IYGB - SYNOPTIC PAPER T - QUESTION 1

a) OBTAIN THE GRADIENT FIRST

$$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{7-1}{5-1} = \frac{6}{4} = \frac{3}{2}$$

FIND THE REQUIRED LINE USING STANDARD FORMULA, USING C(1,1)

$$\Longrightarrow y - y_0 = m(x - x_0)$$
$$\Longrightarrow y - 1 = \tfrac{3}{2}(x-1)$$
$$\Longrightarrow 2y - 2 = 3x - 3$$
$$\Longrightarrow 2y = 3x - 1$$

OR EQUIVALENT.

b) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{ll} l_1: 2y = 3x - 1 & \times 3 \\ l_2: 2x + 3y = 18 & \times 2 \end{array}\right\} \quad \left.\begin{array}{l} 6y = 9x - 3 \\ 4x + 6y = 36 \end{array}\right\} \Longrightarrow 4x + (9x-3) = 36$$
$$\Longrightarrow 13x = 39$$
$$\Longrightarrow x = 3$$

USING $2y = 3x - 1$

$$2y = 8$$
$$y = 4$$

$\therefore C(3,4)$

c) USING THE DISTANCE FORMULA WITH A(1,1) & B(5,7)

WHEN $x = -3 \Longrightarrow 2(-3) + 3y = 18$
$$\Longrightarrow 3y = 24$$
$$\Longrightarrow y = 8 \qquad \text{IE } D(-3,8)$$

- $|AD| = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(-3-1)^2 + (8-1)^2} = \sqrt{16 + 49} = \sqrt{65}$
- $|BD| = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(-3-5)^2 + (8-7)^2} = \sqrt{64 + 1} = \sqrt{65}$

INDEED $|AD| = |BD|$

IYGB - SYNOPTIC PAPER T - QUESTION 2

AREA OF A SECTOR

$$"A = \frac{1}{2}r^2\theta^c"$$

$$45 = \frac{1}{2}r^2 \times 2.5$$

$$90 = \frac{5}{2}r^2$$

$$r^2 = 36$$

$$r = +6 \text{ cm}$$

PERIMETER = ARCLENGTH + 2 RADII

$$P = "r\theta" + 2r$$

$$P = 6 \times 2.5 + 2 \times 6$$

$$P = 15 + 12$$

$$P = 27 \text{ cm}$$

IYGB - SYNOPTIC PAPER T - QUESTION 3

a) USING $S_\infty = \dfrac{a}{1-r}$

$$\Rightarrow 81920 = \frac{20480}{1-r}$$
$$\Rightarrow 4 = \frac{1}{1-r}$$
$\div 20480$

$$\Rightarrow 1-r = \frac{1}{4}$$
$$\frac{3}{4} = r$$

$\therefore r = \frac{3}{4}$ AS REQUIRED

b) USING $u_n = ar^{n-1}$

$$u_5 = 20480 \times \left(\tfrac{3}{4}\right)^4 = 6480$$
$$u_6 = 20480 \times \left(\tfrac{3}{4}\right)^5 = 4860$$

$\therefore$ diff IS 1620

c) USING $S_n = \dfrac{a(1-r^n)}{1-r} = S_\infty(1-r^n)$

$$\Rightarrow S_n > 80000$$
$$\Rightarrow 81920(1-0.75^n) > 80000$$
$$\Rightarrow 1-0.75^n > \frac{125}{128}$$
$$\Rightarrow -0.75^n > -\frac{3}{128}$$
$$\Rightarrow 0.75^n < \frac{3}{128}$$
$$\Rightarrow \log(0.75^n) < \log\left(\tfrac{3}{128}\right)$$
$$\Rightarrow n\log 0.75 < \log\left(\tfrac{3}{128}\right)$$
$$\Rightarrow n > \frac{\log 3/128}{\log 0.75}$$

DIVIDED BY NEGATIVE $\log 0.75 < 0$

$$\Rightarrow n > 13.047\ldots$$

$n = 14$

IYGB - SYNOPTIC PAPER T - QUESTION 4

a) COMPLETING THE SQUARE IN x & y

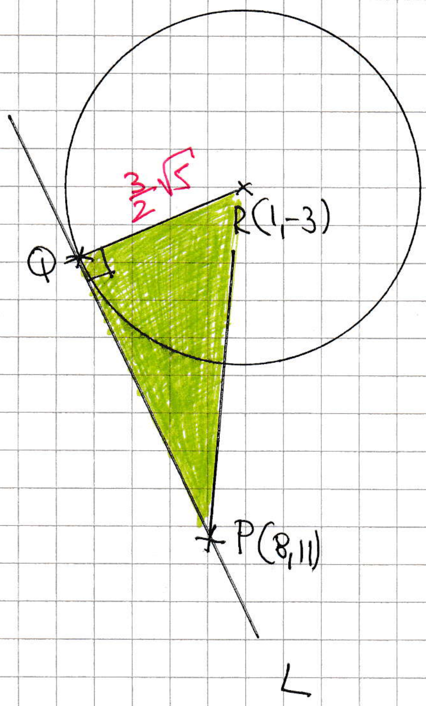
$$\Rightarrow 4x^2+4y^2-8x+24y-5=0$$
$$\Rightarrow x^2+y^2-2x+6y-\frac{5}{4}=0$$
$$\Rightarrow x^2-2x+y^2+6y-\frac{5}{4}=0$$
$$\Rightarrow (x-1)^2-1+(y+3)^2-9-\frac{5}{4}=0$$
$$\Rightarrow (x-1)^2+(y+3)^2=\frac{45}{4}$$

