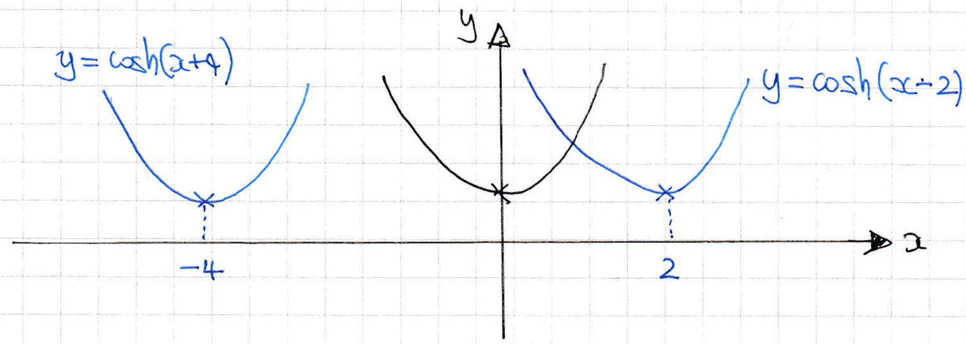
$$\Longrightarrow \operatorname{arcosh} y = \pm x + C$$

$$\Longrightarrow y = \cosh(\pm x + C)$$

BUT cosh IS EVEN SO +C REPRESENTS A HORIZONTAL TRANSLATION

$$y = \cosh(x + C)$$

NOTE THE CONSTANT DOES NOT AFFECT THE SOLUTION, AS IT REPRESENTS A HORIZONTAL TRANSLATION



IYGB - SYNF PAPER C - QUESTION 9

EXAMINING THE DETERMINANT OF $\underline{A}$

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 4 & 5 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix}$$

$$= -8 - 2(2) + 2(6) = -8 - 4 + 12 = 0$$

$\therefore$ NO UNIQUE SOLUTION

WRITE THE SYSTEM AS AN AUGMENTED MATRIX

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 1 & 3 & 3 \\ 4 & 5 & 7 & b \end{array}\right] \xrightarrow[r_{13}(-4)]{r_{12}(-2)} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -3 & -1 & 1 \\ 0 & -3 & -1 & b-4 \end{array}\right]$$

FOR CONSISTENCY WE REQUIRE A ZERO ROW WHICH EVIDENTLY OCCURS WHEN $b-4=1$, IE $b=5$.

CONTINUING THE ROW REDUCTION WITHOUT THE BOTTOM ROW

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -3 & -1 & 1 \end{array}\right] \xrightarrow{r_2(-\frac{1}{3})} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \end{array}\right] \xrightarrow{r_{21}(-2)} \left[\begin{array}{ccc|c} 1 & 0 & \frac{4}{3} & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \end{array}\right]$$

EXTRACTING THE SOLUTION

$$\left.\begin{array}{l} x + \frac{4}{3}z = \frac{5}{3} \\ y + \frac{1}{3}z = -\frac{1}{3} \end{array}\right\} \Longrightarrow \begin{array}{l} x = \frac{5}{3} - \frac{4}{3}z \\ y = -\frac{1}{3} - \frac{1}{3}z \end{array}$$

$\Longrightarrow$ LET $z = t$

$$\Longrightarrow \left\{\begin{array}{l} x = \frac{5}{3} - \frac{4}{3}t \\ y = -\frac{1}{3} - \frac{1}{3}t \\ z = t \end{array}\right.$$

IYGB - SYNF PAPER C - QUESTION 9

LOOKING AT THE REQUIRED FORM OF THE SOLUTION, WE LET $t=-1-3\lambda$ (BY LOOKING AT z)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{3}-\frac{4}{3}t \\ -\frac{1}{3}-\frac{1}{3}t \\ t \end{bmatrix} = \begin{bmatrix} \frac{5}{3}-\frac{4}{3}(-1-3\lambda) \\ -\frac{1}{3}-\frac{1}{3}(-1-3\lambda) \\ -1-3\lambda \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{3}+\frac{4}{3}+4\lambda \\ -\frac{1}{3}+\frac{1}{3}+\lambda \\ -1-3\lambda \end{bmatrix} = \begin{bmatrix} 3+4\lambda \\ \lambda \\ -1-3\lambda \end{bmatrix}$$

