IYGB GCE

Mathematics SYN

Advanced Level

Synoptic Paper R

Difficulty Rating: 3.9475/0.7308

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This synoptic practice paper follows closely the Advanced Level Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 23 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

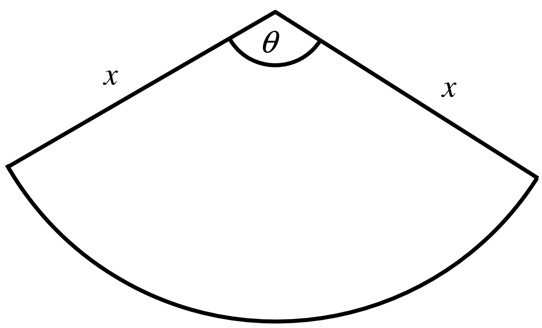
Question 1

Determine the value of the positive constant k given further that

$$\int_{k}^{\frac{1}{2}} \frac{6}{e^{2-3x}}\, dx = 0.1998.$$

Give the value of k to an appropriate degree of accuracy. **(6)**

Question 2



A circular sector of radius x cm subtends an angle of θ radians at the centre.

The area of the sector is 36 cm^2 and its perimeter is P cm.

a) Show clearly that

$$P = 2x + \frac{72}{x}.$$ **(4)**

b) Find the minimum value of P, fully justifying the fact that it is a minimum. **(7)**

c) Deduce the value of θ when P is minimum. **(2)**

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Question 3

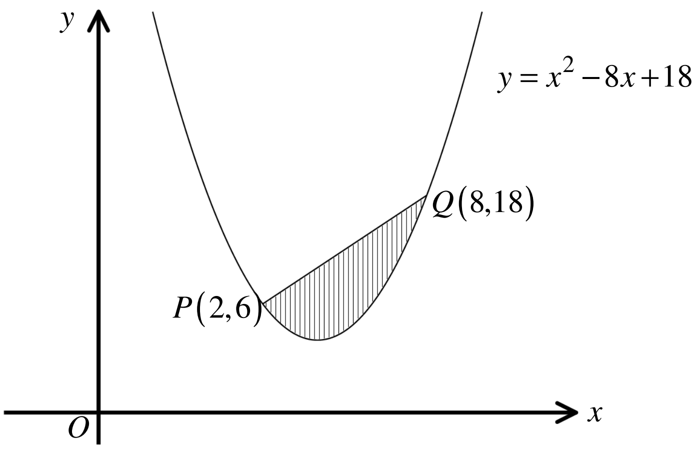
The points A and B have position vectors $\begin{pmatrix} -5 \\ -2 \\ 8 \end{pmatrix}$ and $\begin{pmatrix} 11 \\ 6 \\ 20 \end{pmatrix}$, respectively.

The point M lies on AB so that $|AM|:|MB| = 3:1$

The point P has position vector $\begin{pmatrix} 10 \\ 8 \\ 19 \end{pmatrix}$.

Determine the position vector of the point Q, if M is the midpoint of PQ. **(6)**

Question 4



The figure above shows the parabola with equation

$$y = x^2 - 8x + 18, \; x \in \mathbb{R}.$$

The points $P(3,3)$ and $Q(6,6)$ both lie on the parabola.

Find the exact of the shaded region, bounded by the curve and the straight line segment between P and Q. **(7)**

Question 5

A circle has equation

$$x^2 + y^2 - 8x + cy = 33,$$

where c is a positive constant.

The straight line L, with equation $y = 2x - 12$, intersects the circle at the point with coordinates $(9, k)$.

Find, as an exact surd, the perpendicular distance of L from the centre of the circle. **(10)**

Question 6

In the binomial expansion of $(1+ax)^k$, where a and k are non zero constants, the coefficient of x is 8 and the coefficient of x^2 is 30.

a) Determine the value of a and the value of k. **(8)**

b) Find the coefficient of x^3. **(1)**

Question 7

A polynomial $f(x)$ is defined in terms of the constants a, b and c as

$$f(x) = 2x^3 + ax^2 + bx + c, \quad x \in \mathbb{R}.$$

It is further given that

$$f(2) = f(-1) = 0 \quad \text{and} \quad f(1) = -14.$$

a) Find the values of a, b and c. **(5)**

b) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes. **(5)**

Question 8

$$f(x) = \frac{4x-13}{x-3}, \ x \in \mathbb{R}, \ x \neq 3.$$

a) Show that the equation of $f(x)$ can be written as

$$f(x) = 4 - \frac{1}{x-3}, \ x \in \mathbb{R}, \ x \neq 3. \qquad \textbf{(2)}$$

b) Sketch the graph of $f(x)$.

The sketch must include …

- … the coordinates of the points where $f(x)$ meets the coordinate axes.
- … the equations of any asymptotes of the curve. **(5)**

c) Solve the equation

$$f(x) = \frac{3}{x},$$

giving the answers in the form $a + b\sqrt{7}$, where a and b are constants. **(5)**

Question 9

$$f(n) = n^2 - 2kn + k + 12, \ n \in \mathbb{N},$$

where k is a constant.