$\therefore$ CENTRE AT $(1,-3)$

b) $\text{RADIUS} = \sqrt{\frac{45}{4}} = \frac{\sqrt{45}}{\sqrt{4}} = \frac{\sqrt{45}}{2} = \frac{\sqrt{9}\sqrt{5}}{2} = \frac{3\sqrt{5}}{2}$

ie $k=\frac{3}{2}$

c) LOOKING AT DIAGRAM



- FIND THE DISTANCE PR

$$|PR| = \sqrt{(-3-11)^2+(8-1)^2}$$
$$|PR| = \sqrt{196+49}$$
$$|PR| = \sqrt{245}$$
$$|PR| = 7\sqrt{5}$$

- BY PYTHAGORAS

$$|QR|^2+|QP|^2=|PR|^2$$
$$\left(\tfrac{3}{2}\sqrt{5}\right)^2+|QP|^2=(7\sqrt{5})^2$$
$$\frac{45}{4}+|QP|^2=245$$
$$|QP|^2=\frac{935}{4}$$
$$|QP| = \frac{1}{2}\sqrt{935} \simeq 15.3$$

IYGB - SYN PAPER T - QUESTION 5

MANIPULATE JUST THE LHS OF THE EQUATION

$$\tan x + \cot x = 8\cos 2x$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = 8\cos 2x$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = 8\cos 2x$$

$$\frac{1}{\cos x \sin x} = 8\cos 2x$$

$$\frac{1}{2\sin x\cos x} = 4\cos 2x$$

$$\frac{1}{\sin 2x} = 4\cos 2x$$

$$4\cos 2x \sin 2x = 1$$

$$2\cos 2x \sin 2x = \frac{1}{2}$$

$$\sin 4x = \frac{1}{2}$$

NOW THE $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$$\begin{cases} 4x = \frac{\pi}{6} \pm 2n\pi \\ 4x = \frac{5\pi}{6} \pm 2n\pi \end{cases} \quad n = 0,1,2,3,\ldots$$

$$\begin{cases} x = \frac{\pi}{24} \pm \frac{n\pi}{2} \\ x = \frac{5\pi}{24} \pm \frac{n\pi}{2} \end{cases}$$

$$x_1 = \frac{\pi}{24}$$
$$x_2 = \frac{13\pi}{24}$$
$$x_3 = \frac{5\pi}{24}$$
$$x_4 = \frac{17\pi}{24}$$

IYGB, SYNOPTIC PAPER T – QUESTION 6

a) APPLY $f(2) = g(2) = 0$

$a(2^3+1) - b\times2\times3 = 0$
$9a - 6b = 0$
$3a = 2b$

$b\times2^3 - 5\times2^2 - 2a\times1 = 0$
$8b - 20 - 2a = 0$
$4b - 10 - a = 0$
$4b - 10 = a$

COMBINING THE EXPRESSIONS

$\Rightarrow 3(4b-10) = 2b$
$\Rightarrow 12b - 30 = 2b$
$\Rightarrow 10b = 30$
$\Rightarrow b = 3$
$\Rightarrow a = 2$

$\therefore a = 2,\ b = 3$

b) USING THE VALUES FOUND

$f(x) = 2(x^3+1) - 3x(x+1)$
$f(x) = 2x^3 - 3x^2 - 3x + 2$

$g(x) = 3x^3 - 5x^2 - 2\times2(x-1)$
$g(x) = 3x^3 - 5x^2 - 4x + 4$

BY LONG DIVISION OR MANIPULATIONS GIVEN $x-2$ IS A FACTOR

- $f(x) = 2x^2(x-2) + x(x-2) - (x-2)$
  $f(x) = (x-2)(2x^2+x-1)$
  $f(x) = (x-2)(2x-1)(x+1)$

- $g(x) = 3x^2(x-2) + x(x-2) - 2(x-2)$
  $g(x) = (x-2)(3x^2+x-2)$
  $g(x) = (x-2)(3x-2)(x+1)$

INDEED ANOTHER COMMON FACTOR $(x+1)$

IYGB — SYNOPTIC PAPER T — QUESTION 7

EXPAND BINOMIALLY UP TO x^2

$$\frac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}} = 1 + \frac{-\frac{1}{2}}{1}(-2x) + \frac{-\frac{1}{2}(-\frac{3}{2})}{1\times 2}(-2x)^2 + O(x^3)$$

$$= 1 + x + \frac{3}{2}x^2 + O(x^3)$$

NOW THE GIVEN EQUATION

$$\frac{12}{\sqrt{1-2x}} = 16 - 67x - 2x^2$$

$$12\left(1 + x + \frac{3}{2}x^2\right) \simeq 16 - 67x - 2x^2$$

$$12 + 12x + 18x^2 \simeq 16 - 67x - 2x^2$$

$$20x^2 + 79x - 4 \simeq 0$$

FACTORIZE OR QUADRATIC FORMULA

$$(20x + 1)(x - 4) = 0$$

$$x = \left\langle \begin{matrix} \frac{1}{20} \\ -4 \end{matrix} \right.$$

NOW THE VALIDITY OF THIS EXPANSION IS $-\frac{1}{2} < x < \frac{1}{2}$

$$\therefore \ x = \frac{1}{20}$$

(AS WELL AS $x=4$ CANNOT GO INTO THE RADICAL)