AS REQUIRED

IYGB - SYNF PAPER C - QUESTION 10

a) $x = \frac{3}{2}\left(t + \frac{1}{t}\right)$ $\quad$ $y = \frac{5}{2}\left(t - \frac{1}{t}\right)$

$t + \frac{1}{t} = \frac{2x}{3}$ $\quad$ $t - \frac{1}{t} = \frac{2y}{5}$

ADDING THE EQUATIONS

$2t = \frac{2x}{3} + \frac{2y}{5}$

SUBTRACT THE EQUATIONS

$\frac{2}{t} = \frac{2x}{3} - \frac{2y}{5}$

MULTIPLY THE EXPRESSIONS

$$(2t)\left(\frac{2}{t}\right) = \left(\frac{2x}{3} + \frac{2y}{5}\right)\left(\frac{2x}{3} - \frac{2y}{5}\right)$$

$$4 = \left(\frac{2x}{3}\right)^2 - \left(\frac{2y}{5}\right)^2$$

$$4 = \frac{4x^2}{9} - \frac{4y^2}{25}$$

$$\frac{x^2}{9} - \frac{y^2}{25} = 1$$

AS REQUIRED

b) USING STANDARD RESULTS FOR $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a = 3$ $\quad$ $b = 5$

I) ASYMPTOTES $y = \pm \frac{b}{a}x$

$$y = \pm \frac{5}{3}x$$

II) FIND THE ECCENTRICITY

$$b^2 = a^2(e^2 - 1)$$

$$25 = 9(e^2 - 1)$$

$$\frac{25}{9} = e^2 - 1$$

$$e^2 = \frac{34}{9}$$

$$e = +\frac{\sqrt{34}}{3}$$

FOCI AT $(\pm ae)$

$\pm\left(3 \times \frac{\sqrt{34}}{3}, 0\right)$ I.E. $(\pm\sqrt{34}, 0)$

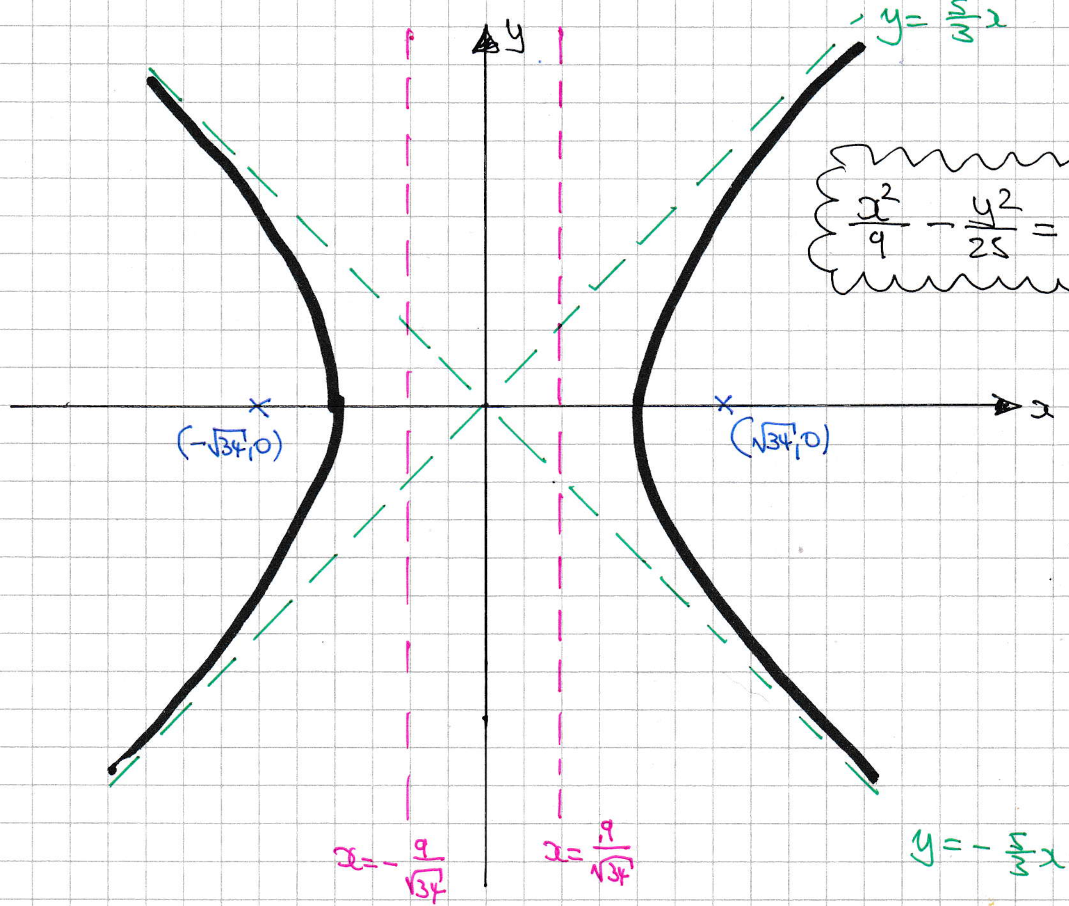
DIRECTRICES AT $x = \pm \frac{a}{e}$

$$x = \pm \frac{3}{\sqrt{34}/3}$$

$$x = \pm \frac{9}{\sqrt{34}}$$

IYGB – SYNF PAPER C – QUESTION 10

c)



IYGB - SYNF PAPER C - QUESTION 11

STARTING WITH THE QUADRATIC

If $x^2+3x+3=0$ $\Rightarrow$ $\alpha+\beta = -\frac{b}{a} = \frac{-3}{1} = -3$

$\Rightarrow$ $\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

NOW FORM THE CUBIC AS FOLLOWS - LET THE ROOTS BE A, B & C

- $A+B+C = \frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\alpha\beta = \frac{\alpha^2+\beta^2}{\alpha\beta}+\alpha\beta = \frac{\alpha^2+\beta^2}{3}+3$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{3}+3 = \frac{(-3)^2-2\times3}{3}+3 = 4$