Given that $f(n) = n^2 - 2kn + k + 12$ is a square number for all values of n, determine the possible values of the constant k. **(5)**

Question 10

$$f(x) = e^{3x} - 4e^{-3x}, \ x \in \mathbb{R} .$$

a) Show that the inequalities

$$f(x) > 0 \ \text{ and } \ f''(x) > 0$$

have the same solution interval. **(4)**

b) Determine, in exact form, the common solution interval. **(3)**

Question 11

A curve has implicit equation

$$8x^4 + 32xy^3 + 16y^4 = 1 .$$

Find the coordinates of any points on the curve whose gradient is $\frac{1}{2}$. **(8)**

Question 12

Show that the following trigonometric equation

$$\tan 2\theta - 3\cot\theta = 0, \quad 0 < \theta < 2\pi, ,$$

has six solutions in the interval $0 < \theta < 2\pi$, giving the answers in terms of π. **(7)**

Question 13

A cubic curve has equation

$$y = x^3 - x^2 + 5, \; x \in \mathbb{R}.$$

The point P lies on the curve where $x = 1$.

Show that the normal to the curve at P does not meet the curve again. **(10)**

Question 14

$$f(x) \equiv \frac{6x^2 - 21x + 17}{(x-3)(x-1)^2}, \quad x \in \mathbb{R}, \; x \neq 3, \quad x \neq 1.$$

a) Express $f(x)$ into partial fractions. **(5)**

$$g(x) \equiv \frac{(x-12)(x+1)}{(x-3)(x-1)^2}, \quad x \in \mathbb{R}, \; x \neq 3, \quad x \neq 1.$$

b) Use the result of part **(a)** to express $g(x)$ into partial fractions.
*No credit will be given in this part by repeating the method used in part **(a)*** **(5)**

Question 15

Find, in its simplest form, the solution of the following equation

$$e^{4x} = 16^{\frac{1}{\ln 2}}.$$

The answer must be supported by detailed workings. **(4)**

Question 16

$$f(x) = \sec x, \qquad 0 \le x < \frac{\pi}{2} \cup \frac{\pi}{2} < x \le \pi .$$

a) Sketch in the same diagram the graphs of $f(x)$ and $f^{-1}(x) = \text{arcsec}\, x$. **(4)**

b) State the domain and range of $f^{-1}(x) = \text{arcsec}\, x$. **(2)**

c) Show clearly that $\text{arcsec}\, x = \arccos\left(\frac{1}{x}\right)$. **(3)**

d) Show further that $\frac{d}{dx}(\text{arcsec}\, x) = \frac{1}{\sqrt{x^4 - x^2}}$. **(5)**

Question 17

Use the substitution $u = \frac{1}{x} + x\mathrm{e}^{x}$ to find an expression for

$$\int \frac{x^3 + x^2 - \mathrm{e}^{-x}}{x^3 + x\mathrm{e}^{-x}}\, dx .$$ **(9)**

Question 18

A geometric progression has first term $\sin\theta$ and common ratio $\cos\theta$.

a) Given the value of θ is such so that the progression converges, show that its sum to infinity is $\cot\frac{\theta}{2}$. **(4)**

A different geometric progression has first term $\cos\theta$ and common ratio $\sin\theta$.

b) Given the value of θ is such so that this progression also converges, show that its sum to infinity is $\sec\theta + \tan\theta$. **(4)**

Question 19

$$f(x) \equiv \frac{x^2}{x-1}, \; x \in \mathbb{R}, \; x \neq -\frac{1}{4}.$$

Use the formal definition of the derivative as a limit, to show that

$$f'(x) = \frac{x(x-2)}{(x-1)^2}.$$ **(7)**

Question 20

During a chemical reaction a compound is formed, whose mass x grams, in time t minutes, satisfies the differential equation

$$\frac{dx}{dt} = k(4+x)(4-x)e^{-t}, \; t \geq 0,$$

where k is a positive constant.

When the chemical reaction started there was no compound present.

The limiting mass of the compound is 2 grams.

Find the value of t, when half the limiting mass of the compound has been produced. **(15)**

Question 21

The straight line l_1 has gradient m and has x intercept 6.

a) Show clearly that the equation of l_1 can be written as $y = m(x-6)$. **(2)**

The straight line l_2, with equation $y = 2x+9$, meets l_1 at the point A.

b) Show that the coordinates of A are $\left(\frac{6m+9}{m-2}, \frac{21m}{m-2}\right)$ **(4)**

The straight line l_3, with equation $y = 2x-3$, meets l_1 at the point B.

c) Show that the distance AB is $\sqrt{\frac{144(1+m^2)}{m^2+4m+4}}$ **(5)**

d) Given further that distance AB is $4\sqrt{2}$ find the two possible equations of l_1. **(4)**

Question 22

An arithmetic progression has first term -10 and common difference 4.

The n^{th} term of the progression is denoted by u_n.

Determine the value of k given that

$$\sum_{n=1}^{2k} u_n - \sum_{n=1}^{k} u_n = 1728.$$ **(8)**