IYGB - SYNOPTIC PAPER T - QUESTION 8

$x = 1 - \cos\theta \quad \bullet \quad y = \sin\theta\sin2\theta \quad \bullet \quad 0 \leq \theta \leq \pi$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta\sin2\theta + 2\sin\theta\cos2\theta}{\sin\theta} = \frac{2\cos^2\theta\,\cancel{\sin\theta} + 2\sin\theta\cos2\theta}{\cancel{\sin\theta}}$$

$$\frac{dy}{dx} = 2\cos^2\theta + 2\cos2\theta$$

SOLVING FOR ZERO

$$\cancel{2}\cos^2\theta + \cancel{2}\cos2\theta = 0$$
$$\cos^2\theta + \cos2\theta = 0$$
$$\frac{1}{2} + \frac{1}{2}\cos2\theta + \cos2\theta = 0$$
$$\frac{3}{2}\cos2\theta = -\frac{1}{2}$$
$$\cos2\theta = -\frac{1}{3} \text{ ETC}$$

OR

$$\cancel{2}\cos^2\theta + \cancel{2}\cos2\theta = 0$$
$$\cos^2\theta + 2\cos^2\theta - 1 = 0$$
$$3\cos^2\theta = 1$$
$$\cos^2\theta = \frac{1}{3}$$
$$\cos\theta = \begin{cases} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \end{cases}$$

NOW AS θ IS BETWEEN 0 & π, $\sin\theta$ MUST BE POSITIVE.

$$\sin\theta = +\sqrt{1-\cos^2\theta}$$
$$\sin\theta = \sqrt{1-\frac{1}{3}}$$
$$\sin\theta = \sqrt{\frac{2}{3}}$$
$$\sin\theta = \sqrt{\frac{6}{9}}$$
$$\sin\theta = \frac{1}{3}\sqrt{6}$$

IYGB - SYNOPTIC PAPER T - QUESTION 8

REWRITE THE PARAMETRICS AS

$$x = 1 - \cos\theta \qquad y = 2\sin^2\theta\cos\theta$$

$\Longrightarrow \cos\theta = \frac{\sqrt{3}}{3}$

$\sin\theta = \frac{\sqrt{6}}{3}$

$x = 1 - \frac{\sqrt{3}}{3} = \frac{3-\sqrt{3}}{3}$

$y = 2 \times \frac{2}{3} \times \frac{\sqrt{3}}{3} = \frac{4}{9}\sqrt{3}$

$\therefore \left(\frac{3-\sqrt{3}}{3}, \frac{4}{9}\sqrt{3}\right)$ //

$\Longrightarrow \cos\theta = -\frac{\sqrt{3}}{3}$

$\sin\theta = \frac{\sqrt{6}}{3}$

$x = 1 + \frac{\sqrt{3}}{3} = \frac{3+\sqrt{3}}{3}$

$y = 2 \times \frac{2}{3} \times \left(-\frac{\sqrt{3}}{3}\right) = -\frac{4}{9}\sqrt{3}$

$\therefore \left(\frac{3+\sqrt{3}}{3}, -\frac{4}{9}\sqrt{3}\right)$ //

IYGB - SYNOPTIC PAPER T - QUESTION 9

FORM SOME EQUATIONS BASED ON THE RECURRENCE FORMULA

- $u_{n+1} = Au_n + B$

$u_3 = Au_2 + B$

$428 = A \times 464 + B$

$464A + B = 428$

- As $n \to \infty$ $u_n \to u_{n+1} \to 320$

$u_{n+1} = Au_n + B$

$320 = A \times 320 + B$

$320A + B = 320$

SUBTRACTING THE EQUATIONS

$144A = 108$

$A = \frac{3}{4}$

AND B CAN NOW BE FOUND

$320A + B = 320$

$320 \times \frac{3}{4} + B = 320$

$240 + B = 320$

$B = 80$

THUS WE NOW HAVE

$u_{n+1} = \frac{3}{4}u_n + 80$

$u_4 = \frac{3}{4}u_3 + 80$

$u_4 = \frac{3}{4} \times 428 + 80$

$u_4 = 401$

IYGB - SYNOPTIC PAPER T - QUESTION 10

MAKE x THE SUBJECT

$$\Rightarrow y = xe^{y}$$
$$\Rightarrow x = \frac{y}{e^{y}}$$

DIFFERENTIATE W.R.T y

$$\Rightarrow \frac{dx}{dy} = \frac{e^{y}\times 1 - y\times e^{y}}{e^{2y}}$$
$$\Rightarrow \frac{dx}{dy} = \frac{\cancel{e^{y}}(1-y)}{e^{\cancel{2y}y}}$$
$$\Rightarrow \frac{dx}{dy} = \frac{1-y}{e^{y}}$$
$$\Rightarrow \frac{dy}{dx} = \frac{e^{y}}{1-y}$$

NOW DIFFERENTIATE WITH RESPECT TO x

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-y)\times e^{y}\frac{dy}{dx} - e^{y}(-1)\times\frac{dy}{dx}}{(1-y)^2}$$
$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-y)e^{y} + e^{y}}{(1-y)^2}\frac{dy}{dx}$$
$$\Rightarrow \frac{d^2y}{dx^2} = \frac{e^{y}(1-y+1)}{(1-y)^2}\frac{dy}{dx}$$
$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2-y}{(1-y)^2}e^{y}\frac{dy}{dx}$$

BUT LOOKING ABOVE $e^{y} = (1-y)\frac{dy}{dx}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2-y}{(1-y)^2}\left[(1-y)\frac{dy}{dx}\right]\frac{dy}{dx}$$
$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2-y}{1-y}\left(\frac{dy}{dx}\right)^2$$
$$\Rightarrow (1-y)\frac{d^2y}{dx^2} = (2-y)\left(\frac{dy}{dx}\right)^2$$