- $AB+BC+CA = \frac{\alpha}{\beta}\times\frac{\beta}{\alpha}+\frac{\beta}{\alpha}(\alpha\beta)+\alpha\beta\times\frac{\alpha}{\beta} = 1+\beta^2+\alpha^2$

$= (\alpha+\beta)^2-2\alpha\beta+1 = (-3)^2-2\times3+1 = 4$

- $ABC = \frac{\alpha}{\beta}\times\frac{\beta}{\alpha}\times\alpha\beta = \alpha\beta = 3$

FINALLY WE HAVE

$$x^3-(4x^2)+(4x)-(3)=0$$

$$x^3-4x^2+4x-3=0$$

IYGB – FURTHER SYNOPTIC PAPER C – QUESTION 12

$$\frac{dy}{dx} = 3x^2y + x^5 \qquad x=0\ ,\ y=1$$

a) WRITE INFORMATION IN THE USUAL NOTATION

$$\Longrightarrow y_{n+1} \approx h y'_n + y_n \qquad x_0=0\ ,\ y_0=1\ ,\ h=0.1$$

$$\Longrightarrow \boxed{y_{n+1} \approx h\left(3x_n^2y_n + x_n^5\right) + y_n}$$

USING THE ABOVE FORMULA TWICE

$$\Longrightarrow y_1 \approx h\left[3\times x_0^2\times y_0 + x_0^5\right] + y_0$$

$$y_1 \approx 0.1\left[3\times 0^2\times 1 + 0^5\right] + 1$$

$$y_1 \approx 1$$

$$\Longrightarrow y_2 \approx h\left[3\times x_1^2\times y_1 + x_1^5\right] + y_1$$

$$y_2 \approx 0.1\left[3\times 0.1^2\times 1 + 0.1^5\right] + 1$$

$$y_2 \approx 1.003001$$

b) WRITE THE O.D.E IN THE USUAL ORDER

$$\Longrightarrow \frac{dy}{dx} - 3x^2y = x^5$$

INTEGRATING FACTOR $= e^{\int -3x^2\,dx} = e^{-x^3}$

$$\Longrightarrow \frac{d}{dx}\left(ye^{-x^3}\right) = x^5e^{-x^3}$$

$$\Longrightarrow ye^{-x^3} = \int x^5e^{-x^3}\,dx$$

IYGB – SYNF PAPER C – QUESTION 12

$$\Longrightarrow \; y e^{-x^3} = \int x^3 \left(x^2 e^{-x^3}\right)\,dx$$

INTEGRATION BY PARTS

$$\Longrightarrow \; y e^{-x^3} = -\frac{1}{3}x^3 e^{-x^3} - \int -x^2 e^{-x^3}\,dx$$