AS REQUIRED

<ALTERNATIVE APPROACH>

DIFFERENTIATE THE EQUATION W.R.T. x

$$\Rightarrow y = xe^{y}$$
$$\Rightarrow \frac{dy}{dx} = 1\times e^{y} + xe^{y}\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx} = e^{y} + xe^{y}\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx} - xe^{y}\frac{dy}{dx} = e^{y}$$
$$\Rightarrow \frac{dy}{dx}(1 - xe^{y}) = e^{y}$$
$$\Rightarrow \frac{dy}{dx}(1 - y) = e^{y}$$

DIFFERENTIATE AGAIN W.R.T x

$$\Rightarrow \frac{d^2y}{dx^2}(1-y) + \frac{dy}{dx}(-1)\frac{dy}{dx} = e^{y}\frac{dy}{dx}$$
$$\Rightarrow \frac{d^2y}{dx^2}(1-y) - \left(\frac{dy}{dx}\right)^2 = e^{y}\frac{dy}{dx}$$

BUT LOOKING AT A FEW LINES ABOVE

$$e^{y} = \frac{dy}{dx}(1-y)$$

THUS WE HAVE

$$\Rightarrow \frac{d^2y}{dx^2}(1-y) - \left(\frac{dy}{dx}\right)^2 = \left[\frac{dy}{dx}(1-y)\right]\frac{dy}{dx}$$
$$\Rightarrow \frac{d^2y}{dx^2}(1-y) - \left(\frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2(1-y)$$
$$\Rightarrow \frac{d^2y}{dx^2}(1-y) = \left(\frac{dy}{dx}\right)^2(1-y) + \left(\frac{dy}{dx}\right)^2$$
$$\Rightarrow \frac{d^2y}{dx^2}(1-y) = \left(\frac{dy}{dx}\right)^2[1-y+1]$$
$$\Rightarrow \frac{d^2y}{dx^2}(1-y) = \left(\frac{dy}{dx}\right)^2(2-y)$$

AS BEFORE

IYGB - SYNOPTIC PAPER T - QUESTION 11

REARRANGE, DIFFERENTIATE USING THE QUOTIENT RULE

$$xy = e^x \implies y = \frac{e^x}{x}$$

$$\implies \frac{dy}{dx} = \frac{xe^x - e^x \times 1}{x^2} = \frac{e^x(x-1)}{x^2}$$

A GENERAL POINT ON THIS CURVE WILL HAVE COORDINATES $\left(a, \frac{e^a}{a}\right), a \neq 0$

AND GRADIENT $\frac{e^a(a-1)}{a^2}$

$$\implies \text{TANGENT}: \quad y - \frac{e^a}{a} = \frac{e^a(a-1)}{a^2}(x-a)$$

$$y - \frac{e^a}{a} = \frac{e^a(a-1)}{a^2}x - \frac{e^a a(a-1)}{a^2}$$

$$y - \frac{e^a}{a} = \frac{e^a(a-1)}{a^2}x - \frac{e^a(a-1)}{a}$$

AS THE TANGENT PASSES THROUGH O

$$\frac{e^a}{a} = \frac{e^a(a-1)}{a}$$

$$1 = a - 1 \qquad \underline{a \neq 0, \; e^a \neq 0}$$

$$a = 2$$

$$\therefore P\left(2, \tfrac{1}{2}e^2\right)$$

IYGB - SYN PAPER T - QUESTION 12

START BY TIDYING UP THE INEQUALITY

$$\Rightarrow x(x-4) < |5x-16| - 4$$
$$\Rightarrow x^2 - 4x + 4 < |5x-16|$$
$$\Rightarrow (x-2)^2 < |5x-16|$$

NOW CONSIDER THE CORRESPONDING EQUATION

$$\Rightarrow (x-2)^2 = |5x-16|$$
$$\Rightarrow (x-4)^4 = |5x-16|^2$$
$$\Rightarrow (x-4)^4 = (5x-16)^2$$

SQUARING BOTH SIDES

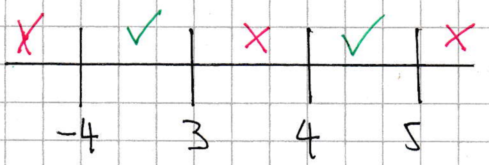
$$\Rightarrow (x-2)^2 = \begin{cases} 5x-16 \\ -5x+16 \end{cases}$$

SOLVING EACH OF THE TWO QUADRATICS

$$\Rightarrow (x-2)^2 = 5x-16$$
$$\Rightarrow x^2 - 4x + 4 = 5x - 16$$
$$\Rightarrow x^2 - 9x + 20 = 0$$
$$\Rightarrow (x-4)(x-5) = 0$$
$$\Rightarrow x = \begin{cases} 4 \\ 5 \end{cases}$$

$$\Rightarrow (x-2)^2 = -5x+16$$
$$\Rightarrow x^2 - 4x + 4 = -5x + 16$$
$$\Rightarrow x^2 + x - 12 = 0$$
$$\Rightarrow (x-3)(x+4) = 0$$
$$\Rightarrow x = \begin{cases} 3 \\ -4 \end{cases}$$

NOW CHECK THE SOLUTION INTERVAL BY TRYING VALUES AGAINST THE ORIGINAL INEQUALITY



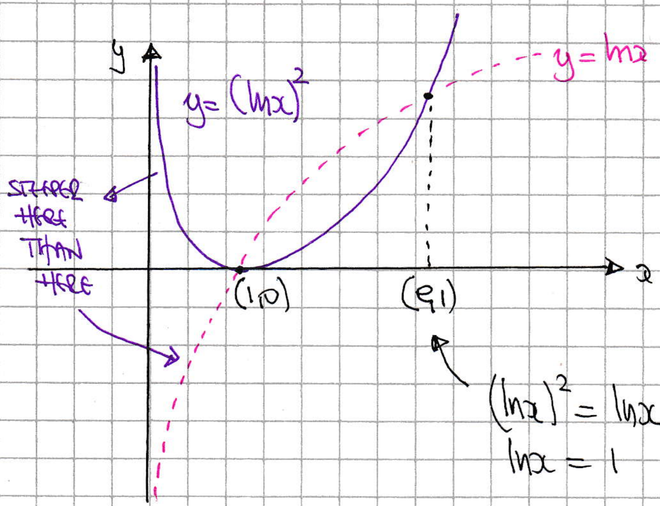
- $x = -10$ $\quad -10(-14) \not< |-66| - 4$
  $140 \not< 62$ ✗
- $x = 0$ $\quad 0 < |-16| - 4$
  $0 < 12$ ✓
- $x = 3.5$ $\quad \frac{7}{2}\left(-\frac{1}{2}\right) < \left|\frac{3}{2}\right| - 4$
  $-\frac{7}{4} < -\frac{5}{2}$ ✗

ETC

$$-4 < x < 3 \quad \text{OR} \quad 4 < x < 5$$

IYGB - SYNOPTIC PAPER T - QUESTION 13

a) SKETCHING THE GRAPH FROM THE GRAPH OF $y = \ln x$



$(\ln x)^2 = \ln x$

$\ln x = 1$ $\quad$ ($\ln x \neq 0$ AT THIS POINT)

$x = e$

b) LOOKING AT THE REQUIRED FINITE AREA IN THE DIAGRAM ABOVE

$$\text{AREA} = \int_1^e \ln x - (\ln x)^2 \, dx = \ldots \text{ INTEGRATION BY PARTS}$$

$\ln x - (\ln x)^2$	$\frac{1}{x} - \frac{2}{x}\ln x$
x	1

$$= \left[x\ln x - x(\ln x)^2 \right]_1^e - \int_1^e 1 - 2\ln x \, dx$$

USING THE RESULT GIVEN FOR $\int \ln x \, dx = x\ln x - x + C$

$$= \left[x\ln x - x(\ln x)^2 - x + 2(x\ln x - x) \right]_1^e$$

$$= \left[x\ln x - x(\ln x)^2 - x + 2x\ln x - 2x \right]_1^e$$

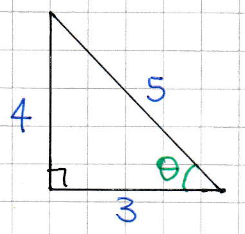
$$= \left[3x\ln x - x(\ln x)^2 - 3x \right]_1^e$$

$$= (\cancel{3e} - e - \cancel{3e}) - (0 - 0 - 3)$$

$$= 3 - e$$

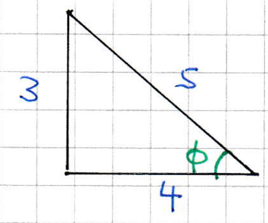
IYGB – SYNOPTIC PAPER T – QUESTION 14

LET $\theta = \arccos\frac{3}{5}$ & $\phi = \arctan\frac{3}{4}$



$\cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$



$\cos\phi = \frac{4}{5}$

$\sin\phi = \frac{3}{5}$

TRANSFORM THE EQUATION

$\Rightarrow \arcsin x + \arccos\frac{3}{5} = 2\arctan\frac{3}{4}$

$\Rightarrow \arcsin x + \theta = 2\phi$

$\Rightarrow \arcsin x = 2\phi - \theta$

$\Rightarrow \sin(\arcsin x) = \sin(2\phi - \theta)$

$\Rightarrow x = \sin 2\phi\cos\theta - \cos 2\phi\sin\theta$

USING DOUBLE ANGLE IDENTITIES $\sin 2\phi \equiv 2\sin\phi\cos\phi$ & $\cos 2\phi = 2\cos^2\phi - 1$

$\Rightarrow x = 2\sin\phi\cos\phi\cos\theta - \sin\theta(2\cos^2\phi - 1)$

$\Rightarrow x = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) - \frac{4}{5}\left(2\times\frac{16}{25} - 1\right)$

$\Rightarrow x = \frac{72}{125} - \frac{28}{125}$

$\Rightarrow x = \frac{44}{125}$

IYGB - SYNOPTIC PAPER T - QUESTION 15

$$\boxed{\frac{\sqrt{2}x+2}{x^2+\sqrt{2}x+1} - \frac{\sqrt{2}x-2}{x^2-\sqrt{2}x+1} = 2}$$

ADD THE FRACTIONS ON THE LEFT (NUMERATOR ONLY)

$$\Rightarrow (\sqrt{2}x+2)(x^2-\sqrt{2}x+1) - (\sqrt{2}x-2)(x^2+\sqrt{2}x+1)$$

$$= \left(\begin{array}{l}\sqrt{2}x^3 - \cancel{2x^2} + \sqrt{2}x \\ \quad \cancel{2x^2} - 2\sqrt{2}x + 2\end{array}\right) - \left(\begin{array}{l}\sqrt{2}x^3 + \cancel{2x^2} + \sqrt{2}x \\ \quad \cancel{-2x^2} - 2\sqrt{2}x - 2\end{array}\right)$$