$$\Longrightarrow \; y e^{-x^3} = -\frac{1}{3}x^3 e^{-x^3} + \int x^2 e^{-x^3}\,dx$$

$$\Longrightarrow \; y e^{-x^3} = -\frac{1}{3}x^3 e^{-x^3} - \frac{1}{3}e^{-x^3} + A$$

$$\Longrightarrow \; \boxed{y = Ae^{x^3} - \frac{1}{3}x^3 - \frac{1}{3}}$$

x^3	$3x^2$
$-\frac{1}{3}e^{-x^3}$	$x^2 e^{-x^3}$

APPLY CONDITIONS $x=0,\ y=1$

$$\Longrightarrow \; 1 = A - \frac{1}{3}$$

$$\Longrightarrow \; A = \frac{4}{3}$$

$$\Longrightarrow \; y = \frac{1}{3}\left[4e^{x^3} - x^3 - 1\right]$$

FINALLY APPLY THE SOLUTION AT $x = 0.2$

$$y\Big|_{x=0.2} = \frac{1}{3}\left[4\times e^{0.2^3} - 0.2^3 - 1\right] = 1.008042781\ldots$$

$$\approx 1.008$$

IYGB - SYNF PAPER C - QUESTION 13

a) PROCEED AS FOLLOWS

$$5\sinh w + 7\cosh w \equiv R\cosh(w+a)$$
$$\equiv R\cosh w\cosh a + R\sinh w\sinh a$$
$$\equiv (R\cosh a)\cosh w + (R\sinh a)\sinh w$$

COMPARING SIDES WE OBTAIN

$$\left.\begin{array}{l} R\cosh a = 7 \\ R\sinh a = 5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} R^2\cosh^2 a = 49 \\ R^2\sinh^2 a = 25 \end{array}\right\} \Rightarrow R^2(\cosh^2 a - \sinh^2 a) = 24$$
$$\Rightarrow R^2 = 24$$
$$\Rightarrow R = +2\sqrt{6}$$

AND BY DIVIDING THE EQUATIONS ABOVE

$$\frac{R\sinh a}{R\cosh a} = \frac{5}{7} \Rightarrow \tanh a = \frac{5}{7}$$
$$\Rightarrow a = \operatorname{artanh}\frac{5}{7}$$
$$\Rightarrow a = \frac{1}{2}\ln\left(\frac{1+\frac{5}{7}}{1-\frac{5}{7}}\right) = \frac{1}{2}\ln\left(\frac{7+5}{7-5}\right) = \frac{1}{2}\ln 6$$
$$\Rightarrow a = \ln\sqrt{6}$$

$$\therefore\ 5\sinh w + 7\cosh w \equiv 2\sqrt{6}\cosh(w + \ln\sqrt{6})$$

b) NOW SOLVING THE EQUATION USING THE RESULT OF PART (a)

$$\Rightarrow 5\sinh w + 7\cosh w = 5$$
$$\Rightarrow 2\sqrt{6}\cosh(w+\ln\sqrt{6}) = 5$$
$$\Rightarrow \cosh(w+\ln\sqrt{6}) = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$$
$$\Rightarrow w + \ln\sqrt{6} = \pm\operatorname{arcosh}\left(\frac{5\sqrt{6}}{12}\right)$$

IYGB – FURTHER SYNOPTIC PAPER C – QUESTION 13

$$\Rightarrow W = \begin{cases} -\ln\sqrt{6} - \operatorname{arcosh}\left(\frac{5\sqrt{6}}{12}\right) \\ -\ln\sqrt{6} + \operatorname{arcosh}\left(\frac{5\sqrt{6}}{12}\right) \end{cases}$$

$$\Rightarrow W = \begin{cases} -\ln\sqrt{6} - \ln\left[\frac{5\sqrt{6}}{12} + \sqrt{\frac{25\times6}{144} - 1}\right] \\ -\ln\sqrt{6} + \ln\left[\frac{5\sqrt{6}}{12} + \sqrt{\frac{25\times6}{144} - 1}\right] \end{cases}$$

$$\Rightarrow W = \begin{cases} -\ln\sqrt{6} - \ln\left(\frac{5\sqrt{6}}{12} + \sqrt{\frac{1}{24}}\right) \\ -\ln\sqrt{6} + \ln\left(\frac{5\sqrt{6}}{12} + \sqrt{\frac{1}{24}}\right) \end{cases}$$