TIDY UP FURTHER THE NUMERATOR

$$= \left[\sqrt{2}x^3 - \sqrt{2}x + 2\right] - \left[\sqrt{2}x^3 - \sqrt{2}x - 2\right]$$

$$= \sqrt{2}x^3 - \sqrt{2}x + 2 - \sqrt{2}x^3 + \sqrt{2}x + 2$$

$$= 4$$

THE COMMON DENOMINATOR OF THE LHS WILL BE

$$(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1) = \begin{array}{l} x^4 - \cancel{\sqrt{2}x^3} + \cancel{x^2} \\ \quad \cancel{\sqrt{2}x^3} - \cancel{2x^2} + \cancel{\sqrt{2}x} \\ \qquad \cancel{x^2} - \cancel{\sqrt{2}x} + 1 \end{array}$$

$$= x^4+1$$

RETURNING TO THE EQUATION

$$\frac{4}{x^4+1} = 2 \quad \Longrightarrow \quad x^4+1 = 2$$

$$\Longrightarrow \quad x^4 = 1$$

$$\Longrightarrow \quad x = \begin{cases} 1 \\ -1 \end{cases}$$

$$\left[\text{OR USING THE "COMPLEXES"} \quad x = \begin{cases} \pm 1 \\ \pm i \end{cases} \right]$$

IYGB - SYNOPTIC PAPER T - QUESTION 16

THIS IS SOME KIND OF QUADRATIC ONCE MANIPULATED

$$\Rightarrow 5 \times 5^{\log x} + 5^{2-\log x} = 30$$

$$\Rightarrow 5 \times 5^{\log x} + 5^{2} \times 5^{-\log x} = 30$$

$$\Rightarrow 5 \times 5^{\log x} + \frac{25}{5^{\log x}} = 30$$

$\div 5$

$$\Rightarrow 5^{\log x} + \frac{5}{5^{\log x}} = 6$$

LET $5^{\log x} = X$

$$\Rightarrow X + \frac{5}{X} = 6$$

$$\Rightarrow X^2 + 5 = 6X$$

$$\Rightarrow X^2 - 6X + 5 = 0$$

$$\Rightarrow (X-1)(X-5) = 0$$

$$\Rightarrow X = \begin{cases} 1 \\ 5 \end{cases}$$

$$\Rightarrow 5^{\log x} = \begin{cases} 1 \\ 5 \end{cases}$$

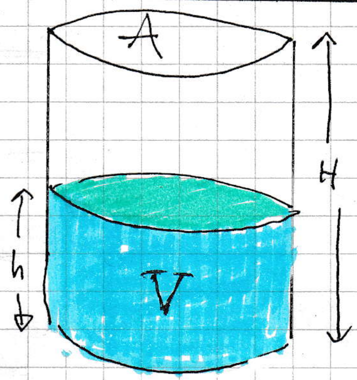
BY INSPECTION & NOTING THAT IN THE ABSENCE OF BASE, THE BASE IS 10

$$\log_{10} x = 0 \qquad\qquad \log_{10} x = 1$$

$$x = 1 \qquad\qquad x = 10$$

IYGB - SYNOPTIC PAPER T - QUESTION 17

FORMING A DIFFERENTIAL EQUATION



$t=0,\ h=H$

$t=T,\ h=0$

$V = Ah$

$\frac{dV}{dh} = A$

$$\Rightarrow \frac{dV}{dt} = -kh^{\frac{1}{2}}$$

$$\Rightarrow \frac{dV}{dh} \times \frac{dh}{dt} = -kh^{\frac{1}{2}}$$

$$\Rightarrow A\frac{dh}{dt} = -kh^{\frac{1}{2}}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{k}{A}h^{\frac{1}{2}}$$

$$\Rightarrow \frac{dh}{dt} = -Bh^{\frac{1}{2}} \qquad \left(B = \frac{k}{A}\right)$$

SOLVING BY SEPARATION OF VARIABLES

$$\Rightarrow \frac{1}{h^{\frac{1}{2}}}\,dh = -B\,dt$$

$$\Rightarrow \int h^{-\frac{1}{2}}\,dh = \int -B\,dt$$

$$\Rightarrow \boxed{2h^{\frac{1}{2}} = -Bt + C}$$

APPLY CONDITION $t=0$ $h=H$

$$\Rightarrow 2H^{\frac{1}{2}} = C$$

$$\Rightarrow \boxed{2h^{\frac{1}{2}} = 2H^{\frac{1}{2}} - Bt}$$

APPLY CONDITION $t=T$, $h=0$

$$\Rightarrow 0 = 2H^{\frac{1}{2}} - BT$$

$$\Rightarrow BT = 2H^{\frac{1}{2}}$$

$$\Rightarrow B = \frac{2H^{\frac{1}{2}}}{T}$$

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$$\Rightarrow 2h^{\frac{1}{2}} = 2H^{\frac{1}{2}} - \frac{2H^{\frac{1}{2}}}{T}t$$

$$\Rightarrow h^{\frac{1}{2}} = H^{\frac{1}{2}} - \frac{H^{\frac{1}{2}}t}{T}$$

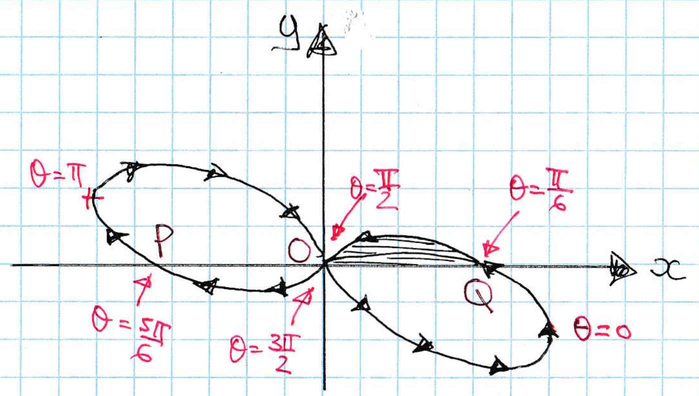
$$\Rightarrow h^{\frac{1}{2}} = H^{\frac{1}{2}}\left(1 - \frac{t}{T}\right)$$