$$\Rightarrow W = \begin{cases} -\ln\sqrt{6} - \ln\left(\frac{5\sqrt{6}}{12} + \frac{\sqrt{6}}{12}\right) \\ -\ln\sqrt{6} + \ln\left(\frac{5\sqrt{6}}{12} + \frac{\sqrt{6}}{12}\right) \end{cases}$$

$$\Rightarrow W = \begin{cases} -\ln\sqrt{6} - \ln\left(\tfrac{1}{2}\sqrt{6}\right) \\ -\ln\sqrt{6} + \ln\left(\tfrac{1}{2}\sqrt{6}\right) \end{cases}$$

$$\Rightarrow W = \begin{cases} -\left[\ln\sqrt{6} + \ln\left(\tfrac{1}{2}\sqrt{6}\right)\right] \\ \ln\left(\frac{\frac{1}{2}\sqrt{6}}{\sqrt{6}}\right) \end{cases}$$

$$\Rightarrow W = \begin{cases} -\ln 3 \\ \ln\frac{1}{2} = -\ln 2 \end{cases}$$

IYGB - SYNF PAPER C - QUESTION 14

OBTAIN THE COMPLEMENTARY FUNCTION — VIA AUXILIARY EQUATION

$$\lambda^2 - 2k\lambda + k^2 = 0$$
$$(\lambda - k)^2 = 0$$
$$\lambda = k \text{ (REPEATED)}$$

$$\therefore\ y = Ae^{kx} + Bxe^{kx}$$

FOR PARTICULAR INTEGRAL TRY $y = P = \text{CONSTANT}$

$$\frac{dy}{dx} = \frac{d^2y}{dx^2} = 0$$

SUB INTO THE O.D.E.

$$0 + 0 + k^2 P \equiv \frac{1}{4}$$
$$P = \frac{1}{4k^2}$$

$\therefore$ GENERAL SOLUTION

$$y = Ae^{kx} + Bxe^{kx} + \frac{1}{4k^2}$$

IYGB - SYNOPTIC FURTHER MATHS PAPER C - QUESTION 15

a) WRITE THE EQUATIONS IN PARAMETRIC FORM

$$\underline{r}_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \begin{pmatrix} a & +\lambda \\ -4 & -4\lambda \\ 0 & -2\lambda \end{pmatrix} \qquad \underline{r}_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a & +2\mu \\ -1 & -5\mu \\ 1 & -3\mu \end{pmatrix}$$

EQUATE $\underline{j}$ & $\underline{k}$ COMPONENTS

$$\left.\begin{array}{l} \underline{j}: \ -4-4\lambda = -1-5\mu \quad \times 1 \\ \underline{k}: \ -2\lambda = 1-3\mu \quad \times(-2) \end{array}\right\} \Longrightarrow \left.\begin{array}{r} -4-4\lambda = -1-5\mu \\ 4\lambda = -2+6\mu \end{array}\right\} \text{ADD}$$

$$-4 = -3+\mu$$
$$\underline{\mu = -1}$$

$$\& \quad 4\lambda = -2-6$$
$$\underline{\lambda = -2}$$

CHECKING $\underline{i}$ FOR CONSISTENCY

$$a+\lambda = a-2$$
$$a+2\mu = a+2(-1) = a-2$$

$\therefore$ IF $\lambda=-2$, $\mu=-2$ ALL 3 COMPONENTS AGREE, SO LINES INTERSECT FOR ALL a

b) USING PART (a)

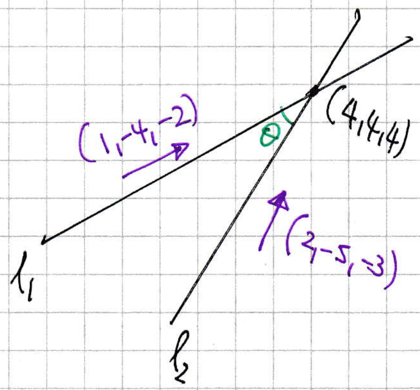
IF $\lambda=-2$ & $\mu=-1$ THE INTERSECTION WILL BE