$$\Rightarrow h = H\left(1 - \frac{t}{T}\right)^2$$

AS REQUIRED

IYGB – SYNOPTIC PAPER T – QUESTION 18

DETERMINE, BY INSPECTION, THE DIRECTION IN WHICH THE CURVE IS TRACED



$y=0$, $\sin 2\theta - \cos\theta = 0$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1)$

- $\cos\theta = 0$ & • $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{2}$ $\qquad \theta = \frac{\pi}{6}$

$\theta = \frac{3\pi}{2}$ $\qquad \theta = \frac{5\pi}{6}$

a) SETTING UP AN INTEGRAL TO FIND THE AREA OF THE SHADED REGION

$\Rightarrow \text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{\theta_1}^{\theta_2} y(\theta)\frac{dx}{d\theta}\,d\theta$

$\Rightarrow \text{AREA} = \int_{\theta=\frac{\pi}{2}}^{\theta=\frac{\pi}{6}} (\sin 2\theta - \cos\theta)(-\sin\theta)\,d\theta$

$\Rightarrow \text{AREA} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2\sin\theta\cos\theta - \cos\theta)(+\sin\theta)\,d\theta$

$\Rightarrow \text{AREA} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^2\theta\cos\theta - \cos\theta\sin\theta\,d\theta$

BY REVERSE CHAIN RULE (INSPECTION)

$\Rightarrow \text{AREA} = \left[\frac{2}{3}\sin^3\theta - \frac{1}{2}\sin^2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$\Rightarrow \text{AREA} = \left[\frac{2}{3} - \frac{1}{2}\right] - \left[\frac{1}{12} - \frac{1}{8}\right]$

$\Rightarrow \text{AREA} = \frac{5}{24}$

AS REQUIRED

IYGB – SYNOPTIC PAPER T – QUESTION 18

b) NEXT FIND THE AREA FOR WHICH $x \geq 0,\ y \leq 0$

$$\text{AREA} = \int_{\theta=\frac{3\pi}{2}}^{\theta=\frac{\pi}{6}} \underbrace{2\sin^2\theta\cos\theta - \sin\theta\cos\theta}_{\text{MANIPULATED EARLIER}}\ d\theta$$

THE AREA IS BELOW THE x AXIS, THIS HOWEVER WILL TURN OUT POSITIVE IN PARAMETRIC

$$\Rightarrow \text{"AREA UNDER"} = \int_{\frac{3\pi}{2}}^{\frac{5\pi}{6}} 2\sin^2\theta\cos\theta - \sin\theta\cos\theta\ d\theta$$

$$\Rightarrow \text{"AREA UNDER"} = \left[\frac{2}{3}\sin^3\theta - \frac{1}{2}\sin^2\theta\right]_{\frac{3\pi}{2}}^{\frac{5\pi}{6}}$$

$$\Rightarrow \text{"AREA UNDER"} = \left(\frac{1}{12} - \frac{1}{8}\right) - \left(-\frac{2}{3} - \frac{1}{2}\right)$$

$$\Rightarrow \text{"AREA UNDER"} = \frac{9}{8}$$

[WE WOULD HAVE GOT THE SAME IF WE INTEGRATED $\frac{5\pi}{6}$ TO $\frac{3\pi}{2}$]

$\therefore$ AREA OF ONE LOOP IS $\frac{9}{8} + \frac{5}{24} = \frac{4}{3}$

TOTAL AREA IS $2 \times \frac{4}{3} = \frac{8}{3}$

AS REQUIRED

c) PROCEED AS FOLLOWS

$$y = \sin 2\theta - \cos\theta$$
$$y = 2\sin\theta\cos\theta - \cos\theta$$
$$y = 2\sin\theta\cos\theta - x$$
$$y + x = 2\sin\theta\cos\theta$$
$$(y+x)^2 = 4\sin^2\theta\cos^2\theta$$
$$(y+x)^2 = 4\cos^2\theta(1-\cos^2\theta)$$
$$(y+x)^2 = 4x^2(1-x^2)$$

IYGB – SYNOPTIC PAPER T – QUESTION 14

$$y = x^3 - 3x^2 - 9x + 3, \quad x \in \mathbb{R}$$

USING THE FACT THAT ALL CUBICS HAVE ROTATIONAL SYMMETRY ABOUT THEIR POINT OF INFLEXION WE PROCEED AS FOLLOWS

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

$$\frac{d^2y}{dx^2} = 6x - 6$$

BY INSPECTION THE CURVE HAS A POINT OF INFLEXION AT $x = 1$

$$\therefore y = 1 - \cancel{3} - 9 + \cancel{3} = -8$$

$$\therefore P(1, -8)$$

TO JUSTIFY THE ODDITY ABOUT P, TRANSLATE THE CURVE TO THE ORIGIN & INVESTIGATE ODDITY ABOUT O

- "UP BY 8" $\Longrightarrow y = (x^3 - 3x^2 - 9x + 3) + 8$
  $\Longrightarrow y = x^3 - 3x^2 - 9x + 11$
- "LEFT BY 1" $\Longrightarrow y = (x+1)^3 - 3(x+1)^2 - 9(x+1) + 11$
  $\Longrightarrow y = x^3 + \cancel{3x^2} + 3x + \cancel{1} - \cancel{3x^2} - 6x - \cancel{3} - 9x - \cancel{9} + \cancel{11}$
  $\Longrightarrow y = x^3 - 12x$