$$(a-2, 4, 4)$$

$\therefore$ $\underline{a=6}$ & $\underline{b=4}$

IYGB - SYNF PAPER C - QUESTION 15

c) DOTTING THE DIRECTION VECTORS OF l_1 & l_2



$\Longrightarrow (1,-4,-2)\cdot(2,-5,-3) = |1,-4,-2||2,-5,-3|\cos\theta$

$\Longrightarrow 2+20+6 = \sqrt{1+16+4}\sqrt{4+25+9}\cos\theta$

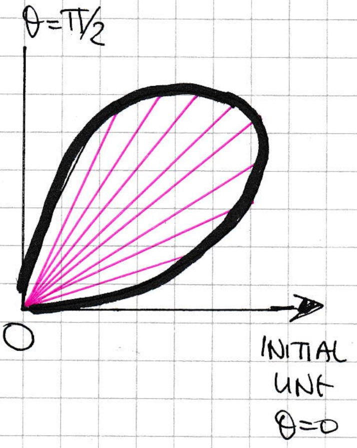
$\Longrightarrow 28 = \sqrt{21}\sqrt{38}\cos\theta$

$\Longrightarrow \cos\theta = \dfrac{28}{\sqrt{21}\sqrt{38}}$

$\Longrightarrow \theta = 7.6°$

IYGB - SYNF PAPER C - QUESTION 16

LOOKING AT THE LOOP ON THE RIGHT



$$\text{AREA} = \frac{1}{2}\int_{\theta_1}^{\theta_2} (r(\theta))^2 \, d\theta$$

$$\text{AREA} = \frac{1}{2}\int_{\theta=0}^{\theta=\pi/2} \left[2\sin 2\theta \sqrt{\cos\theta}\,\right]^2 d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} 4\sin^2 2\theta \cos\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} 4(2\sin\theta\cos\theta)^2 \cos\theta \, d\theta$$

$$= \int_0^{\pi/2} 8\sin^2\theta \cos^2\theta \cos\theta \, d\theta$$

MANIPULATE AS FOLLOWS, OR USE THE SUBSTITUTION $u=\sin\theta$

$$= \int_0^{\pi/2} 8\sin^2\theta(1-\sin^2\theta)\cos\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} 8\sin^2\theta\cos\theta - 8\sin^4\theta\cos\theta \, d\theta$$

BY RECOGNITION WE HAVE

$$= \left[\frac{8}{3}\sin^3\theta - \frac{8}{5}\sin^5\theta\right]_0^{\pi/2}$$

$$= \left(\frac{8}{3}-\frac{8}{5}\right) - (0-0)$$

$$= 8\left(\frac{1}{3}-\frac{1}{5}\right)$$

$$= \frac{16}{15}$$

AS REQUIRED

IYGB – SYNF PAPER C – QUESTION 17

a) LOOKING AT THE EQUATION GIVEN

$\Longrightarrow z^7 - 1 = 0$

$\Longrightarrow z^7 = 1$

$\Longrightarrow z^7 = e^{2k\pi i} \qquad k = 0, 1, 2, \dots, 6$

$\Longrightarrow z = e^{\frac{2k\pi}{7} i}$

$\therefore \omega = z_1 = e^{\frac{2\pi}{7} i}$

b) LOOKING AGAIN AT THE POLYNOMIAL GIVEN

$$z^7 - 1 = 0$$
$$(z-1)(z^6 + z^5 + z^4 + z^3 + z^2 + z + 1) = 0$$

EITHER $z = 1$ OR $1 + z + z^2 + z^3 + z^4 + z^5 + z^6 = 0$

OR WRITTEN IN ω

$$1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = 0$$

c) FROM PART (a) WE HAVE

$$\omega^2 + \omega^5 = \left(e^{i\frac{2\pi}{7}}\right)^2 + \left(e^{i\frac{2\pi}{7}}\right)^5 = e^{i\frac{4\pi}{7}} + e^{i\frac{10\pi}{7}}$$
$$= e^{i\frac{4\pi}{7}} + e^{-\frac{4\pi}{7}} = 2\cosh\left(i\frac{4\pi}{7}\right) = 2\cos\left(\frac{4\pi}{7}\right)$$