EVIDENTLY THIS IS ODD, AS

$$f(x) = x^3 - 12x$$
$$f(-x) = (-x)^3 - 12(-x)$$
$$= -x^3 + 12x$$
$$= -f(x)$$

CONSEQUENTLY "OUR CURVE" IS ODD ABOUT $P(1, -8)$

IYGB – SYNOPTIC PAPER T – QUESTION 20

- START BY CARRYING THE INTEGRATION IN TERMS OF k

$$\int_1^k \frac{(\sqrt{3}k+\sqrt{3x})^2}{kx^3}\,dx = \int_1^k \frac{3k^2+2k\sqrt{3}\sqrt{3x}+3x}{kx^3}\,dx$$

$$= \int_1^k \frac{3k^2}{kx^3}+\frac{6k\sqrt{x}}{kx^3}+\frac{3x}{kx^3}\,dx = \int_1^k 3kx^{-3}+6x^{-\frac{5}{2}}+\frac{3}{k}x^{-2}\,dx.$$

$$= \left[-\frac{3k}{2}x^{-2}-4x^{-\frac{3}{2}}-\frac{3}{k}x^{-1}\right]_1^k = \left[\frac{3k}{2x^2}+\frac{4}{x^{\frac{3}{2}}}+\frac{3}{kx}\right]_k^1$$

$$= \left(\frac{3k}{2}+4+\frac{3}{k}\right)-\left(\frac{3k}{2k^2}+\frac{4}{k^{3/2}}+\frac{3}{k^2}\right)$$

$$= \frac{3}{2}k+4+\frac{3}{k}-\frac{3}{2k}-\frac{4}{k^{3/2}}-\frac{3}{k^2}$$

$$= \left(\frac{3}{2}k+4+\frac{3}{2k}-\frac{3}{k^2}\right)-\frac{4}{k^{\frac{3}{2}}} \longleftarrow \boxed{a-\sqrt{k}}$$

- AS $k \in \mathbb{Q}$, $\frac{3}{2}k+4+\frac{3}{2k}-\frac{3}{k^2} \in \mathbb{Q}$

- HENCE

$$-\frac{4}{k^{3/2}} = -\sqrt{k}$$

$$4 = k^2$$

$$k = \begin{cases} 2 \\ \cancel{-2} \end{cases}$$

OTHERWISE $\sqrt{k}$ IS NOT DEFINED

- FINALLY WE CAN FIND a

$$a = \frac{3}{2}k+4+\frac{3}{2k}-\frac{3}{k^2} = \frac{3}{2}\times2+4+\frac{3}{2\times2}-\frac{3}{4}$$

$$a = 3+4+\frac{3}{4}-\frac{3}{4}$$

$$a = 7$$

IYGB – SYNOPTIC PAPER T – QUESTION 21

START BY OBTAINING THE GRADIENT FUNCTION IN TERMS OF k

$$y = \frac{k + 8x\sqrt{x}}{12x} = \frac{k + 8x^{\frac{3}{2}}}{12x} = \frac{k}{12x} + \frac{8x^{\frac{3}{2}}}{12x} = \frac{k}{12}x^{-1} + \frac{2}{3}x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{k}{12}x^{-2} + \frac{1}{3}x^{-\frac{1}{2}} = \frac{1}{3\sqrt{x}} - \frac{k}{12x^2}$$

PROCEED AS FOLLOWS

$$6x + y = 17$$
$$y = -6x + 17$$

$$\therefore \left.\frac{dy}{dx}\right|_{y=2} = -6$$

$$\frac{1}{3\sqrt{x}} - \frac{k}{12x^2} = -6 \quad \times 12x^2$$

$$4x^{\frac{3}{2}} - k = -72x^2$$

$$k = 4x^{\frac{3}{2}} + 72x^2$$

ALSO WE HAVE $y = 2$

$$2 = \frac{k + 8x\sqrt{x}}{12x}$$

$$24x = k + 8x\sqrt{x}$$

$$k = 24x - 8x\sqrt{x}$$

$$k = 24x - 8x^{\frac{3}{2}}$$

IYGB – SYNOPTIC PAPER T – QUESTION 21

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} k = 4x^{\frac{3}{2}} + 72x^2 \\ k = 24x - 8x^{\frac{3}{2}} \end{array}\right\}$$

$$72x^2 + 4x^{\frac{3}{2}} = 24x - 8x^{\frac{3}{2}}$$

$$72x^2 + 12x^{\frac{3}{2}} - 24x = 0$$

$$12x\left(6x + x^{\frac{1}{2}} - 2\right) = 0$$

$$12x\left(3x^{\frac{1}{2}} + 2\right)\left(2x^{\frac{1}{2}} - 1\right) = 0$$

As $x \neq 0$

$$x^{\frac{1}{2}} = \begin{cases} \cancel{-\frac{2}{3}} \\ \frac{1}{2} \end{cases}$$

$\therefore \ x = \frac{1}{4}$

$$\therefore \ k = 24\left(\tfrac{1}{4}\right) - 8\left(\tfrac{1}{4}\right)^{\frac{3}{2}} = 6 - 8 \times \tfrac{1}{8} = 6 - 1$$

$\therefore \ k = 5$