AS REQUIRED

d) SIMILARLY WE ALSO HAVE

- $\omega + \omega^6 = e^{i\frac{2\pi}{7}} + \left(e^{i\frac{2\pi}{7}}\right)^6 = e^{i\frac{2\pi}{7}} + e^{\frac{12\pi}{7} i} = e^{i\frac{2\pi}{7}} + e^{-\frac{2\pi}{7} i}$

$$= 2\cosh\left(i\frac{2\pi}{7}\right) = 2\cos\left(\frac{2\pi}{7}\right)$$

- $\omega^3 + \omega^4 = \left(e^{i\frac{2\pi}{7}}\right)^3 + \left(e^{i\frac{2\pi}{7}}\right)^4 = e^{i\frac{6\pi}{7}} + e^{\frac{8\pi}{7} i} = e^{i\frac{6\pi}{7}} + e^{-\frac{6\pi}{7} i}$

$$= 2\cosh\left(i\frac{6\pi}{7}\right) = 2\cos\left(\frac{6\pi}{7}\right)$$

IYGB - SYNF PAPER C - QUESTION 17

FINALLY WE HAVE

$\Longrightarrow \ w^6 + w^5 + w^4 + w^3 + w^2 + w + 1 = 0$

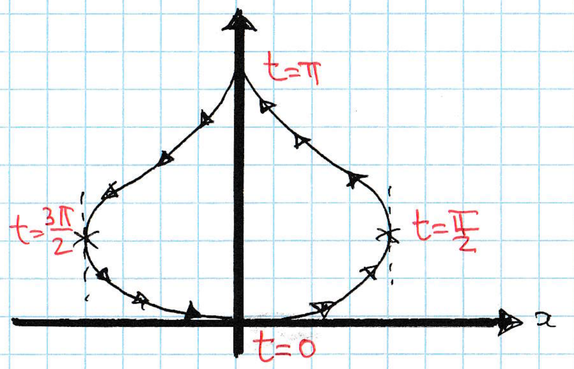
$\Longrightarrow \ (w^6 + w) + (w^5 + w^2) + (w^4 + w^3) = -1$

$\Longrightarrow \ 2\cos\frac{2\pi}{7} + 2\cos\frac{4\pi}{7} + 2\cos\frac{6\pi}{7} = -1$

$\Longrightarrow \ \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$ //

IYGB – FURTHER SYNOPTIC PAPER C – QUESTION 18

START BY TRACING THE CURVE, TO OBTAIN THE VALUES OF t AT DIFFERENT POINTS



SET A VOLUME INTEGRAL IN y (PARAMETRIC) BY REVOLVING THE "R.H.S" OF THE CURVE.

$$V = \pi\int_{y_1}^{y_2} [x(y)]^2\,dy = \pi\int_{t_1}^{t_2} [x(t)]^2 \frac{dy}{dt}\,dt$$

$$= \pi\int_0^{\pi} (\sin t)^2 (2t)\,dt = \pi\int_0^{\pi} 2t\sin^2 t\,dt$$

PROCEED BY TRIGONOMETRIC IDENTITIES, FOLLOWED BY INTEGRATION BY PARTS

$$\dots = \pi\int_0^{\pi} 2t\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2t\right)dt$$

$$= \pi\int_0^{\pi} t - t\cos 2t\,dt$$

$$= \pi\int_0^{\pi} t\,dt + \pi\int_0^{\pi} -t\cos 2t\,dt$$

$-t$	-1
$\frac{1}{2}\sin 2t$	$\cos 2t$

$$= \pi\left[\left[\tfrac{1}{2}t^2\right]_0^{\pi} + \left[-\tfrac{1}{2}t\sin 2t\right]_0^{\pi} - \int_0^{\pi} -\tfrac{1}{2}\sin 2t\,dt\right]$$

$$= \pi\left[\left[\tfrac{1}{2}t^2 - \tfrac{1}{2}t\sin 2t\right]_0^{\pi} + \int_0^{\pi} \tfrac{1}{2}\sin 2t\,dt\right]$$

IYGB - SYNF PAPER C - QUESTION 18

FINISHING OFF THE LAST INTEGRATION & EVALUATING

$$= \pi \left[\frac{1}{2}t^2 - \frac{1}{2}t\sin 2t - \frac{1}{4}\cos 2t \right]_0^{\pi}$$

$$= \pi \left[\left(\frac{1}{2}\pi^2 - 0 \cancel{- \frac{1}{4}}\right) - \left(0 - 0 \cancel{- \frac{1}{4}}\right) \right]$$

$$= \frac{1}{2}\pi^3$$

IYGB - SYNF PAPER C - QUESTION 19

LET THE THREE CONSECUTIVE INTEGERS BE

$$k-1, \quad k, \quad k+1$$

CUBING & ADDING GIVES

$$f(k) = (k-1)^3 + k^3 + (k+1)^3$$
$$= (k^3 \cancel{-3k^2} + 3k \cancel{-1}) + k^3 + (k^3 \cancel{+3k^2} + 3k \cancel{+1})$$
$$= 3k^3 + 6k$$
$$= 3k(k^2+2)$$

NOW k CAN TAKE ONE OF THE FOLLOWING 3 FORMS

$$k = 3n, \; 3n+1, \; 3n+2$$

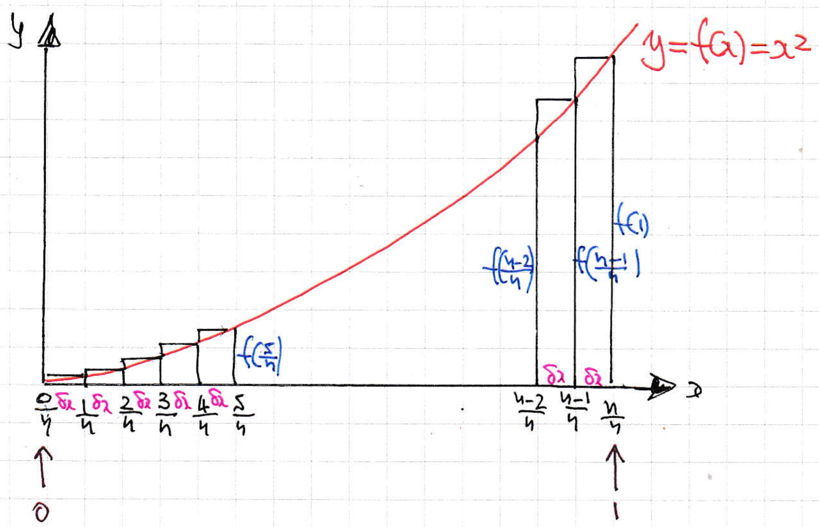
EXAMINING EACH CASE

- $f(k) = f(3n) = 3(3n)\left[(3n)^2+2\right] = 9n(9n^2+2)$
- $f(k) = f(3n+1) = 3(3n+1)\left[(3n+1)^2+2\right] = 3(3n+1)(9n^2+6n+1+2)$
  $= 3(3n+1)(9n^2+6n+3) = 9(3n+1)(3n^2+2n+1)$
- $f(k) = f(3n+2) = 3(3n+2)\left[(3n+2)^2+2\right] = 3(3n+2)\left[9n^2+12n+4+2\right]$
  $= 3(3n+2)(9n^2+12n+6) = 9(3n+2)(3n^2+4n+2)$

$\therefore$ THE SUM OF CUBES OF ANY 3 CONSECUTIVE POSITIVE INTEGERS WILL BE A MULTIPLE OF 9

IYGB – SYNF PAPER C – QUESTION 20

LOOKING AT THE DIAGRAM BELOW



- $\delta x = \frac{a-b}{n} = \frac{1-0}{n} = \frac{1}{n}$
- $x_i = a + i\,\delta x$
  $x_i = 0 + i\left(\frac{1}{n}\right)$
  $x_i = \frac{i}{n}$
- $f(x_i) = \frac{i^2}{n^2}$

NOW WE HAVE TO COMPUTE A LIMIT, AS $n \to \infty$

$$\int_0^1 x^2\,dx = \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left[\underbrace{\left(\frac{i^2}{n^2}\right)}_{f(x_i)}\underbrace{\left(\frac{1}{n}\right)}_{\delta x}\right]\right] = \lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{i^2}{n^3}\right]$$

AS THE SUM HAS i DEPENDENCE ONLY, I.E n IS A CONSTANT AS FAR AS THE SUMMATION GOES, WE HAVE

$$\int_0^1 x^2\,dx = \lim_{n\to\infty}\left[\frac{1}{n^3}\sum_{i=1}^{n} i^2\right] = \lim_{n\to\infty}\left[\frac{1}{n^3}\times\frac{1}{6}n(n+1)(2n+1)\right]$$

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

$$= \lim_{n\to\infty}\left[\frac{n(n+1)(2n+1)}{6n^3}\right] = \lim_{n\to\infty}\left[\frac{2n^3+3n^2+n}{6n^3}\right]$$

$$= \lim_{n\to\infty}\left[\frac{2+\frac{3}{n}+\frac{1}{n^2}}{6}\right] = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \int_0^1 x^2\,dx = \frac{1}{3}$$